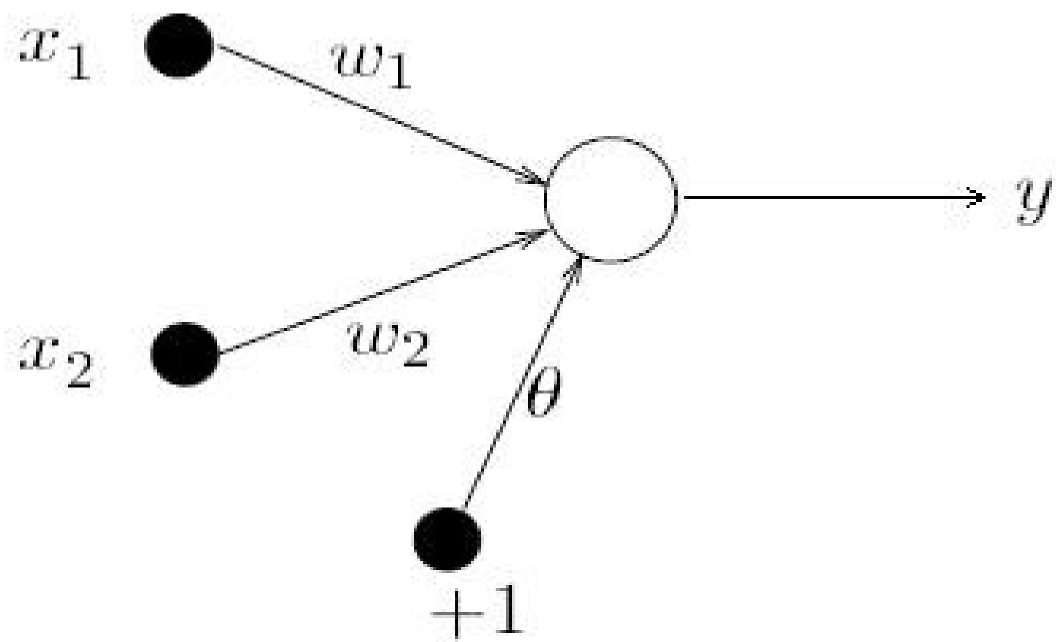
The Simple Perceptron

Simple Perceptron

- The perceptron is a single layer feed-forward neural network.



Simple Perceptron

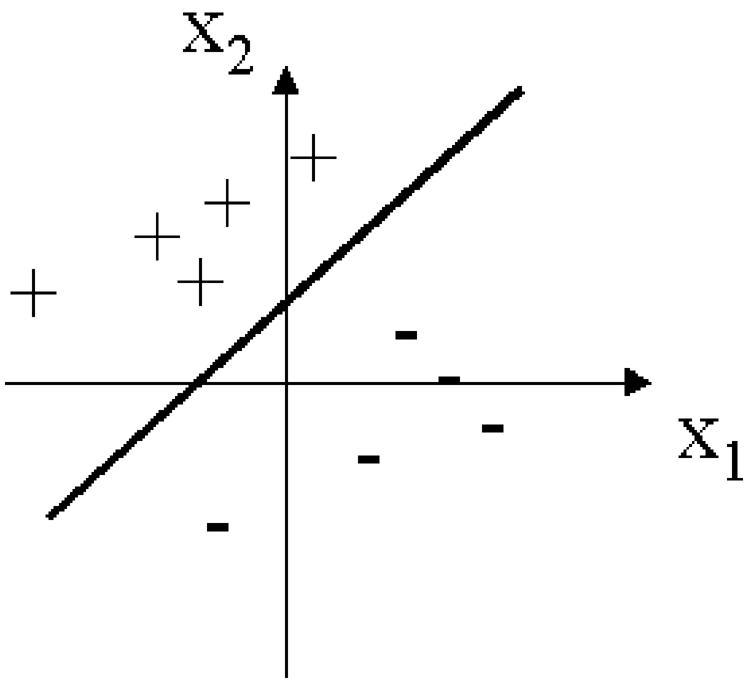
- Simplest output function

$$y = \operatorname{sgn}\left(\sum_{i=1}^{2} w_i x_i + \theta\right)$$

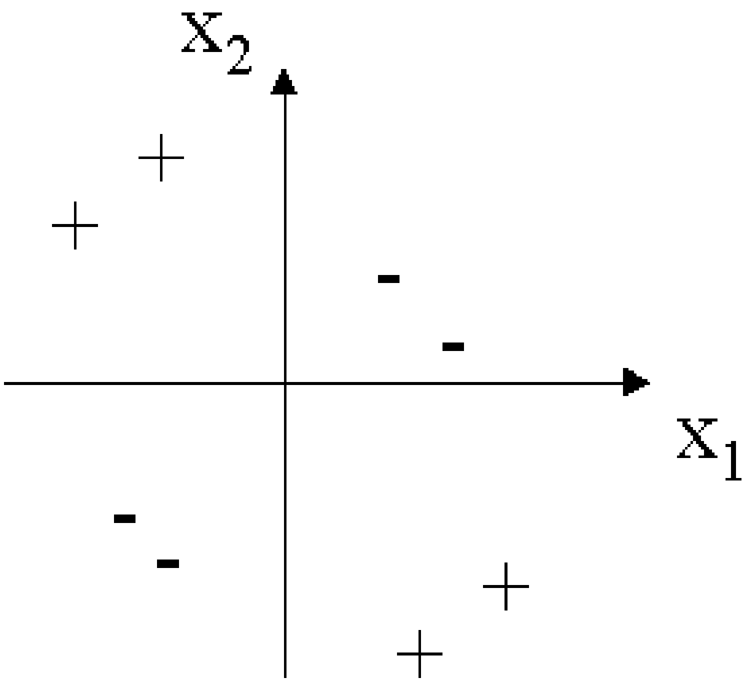
$$\operatorname{sgn}(s) = \begin{cases} 1 & \text{if } s > 0 \\ -1 & \text{otherwise.} \end{cases}$$

