Physics Reference

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Contents

Chapter 1

Introduction

This chapter will offer reference and information that applies to the entire book.

1.1 Standard Units

In physics it's often important to have precisely defined units for the purposes of making very accurate measurements or simply having a coherent unit system. It's possible to derive all necessary units from five measurements of **length, mass, time, current, and temperature**. The standard SI units for these properties are listed bellow:

Type	Unit	Definition
Length	Meter(m)	Length of distance light in a vacuum travels in $\frac{1}{299792458}$ seconds
Mass	Kilogram(kg)	Defined by fixing the Planck's constant $h = 6.62607015 \times 10^{-34} kg\ m^2 s^{-1}$
Time	Second(s)	Defined by fixing the ground-state hyperfine transition frequency of the caesium-133 atom, to be $9192631770 s^{-1}$
Current	Ampere(A)	Defined by fixing the charge of an electron as $1.602176634 \times 10^{-19} A \cdot s$
Temperature	Kelvin(K)	Defined by fixing the value of the Boltzmann constant k to $1.380649 \times 10^{-23} kg \cdot m^2 s^{-2} K^{-1}$

Common prefixes are listed bellow:

Prefix	Symbol	Definition
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

Chapter 2

Electricity and Magnetism

Law 2.0.1. Maxwells Equations the general laws of electricity and magnetism.

$$\nabla \times \vec{E} = -\vec{B}$$

$$\nabla \times \vec{H} = \vec{J} + \vec{D}$$

$$\nabla \cdot \vec{D} = S$$

$$\nabla \cdot \vec{B} = 0$$

2.1 Electronics

2.1.1 General Components

Definition 2.1.1. Electric Field force per unit charge or N/C

Definition 2.1.2. Voltage or *Potential* is the change in energy per unit charge brought on by traveling through an electric field or J/C or simply V

Remark. The units N/C is equivalent to V/m

Definition 2.1.3. Power can be derived from units of current and voltage for the following formula where P is power(W), V is voltage(V), and I is current(A).

$$P = VI \tag{2.1.1}$$

Law 2.1.1. Ohms Law models the voltage drop across a purely resistive load with the following formula where V is voltage(V), I is current(A), and R is resistence(Ω).

$$V = IR \tag{2.1.2}$$

Definition 2.1.4. A **Resistor** a device that will produce a voltage drop according to ohms law with a resistance as V/A

Definition 2.1.5. Resistivity($\Omega\,$m) is used to calculate how much resistance we expect from a material use the following formula where R is resistence(Ω), ρ is **Resistivity**($\Omega\,$m), l is length(m), and A is cross sectional area(m^2).

$$R = \rho\frac{l}{A} \tag{2.1.3}$$

Law 2.1.2. Kirchoff's Voltage Law states precisely that the algebraic sum of all voltages around a closed path is zero.

$$\sum V_n = 0 \tag{2.1.4}$$

Law 2.1.3. Kirchoff's Current Law states precisely that the algebraic sum of all currents entering a node is zero.

$$\sum I_n = 0 \tag{2.1.5}$$

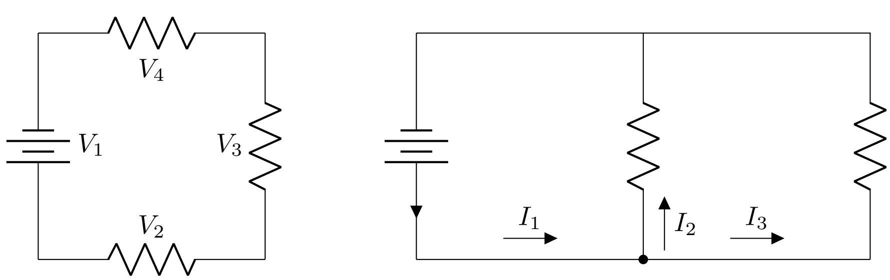
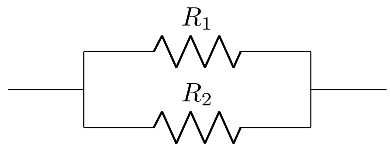


Figure 2.1: Two circuits to demonstrate Krichhoff's Laws

Law 2.1.4. Resistors in Series will simply be the sum of the individual resistance because we are adding the length of the resistors together.

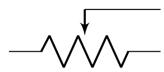
$$R_T = \sum R_i \tag{2.1.6}$$

Law 2.1.5. Resistors in Parallel will decrease the overall resistance as indicated by the definition of resistivity 2.1.5.

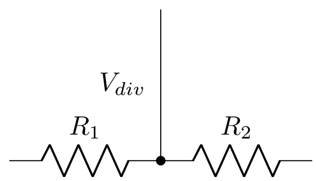


$$\frac{1}{R_T} = \sum \frac{1}{R_i} \tag{2.1.7}$$

Definition 2.1.6. Voltage Divider or **Potentiometer** is a arangement of two resistors with a connection between them. A potentiometer refers to a voltage divider where the resistance ratio between the two resistors can be adjusted.



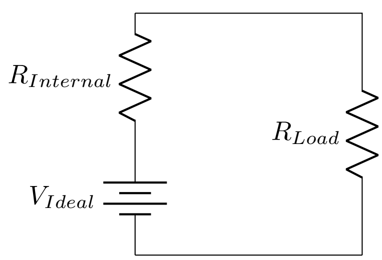
To calculate the voltage we expect at the divider we need to know the voltage across the whole potentiometer V and the ratio between the two resistors $\frac{R_2}{R_1}$.



$$V_{div} = V\frac{R_2}{R_1 + R_2} \tag{2.1.8}$$

Definition 2.1.7. Ideal Battery - a battery that always produces the same potential difference.

Definition 2.1.8. Real Battery - a battery with some internal resistance that reduces the potential difference across the terminals depending on the amount of current.

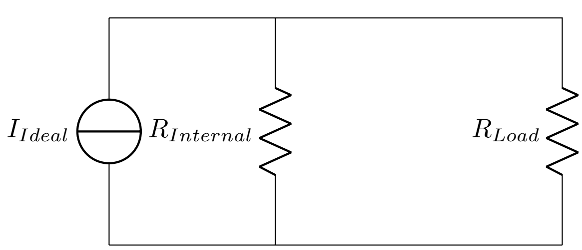


$$V_{Real} = V_{Ideal} - IR_{Internal}$$

$$V_{Real} = V_{Ideal}\frac{R_{Load}}{R_{Internal} + R_{Load}} \tag{2.1.9}$$

Definition 2.1.9. Ideal Current Source - a device that always provides that same current to a load.

Definition 2.1.10. Real Current Source - a current source with an internal resistance connected in parallel that reduces the current produced when the load resistance is high.



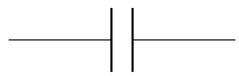
$$I_{Real} = I_{Ideal} - \frac{V}{R_{Internal}}$$

$$I_{Real} = I_{Ideal}\frac{R_{Internal}}{R_{Internal} + R_{Load}} \tag{2.1.10}$$

Definition 2.1.11. A **Capacitor** is a device that accumulates a charge q when a voltage v is applied with a proportionality constant C with units $\mathrm{F(farad)} = \mathrm{C\,V^{-1}} = \mathrm{C^2\,J^{-1}}$

$$q = Cv \tag{2.1.11}$$

$$i = C\frac{dv}{dt} \tag{2.1.12}$$



Remark. The energy stored in a capacitor can derived by integrating power over time:

$$\omega_C = \frac{1}{2}Cv^2 \tag{2.1.13}$$

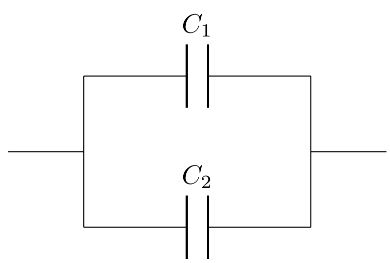
Definition 2.1.12. A **Plate Capacitor** is a capacitor made of two conductive plates with area A separated by distance l.

$$C = \frac{\varepsilon_0 \kappa A}{l}$$

Law 2.1.6. Capacitors in Series will decrease the overall capacitance.

