Physics Reference

Kaedon Cleland-Host

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Contents

Chapter 1

Introduction

This chapter will offer reference and information that applies to the entire book.

1.1 Standard Units

In physics it's often important to have precisely defined units for the purposes of making very accurate measurements or simply having a coherent unit system. It's possible to derive all necessary units from five measurements of **length, mass, time, current, and temperature**. The standard SI units for these properties are listed bellow:

Type	Unit	Definition
Length	Meter(m)	Length of distance light in a vacuum travels in $\frac{1}{299792458}$ seconds
Mass	Kilogram(kg)	Defined by fixing the Planks constant $h = 6.62607015 \times 10^{-34} kg\ m^2 s^{-1}$
Time	Second(s)	Defined by fixing the ground-state hyperfine transition frequency of the caesium-133 atom, to be $9192631770 s^{-1}$
Current	Ampere(A)	Defined by fixing the charge of an electron as $1.602176634 \times 10^{-19} A \cdot s$
Temperature	Kelvin(K)	Defined by fixing the value of the Boltzmann constant k to $1.380649 \times 10^{-23} kg \cdot m^2 s^{-2} K^{-1}$

Common prefixes are listed bellow:

Prefix	Symbol	Definition
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

Chapter 2

Electricity and Magnetism

Law 2.0.1. Maxwells Equations the general laws of electricity and magnetism.

$$\nabla \times \vec{E} = -\vec{B}$$

$$\nabla \times \vec{H} = \vec{J} + \vec{D}$$

$$\nabla \cdot \vec{D} = S$$

$$\nabla \cdot \vec{B} = 0$$

2.1 Electronics

2.1.1 General Components

Definition 2.1.1. Electric Field force per unit charge or N/C

Definition 2.1.2. Voltage or *Potential* is the change in energy per unit charge brought on by traveling through an electric field or J/C or simply V

Remark. The units N/C is equivalent to V/m

Definition 2.1.3. Power can be derived from units of current and voltage for the following formula where P is power(W), V is voltage(V), and I is current(A).

$$P = VI \tag{2.1.1}$$

Law 2.1.1. Ohms Law models the voltage drop across a purely resistive load with the following formula where V is voltage(V), I is current(A), and R is resistence(Ω).

$$V = IR \tag{2.1.2}$$

Definition 2.1.4. A **Resistor** a device that will produce a voltage drop according to ohms law with a resistance as V/A

Definition 2.1.5. Resistivity(Ω m) is used to calculate how much resistance we expect from a material use the following formula where R is resistence(Ω), ρ is **Resistivity**(Ω m), l is length(m), and A is cross sectional area(m^2).

$$R = \rho\frac{l}{A} \tag{2.1.3}$$

Law 2.1.2. Kirchoff's Voltage Law states precisely that the algebraic sum of all voltages around a closed path is zero.

$$\sum V_n = 0 \tag{2.1.4}$$

Law 2.1.3. Kirchoff's Current Law states precisely that the algebraic sum of all currents entering a node is zero.

$$\sum I_n = 0 \tag{2.1.5}$$

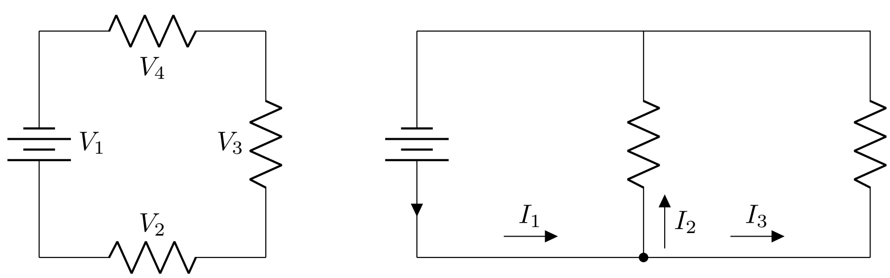
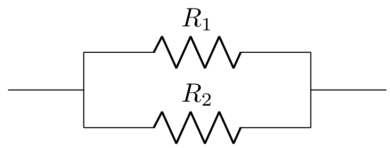


Figure 2.1: Two circuits to demonstrate Krichhoff's Laws

Law 2.1.4. Resistors in Series will simply be the sum of the individual resistance because we are adding the length of the resistors together.