- Used to classify patterns said to be linearly separable

Linearly Separable



Linearly Separable



Not Linearly Separable

$$w_1 x_1 + w_2 x_2 + \theta = 0$$

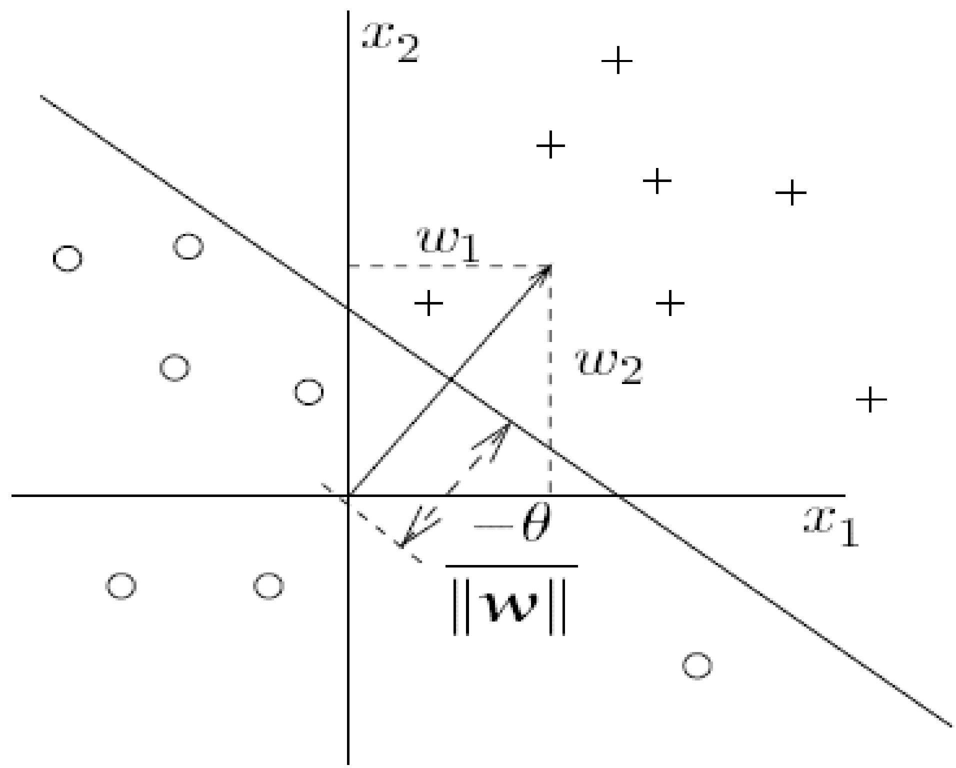
Linearly Separable

The bias is proportional to the offset of the plane from the origin

The weights determine the slope of the line

The weight vector is perpendicular to the plane

$$x_2 = -\frac{w_1}{w_2}x_1 - \frac{\theta}{w_2}$$



Perceptron Learning Algorithm

- We want to train the perceptron to classify inputs correctly
- Accomplished by adjusting the connecting weights and the bias
- Can only properly handle linearly separable sets

Perceptron Learning Algorithm

- We have a "training set" which is a set of input vectors used to train the perceptron.
- During training both w_i and θ *(bias)* are modified for convenience, let $w_0 = \theta$ and $x_0 = 1$
- *Let,* η*,* the *learning rate*, be a small positive number (small steps lessen the possibility of destroying correct classifications)
- Initialise w_i to some values

Perceptron Learning Algorithm

$$\textit{Desired output} \qquad d(n) = \begin{cases} +1 & if\ x(n) \in set\ A \\ -1 & if\ x(n) \in set\ B \end{cases}$$

1. Select random sample from training set as input
2. If classification is correct, do nothing
3. If classification is incorrect, modify the weight vector *w* using

$$w_i = w_i + \eta d(n) x_i(n)$$

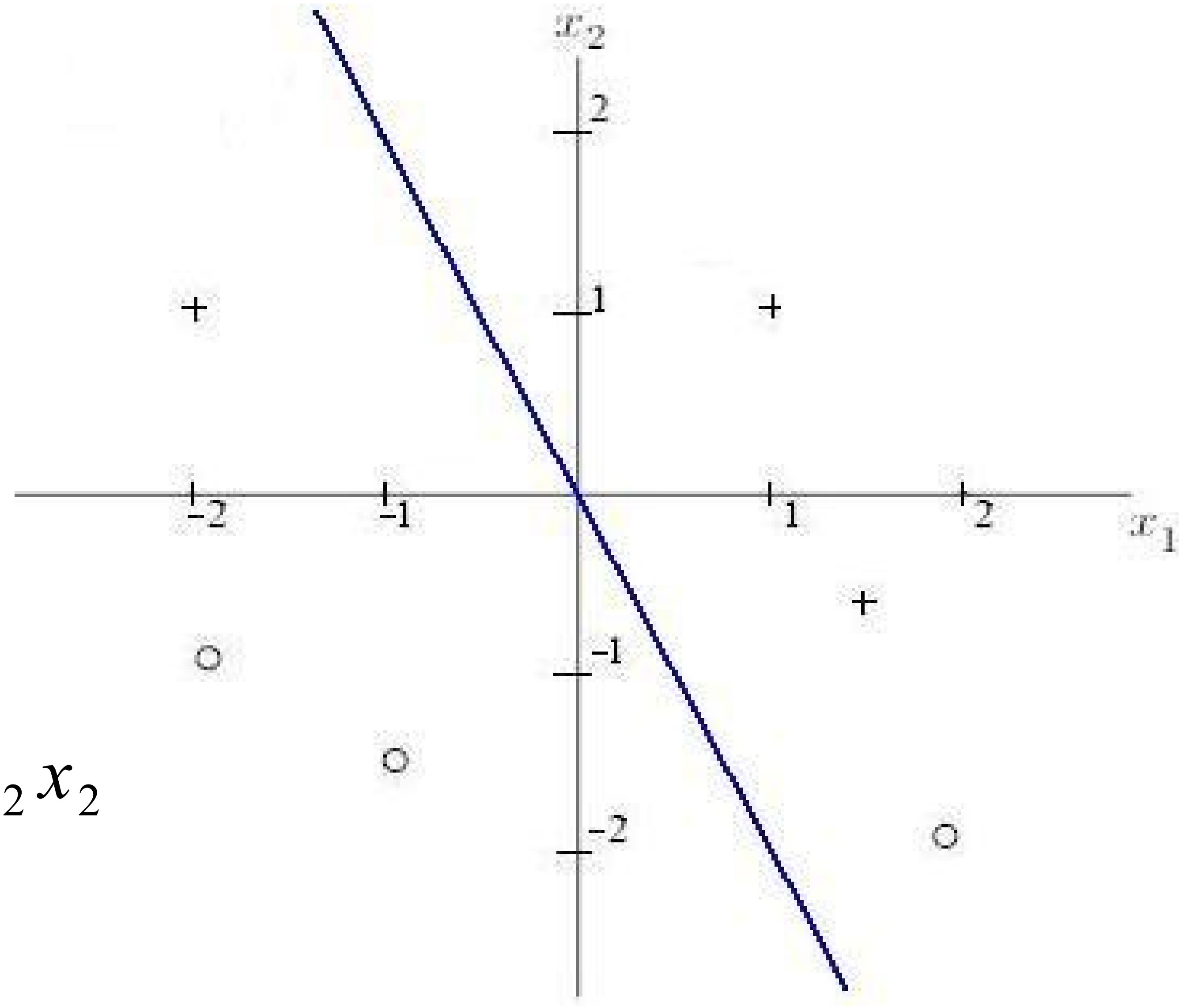
Repeat this procedure until the entire training set is classified correctly