$$\frac{1}{C_T} = \sum \frac{1}{C_i} \tag{2.1.14}$$

Law 2.1.7. Capacitors in Parallel is simply the sum of the individual capacitance.



$$C_T = \sum C_i \tag{2.1.15}$$

Definition 2.1.13. An **Inductor** is a device which accumulates a magnetic flux $\phi = BA$ when a current is applied with a proportionality constant L with units $\mathrm{H(henry)} = \mathrm{J\,A^{-2}}$

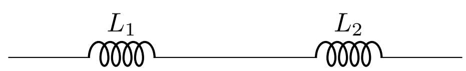
$$\phi = Li \tag{2.1.16}$$

$$v = L\frac{di}{dt} \tag{2.1.17}$$

Remark. The energy stored in an inductor can derived by integrating power over time:

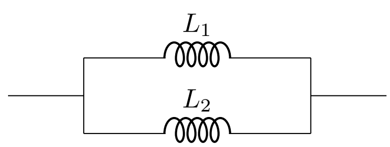
$$\omega_L = \frac{1}{2}Li^2 \tag{2.1.18}$$

Law 2.1.8. Inductors in Series is simply the sum of the individual inductance.



$$L_T = \sum L_i \tag{2.1.19}$$

Law 2.1.9. Inductors in Parallelwill decrease the overall inductance.



$$\frac{1}{L_T} = \sum \frac{1}{L_i} \tag{2.1.20}$$

2.1.2 RC and RL Circuits

Definition 2.1.14. Charging RC Circuit is a circuit with a resistor, capacitor and voltage source to charge the capacitor. The voltage equation around the loop can be written as

$$V = Ri + \frac{1}{C}\int_0^t i(\sigma)d\sigma$$

$$i(t) = \frac{V}{R}e^{-t/RC}$$

$$v_c(t) = V(1 - e^{-t/RC})$$

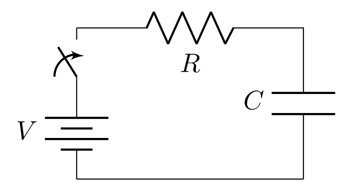


Figure 2.2: a basic RC circuit in the charging arrangement

Definition 2.1.15. Discharging RC Circuit is a circuit with a capacitor and resistor to discharge the capacitor. The current relation during discharge is symmetric to charge so we can rewrite the equations

$$i(t) = \frac{V}{R}(1 - e^{-t/RC})$$

$$v_c(t) = Ve^{-t/RC}$$

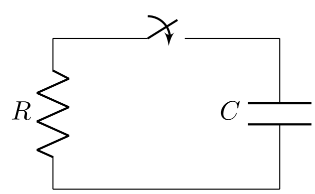


Figure 2.3: a basic RC circuit in the discharging arrangement

Definition 2.1.16. Charging RL Circuit is a circuit with a resistor, inductor and voltage source to charge the inductor. The current equation can be written as

$$i(t) = \frac{V}{R}(1 - e^{-tR/L})$$

$$v_l(t) = Ve^{-tR/L}$$

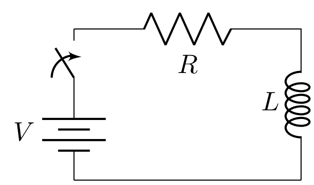


Figure 2.4: a basic RL circuit in the charging arrangement

Definition 2.1.17. Discharging RL Circuit is a circuit with a inductor and resistor to discharge the inductor. The current relation during discharge is symmetric to charging so we can rewrite the equations

$$i(t) = \frac{V}{R}e^{-tR/L}$$

$$v_l(t) = Ve^{-tR/L}$$

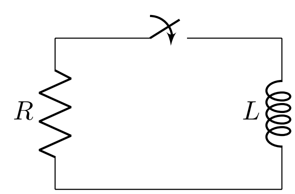


Figure 2.5: a basic RL circuit in the discharging arrangement

2.1.3 AC Circuits

Definition 2.1.18. An AC Voltage source is a voltages source with a alternating voltage. Typically the voltage changes according to the following function:

$$V(t) = V_p \sin(\omega t)$$

Definition 2.1.19. Effective Voltage or **Root Mean Square Voltage** is the voltage $V_p/\sqrt{2}$ where V_p is the peak sinusoidal voltage. This represents the DC voltage that would produce the same power draw as the AC voltage.

Example. **Resistor in AC Circuit** Bellow is a AC Circuit with a resistor. The current is determined by the following function

$$i(t) = \frac{V_p}{R}\sin\omega t$$

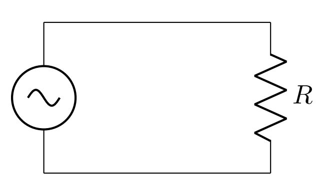


Figure 2.6: a basic AC circuit connected to a resistive load

Definition 2.1.20. Effective Current or **Root Mean Square Current** is the current $I_p/\sqrt{2}$ where I_p is the peak sinusoidal current. This represents the DC current that would produce the same power draw as the AC current.

Example. **Capacitor in AC Circuit** Bellow is a AC Circuit with a capacitor. The current is determined by the following function

$$i(t) = -\omega C V_p \sin \omega t$$

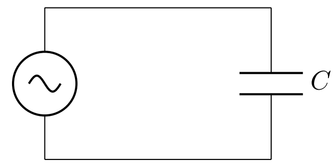


Figure 2.7: a basic AC circuit connected to a capacitive load

Definition 2.1.21. A Phasor Signal is a complex number that we use to represent the output of a wavelike function.

$$Re^{i\theta} = R\cos(\theta) + Ri\sin(\theta)$$

Law 2.1.10. To convert froma phasor signal to the observed signal the following formula applies where $x(t)$ si the observed signal and c is the complex phasor signal.

$$x(t) = Real(c \cdot e^{i\omega t})$$

Law 2.1.11. Generalized Ohm's Law is the complex version of ohm's law that includes complex impedance.

$$\tilde{V} = \tilde{I}\tilde{Z}$$

$$\tilde{Z} = R + iX$$

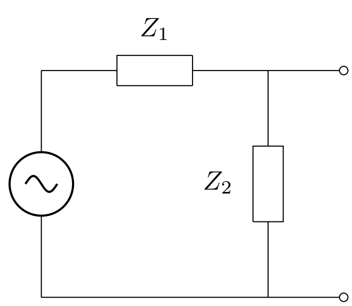
Where R is circuit resistance and X is circuit reactance.

Example. **Impotence of basic components**:

- **Resistor** $\tilde{Z} = \frac{\tilde{V}}{\tilde{I}} = R$
- **Capacitor** $\tilde{Z} = \frac{\tilde{V}}{\tilde{I}} = \frac{1}{i\omega C}$
- **Inductor** $\tilde{Z} = \frac{\tilde{V}}{\tilde{I}} = i\omega L$

Law 2.1.12. Generalized Voltage Divider is the complex version of the voltage divider formula that includes complex impedance.

$$\tilde{V_{out}} = \tilde{V_{in}} \frac{\tilde{Z}_2}{\tilde{Z}_1 + \tilde{Z}_2}$$



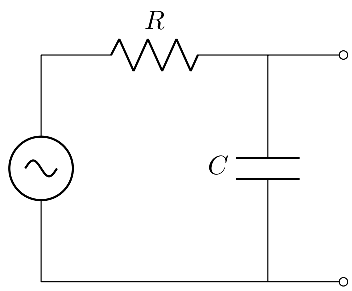
2.1.4 AC Filters

Definition 2.1.22. A **Low Pass Filter** is a circuit that allows low frequency signals through, but blocks out higher frequency signals.

$$\tilde{V_{out}} = \tilde{V_{in}} \frac{1}{1 + i\omega RC}$$

$$V_{out} = V_{in} \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\phi = \tan(-\omega RC)$$

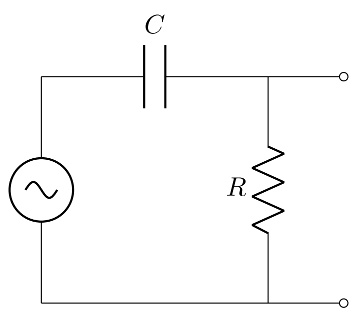


Definition 2.1.23. A **High Pass Filter** is a circuit that allows high frequency signals through, but blocks out lower frequency signals.

$$\tilde{V_{out}} = \tilde{V_{in}} \frac{1}{1 - \frac{i}{\omega RC}}$$

$$V_{out} = V_{in} \frac{1}{\sqrt{1 + (\frac{1}{\omega RC})^2}}$$

$$\phi = \tan(\frac{1}{\omega RC})$$



Definition 2.1.24. The **Breakpoint Frequency** is the frequency of a filter that produces an inflection point on the frequency response of that circuit. For simple low and high pass filters it can be represented as the following:

$$\omega_c = \frac{1}{RC}$$

$$f_c = \frac{1}{2\pi RC}$$

Definition 2.1.25. A **Decibel** scale is used to measure a ratio that can represent very large and very small values.

