

**Math 347H      Review Problems for Examination I**  
**February 2020**

**Problem 1.** a). Find the general solution, in explicit form, to the differential equation

$$xy' + 3y = \frac{\sin x}{x^2}, \quad x > 0.$$

b). Verify by substitution that your answer to part a) is a solution of the original equation.

**Problem 2.** Find the general solution of  $y' = (3x^2 + \cos x)(1 + y^2)$ .

**Problem 3.** a). Find the general solution to the differential equation

$$x^2y' = xy + 2\frac{x^2}{\cos \frac{y}{x}}.$$

You may leave your solution in implicit form (Hint: The equation is homogeneous).

b). Find the particular solution satisfying  $y(1) = 0$  for the equation in part a).

**Problem 4.** Consider the autonomous equation  $y' = e^y y^2(4 - y^2)$ .

a). Draw the phase line.

b). Find all equilibrium solutions and sketch several graphs of solutions in  $xy$ - plane.

c). If  $y(0) = 1$ , what is  $\lim_{x \rightarrow \infty} y(x)$ ?

**Problem 5.** Consider the equation

$$2xy^2 + y + (2x^2y + x + 2y)y' = 0.$$

a). Is this equation exact?

b). Find the general solution to this equation. You may leave your answer in implicit form.

**Problem 6.** Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of  $190^\circ\text{F}$  when Freshly poured, and 5 minutes later has cooled to  $130^\circ\text{F}$  in a room at  $70^\circ\text{F}$ . When does the coffee reach  $100^\circ\text{F}$ ? (Hint:  $\frac{dT}{dt} = k(70 - T)$ , where  $T$  is the temperature at time  $t$ ).

**Problem 7.** Find the solutions to the differential equations

a).  $y'' - 2y' + 5y = 0$ ,  $y(0) = 1$ ,  $y'(0) = 3$ ;

b).  $4y'' - 4y' + y = 0$ ,  $y(0) = 1$ ,  $y'(0) = 2$ .

**Problem 8.** a). Check that  $y_1(t) = t^{-1}$  is a solution of

$$t^2y'' - ty' - 3y = 0, \quad t > 0.$$

b). Find another linearly independent solution of the given equation.

c). Use the method of variation of parameters to find a particular solution of

$$t^2y'' - ty' - 3y = 2t^2, \quad t > 0.$$

**Problem 9.** a). Find the general solution of  $y'' + 2y' - 3y = 0$ .

b). Find the general solution of the following equation

$$y'' + 2y' - 3y = 2e^t.$$

c). Give a suitable form for a particular solution  $Y(t)$  for the following equation if the method of undetermined coefficients is to be used (you don't need to determine the coefficients):

$$y'' + 2y' - 3y = 2t^4 + t^2e^{-3t} + \sin 3t.$$

**Problem 10.** Consider an undamped, forced vibrating system described by the initial value problem

$$u'' + 4u = 2\cos(\omega t), \quad u(0) = 1, \quad u'(0) = 0.$$

a). Find the solution for  $\omega = 1$ .

b). Find a particular frequency such that the solution is unbounded. What is the solution in this case?