



PHY 410 - Reference Sheet

Boltzmann's constant

$$k_B = 1.380649 \times 10^{-23} \text{ m}^2 \text{ s}^{-2} \text{ K}^{-1}$$

Entropy $S = k_B \sigma$

Temperature $T = \tau / k_B$

Canonical Ensemble

Partition Function - partition by energy levels for a fixed temperature

$$z = \sum_n e^{-\varepsilon_n / \tau}, \quad \mathcal{P}(n) = \frac{1}{z} e^{-\varepsilon_n / \tau}$$

Expected Value of \mathbb{X} is the average across all energies (Thermal Average).

$$\langle \mathbb{X} \rangle = \sum_n \mathbb{X}(n) \mathcal{P}(n) = \frac{1}{z} \sum_n \mathbb{X}(n) e^{-\varepsilon_n / \tau}$$

Expected Energy in the canonical ensemble is

$$U = \langle \varepsilon \rangle = \frac{1}{z} \sum_n \varepsilon_n e^{-\varepsilon_n / \tau}$$

$$U = \langle \varepsilon \rangle = \tau^2 \frac{1}{z} \frac{\partial z}{\partial \tau} = \tau^2 \frac{\partial}{\partial \tau} \log z$$

The total partition function and expected value for N non-interacting particles is simply

$$z_N = z_1^N$$

$$\langle \mathbb{X} \rangle_N = N \langle \mathbb{X} \rangle_1 \Rightarrow U_N = N U_1$$

Helmholtz Free Energy

$$F = U - \tau \sigma = U - S T = -\tau \log z$$

$$dF = -\sigma d\tau - P dV$$

Entropy $\sigma = - \left(\frac{\partial F}{\partial \tau} \right)_V$, $S = k_B \sigma$

Pressure

$$P = - \left(\frac{\partial U}{\partial V} \right)_\sigma = \tau \left(\frac{\partial \sigma}{\partial V} \right)_U = - \left(\frac{\partial F}{\partial V} \right)_\tau$$

Energy $U = -\tau^2 \frac{\partial}{\partial \tau} \left(\frac{F}{\tau} \right)$

Thermal Radiation

Single Frequency Photon Gas is a system in the canonical ensemble that considers photons of a specific frequency ω .

$$\varepsilon = s \hbar \omega, \quad s = 0, 1, 2, 3, \dots$$

$$z = \sum_{s=0}^{\infty} e^{-s \hbar \omega / \tau} = \frac{1}{1 - e^{-\hbar \omega / \tau}}$$

$$\mathcal{P}(s) = \frac{e^{-s \hbar \omega / \tau}}{z}$$

$$\langle s \rangle = \frac{1}{z} \sum_{s=0}^{\infty} s e^{-s \hbar \omega / \tau} = \frac{1}{e^{\hbar \omega / \tau} - 1}$$

Photon Gas is an expansion of the single frequency photon gas that considers all the possible cavity modes. The modes are 2 fold degenerate for the 2 independent polarizations.

$$\omega_n = \frac{c\pi}{L} \sqrt{n_x^2 + n_y^2 + n_z^2} = \frac{c\pi n}{L}$$

$$U = \langle \varepsilon \rangle = 2 \sum_n \frac{\hbar \omega_n}{e^{\hbar \omega_n / \tau} - 1} = \frac{\pi^2 V}{15 (\hbar c)^3} \tau^4$$

Stefan-Boltzmann Law

$$\frac{U}{V} = \frac{\hbar}{\pi^2 c^3} \int_0^{\infty} \frac{\omega^3}{e^{\hbar \omega / \tau} - 1} d\omega = \frac{\pi^2}{15 (\hbar c)^3} \tau^4$$

Spectral Density Function

$$\frac{\partial U}{\partial \omega} \frac{1}{V} = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\hbar \omega / \tau} - 1}$$

Flux Density (σ_B = Stefan-Boltzmann constant)

$$J_\mu = \frac{1}{4} \frac{cU}{V} = \sigma_B \tau^4 = \frac{\pi^2}{60 (\hbar c)^3} \tau^4$$

Phonons in a Solid (Debye Model)

Phonons in a solid is a system in the canonical ensemble that is very similar to thermal radiation except there is 3 fold degeneracy from 3 polarizations of phonons and an upper cutoff frequency ω_D due to the separation distance between atoms.

$$\omega_n = \frac{\pi c_S}{L} \sqrt{n_x^2 + n_y^2 + n_z^2} = \frac{\pi c_S}{L} n$$

Debye cutoff frequency

$$\omega_D = c_S \left(\frac{6\pi^2 N}{V} \right)^{1/3}, \quad \omega_D = \frac{\pi c_S}{L} n_D$$

Grand Canonical Ensemble

Chemical Potential

$$\mu = \left(\frac{\partial F}{\partial N} \right)_{\tau, V}$$

$$\mu = \tau \log \left(\frac{N \lambda_T^3}{V} \right) = \tau \log \left(\frac{n}{n_Q} \right)$$

$$\mu = \left(\frac{\partial U}{\partial N} \right)_{\sigma, V} = -\tau \left(\frac{\partial \sigma}{\partial N} \right)_{U, V}$$

Grand Partition Function - partition by energy levels for a fixed temperature and all possible values of N

$$z_\varepsilon = \sum_N \sum_{n(N)} e^{(\varepsilon_n - \mu N) / \tau}$$

$$\mathcal{P}(N, \varepsilon_n) = \frac{1}{z_\varepsilon} e^{-(\varepsilon_n - \mu N) / \tau}$$

