Electricity and Magnetism

from the context of the course
PHY 481: Electricity and Magnetism

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Chapter 1

Introduction

1.1 The SI System

In physics it's often important to have precisely defined units for the purposes of making very accurate measurements or simply having a coherent unit system. It's possible to derive all necessary units from five measurements of **length, mass, time, current, and temperature**. The standard SI units for these properties are listed bellow:

Type	Unit	Definition
Length	Meter(m)	Length of distance light in a vacuum travels in $\frac{1}{299792458}$ seconds
Mass	Kilogram(kg)	Defined by fixing the Planck's constant $h = 6.62607015 \times 10^{-34} kg\ m^2 s^{-1}$
Time	Second(s)	Defined by fixing the ground-state hyperfine transition frequency of the caesium-133 atom, to be $9192631770 s^{-1}$
Current	Ampere(A)	Defined by fixing the charge of an electron as $1.602176634 \times 10^{-19} A \cdot s$
Temperature	Kelvin(K)	Defined by fixing the value of the Boltzmann constant k to $1.380649 \times 10^{-23} kg \cdot m^2 s^{-2} K^{-1}$

Common prefixes are listed bellow:

Prefix	Symbol	Definition
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

1.2 Physical Constants

In the process of understanding our universe, we establish some physical constants that are determined through measurements. Here is a table of physical constants relevant to Electricity and Magnetism.

Physical Constant	Symbol	Value
Vacuum Permittivity	ϵ_0	$8.8541878128(13) * 10^{-12} \mathrm{C}^2\,\mathrm{N}^{-1}\,\mathrm{m}^2$

1.3 Notation

Definition 1.3.1. Electrostatic Maxwell's Equations - Maxwell's equations with no time-dependence.

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\nabla \times E = 0$$

$$\nabla \cdot E = 0$$

$$\nabla \times E = \upsilon_0 J$$

Definition 1.3.2. Cartesian Coordinates are a simple grid coordinate system using three coordinates (x, y, z) to define a point in space.

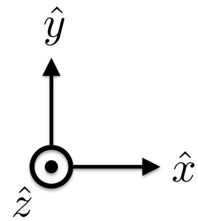

Figure 1.1: Diagram of Cartesian Coordinates

$$\hat{\mathbf{x}} = (1, 0, 0), \hat{\mathbf{y}} = (0, 1, 0), \hat{\mathbf{z}} = (0, 0, 1)$$

Definition 1.3.3. Griffiths script-r denoted $\boldsymbol{\mathscr{r}}$ is the displacement vector from the source location $\mathbf{r}'$ to the observation location $\mathbf{r}$.

$$\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$$

$$\mathbf{r}' = x'\hat{\mathbf{x}} + y'\hat{\mathbf{y}} + z'\hat{\mathbf{z}}$$

$$\boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}' = (x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}}$$

The unit vector for Griffiths script-r is

$$\hat{\boldsymbol{\mathscr{r}}} = \frac{\boldsymbol{\mathscr{r}}}{\mathscr{r}} = \frac{(x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}}}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}}$$

Definition 1.3.4. Spherical Coordinates is an extension of the standard polar coordinates. It uses three coordinates (r, θ, ϕ) to describe a point in space. $\mathbf{r}$ is the distance from the origin to the point, θ is the angle from the z axis and ϕ is the "azimuthal" angle from the x axis similar to 2D polar coordinates.

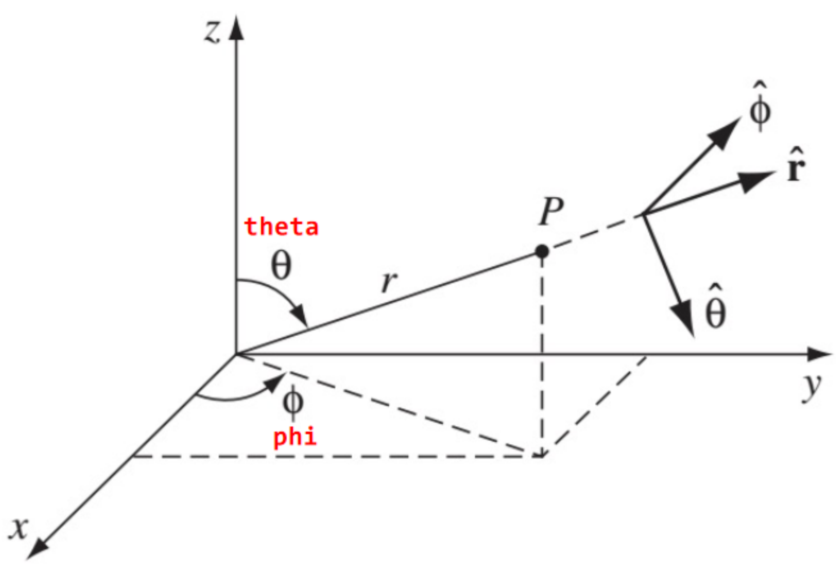

Figure 1.2: Diagram of Spherical Coordinates

$$\begin{aligned} x &= r \sin\theta \cos\phi & \hat{\mathbf{x}} &= \sin(\theta)\cos(\phi)\hat{\mathbf{r}} + \cos(\theta)\cos(\phi)\hat{\theta} - \sin(\phi)\hat{\phi} \\ y &= r \sin\theta \sin\phi & \hat{\mathbf{y}} &= \\ z &= r\cos\theta & \hat{\mathbf{z}} &= \end{aligned}$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} & \hat{\mathbf{r}} &= \sin(\theta)\cos(\phi)\mathbf{x} + \sin(\theta)\sin(\phi)\mathbf{y} + \cos(\theta)\mathbf{z} \\ \theta &= \tan^{-1}(\sqrt{x^2 + y^2}/2) & \hat{\theta} &= \cos(\theta)\cos(\phi)\mathbf{x} + \cos(\theta)\sin(\phi)\mathbf{y} - \sin(\theta)\mathbf{z} \\ \phi &= \tan^{-1}(y/x) & \hat{\phi} &= -\sin(\phi)\mathbf{x} + \cos(\phi)\mathbf{y} \end{aligned}$$

Definition 1.3.5. Cylindrical Coordinates use s and ϕ as in 2D polar coordinates with a z coordinate to extend into the third dimension.

