

Quantum Mechanics II  
from the context of the courses  
PHY 851-852: Quantum Mechanics

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## 0.1 The SI System

In physics it's often important to have precisely defined units for the purposes of making very accurate measurements or simply having a coherent unit system. It's possible to derive all necessary units from five measurements of **length, mass, time, current, and temperature**. The standard SI units for these properties are listed below:

Type	Unit	Definition
Length	Meter( $m$ )	Length of distance light in a vacuum travels in $\frac{1}{299792458}$ seconds
Mass	Kilogram( $kg$ )	Defined by fixing the Planck's constant $h = 6.62607015 \times 10^{-34} kg \cdot m^2 s^{-1}$
Time	Second( $s$ )	Defined by fixing the ground-state hyperfine transition frequency of the caesium-133 atom, to be $9192631770 s^{-1}$
Current	Ampere( $A$ )	Defined by fixing the charge of an electron as $1.602176634 \times 10^{-19} A \cdot s$
Temperature	Kelvin( $K$ )	Defined by fixing the value of the Boltzmann constant $k$ to $1.380649 \times 10^{-23} kg \cdot m^2 s^{-2} K^{-1}$

Common prefixes are listed below:

Prefix	Symbol	Definition
mega	M	$10^6$
kilo	k	$10^3$
milli	m	$10^{-3}$
micro	$\mu$	$10^{-6}$
nano	$n$	$10^{-9}$
pico	$p$	$10^{-12}$
femto	$f$	$10^{-15}$

Additionally, the following are defined constants:

Symbol	Definition
$\hbar$	$\hbar = \frac{h}{2\pi} \approx 1.0546 \times 10^{-34} kg \cdot m^2 s^{-1}$
$e$	Charge of an electron $e = 1.602176634 \times 10^{-19} C$

## 0.2 Why make a second book?

I've found that in the course of study it is incredibly difficult to make a universal reference book across classes. Maybe that is something I will make in the future. Until I know the exact topics that are covered in this course and the depth at which they are covered, I will keep the two books separate to make studying for the current course simpler.

Another major difference between these two books is the notation. The first book used Schrödinger notation with a few examples of Heisenberg notation. With the hope of standardizing notation and consistently representing quantum systems, this book will strictly use Heisenberg notation.

## 0.3 Stern-Gerlach Experiments

The Stern-Gerlach experiments are a great example of a system that cannot be accurately described by classical mechanics.

**Definition 0.3.1.** Recall from classical mechanics that **Classical Magnetic Moment** is defined using the following formula

$$\mu = \frac{q}{2m} \mathbf{L}$$

$$\mathbf{L} = r m v$$

$r$  is radius,  $m$  is mass,  $v$  is tangential velocity,  $q$  is charge,  $\mathbf{L}$  is angular momentum, and  $\mu$  is magnetic moment.

**Definition 0.3.2.** Electron, Protons, and Neutrons all have an **intrinsic angular momentum** called **spin** denoted  $\mathbf{S}$ .

**Definition 0.3.3.** Electrons, Protons, and Neutrons also have an **intrinsic magnetic moment** defined by

$$\mu = g \frac{q}{2m} \mathbf{S}$$

$g$  is the dimensionless gyroscopic ratio or  $g$ -factor which can be derived using quantum mechanics.

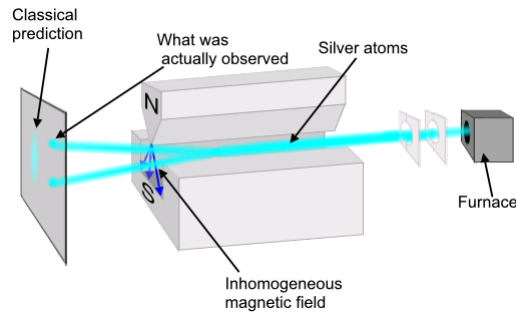


Figure 1: Diagram of the Stern-Gerlach experiment

The first Stern-Gerlach experiment seeks to measure the magnetic moment of the valence electron. A silver atom has 47 electrons and 47 protons. The magnetic moments depends on the inverse of mass, so we can neglect heavy protons and neutrons. Silver has an electron configuration of  $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 4d^{10} 5s^1$ , so the only electron that contributes to the magnetic moment is the valence electron  $5s^1$ . Knowing this we expect the magnetic moment of the silver atom to be

$$\mu = -g_e \frac{e}{2m_e} \mathbf{S}$$

Following the laws of electromagnetism the force in the  $z$  direction is

$$F_z = -g_e \frac{e}{2m_e} S_z \frac{\partial B_z}{\partial z}$$

The deflection of the beam is therefore a measurement of the spin of the valence electron of the silver atoms in the  $z$ -direction. Classically, we would expect the magnetic moment to be aligned in random directive and to observe a continuous range of deflection. Instead we observe two distinct magnetic moments. The magnitudes of these deflections are consistent with the spins of

$$S_z = \pm \frac{\hbar}{2}$$

This is called **quantization** of the electron's spin angular momentum component. The factor  $\frac{1}{2}$  in the equation is why we refer to electrons as having **spin-1/2**.

# Chapter 1

## Quantum Systems

This chapter will outline our system of notation and the fundamental concepts of quantum mechanics.