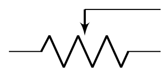
$$R_T = \sum R_i \tag{2.1.6}$$

Law 2.1.5. Resistors in Parallel will decrease the overall resistance as indicated by the definition of resistivity 2.1.5.

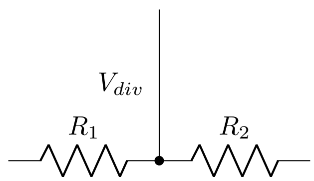


$$\frac{1}{R_T} = \sum \frac{1}{R_i} \tag{2.1.7}$$

Definition 2.1.6. Voltage Divider or **Potentiometer** is a arangement of two resistors with a connection between them. A potentiometer refers to a voltage divider where the resistance ratio between the two resistors can be adjusted.



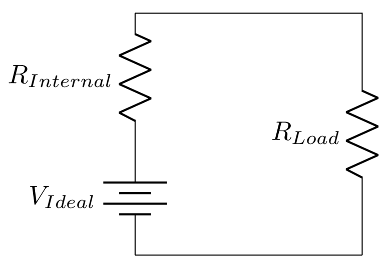
To calculate the voltage we expect at the divider we need to know the voltage across the whole potentiometer V and the ratio between the two resistors $\frac{R_2}{R_1}$.



$$V_{div} = V\frac{R_2}{R_1 + R_2} \tag{2.1.8}$$

Definition 2.1.7. Ideal Battery - a battery that always produces the same potential difference.

Definition 2.1.8. Real Battery - a battery with some internal resistance that reduces the potential difference across the terminals depending on the amount of current.

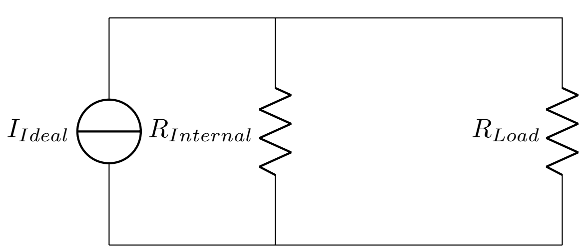


$$V_{Real} = V_{Ideal} - IR_{Internal}$$

$$V_{Real} = V_{Ideal}\frac{R_{Load}}{R_{Internal} + R_{Load}} \tag{2.1.9}$$

Definition 2.1.9. Ideal Current Source - a device that always provides that same current to a load.

Definition 2.1.10. Real Current Source - a current source with an internal resistance connected in parallel that reduces the current produced when the load resistance is high.



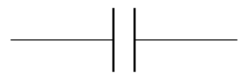
$$I_{Real} = I_{Ideal} - \frac{V}{R_{Internal}}$$

$$I_{Real} = I_{Ideal}\frac{R_{Internal}}{R_{Internal} + R_{Load}} \tag{2.1.10}$$

Definition 2.1.11. A **Capacitor** is a device that accumulates a charge q when a voltage v is applied with a proportionality constant C with units $\mathrm{F(farad)} = \mathrm{C\,V^{-1}} = \mathrm{C^2\,J^{-1}}$

$$q = Cv \tag{2.1.11}$$

$$i = C\frac{dv}{dt} \tag{2.1.12}$$



Remark. The energy stored in a capacitor can derived by integrating power over time:

$$\omega_C = \frac{1}{2}Cv^2 \tag{2.1.13}$$

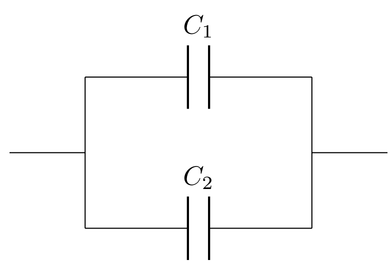
Definition 2.1.12. A **Plate Capacitor** is a capacitor made of two conductive plates with area A separated by distance l.