Learning Example

Initial Values:

$\eta = 0.2$

$$w = \begin{pmatrix} 0 \\ 1 \\ 0.5 \end{pmatrix}$$



$$0 = w_0 + w_1 x_1 + w_2 x_2$$

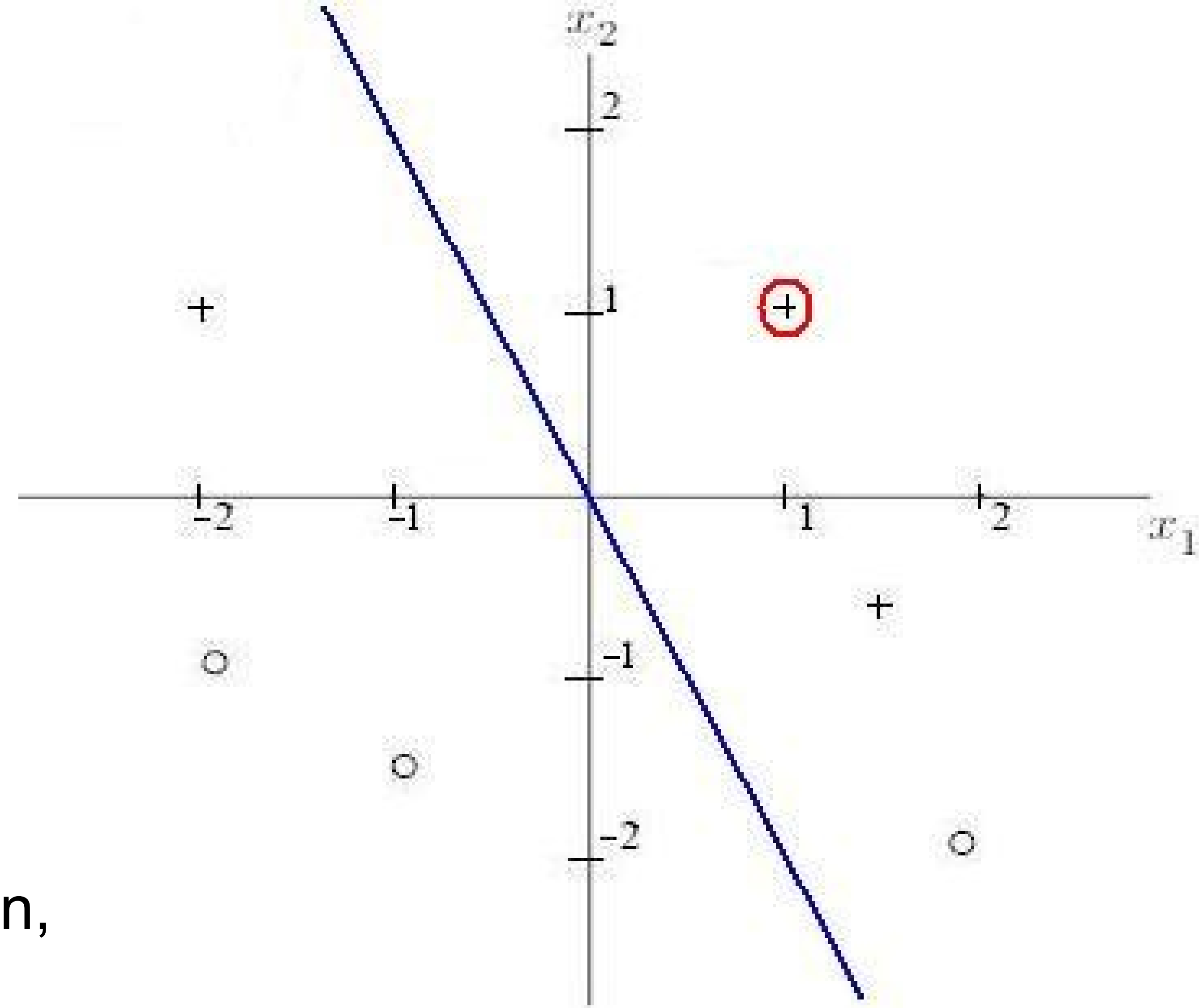
$$= 0 + x_1 + 0.5x_2$$

$$\Rightarrow \quad x_2 = -2x_1$$

Learning Example

$\eta = 0.2$

$$w = \begin{pmatrix} 0 \\ 1 \\ 0.5 \end{pmatrix}$$



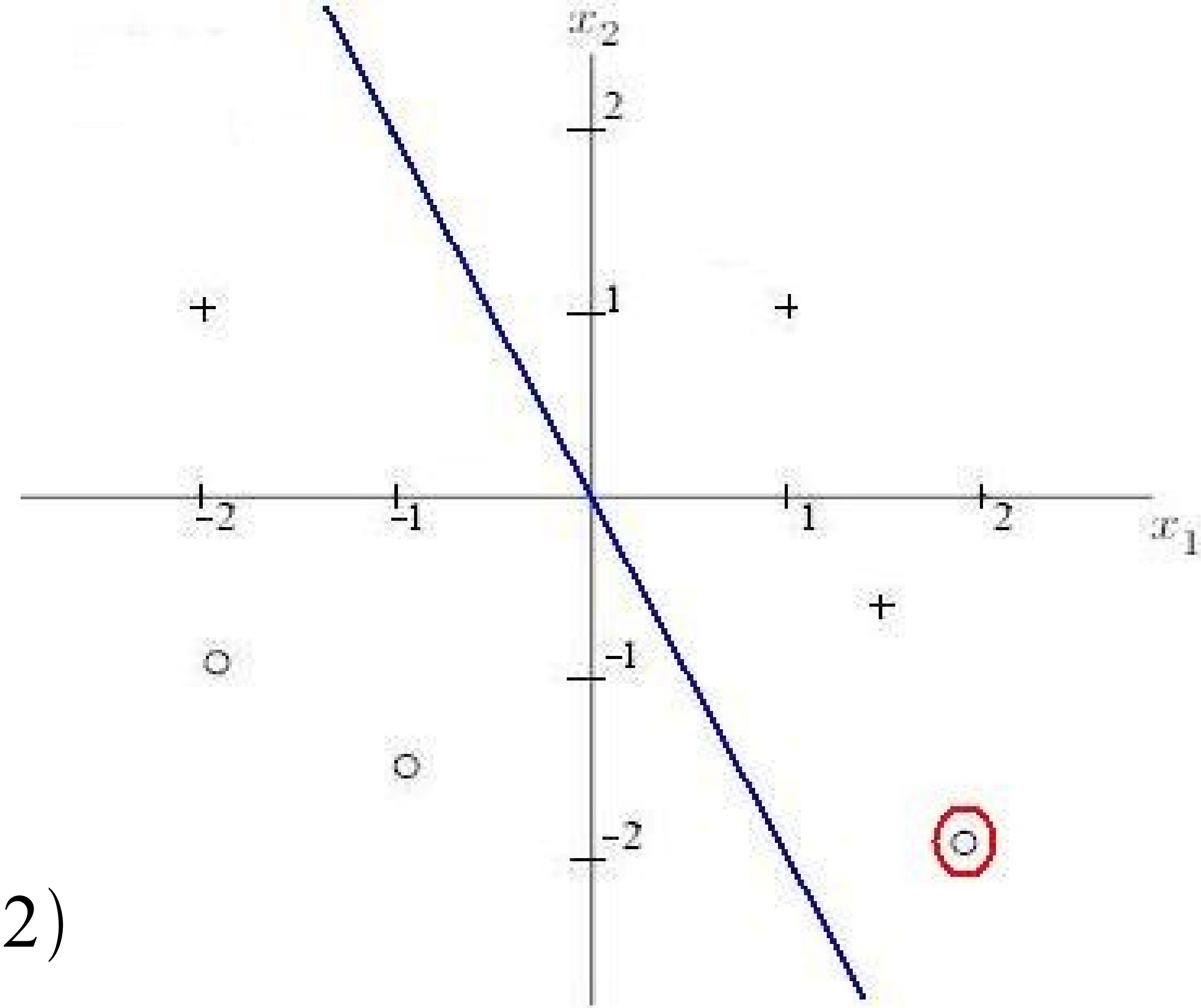
$x_1 = 1$, $x_2 = 1$