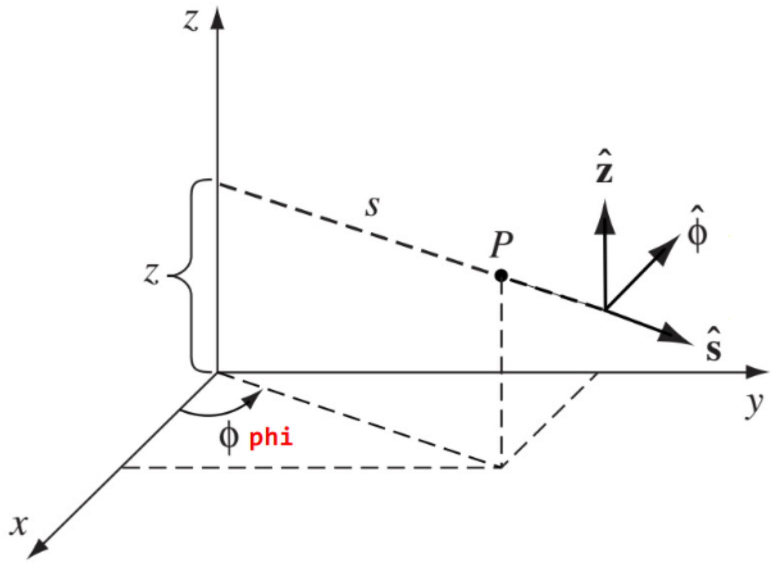

Figure 1.3: Diagram of Cylindrical Coordinates

$$
\begin{array}{ll}
x = s\cos\phi & \hat{\mathbf{x}} = \cos(\phi)\hat{\mathbf{s}} - \sin(\phi)\hat{\phi} \\
y = s\sin\phi & \hat{\mathbf{y}} = \sin(\phi)\hat{\mathbf{s}} + \cos(\phi)\hat{\phi} \\
z = z & \hat{\mathbf{z}} = \\
 & \\
s = \sqrt{x^2+y^2} & \hat{\mathbf{s}} = \cos(\phi)\hat{\mathbf{x}} + \sin(\phi)\hat{\mathbf{y}} \\
\phi = \tan^{-1}(y/x) & \hat{\phi} = -\sin(\phi)\hat{\mathbf{x}} + \cos(\phi)\hat{\mathbf{y}} \\
z = z & \hat{\mathbf{z}} = \hat{\mathbf{z}}
\end{array}
$$

Definition 1.3.6. Nabla is the gradient operator defined as

$$\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial x}$$

It can be used to right gradient and divergence.

Definition 1.3.7. Gradient is a vector field that represented the direction of increasing value of a scalar field.

$$\nabla F = \frac{\partial F}{\partial x}\hat{\mathbf{x}} + \frac{\partial F}{\partial y}\hat{\mathbf{y}} + \frac{\partial F}{\partial z}\hat{\mathbf{z}} = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right)$$

Definition 1.3.8. Divergence is a scalar field that represents the total flux per unit volume.

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Definition 1.3.9. Curl is a vector field that represents the total circulation per unit of enclosed area of a vector field.

$$\nabla \times \mathbf{F} = \det\begin{pmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{pmatrix} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{\mathbf{z}}$$

Theorem 1.3.1. Special Second Derivatives

$$\nabla \times (\nabla F) = 0$$

$$\nabla \cdot (\nabla \times \mathbf{F}) = 0$$

Theorem 1.3.2. Gradient Theorem

$$\int_a^b (\nabla f) \cdot d\mathbf{l} = f(b) - f(a)$$

Theorem 1.3.3. Gauss's Theorem (Divergence Theorem)

$$\int (\nabla \cdot A) d\tau = \oint A \cdot d\mathbf{a}$$

Theorem 1.3.4. Stokes' Theorem (Curl Theorem)

$$\int (\nabla \times \mathbf{A}) \cdot \mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$$

Chapter 2

Electrostatics

2.1 The Electric Field

Law 2.1.1. Coulomb's Law gives to force on an observation charge q_o due to a single points charge q_s.

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_o q_s}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}$$

Definition 2.1.1. The **Electric Field** is a vector field with units of $\mathrm{N\,C^{-1}}$ that represents the force on a particle with a charge of 1C. From coulombs law the electric field of a group of n charges is described by the following equation.

$$\mathbf{F} = q_o\mathbf{E}$$

Theorem 2.1.1. Point Charge Electric Field The electric field of a group of n charges is described by the following equation.

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^2}\hat{\boldsymbol{\mathscr{r}}}_i$$

Theorem 2.1.2. Continuous Charge Electric Field The electric field produced by charge distributed continuously over some region is described by the following equation:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{1}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}dq$$

Line Charge The electric field produced by a continuous line of charge with charge per unit distance λ is described by the following equation:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\lambda(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}dl$$

Surface Charge The electric field produced by a continuous surface of charge with charge per unit area σ is described by the following equation:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\sigma(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}da$$

Volume Charge The electric field produced by a continuous surface of charge with charge per unit area ρ is described by the following equation:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}d\tau$$