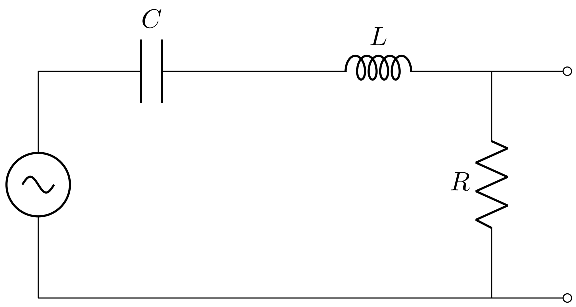
$$\text{Decibal} = 20\log_{10}(|\frac{V_{out}}{V_{in}}|) = 10\log_{10}(|\frac{P_{out}}{P_{in}}|)$$

Definition 2.1.26. A **Band Pass Filter** is a circuit that only allows mid range frequency signals through, but blocks out lower and higher frequency signals.

$$\tilde{V_{out}} = \tilde{V_{in}} \frac{1}{1 - \frac{i}{\omega RC} + i\omega L}$$

$$V_{out} = V_{in} \frac{1}{\sqrt{1 + (\omega \frac{L}{R} - \frac{1}{\omega RC})^2}}$$

$$\phi = \tan(\frac{1}{\omega RC} - \omega \frac{L}{R})$$



Definition 2.1.27. The **Resonance Frequency** is the frequency of resonance or max output for a band pass filter.

$$\omega_0 = \frac{1}{\sqrt{RL}}$$

$$f_0 = \frac{1}{2\pi\sqrt{RL}}$$

Definition 2.1.28. The **Band Width** is the difference between the two cutoff frequencies for a band pass filter.

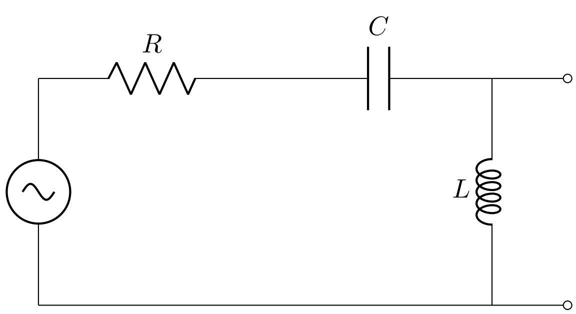
$$\omega_{c1} - \omega_{c2} = \frac{R}{L}$$

$$f_{c1} - f_{c2} = \frac{R}{2\pi L}$$

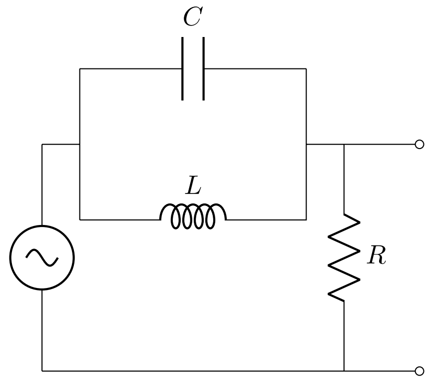
Definition 2.1.29. The **Quality Factor** is a measure of the sharpness of the resonance peak of a band pass filter.

$$Q = \frac{f_0}{f_{c1} - f_{c2}} = \frac{\omega_0}{\omega_{c1} - \omega_{c2}}$$

Example. **Resonant High Pass Filter**:



Example. **Notch Filter**:

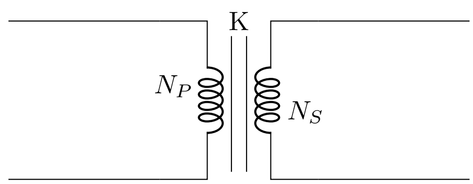


2.1.5 Transformers

Definition 2.1.30. A **Transformer** is a primary and secondary connected by a ferromagnetic core such that the primary coil induces voltage in the secondary.

$$\frac{V_S}{V_P} = \frac{V_S}{V_P}$$

$$\frac{I_S}{I_P} = \frac{V_P}{V_S}$$

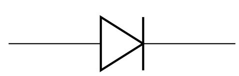


2.1.6 Silicon-based Components

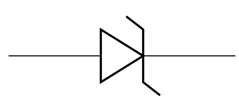
Definition 2.1.31. **N-type silicon** is silicon doped with an element like Phosphorous which produces free electrons in the material.

Definition 2.1.32. **P-type silicon** is silicon doped with an element like Boron which produces electron holes in the material.

Definition 2.1.33. A **Diode(PN Junction)** is a device that has a very low resistance for current in one direction but a very high resistance for current in the other direction.



Definition 2.1.34. A **Zener Diode** is a diode with extremely high doping that operates like a standard diode in the forward direction, but in a situation of very high (4-500V) reverse potential the diode very quickly becomes conducting without damage.



Definition 2.1.35. **Half Bridge Rectifier** is a diode that converts alternating current into directed current with changing magnitude. A capacitor is often included across the output leads to smooth out the output. The two following equation model the output voltage and ripple voltage of a full bridge rectifier.

$$V_{out} = V_s - (D_v)$$

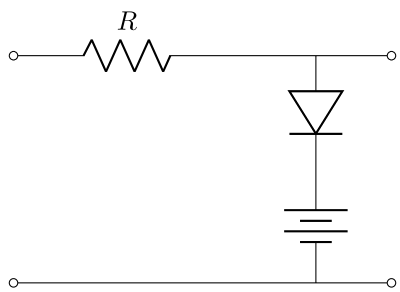
$$\Delta V = \frac{V_o ut}{f R_L C}$$

Definition 2.1.36. Full Bridge Rectifier is a system of diodes that converts alternating current into directed current with changing magnitude. A capacitor is often included across the output leads to smooth out the output. The two following equation model the output voltage and ripple voltage of a full bridge rectifier.

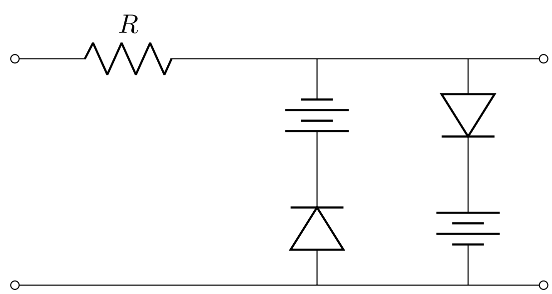
$$V_{out} = V_s - 2(D_v)$$

$$\Delta V = \frac{V_o ut}{2fR_LC}$$

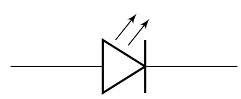
Definition 2.1.37. Half Bridge Diode Clamp - limits the positive voltage using a battery and a diode. The voltage cannot rise higher than the voltage of the battery plus the activation voltage of the diode.



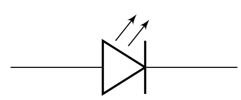
Definition 2.1.38. Full Bridge Diode Clamp - limits the positive and negative voltage using batteries and diodes. The voltage cannot rise higher or lower than the voltage of the battery plus the activation voltage of the diode.