$w^T x > 0$

Correct classification, no action

Learning Example

$\eta = 0.2$

$$w = \begin{pmatrix} 0 \\ 1 \\ 0.5 \end{pmatrix}$$



$x_1 = 2$, $x_2 = -2$

$$w_0 = w_0 - 0.2 * 1$$

$$w_1 = w_1 - 0.2 * 2$$

$$w_2 = w_2 - 0.2 * (-2)$$

Learning Example

$\eta = 0.2$

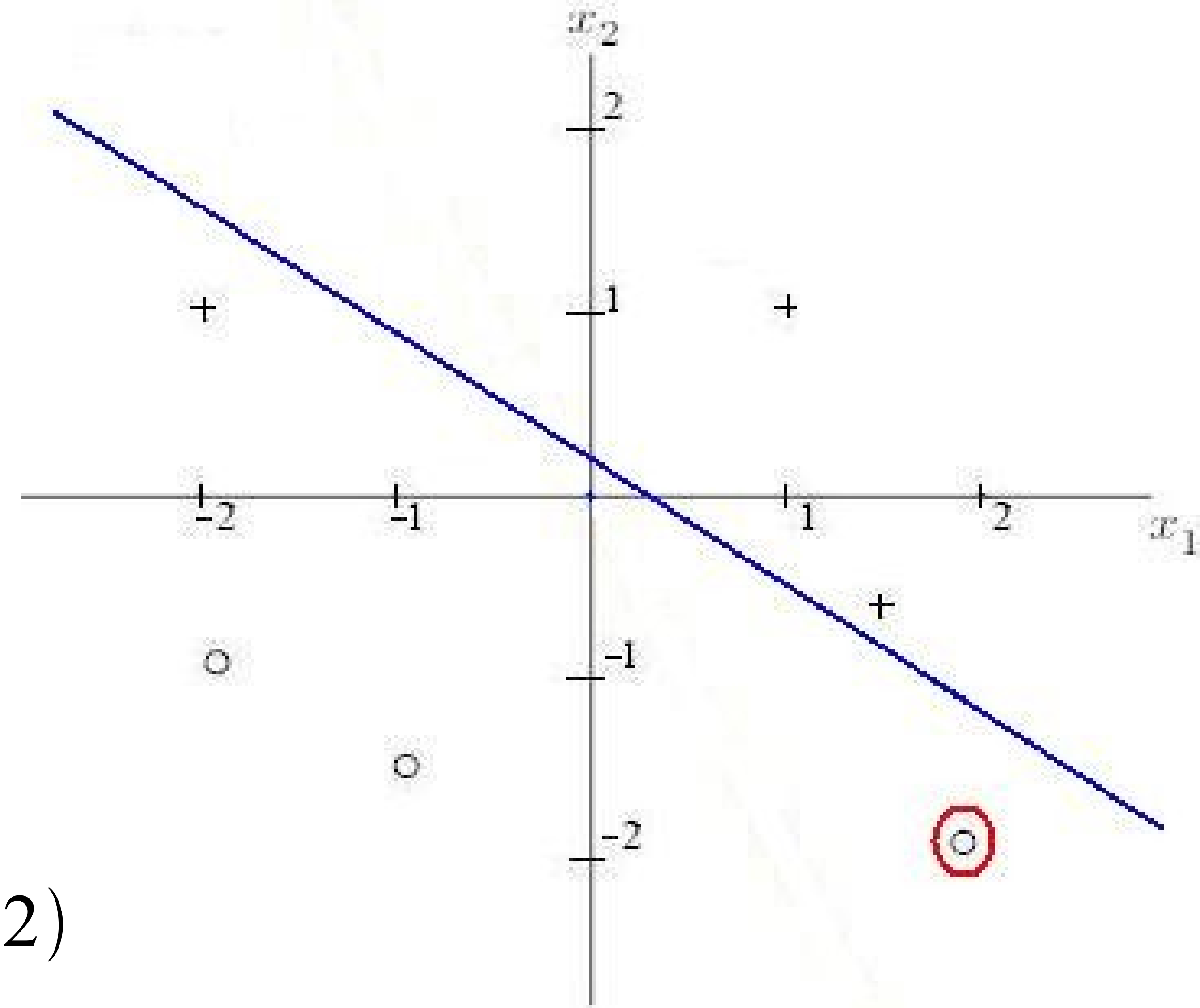
$$w = \begin{pmatrix} -0.2 \\ 0.6 \\ 0.9 \end{pmatrix}$$