Fugacity

$$\lambda = e^{\mu / \tau}$$

$$z_\varepsilon = \sum_N \lambda^N \sum_{s(N)} e^{-\varepsilon_s / \tau} = \sum_N \lambda^N z_N$$

Expected Value of \mathbb{X} is the average across all energies (Diffusive Average).

$$\langle \mathbb{X} \rangle = \frac{1}{z_\varepsilon} \sum_N \sum_s \mathbb{X}(N, s) e^{-(\varepsilon_s - \mu N) / \tau}$$

Expected Number of Particles in the grand canonical ensemble is

$$N = \langle N \rangle = \tau \frac{\partial}{\partial \mu} \log z_\varepsilon$$

$$N = \langle N \rangle = \lambda \frac{\partial}{\partial \lambda} \log z_\varepsilon$$

Expected Energy in the grand canonical ensemble is

$$U = \langle \varepsilon \rangle = \frac{1}{z_\varepsilon} \sum_N \sum_{n(N)} \varepsilon_n e^{-(\varepsilon_n - \mu N) / \tau}$$

$$U = \langle \varepsilon \rangle = \tau^2 \left(\frac{\partial}{\partial \tau} \log z_\varepsilon \right)_\lambda$$

Grand Potential

$$\Omega = U - \sigma \tau - \mu N$$

$$\Omega = -\tau \log z_\varepsilon$$

$$\sigma = - \left(\frac{\partial \Omega}{\partial \tau} \right)_{V, \mu}$$

$$P = - \left(\frac{\partial \Omega}{\partial V} \right)_{\tau, \mu}$$

$$N = - \left(\frac{\partial \Omega}{\partial \mu} \right)_{\tau, V}$$

System of Non-interacting particles

The grand partition function for a system with M energy states where n_α is the number of particles occupying a state is

$$z_\varepsilon = \prod_{\alpha=1}^M z_{\alpha}, \quad z_{\alpha} = \sum_{n_{\alpha}} e^{-n_{\alpha}(\varepsilon_{\alpha} - \mu) / \tau}$$

$$U = \sum_{\alpha=1}^M \varepsilon_{\alpha} f(\varepsilon_{\alpha}), \quad N = \sum_{\alpha=1}^M f(\varepsilon_{\alpha})$$

Fermions

$$n_{\alpha} = 0, 1$$

$$z_{\alpha} = 1 + e^{-(\varepsilon_{\alpha} - \mu) / \tau} = 1 + \lambda e^{-\varepsilon_{\alpha} / \tau}$$

Fermi-Dirac Distribution is the expected number of a particles in a particular energy ε_{α} .

$$\langle n_{\alpha} \rangle = f(\varepsilon_{\alpha}) = \frac{1}{e^{-(\varepsilon_{\alpha} - \mu) / \tau} + 1} = \frac{1}{\lambda e^{-\varepsilon_{\alpha} / \tau} + 1}$$

For $\tau \rightarrow 0$: $f(\varepsilon_{\alpha}) = \theta(\varepsilon_{\alpha} - \mu)$

Bosons (**Bonsons**)

$$n_{\alpha} = 0, 1, 2, 3, \dots$$

$$z_{\alpha} = \frac{1}{1 - e^{-(\varepsilon_{\alpha} - \mu) / \tau}} = \frac{1}{1 - \lambda e^{-\varepsilon_{\alpha} / \tau}}$$

Boson Distribution is the expected number of a particles in a particular energy ε_{α} .

$$\langle n_{\alpha} \rangle = f(\varepsilon_{\alpha}) = \frac{1}{e^{(\varepsilon_{\alpha} - \mu) / \tau} - 1} = \frac{1}{\lambda^{-1} e^{\varepsilon_{\alpha} / \tau} - 1}$$

Ideal Gas

Both fermions and bosons behave identically at the classical limit $\varepsilon_{\alpha} - \mu \gg \tau$.

$$\langle n_{\alpha} \rangle = e^{-(\varepsilon_{\alpha} - \mu) / \tau}$$

$$z_{\varepsilon} = \sum_N \lambda^N z_N = \sum_N \lambda^N \frac{1}{N!} z_1^N$$

$$z_{\varepsilon} = e^{\lambda z_1}, \quad \log z_{\varepsilon} = \lambda z_1$$

$$PV = N\tau, \quad U = \frac{3}{2} N\tau$$

$$\sigma = N \left[\log \frac{n_Q}{n} + \frac{5}{2} \right], \quad \mu = \tau \log \frac{n}{n_Q}$$

Internal Excitations

Expansion of the ideal gas to take into account the additional energy states from internal excitations.

$$z_{int} = \sum_{\alpha} e^{-\varepsilon_{\alpha} / \tau}$$

$$z_{\varepsilon} = 1 + \lambda z_{int} e^{-\varepsilon_n / \tau}$$

Internal Excitation Corrections

$$\lambda = \frac{n}{n_Q z_{int}}$$

$$\mu = \tau \left(\log \frac{n}{n_Q} - \log z_{int} \right)$$

$$F = N\tau \left[\log \frac{n}{n_Q} - 1 \right] - N\tau \log z_{int}$$

$$\sigma = N \left[\log \frac{n}{n_Q} + \frac{5}{2} \right] - \left(\frac{\partial F_{int}}{\partial \tau} \right)_V$$

DOG (bork)



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