Definition 2.1.39. A **Photo-diode** is a component made by a pin-junction that works like a normal diode unless it is exposed to light. When exposed to light it allows current in the reverse direction.



Definition 2.1.40. A **Light emitting diode** is a diode made with a direct bandgap semiconductor that converts some of the energy used in it's voltage drop to produce light.



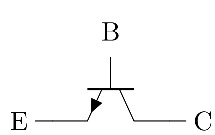
Definition 2.1.41. A **NPN Transistor** is an active component with three contacts on n-type, p-type, and n-type semiconductors. α represents the ratio between the collector current and the current through the emitter. β represents the ratio between the collector current and the base current. β is usually large which allows a very small current to create a very large current through the collector.

$$I_E = I_B + I_C$$

$$\beta = \frac{\alpha}{1 - \alpha}$$

$$I_C = \alpha I_E$$

$$I_C = \beta I_B$$



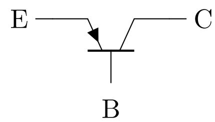
Definition 2.1.42. A **PNP Transistor** is an active component with three contacts on p-type, n-type, and p-type semiconductors. A PNP is identical to the NPN transistor but the current flows in the opposite direction.

$$I_E = I_B + I_C$$

$$\beta = \frac{\alpha}{1-\alpha}$$

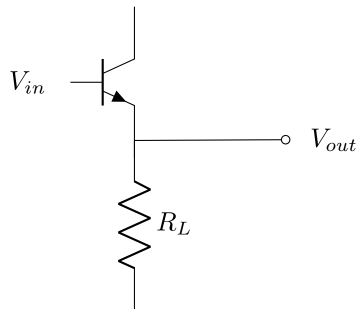
$$I_C = \alpha I_E$$

$$I_C = \beta I_B$$



Example. **Emitter-follower circuit** is a circuit that uses a transistor to apply the same voltage as the the base to a load at the emitter but at a much higher current.

$$V_{out} = V_{in} - 0.6$$



Example. **Common Emitter Amplifier circuit** is a circuit that uses a transistor to an resistor network to amplify a voltage.

$$V_{out} = V_{cc} - R_c I_c$$

$$V_B = V_E + 0.6$$

$$I_E = I_B + I_C$$

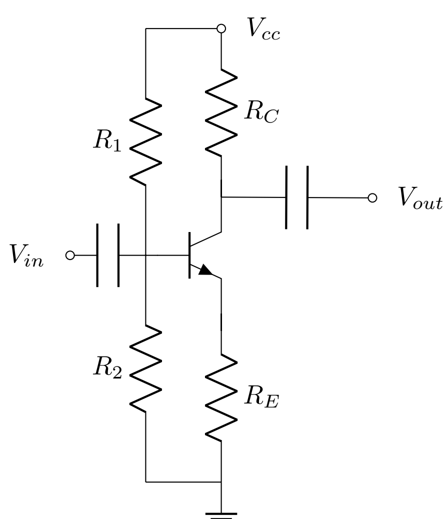
$$\frac{\Delta V_{out}}{\Delta V_{in}} = -\frac{R_C}{R_E}$$

$$V_{cc} - (R_C + R_E)I_C - V_{CE} = 0$$

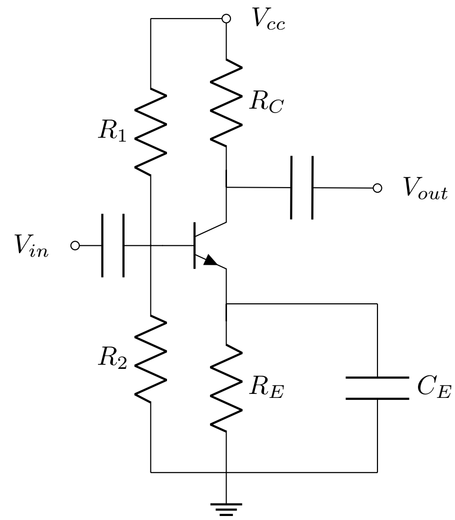
$$\bar{V_B} = (\frac{R_2}{R_1 + R_2})V_{cc}$$

$$\bar{V_C} = \frac{1}{2}V_{cc}$$

$$\frac{V_{cc}}{R_1 + R_2} = \frac{R_C}{R_E}\frac{I_C}{\beta}$$

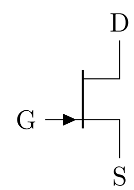


Example. **Emitter Bypass**



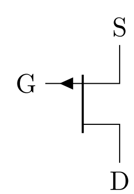
Definition 2.1.43. N-Channel Field Effect Transistor is a n-type channel with a p-type gate. The voltage on the gate determines the resistance of the channel.

$$\frac{\Delta I_D}{\Delta V_{GS}} = g_m$$



Definition 2.1.44. P-Channel Field Effect Transistor is a p-type channel with a n-type gate. The voltage on the gate determines the resistance of the channel.

$$\frac{\Delta I_D}{\Delta V_{GS}} = g_m$$



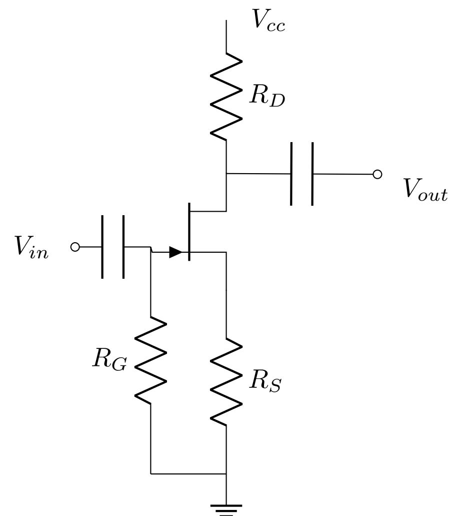
Example. **Common Source JFET Amplifier** is a circuit that uses a JFET to amplify an AC signal.

$$\bar{V_G} - \bar{V_S} = -R_S \bar{I_D}$$

$$\bar{V_S} = R_S \bar{I_D}$$

$$R_S = \frac{-(\bar{V_G} - \bar{V_S})}{\bar{I_D}}$$

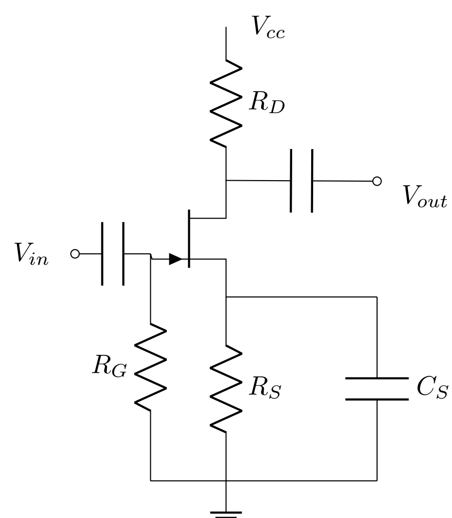
$$\frac{\Delta V_{out}}{\Delta V_{in}} = \frac{-R_D(\frac{\Delta I_D}{\Delta V_{GS}})}{1 + R_S(\frac{\Delta I_D}{\Delta V_{GS}})} = \frac{-R_D(g_m)}{1 + R_S(g_m)}$$



Example. **Common Source JFET Amplifier Source Bypass** is a circuit that uses a JFET to amplify an AC signal. The capacitor increases the gain for high frequencies.