$x_1 = 2,\ x_2 = -2$

$$w_0 = w_0 - 0.2 * 1$$

$$w_1 = w_1 - 0.2 * 2$$

$$w_2 = w_2 - 0.2 * (-2)$$



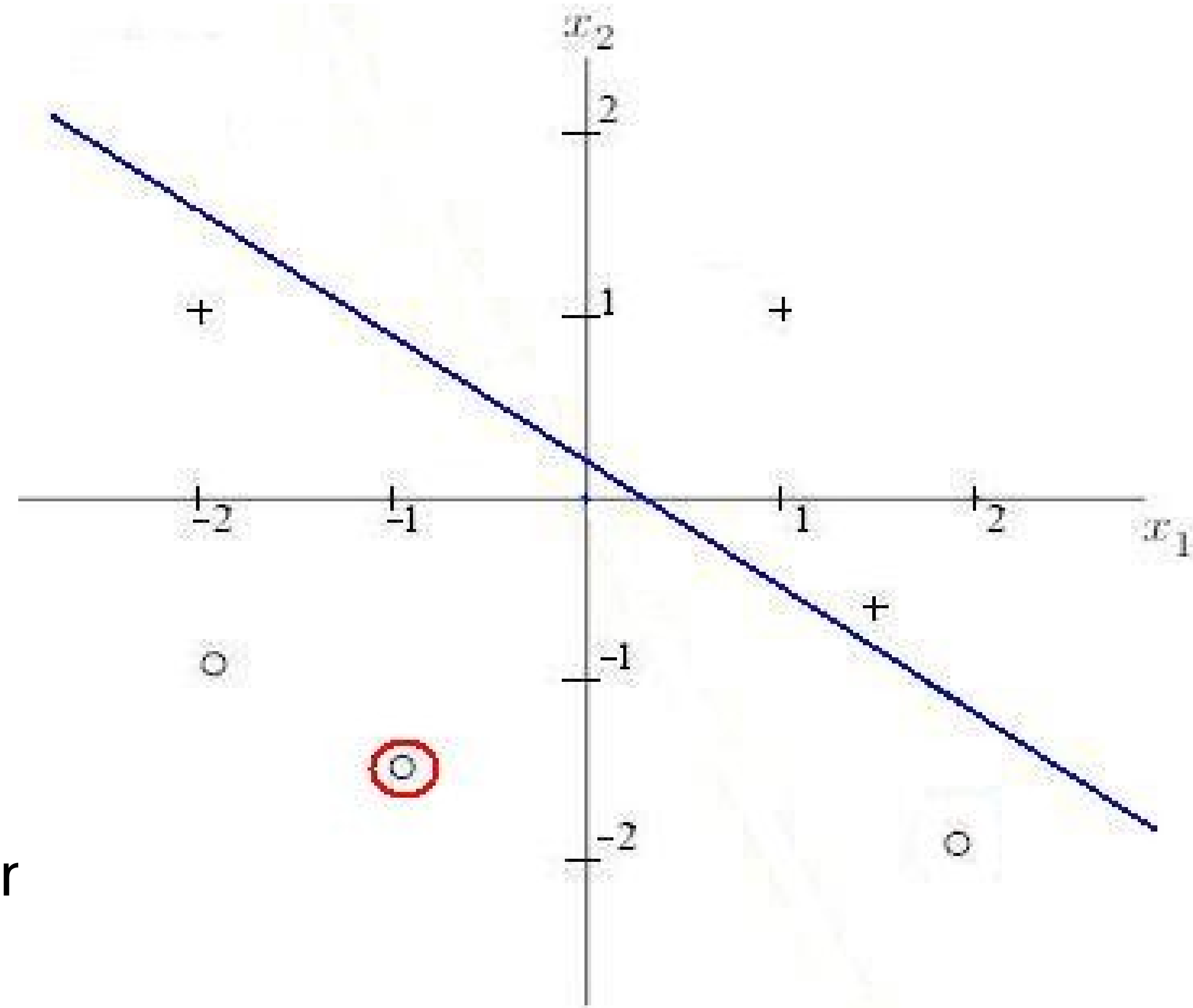
Learning Example

$\eta = 0.2$

$$w = \begin{pmatrix} -0.2 \\ 0.6 \\ 0.9 \end{pmatrix}$$

$x_1 = -1,\ x_2 = -1.5$
$w^T x < 0$

Correct classificatior