$$C = \frac{\varepsilon_0 \kappa A}{l}$$

Law 2.1.6. Capacitors in Series will decrease the overall capacitance.

$$\frac{1}{C_T} = \sum \frac{1}{C_i} \tag{2.1.14}$$

Law 2.1.7. Capacitors in Parallel is simply the sum of the individual capacitance.



$$C_T = \sum C_i \tag{2.1.15}$$

Definition 2.1.13. An **Inductor** is a device which accumulates a magnetic flux $\phi = BA$ when a current is applied with a proportionality constant L with units $\mathrm{H(henry)} = \mathrm{J\,A^{-2}}$

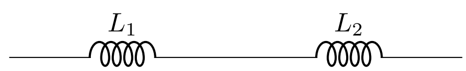
$$\phi = Li \tag{2.1.16}$$

$$v = L\frac{di}{dt} \tag{2.1.17}$$

Remark. The energy stored in an inductor can derived by integrating power over time:

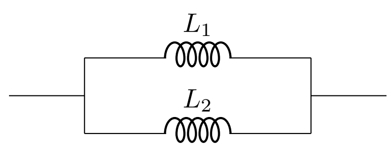
$$\omega_L = \frac{1}{2}Li^2 \tag{2.1.18}$$

Law 2.1.8. Inductors in Series is simply the sum of the individual inductance.



$$L_T = \sum L_i \tag{2.1.19}$$

Law 2.1.9. Inductors in Parallelwill decrease the overall inductance.



$$\frac{1}{L_T} = \sum \frac{1}{L_i} \tag{2.1.20}$$

2.1.2 RC and RL Circuits

Definition 2.1.14. Charging RC Circuit is a circuit with a resistor, capacitor and voltage source to charge the capacitor. The voltage equation around the loop can be written as

$$V = Ri + \frac{1}{C}\int_0^t i(\sigma)d\sigma$$

$$i(t) = \frac{V}{R}e^{-t/RC}$$

$$v_c(t) = V(1 - e^{-t/RC})$$

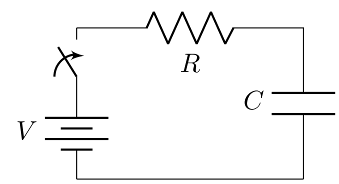


Figure 2.2: a basic RC circuit in the charging arrangement

Definition 2.1.15. Discharging RC Circuit is a circuit with a capacitor and resistor to discharge the capacitor. The current relation during discharge is symmetric to charge so we can rewrite the equations

$$i(t) = \frac{V}{R}(1 - e^{-t/RC})$$

$$v_c(t) = Ve^{-t/RC}$$

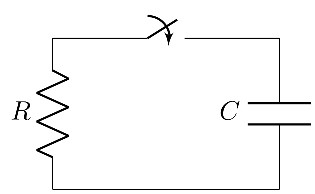


Figure 2.3: a basic RC circuit in the discharging arrangement

Definition 2.1.16. Charging RL Circuit is a circuit with a resistor, inductor and voltage source to charge the inductor. The current equation can be written as

$$i(t) = \frac{V}{R}(1 - e^{-tR/L})$$
$$v_l(t) = Ve^{-tR/L}$$

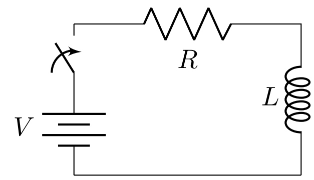


Figure 2.4: a basic RL circuit in the charging arrangement

Definition 2.1.17. Discharging RL Circuit is a circuit with a inductor and resistor to discharge the inductor. The current relation during discharge is symmetric to charging so we can rewrite the equations

$$i(t) = \frac{V}{R}e^{-tR/L}$$
$$v_l(t) = Ve^{-tR/L}$$

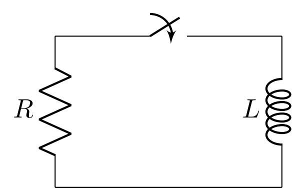


Figure 2.5: a basic RL circuit in the discharging arrangement

2.1.3 AC Circuits

Definition 2.1.18. An AC Voltage source is a voltages source with a alternating voltage. Typically the voltage changes according to the following function:

$$V(t) = V_p \sin(\omega t)$$

Definition 2.1.19. Effective Voltage or **Root Mean Square Voltage** is the voltage $V_p/\sqrt{2}$ where V_p is the peak sinusoidal voltage. This represents the DC voltage that would produce the same power draw as the AC voltage.