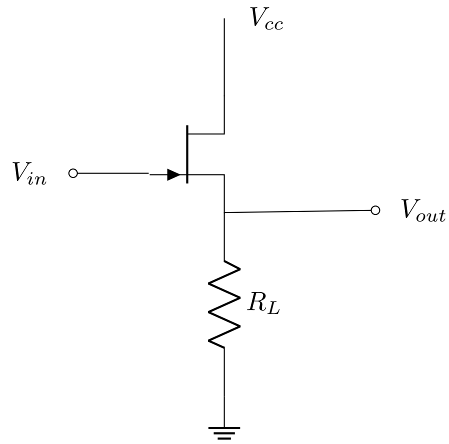
$$\frac{1}{\omega C_s} = \frac{R_S}{10}$$

$$\frac{\Delta V_{out}}{\Delta V_{in}} = -R_D g_m$$



Example. **FET follower** is a circuit that uses a JFET to following an AC input voltage. The output voltage follows the input if $R_L g_m >> 1$

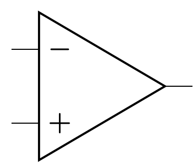
$$\Delta V_{out} = R_L g_m (\Delta V_{in} - \Delta v_{out})$$



2.1.7 Operational Amplifiers

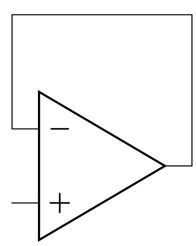
Definition 2.1.45. Ideal Operational Amplifier is an amplifier with ∞ input impedance, 0 output impedance, and a gain(A) of ∞.

$$V_{out} = A(V_+ - V_-)$$



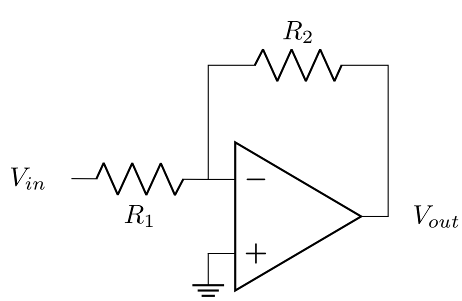
Example. **Operational Amplifier Follower**

$$V_+ = V_-$$
$$\frac{V_o ut}{V_i n} = 1$$



Example. **Inverting Operational Amplifier**

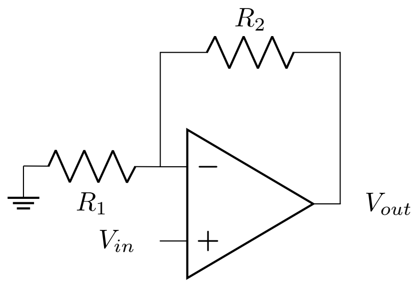
$$V_+ = V_- = 0$$
$$\frac{V_o ut}{V_i n} = -\frac{R_2}{R_1}$$



Example. **Non-inverting Operational Amplifier**

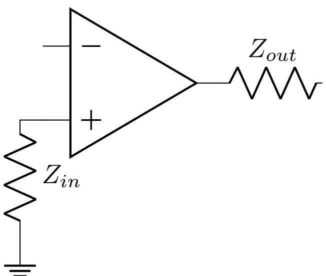
$$V_+ = V_- = V_{in}$$

$$\frac{V_out}{V_in} = \frac{R_2}{R_1}$$



Definition 2.1.46. Real Operational Amplifier is an amplifier with large input impedance, small output impedance, and a large gain(A).

$$V_{out} = A(V_+ - V_-)$$

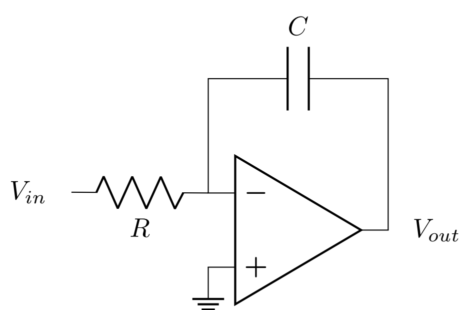


Definition 2.1.47. Slew rate represents how fast the output of an op-amp can respond to changes in the input voltage.

$$\frac{dV_{out}}{dt} = 110\,\mathrm{V\,s^{-1}}$$

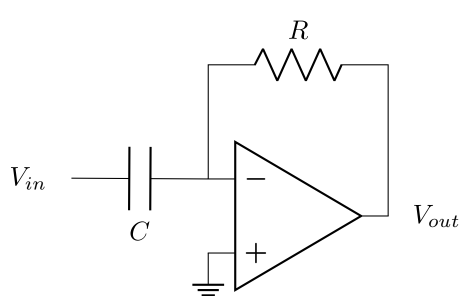
Example. **Integrator**

$$V_{out} = \frac{-1}{RC}\int V_{in}dt$$



Example. **Differentiator**

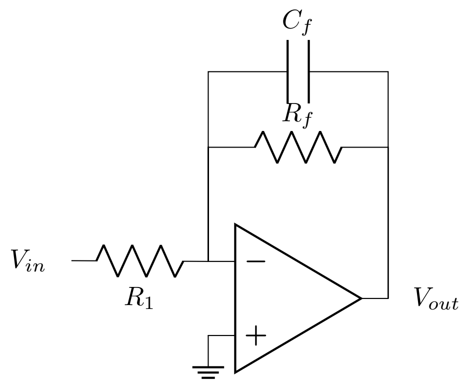
$$V_{out} = -CR\frac{dV_{in}}{dt}$$



Example. **Low-pass Filter**

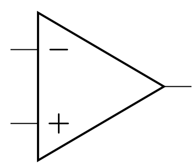
$$\frac{\tilde{V_o}ut}{\tilde{V_i}n} = -\tilde{Z}_f R_1 = -\frac{\frac{R_f}{R_1}}{1 + i\omega C_f R_f}$$

$$\omega_{bp} = \frac{1}{R_f C_f}$$



2.1.8 Digital Circuits

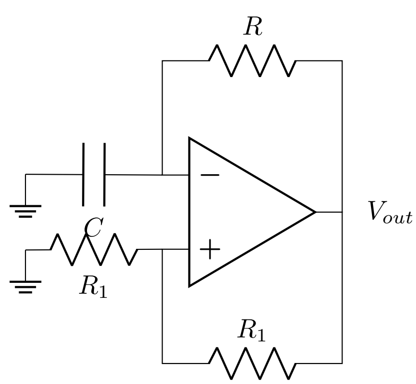
Definition 2.1.48. Comparator - if V_+ is greater than V_- then the output is V_{CC} otherwise it is $-V_{CC}$.



Example. **Timer Circuit**

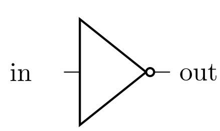
$$V_{out} = A(V_+ - V_-)$$

However V_{out} is limited by $+V_{CC}$ and $-V_{CC}$. So V_{out} is always either $+V_{CC}$ or $-V_{CC}$. The capacitor will charge until it reaches half of the output voltage and then it will start discharging until it reaches half the output voltage and the cycle continuous.



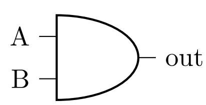
Definition 2.1.49. Inverter Takes a logical input and returns the opposite. It is constructed using a npn bipolar junction transistor.

$$\text{out} = \overline{\text{in}}$$



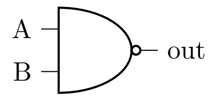
Definition 2.1.50. AND Gate Takes two logical input and returns true if and only if both inputs are true. It is constructed using two npn bipolar junction transistor in series.

$$\text{out} = A \cdot B$$



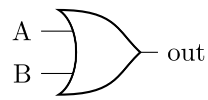
Definition 2.1.51. NAND Gate (Not AND) Takes two logical input and returns false only if and only if both inputs are true. It is constructed using two npn bipolar junction transistor in series.

$$\text{out} = \overline{A \cdot B}$$



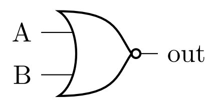
Definition 2.1.52. OR Gate Takes two logical input and returns true if and only if any of the inputs are true. It is constructed using two npn bipolar junction transistor in parallel.

$$\text{out} = A + B$$



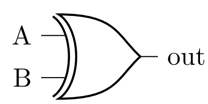
Definition 2.1.53. NOR Gate (Not OR) Takes two logical input and returns false if and only if any of the inputs are true. It is constructed using two npn bipolar junction transistor in parallel.

$$\text{out} = \overline{A + B}$$



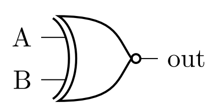
Definition 2.1.54. XOR Gate (eXclusive OR) Takes two logical input and returns true if and only if exactly one of the two inputs is true. It is constructed using bipolar junction transistors.