no action



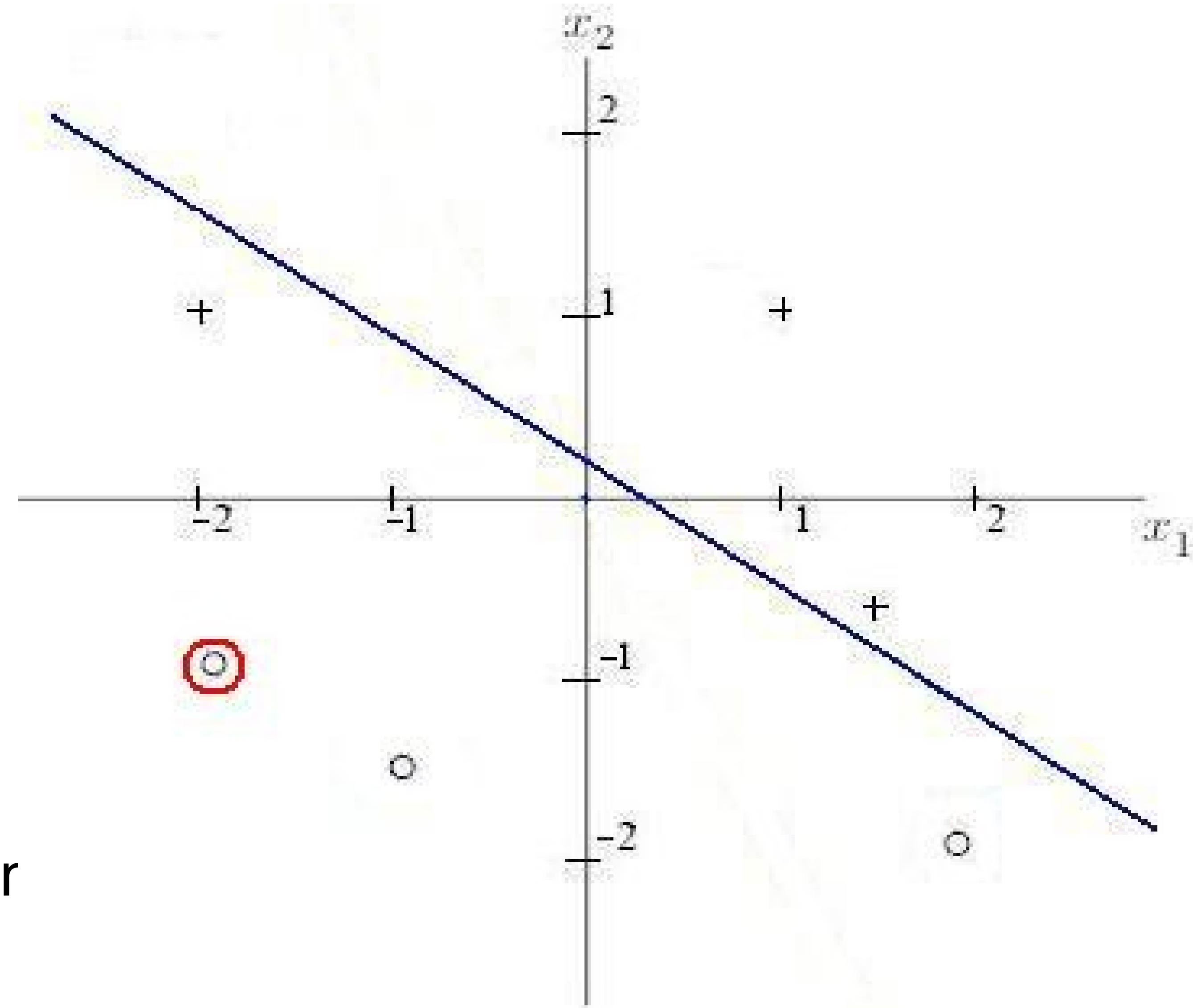
Learning Example

$\eta = 0.2$

$$w = \begin{pmatrix} -0.2 \\ 0.6 \\ 0.9 \end{pmatrix}$$

$x_1 = -2$, $x_2 = -1$
$w^T x < 0$

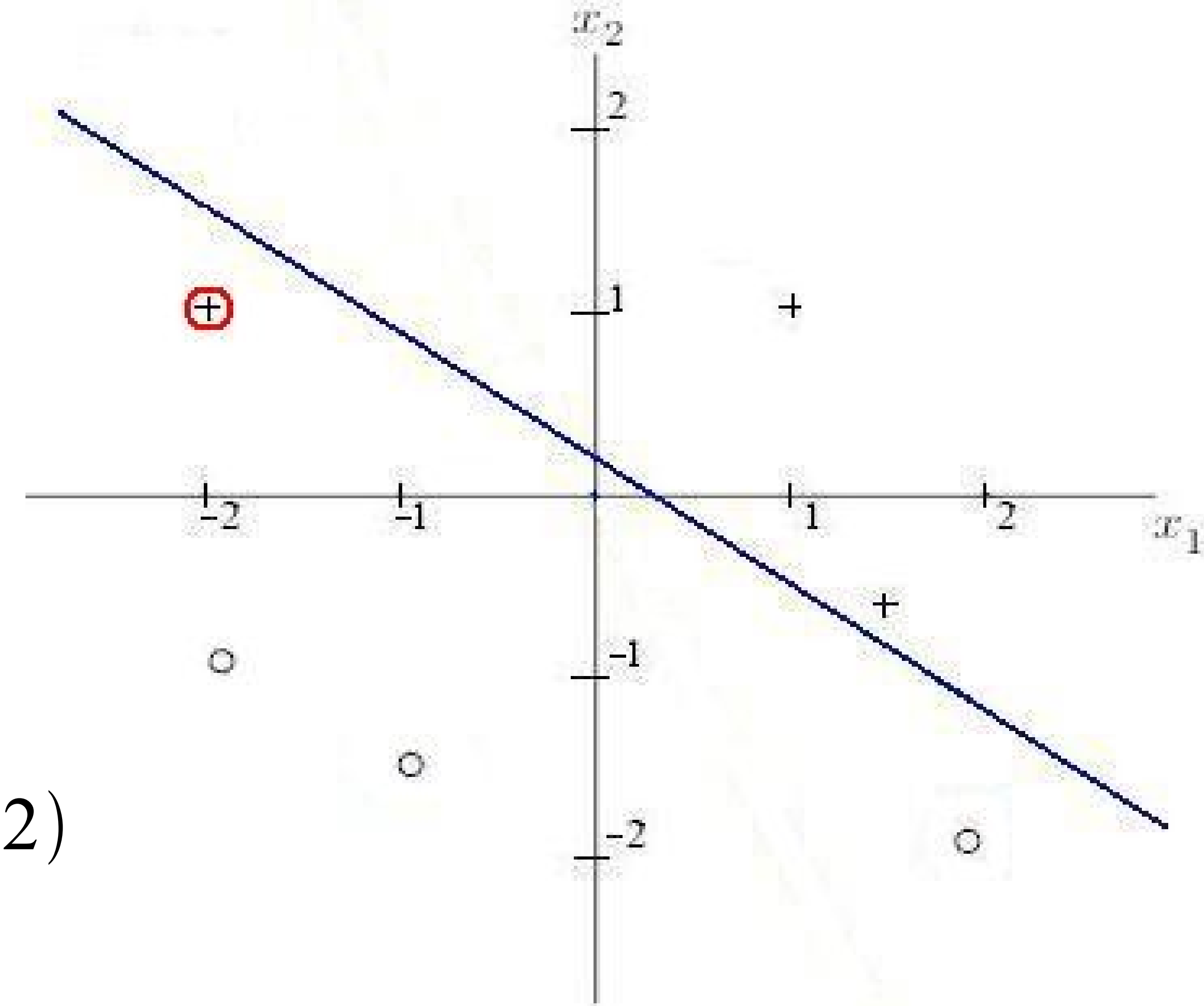
Correct classificatior
no action



Learning Example

$\eta = 0.2$