Example. **Resistor in AC Circuit** Bellow is a AC Circuit with a resistor. The current is determined by the following function

$$i(t) = \frac{V_p}{R} \sin \omega t$$

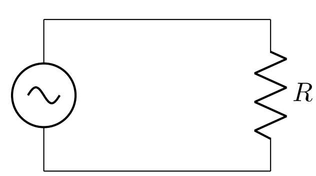


Figure 2.6: a basic AC circuit connected to a resistive load

Definition 2.1.20. Effective Current or **Root Mean Square Current** is the current $I_p/\sqrt{2}$ where I_p is the peak sinusoidal current. This represents the DC current that would produce the same power draw as the AC current.

Example. **Capacitor in AC Circuit** Bellow is a AC Circuit with a capacitor. The current is determined by the following function

$$i(t) = -\omega C V_p \sin \omega t$$

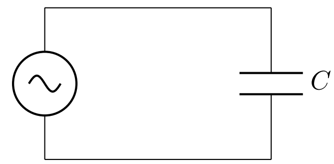


Figure 2.7: a basic AC circuit connected to a capacitive load

Definition 2.1.21. A Phasor Signal is a complex number that we use to represent the output of a wavelike function.

$$Re^{i\theta} = R\cos(\theta) + Ri\sin(\theta)$$

Law 2.1.10. To convert froma phasor signal to the observed signal the following formula applies where $x(t)$ si the observed signal and c is the complex phasor signal.

$$x(t) = Real(c \cdot e^{i\omega t})$$

Law 2.1.11. Generalized Ohm's Law is the complex version of ohm's law that includes complex impedance.

$$\tilde{V} = \tilde{I}\tilde{Z}$$

$$\tilde{Z} = R + iX$$

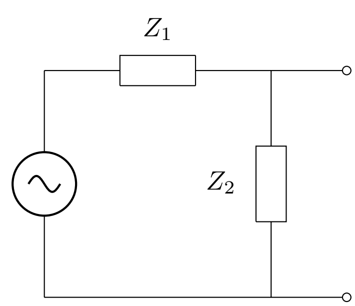
Where R is circuit resistance and X is circuit reactance.

Example. **Impotence of basic components**:

- **Resistor** $\tilde{Z} = \frac{\tilde{V}}{\tilde{I}} = R$
- **Capacitor** $\tilde{Z} = \frac{\tilde{V}}{\tilde{I}} = \frac{1}{i\omega C}$
- **Inductor** $\tilde{Z} = \frac{\tilde{V}}{\tilde{I}} = i\omega L$

Law 2.1.12. Generalized Voltage Divider is the complex version of the voltage divider formula that includes complex impedance.

$$\tilde{V_{out}} = \tilde{V_{in}} \frac{\tilde{Z}_2}{\tilde{Z}_1 + \tilde{Z}_2}$$



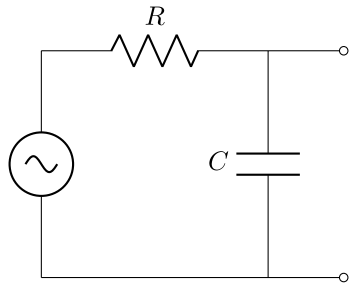
2.1.4 AC Filters

Definition 2.1.22. A **Low Pass Filter** is a circuit that allows low frequency signals through, but blocks out higher frequency signals.

$$\tilde{V_{out}} = \tilde{V_{in}} \frac{1}{1 + i\omega RC}$$

$$V_{out} = V_{in} \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\phi = \tan(-\omega RC)$$