$$\text{out} = (A + B) \cdot (\overline{A} + \overline{B})$$



Definition 2.1.55. XNOR Gate (eXclusive NOR) Takes two logical input and returns false if and only if exactly one of the two inputs is true. It is constructed using bipolar junction transistors.

$$\text{out} = (A \cdot B) + (\overline{A} \cdot \overline{B})$$



Theorem 2.1.1. Commutative Property of Boolean Algebra

$$A \cdot B = B \cdot A$$

$$A + B = B + A$$

Theorem 2.1.2. Associative Property of Boolean Algebra

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A + B) + C = A + (B + C)$$

Theorem 2.1.3. Distributive Property of Boolean Algebra

$$A \cdot (B + C) = A \cdot B + A \cdot C$$

$$A + (B \cdot C) = (A + B) \cdot (A + C)$$

Theorem 2.1.4. Absorption Theorem

$$A \cdot (A + B) = A$$

$$A + (A \cdot B) = A$$

Theorem 2.1.5. Demorgan's Theorems

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

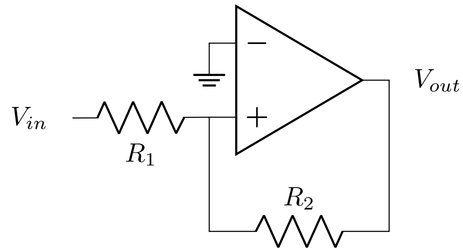
$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

Definition 2.1.56. Schmitt Trigger - is a component of a gate that reduces noise in the undefined section of a gates output(in between 1 and 0). As long as the noise is within the range $[\frac{R_1}{R_2}(-V_{cc}), \frac{R_1}{R_2}(+V_{cc})]$.

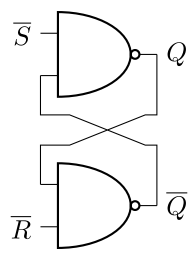
$$V_+ = \frac{V_{out}R_1 + V_{in}R_2}{R_1 + R_2}$$

$$V_{in} < \frac{R_1}{R_2}(-V_{cc}) \Rightarrow V_{out} = -V_{cc}$$

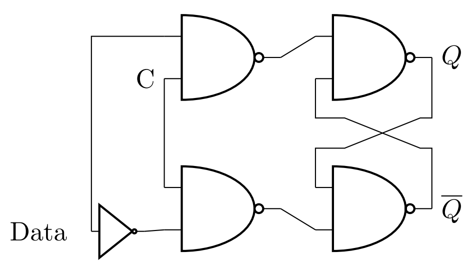
$$V_{in} > \frac{R_1}{R_2}(+V_{cc}) \Rightarrow V_{out} = +V_{cc}$$



Definition 2.1.57. RS flip-flop is a circuit of gates that can store a bit of information.



Definition 2.1.58. Data flip-flop is a circuit of gates that can store a bit of information with a ciruit to allow for a single input that reads according to a clock cycle. If the clock input is 0 then the state of the flip flop cannot be changed.



Chapter 3

Thermodynamics

Definition 3.0.1. Thermal Expansion is a the physical expansion or retraction of materials under temperature changes. Where $\alpha =$ coefficient of linear expansion.
Linear Thermal Expansion:

$$L = L_0(1 + \alpha\Delta T)$$

Volumetric Thermal Expansion:

$$V = V_0(1 + 3\alpha\Delta T + 3(\alpha\Delta T)^2 + (\alpha\Delta T)^3)$$
$$V \approx V_0(1 + 3\alpha\Delta T)$$

Law 3.0.1. Young's Modulus is the constant for the relationship between force applied and a change in length.

$$E = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

Where E is the young's modulus, F is the force applied, A is the area over which the force is applied, ΔL is the change in length, and L is the original length.

Law 3.0.2. Heat Capacity Relation is the relationship between thermal energy in materials and there temperature.

$$E = qm\Delta T$$

Where q is the specific heat of the material, m is mass, ΔT is the change in temperature, and E is the amount of energy required or released.

Law 3.0.3. Heat of Phase Transitions - is the energy required/released to break/form inter-molecular bonds that influence the phases of matter.

$$E = L_f m$$

Where L_f is the heat of fusion, m is the mass, and E is the energy required.

Definition 3.0.2. Conduction is the rate of heat energy transfer through a material.

$$E = k\frac{A}{L}\Delta T t$$

Where E is the energy transferred, A is area, L is the length, ΔT is the temperature difference, and t is time.

Law 3.0.4. Unified Gas Law models the behavior of a a fixed amount of gas.

$$\frac{PV}{T} = \frac{PV}{T}$$

Where P is pressure, V is volume, and T is temperature.

Law 3.0.5. Ideal Gas Law models the behavior of an ideal gas in a closed container.

$$PV = nRT$$

Where P is pressure, V is volume, n is moles of gass in the material, R is the ideal gas constant $8.31446261815324\,\mathrm{m}^3\,\mathrm{Pa}\,\mathrm{K}^{-1}\,\mathrm{mol}^{-1}$, T is temperature.

Remark. To convert from number of particles to moles use the following formula.

$$N = n * 6.022 * 10^{23}$$

Definition 3.0.3. The standard temperature and pressure is a particular temperature and pressure that is used for situations near sea level.

- $T = 273\mathrm{K}$
- $P = 1atm = 101300\mathrm{Pa}$
- $V = 22.4\mathrm{L} = 0.00224m^3$

Law 3.0.6. The **Boltzmann Distribution** describes how the velocities of particles at a particular temperature behave.

$$P(v) = Cv^2e^{-v^2}$$

$$v_p = \sqrt{2}\sqrt{\frac{RT}{M}}$$

$$v_{avg} = \sqrt{\frac{8}{\pi}}\sqrt{\frac{RT}{M}}$$

$$v_{rms} = \sqrt{3}\sqrt{\frac{RT}{M}}$$

Law 3.0.7. The **Internal Energy** stored by the temperature of a system is modeled by the following formula

$$U = \frac{f}{2}nRT$$

Where f is the degrees of freedom: 3 for monoatomic gasses, 5 for linear molecules, and 6 for asymmetric molecules.

3.1 The Laws of Thermodynamics

Law 3.1.1. The **First Law of Thermodynamics** states that the change in internal energy is given by the sum of change in heat energy times that work done on the system.

$$\Delta U = Q + W$$

Work done to the gas is modeled by the following formula:

$$W = -\int P\ dV$$

Example. For **Isothermal** processes the temperature is constant so pressure follows $P = \frac{nRT}{V}$ so we can calculate the work done on the system:

$$W = -\int \frac{nRT}{V}\ dV = -nRT \ln \frac{V_f}{V_i}$$

And from the first law of thermodynamics we can calculate the heat absorbed by the system:

$$Q = -W = nRT \ln \frac{V_f}{V_i}$$

Example. For **Isobaric** processes the pressure is constant so we can calculate the work done on the system:

$$W = -\int P_0\ dV = -P_0 \Delta V$$

And from the first law of thermodynamics we can calculate the heat absorbed by the system:

$$Q = \Delta U - W = \frac{f}{2} nR\Delta T + P_0 \Delta V$$

Example. For **Isochoric(Isovolumetric)** processes the volume is constant so we can calculate the work done on the system:

$$W = 0$$

So clearly the change in internal energy is equal to the heat absorbed by the system:

$$Q = \Delta U = \frac{f}{2} nR\Delta T$$

Example. For **Adiabatic** processes the heat transferred is zero so we can calculate the work done on the system:

$$W = \Delta U = \frac{f}{2} nR\Delta T$$

By definition heat absorbed is zero:

$$Q = 0$$

We can also derive the relationship of pressure and volume:

$$P_1 V_1^{\frac{f+2}{f}} = P_2 V_2^{\frac{f+2}{f}}$$

Law 3.1.2. Second Law of Thermodynamics states that the change in heat energy divided by the temperature is less than or equal to the change in entropy:

$$\frac{\Delta Q}{T} \leq \Delta S$$

Remark. For reversible changes the change in entropy is equal to change in heat.