$$w = \begin{pmatrix} -0.2 \\ 0.6 \\ 0.9 \end{pmatrix}$$



x_1 = -2, x_2 = 1

$$w_0 = w_0 + 0.2 * 1$$

$$w_1 = w_1 + 0.2 * (-2)$$

$$w_2 = w_2 + 0.2 * 1$$

Learning Example

$\eta = 0.2$

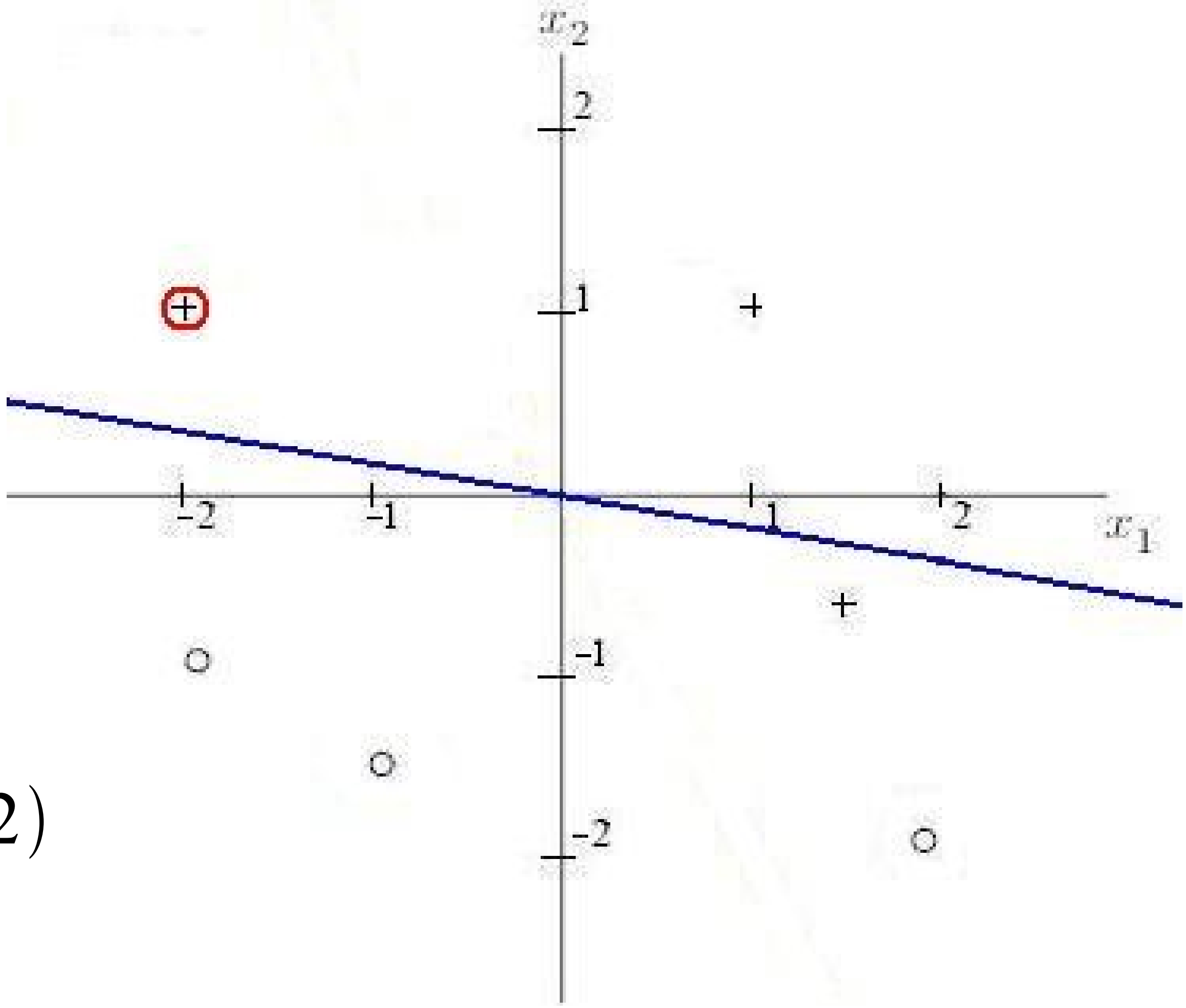
$$w = \begin{pmatrix} 0 \\ 0.2 \\ 1.1 \end{pmatrix}$$