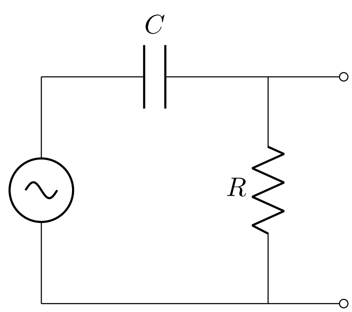


Definition 2.1.23. A **High Pass Filter** is a circuit that allows high frequency signals through, but blocks out lower frequency signals.

$$\tilde{V_{out}} = \tilde{V_{in}} \frac{1}{1 - \frac{i}{\omega RC}}$$

$$V_{out} = V_{in} \frac{1}{\sqrt{1 + (\frac{1}{\omega RC})^2}}$$

$$\phi = \tan(\frac{1}{\omega RC})$$



Definition 2.1.24. The **Breakpoint Frequency** is the frequency of a filter that produces an inflection point on the frequency response of that circuit. For simple low and high pass filters it can be represented as the following:

$$\omega_c = \frac{1}{RC}$$

$$f_c = \frac{1}{2\pi RC}$$

Definition 2.1.25. A **Decibel** scale is used to measure a ratio that can represent very large and very small values.

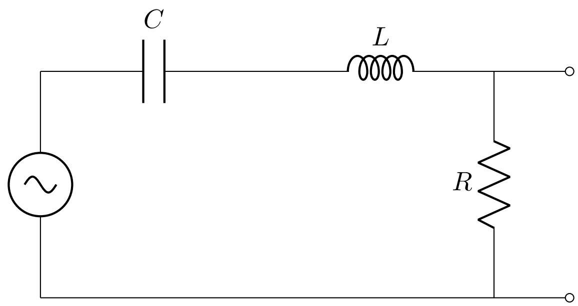
$$\text{Decibal} = 20 \log_{10}(|\frac{V_{out}}{V_{in}}|) = 10 \log_{10}(|\frac{P_{out}}{P_{in}}|)$$

Definition 2.1.26. A **Band Pass Filter** is a circuit that only allows mid range frequency signals through, but blocks out lower and higher frequency signals.

$$\tilde{V_{out}} = \tilde{V_{in}} \frac{1}{1 - \frac{i}{\omega RC} + i\omega L}$$

$$V_{out} = V_{in} \frac{1}{\sqrt{1 + (\omega \frac{L}{R} - \frac{1}{\omega RC})^2}}$$

$$\phi = \tan(\frac{1}{\omega RC} - \omega \frac{L}{R})$$



Definition 2.1.27. The **Resonance Frequency** is the frequency of resonance or max output for a band pass filter.

$$\omega_0 = \frac{1}{\sqrt{RL}}$$

$$f_0 = \frac{1}{2\pi\sqrt{RL}}$$

Definition 2.1.28. The **Band Width** is the difference between the two cutoff frequencies for a band pass filter.

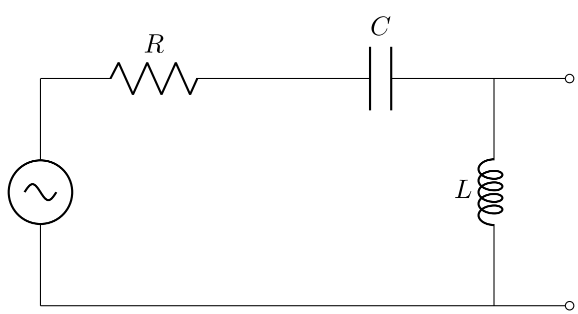
$$\omega_{c1} - \omega_{c2} = \frac{R}{L}$$

$$f_{c1} - f_{c2} = \frac{R}{2\pi L}$$

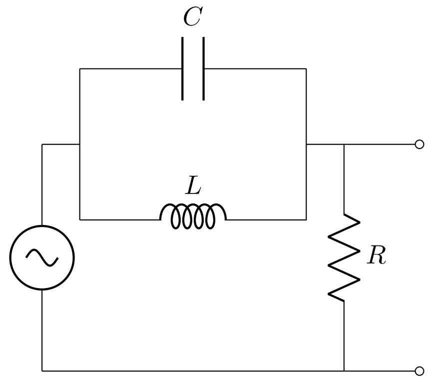
Definition 2.1.29. The **Quality Factor** is a measure of the sharpness of the resonance peak of a band pass filter.