Definition 3.1.1. The **Efficiency** of a heat pump is the work required to run the pump per heat output.

$$E = \frac{W}{Q_{out}}$$

The **Coefficient of Performance** of a refrigerator is the heat input per work required to run the pump.

$$\kappa = \frac{Q_{in}}{W}$$

Definition 3.1.2. Under the second law of thermodynamic the **Carnot Efficiency** and **Carnot Coefficient of Performance** are the maximum possible efficiency and coefficient of performance for any pump at that temperature.

$$E_C = \frac{T_H - T_C}{T_H}$$

$$\kappa_C = \frac{T_C}{T_H - T_C}$$

Chapter 4

Relativity

Theorem 4.0.1. Principle of Relativity states that the laws of physics are the same in all inertial frames. There is no way to detect absolute motion and there is no preferred inertial frame.

Theorem 4.0.2. Constancy of the Speed of Light states that all observers from any inertial frames measure the speed of light as 299792458m/s.

Theorem 4.0.3. Gravitational Equivalence Principle state that inertia mass and gravitational mass are equal.

Definition 4.0.1. Inertia Frames are reference frames where Newtons first law applies.

Law 4.0.1. Lorentz-Fitzgerald's Change of Reference Frame When changing between a reference frame with coordinate x, y, z, t to a reference frame with a relative velocity v in the x direction and coordinates x', y', z', t' the following equations apply:

$$x' = \gamma(x - \beta ct)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \beta\frac{x}{c})$$

with β and γ defined as

$$\beta = \frac{v}{c}$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Remark. From this law we can derive an understanding of time dilation and length contraction:

$$T' = \frac{T}{\sqrt{1-\beta^2}}$$
$$L' = L\sqrt{1-\beta^2}$$

We can also derive an equation for converting velocities to difference reference frames:

$$U' = \frac{v+U}{1+\frac{vU}{c^2}}$$

Law 4.0.2. Space-time Interval is the the distance than an object travels through space-time.

$$(\Delta s)^2 = (\Delta r)^2 - (c\Delta t)^2 = (x, y, z, ict)$$

The space-time interval will be the same independent to the reference frame.

Definition 4.0.2. Relativistic Doppler Effect is the effect observed from moving sources of waves. Let f_0 be the original frequency, f be the observed frequency, $v_{\parallel}$ be the component of the relative velocity parallel to the observer, and $v_{\perp}$ be the component of the relative velocity away from the observer.

$$f = f_0\sqrt{1-\frac{v_{\parallel}^2}{c^2}}\frac{\sqrt{1-\frac{v_{\perp}}{c}}}{\sqrt{1+\frac{v_{\perp}}{c}}}$$

Definition 4.0.3. Relativistic Momentum the momentum of an object according to relativity.

$$\rho = \gamma m\vec{v} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Definition 4.0.4. Relativistic Energy is the energy of objects according to relativity

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

Definition 4.0.5. Relativistic Energy and Momentum Relations the following equation describes how momentum and energy are related

$$E^2 = \rho^2c^2 + m^2c^2$$
$$\frac{\rho}{E} = \frac{\beta}{c}$$

Definition 4.0.6. Relativistic Mass is the mass of objects according to relativity

$$m = \gamma m_0 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Chapter 5

Quantum Physics

5.1 Introduction

5.1.1 Optical Spectroscopy

Definition 5.1.1. Hydrogen Spectrum: J. Rydberg is a model of the emission lines for hydrogen, where n is the final state of the electron, $k > n$ is the initial state ($n, k \in \{1, 2, 3, 4, 5\}$) and $R_H = 1.096776 * 10^7\text{m}^{-1}$.

$$\frac{1}{\lambda} = R_H(\frac{1}{n^2} - \frac{1}{k^2})$$

5.1.2 Black body Radiation

Law 5.1.1. Stefan-Boltzmann Law models the power of thermal radiation released by an object.

$$P = \sigma e A T^4$$

Where σ is the Stefan-Boltzmann constant ($5.6703 * 10^{-8}\text{W/m}^2\text{K}^4$), e is the emissivity, A is the area, and T is the temperature in Kelvin.

Law 5.1.2. Wien's displacement Law models peak wavelength of light emitted by an object as a specific temperature.

$$\lambda_{max} = b/T$$

Where λ_{max} is the wavelength of light at the peak thermal radiation, b is wien's constant ($2.898 * 10^{-3}$), and T is temperature in Kelvin.

Law 5.1.3. The **Planck Distribution** models the derivative of power output for a black body with frequency or wavelength of radiated light. It was created under the assumption that light has quantized units.

Law 5.1.4. Planck's constant is the coeficient that relates the frequency of a photon to it's energy, recall that this number is one of our fundamentally defined constants as $h = 6.62607015 \times 10^{-34} kg\ m^2 s^{-1}$.

$$E_{quanta} = hf$$

5.1.3 Structure of an Atom

Law 5.1.9. Rutherford Scattering is a model for predicting the scattering of an alpha particle off a nucleus derived from Coulomb's law.

$$b = \frac{1}{2}\frac{1}{4\pi\epsilon_0}(z_1 e)(z_2 e)\frac{1}{KE}\cot(\frac{\theta}{2})$$

Law 5.1.5. The **Law of Particles in perpendicular Magnetic and Electric fields** is derived from Maxwell's Equations. It described the charge mass ratio of a particle that is deflected at a particular angle in perpendicular Magnetic and Electric fields. There are two possible configurations: a velocity selector where the magnetic and electric field are equal or a spectrometer which seperates particles based on mass-charge ratio.

$$\frac{q}{m} = \frac{E\tan\theta}{B^2 l}$$

$$\frac{R}{B} = v_0$$

Law 5.1.6. Photoelectric Effect is the effect observed by shining light on a voltage gap in a vacuum. The energy of one photon is used to accelerate one electron. Let ϕ be the energy required to liberate one electron (work function) from the cathode. We can derive the following equations:

$$hf = \phi + \frac{1}{2}mv^2$$

$$hf = \phi + ev_0$$

Law 5.1.7. Wilhelm Conrad Routgen discovered the production of x-rays through firing electrons at an cathode in a vacuum. He developed the **Duane-Hunt rule** while models the minimum wavelength photon or highest energy photon that can be produced with this method.

$$\lambda_{min} = \frac{hc}{ev_0}$$

Law 5.1.8. The **Compton Effect** a model that predicts how a photon will scatter off an electron. Let ϕ represents the deflected angle of the electron and θ be the deflected angle of the photon.

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos(\theta))$$

where b is the impact parameter and θ is the angle of scattering.

$$f = \frac{\pi}{4} nt(kz_1 e z_2 z)^2 \frac{1}{KE^2} \cot^2(\frac{\theta}{2})$$

where f is the fraction of particles that are scattered with an angle greater than θ, n is the number of atoms per unit volume, and t is the thickness.
Differential form:

$$\frac{N(\theta)}{N_i} = \frac{1}{16} nt(kz_1 e z_2 z)^2 \frac{1}{KE^2} \frac{1}{r^2} \frac{1}{\sin^4(\frac{\theta}{2})}$$

where $N(\theta)$ is the number of scattered particles per unit area, n is the number of atoms per unit volume, t is the thickness, and r is the radius of the detector.

Law 5.1.10. The Bohr Model of a Hydrogen Atom is a classical model of an atom. In order to complete the model assumptions about the energy states of electron orbitals, and the angular momentum of a state is quantized.

