PHY 410 - Reference Sheet

Boltzmann's constant
$k_B = 1.380649 \times 10^{-23} \mathrm{m}^2\,\mathrm{s}^{-2}\,\mathrm{K}^{-1}$
Entropy $S = k_B \sigma$
Temperature $T = \tau / k_B$

Canonical Ensemble

Partition Function - partition by energy levels for a fixed temperature

$$z = \sum_n e^{-\varepsilon_n/\tau}, \quad \mathcal{P}(n) = \frac{1}{z} e^{-\varepsilon_n/\tau}$$

Expected Value of $\mathbb{X}$ is the average across all energies (Thermal Average).

$$\langle \mathbb{X} \rangle = \sum_n \mathbb{X}(n) \mathcal{P}(n) = \frac{1}{z} \sum_n \mathbb{X}(n) e^{-\varepsilon_n/\tau}$$

Expected Energy in the canonical ensemble is

$$U = \langle \varepsilon \rangle = \frac{1}{z} \sum_n \varepsilon_n e^{-\varepsilon_n/\tau}$$

$$U = \langle \varepsilon \rangle = \tau^2 \frac{1}{z} \frac{\partial z}{\partial \tau} = \tau^2 \frac{\partial}{\partial \tau} \log z$$

The total partition function and expected value for N non-interacting particles is simply

$$z_N = z_1^N$$

$$\langle \mathbb{X} \rangle_N = N \langle \mathbb{X} \rangle_1 \quad \Rightarrow \quad U_N = N U_1$$

Helmholtz Free Energy

$$F = U - \tau\sigma = U - ST = -\tau \log z$$

$$dF = -\sigma d\tau - P dV$$

Entropy $\sigma = -\left(\frac{\partial F}{\partial \tau}\right)_V, \quad S = k_B \sigma$

Pressure

$$P = -\left(\frac{\partial U}{\partial V}\right)_\sigma = \tau \left(\frac{\partial \sigma}{\partial V}\right)_U = -\left(\frac{\partial F}{\partial V}\right)_\tau$$

Energy $U = -\tau^2 \frac{\partial}{\partial \tau} \left(\frac{F}{\tau}\right)$

Thermal Radiation

Single Frequency Photon Gas is a system in the canonical ensemble that considers photons of a specific frequency ω.

$$\varepsilon = s\hbar\omega, \quad s = 0, 1, 2, 3, \dots$$

$$z = \sum_{s=0}^{\infty} e^{-s\hbar\omega/\tau} = \frac{1}{1 - e^{-\hbar\omega/\tau}}$$

$$\mathcal{P}(s) = \frac{e^{-s\hbar\omega/\tau}}{z}$$

$$\langle s \rangle = \frac{1}{z} \sum_{s=0}^{\infty} s e^{-s\hbar\omega/\tau} = \frac{1}{e^{\hbar\omega/\tau} - 1}$$

Photon Gas is an expansion of the single frequency photon gas that considers all the possible cavity modes. The modes are 2 fold degenerate for the 2 independent polarizations.

$$\omega_n = \frac{c\pi}{L} \sqrt{n_x^2 + n_y^2 + n_z^2} = \frac{c\pi n}{L}$$

$$U = \langle \epsilon \rangle = 2 \sum_n \frac{\hbar\omega_n}{e^{\hbar\omega_n/\tau} - 1}$$

Stefan-Boltzmann Law

$$U = \langle \epsilon \rangle = \frac{\hbar V}{\pi^2 c^3} \int_0^\infty \frac{\omega^3}{e^{\hbar\omega/\tau} - 1} d\omega = \frac{\pi^2 V}{15(\hbar c)^3} \tau^4$$

Phonons in a Solid

Grand Canonical Ensemble

Chemical Potential

Grand Potential

Fermions and Bosons

Ideal Gas

DOG (bork)

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