$$Q = \frac{f_0}{f_{c1} - f_{c2}} = \frac{\omega_0}{\omega_{c1} - \omega_{c2}}$$

Example. **Resonant High Pass Filter**:



Example. **Notch Filter**:

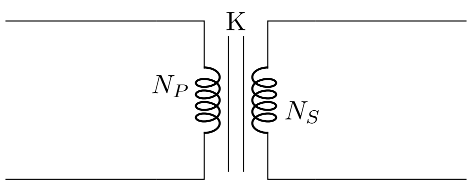


2.1.5 Transformers

Definition 2.1.30. A **Transformer** is a primary and secondary connected by a ferromagnetic core such that the primary coil induces voltage in the secondary.

$$\frac{V_S}{V_P} = \frac{V_S}{V_P}$$

$$\frac{I_S}{I_P} = \frac{V_P}{V_S}$$

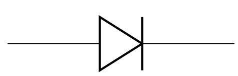


2.1.6 Silicon-based Components

Definition 2.1.31. **N-type silicon** is silicon doped with an element like Phosphorous which produces free electrons in the material.

Definition 2.1.32. **P-type silicon** is silicon doped with an element like Boron which produces electron holes in the material.

Definition 2.1.33. A **Diode(PN Junction)** is a device that has a very low resistance for current in one direction but a very high resistance for current in the other direction.



Chapter 3

Thermodynamics

Definition 3.0.1. Thermal Expansion is a the physical expansion or retraction of materials under temperature changes. Where $\alpha =$ coefficient of linear expansion.

Linear Thermal Expansion:

$$L = L_0(1 + \alpha\Delta T)$$

Volumetric Thermal Expansion:

$$V = V_0(1 + 3\alpha\Delta T + 3(\alpha\Delta T)^2 + (\alpha\Delta T)^3)$$

$$V \approx V_0(1 + 3\alpha\Delta T)$$

Law 3.0.1. Young's Modulus is the constant for the relationship between force applied and a change in length.

$$E = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

Where E is the young's modulus, F is the force applied, A is the area over which the force is applied, ΔL is the change in length, and L is the original length.

Law 3.0.2. Heat Capacity Relation is the relationship between thermal energy in materials and there temperature.

$$E = qm\Delta T$$

Where q is the specific heat of the material, m is mass, ΔT is the change in temperature, and E is the amount of energy required or released.

Law 3.0.3. Heat of Phase Transitions - is the energy required/released to break/form inter-molecular bonds that influence the phases of matter.

$$E = L_f m$$

Where L_f is the heat of fusion, m is the mass, and E is the energy required.

Definition 3.0.2. Conduction is the rate of heat energy transfer through a material.

$$E = k\frac{A}{L}\Delta T t$$

Where E is the energy transferred, A is area, L is the length, ΔT is the temperature difference, and t is time.

Law 3.0.4. Stefan-Boltzmann Law models the power of thermal radiation released by an object.

$$P = \sigma e A T^4$$

Where σ is the Stefan-Boltzmann constant ($5.6703 * 10^{-8}\mathrm{W/m^2K^4}$), e is the emissivity, A is the area, and T is the temperature in Kelvin.

Law 3.0.5. Wien's displacement Law models peak wavelength of light emitted by an object as a specific temperature.

$$\lambda_{max} = b/T$$

Where λ_{max} is the wavelength of light at the peak thermal radiation, b is wien's constant ($2.898 * 10^{-3}$), and T is temperature in Kelvin.

Law 3.0.6. Unified Gas Law models the behavior of a a fixed amount of gas.

$$\frac{PV}{T} = \frac{PV}{T}$$

Where P is pressure, V is volume, and T is temperature.