1. **Radii of orbits**

$$r_n = a_0 \frac{n^2}{z} = n^2 \frac{\epsilon_0 h^2}{\pi m_e e^2 z}$$

$$a_0 \approx 0.529\text{Å}$$

2. **Energies of orbits**

$$E_n = -E_0 \frac{z^2}{n^2} = -\frac{me^4 z^2}{8h^2 \epsilon_0^2 n^2}$$

$$E_0 \approx 13.6\text{eV}$$

3. **Velocity of orbits**

$$\beta_n = \frac{z}{n}\alpha$$

$$\beta_n = \frac{v_n}{c} = \frac{z}{n}\beta_1 = \frac{e^2}{2\epsilon_0 hc}\frac{z}{n}$$

$$\alpha \approx \frac{1}{137}$$

5.1.4 X-Ray Diffraction

Law 5.1.11. Bragg's Law is a simple model that predicts the scattering angle of x-rays through a crystal with planes separated by a distance d.

$$2d\sin(\theta) = n\lambda$$

5.2 Particle Wave Duality

Law 5.2.1. De Broglie Waves predicts the wavelength of both matter and energy (photons) where h is Planck's constant and λ is the wavelength.

$$hf = pc = p\lambda f$$

$$\frac{hc}{pc} = \frac{h}{p} = \lambda$$

Law 5.2.2. Bohr's condition for angular momentum Is the definition of angular momentum of an electron in an orbital it was necessary for the Bohr model of the atom.

$$L = n\frac{h}{2\pi}$$

Law 5.2.3. Clinton Joseph Davisson Electron scattering models the scattering of electrons in a material with a lattice constant D.

$$D\sin(\phi) = n\lambda$$

5.2.1 Wave mechanics

Definition 5.2.1. A **Wave Function** is a periodic function with an amplitude. The following is a a simple formalism.

$$E(x,t) = A\sin(kx - \omega t)$$

Where x and t are position and time respectively. k is the wave number, ω is the frequency in rad/s, and A is the amplitude.

Example. **Wave Functions in Electrodynamics**
Waves: $\vec{E}, \vec{B} \in \mathbb{R}^3$
Intensity: $I \sim \vec{E}^2$

Example. **Wave Functions in Quantum Mechanics**
Waves: $\psi \in \mathbb{C}$
Probability density: $S \sim |\psi|^2$

Definition 5.2.2. Fourier Transformation is a bijection between that translates periodic functions in none periodic functions. In quantum mechanics it used to translated between position and frequency.

Law 5.2.4. Heisenburg's Uncertainty Principle states that there are conjugate variables that we cannot reduced the uncertainty of both below a ratio.

$$\delta x \cdot \delta p_x \geq \frac{h}{4\pi}$$

$$\delta y \cdot \delta p_y \geq \frac{h}{4\pi}$$

$$\delta z \cdot \delta p_z \geq \frac{h}{4\pi}$$

$$\delta t \cdot \delta E \geq \frac{h}{4\pi}$$

$$\delta \phi \cdot \delta L \geq \frac{h}{4\pi}$$

5.3 Quantum Mechanics

Wave functions must be single values, with finite values where the limit: $\lim_{x\to\infty} \psi = 0$ and they must be continuously differentiable.

Definition 5.3.1. A **Wave function** represents the probability density of a physical wavelike particle.

$$\Psi(\vec{r}, t) =$$

Definition 5.3.2. An **Operator** is a transformation that extracts a physics property from a wave function.

Theorem 5.3.1. Expectation value can be found for a particular operator by integrating the wave function over all space.

$$\langle \hat{O} \rangle = \int_{-\infty}^{+\infty} \Psi^* \hat{O} \Psi = \langle \Psi^*, \hat{O}\Psi \rangle$$

Definition 5.3.3. Schrodinger equation is the wave function written by Schrodinger to model quantum measurements.

$$\hat{E}\Psi(x,t) = \hat{K}\Psi(x,t) + \hat{V}\Psi(x,t)$$

Where $\hat{E}$ is the energy operator, $\hat{K}$ is the kinetic energy operator, and $\hat{V}$ is the potential energy operator.

In one dimension we can derive the operators for position, momentum, and energy.

Example. **Momentum Operator**

$$\hat{P}\Psi(x,t) = -i\frac{h}{2\pi}\frac{\partial}{\partial x}\Psi(x,t)$$

Example. **Kinetic Energy Operator**

$$\hat{K}\Psi(x,t) = -\frac{h^2}{8\pi^2 m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

Example. **Total Energy Operator**

$$\hat{E}\Psi(x,t) = i\frac{h}{2\pi}\frac{\partial}{\partial t}\Psi(x,t)$$

Now lets consider an example of a wave function for a one dimensional particle.

Example. **Infinite Square Well Wave Function** is the one dimensional wave function for a particle in a gap.

$$\Psi(x) = A\sin(kx)$$

$$k = \frac{n\pi}{L}$$

$$A = \sqrt{\frac{2}{L}}$$

Now lets apply the energy operator to find the energy spectrum.

$$E = n^2\frac{h^2}{8mL^2}$$

This example can also be generalized to 3 dimensions. Now we have:

$$\Psi(\vec{x}) = A\sin(k_1x_1)\sin(k_2x_2)\sin(k_3x_3)$$

Note that each dimension can have a different state. So lets find the energy for this 3 dimensional case.

$$E_{n_1,n_2,n_3} = \frac{h^2}{8m}\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2}\right)$$

$$E_{n_1,n_2,n_3} = \frac{h^2}{8mL^2}(n_1^2 + n_2^2 + n_3^2)$$

Example. **Finite Square Well Wave Function** is the one dimensional wave function for a particle outside of a gap. In this example the positentals are non-zero for the regions 1 and 3 and zero for the region 2.

$$\Psi_1(x) = Ae^{\alpha x}$$

$$\Psi_2(x) = Ee^{ikx} + Fe^{-ikx}$$

$$\Psi_3(x) = De^{-\alpha x}$$

Example. **Simple Oscillator** can be modeled using a wave function.

$$\epsilon = \sqrt{\frac{m\omega}{\bar{h}}}\sqrt{\frac{k}{\omega\bar{h}}}$$

$$\Psi(\epsilon) = \frac{1}{\sqrt{2^n n!\sqrt{\pi}x_0}}H_n(\epsilon)e^{\frac{1}{2}\epsilon^2}$$

$$E_n = (\frac{1}{2}+n)\frac{h}{2\pi}\omega = (\frac{1}{2}+n)hf$$

5.3.1 The Hydrogen Atom

Definition 5.3.4. The Schrodinger Hydrogen Atom Model uses the techniques of the Schrodinger equation to model the properties of the hydrogen atom.

- **Radial equation**

$$\frac{1}{r^2}\cdot\frac{d}{dr}(r^2\frac{dR}{dv}) + \frac{2\mu}{\hbar^2}[E - V - \frac{\hbar^2}{2\mu}\cdot\frac{\ell(\ell+1)}{r^2}]R = 0$$

- **Polar equation**

$$\frac{1}{\sin(\theta)}\cdot\frac{d}{d\theta}(\sin(\theta)\cdot\frac{df}{d\theta}) + [\ell(\ell+1) - \frac{m_\ell^2}{\sin^2(\theta)}]f = 0$$

- **Azimuthal equation**

$$\frac{d^2g}{d\phi^2} = -m_\ell^2\cdot g$$

Quantum numbers

- principal: $n = 1, 2, 3, ...$
- orbital angular momentum: $\ell = 0, 1, 2, ...(n-1)$
- magnetic: $m_\ell = -\ell, ..., -1, 0, 1, ..., \ell$
- spin: $s = \frac{1}{2}$

Wave function

$$\Psi(r,\theta,\phi) = R(r)\cdot f(\theta)\cdot g(\phi)$$

Total Energy

$$E_n = \frac{-13.6\text{eV}}{n^2}$$

Total Orbital angular momentum

$$L = \sqrt[2]{\ell(\ell+1)}\cdot\hbar$$

$$L_z = m_\ell\cdot\hbar$$

Total Spin

$$S = \sqrt[2]{s(s+1)}\cdot\hbar = \frac{\sqrt{3}}{2}\hbar$$

$$S_z = -\frac{1}{2}, \frac{1}{2}$$

Definition 5.3.5. Magnetic moment is defined for an electron by the momentum crossed with the radius.

$$\vec{\mu} = \frac{-e}{2m}\vec{L}$$

$$\mu_z = \frac{-e\hbar}{2m}m_\ell$$

Bohr magneton:

$$\mu_B = \frac{e\hbar}{2m}$$

Definition 5.3.6. Zeeman Effect is the observed effect where applying a magnetic field to an atom splits a state into its magnetic states. Additionally it is possible for the internal magnetic field from the elections spin. The energy difference between aligned spins and misaligned spins. This is known as the Hyperfine transition.