Law 3.0.7. Ideal Gas Law models the behavior of an ideal gas in a closed container.

$$PV = nRT$$

Where P is pressure, V is volume, n is moles of gass in the material, R is the ideal gas constant $8.31446261815324\mathrm{m^3\,Pa\,K^{-1}\,mol^{-1}}$, T is temperature.

Remark. To convert from number of particles to moles use the following formula.

$$N = n * 6.022 * 10^{23}$$

Definition 3.0.3. The standard temperature and pressure is a particular temperature and pressure that is used for situations near sea level.

- $T = 273\text{K}$
- $P = 1atm = 101300\text{Pa}$
- $V = 22.4\text{L} = 0.00224m^3$

Law 3.0.8. The **Boltzmann Distribution** describes how the velocities of particles at a particular temperature behave.

$$P(v) = Cv^2e^{-v^2}$$

$$v_p = \sqrt{2}\sqrt{\frac{RT}{M}}$$

$$v_avg = \sqrt{\frac{8}{\pi}}\sqrt{\frac{RT}{M}}$$

$$v_rms = \sqrt{3}\sqrt{\frac{RT}{M}}$$

Law 3.0.9. The **Internal Energy** stored by the temperature of a system is modeled by the following formula

$$E_{th} = \frac{f}{2}nRT$$

Where f is the degrees of freedom: 3 for monoatomic gasses, 5 for linear molecules, and 6 for chiral molecules.

Chapter 4

Relativity

Theorem 4.0.1. Principle of Relativity states that the laws of physics are the same in all inertial frames. There is no way to detect absolute motion and there is no preferred inertial frame.

Theorem 4.0.2. Constancy of the Speed of Light states that all observers from any inertial frames measure the speed of light as 299792458m/s.

Theorem 4.0.3. Gravitational Equivalence Principle state that inertia mass and gravitational mass are equal.

Definition 4.0.1. Inertia Frames are reference frames where Newtons first law applies.

Law 4.0.1. Lorentz-Fitzgerald's Change of Reference Frame When changing between a reference frame with coordinate x, y, z, t to a reference frame with a relative velocity v in the x direction and coordinates x', y', z', t' the following equations apply:

$$x' = \gamma(x - \beta ct)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \beta\frac{x}{c})$$

with β and γ defined as

$$\beta = \frac{v}{c}$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Remark. From this law we can derive an understanding of time dilation and length contraction:

$$T' = \frac{T}{\sqrt{1-\beta^2}}$$
$$L' = L\sqrt{1-\beta^2}$$

We can also derive an equation for converting velocities to difference reference frames:

$$U' = \frac{v+U}{1+\frac{vU}{c^2}}$$

Law 4.0.2. Space-time Interval is the the distance than an object travels through space-time.

$$(\Delta s)^2 = (\Delta r)^2 - (c\Delta t)^2 = (x, y, z, ict)$$

The space-time interval will be the same independent to the reference frame.

Definition 4.0.2. Relativistic Doppler Effect is the effect observed from moving sources of waves. Let f_0 be the original frequency, f be the observed frequency, $v_{\|}$ be the component of the relative velocity parallel to the observer, and $v_{\perp}$ be the component of the relative velocity away from the observer.

$$f = f_0\sqrt{1-\frac{v_{\|}^2}{c^2}}\frac{\sqrt{1-\frac{v_{\perp}}{c}}}{\sqrt{1+\frac{v_{\perp}}{c}}}$$

Definition 4.0.3. Relativistic Momentum the momentum of an object according to relativity.

$$\rho = \gamma m\vec{v} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Definition 4.0.4. Relativistic Energy is the energy of objects according to relativity

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

Definition 4.0.5. Relativistic Energy and Momentum Relations the following equation describes how momentum and energy are related

$$E^2 = \rho^2c^2 + m^2c^2$$
$$\frac{\rho}{E} = \frac{\beta}{c}$$

Definition 4.0.6. Relativistic Mass is the mass of objects according to relativity

$$m = \gamma m_0 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$