$x_1 = -2,\ x_2 = 1$

$$w_0 = w_0 + 0.2 * 1$$

$$w_1 = w_1 + 0.2 * (-2)$$

$$w_2 = w_2 + 0.2 * 1$$



Learning Example

$\eta = 0.2$

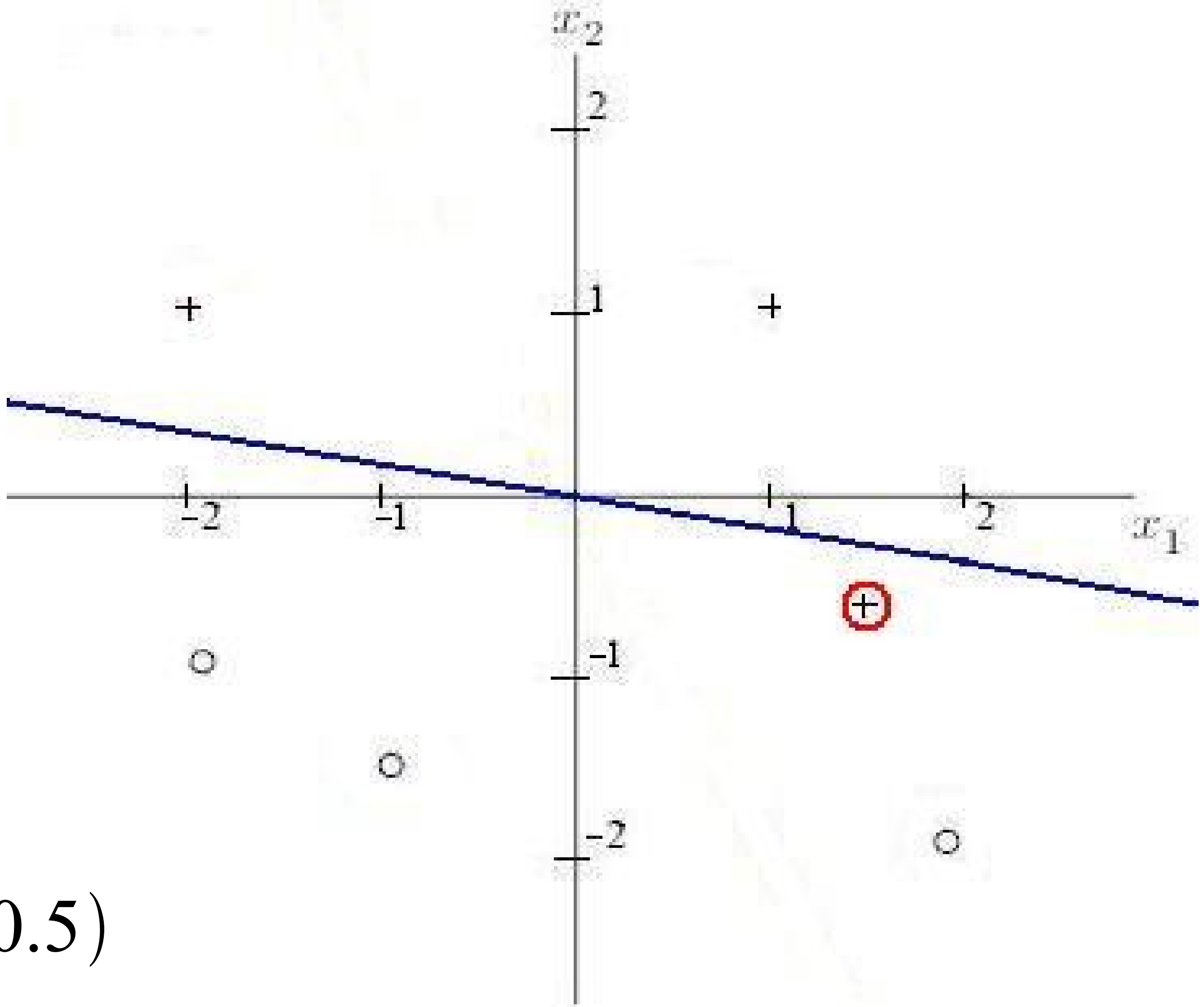
$$w = \begin{pmatrix} 0 \\ 0.2 \\ 1.1 \end{pmatrix}$$

$x_1 = 1.5,\ x_2 = -0.5$

$$w_0 = w_0 + 0.2 * 1$$

$$w_1 = w_1 + 0.2 * 1.5$$

$$w_2 = w_2 + 0.2 * (-0.5)$$



Learning Example

$\eta = 0.2$

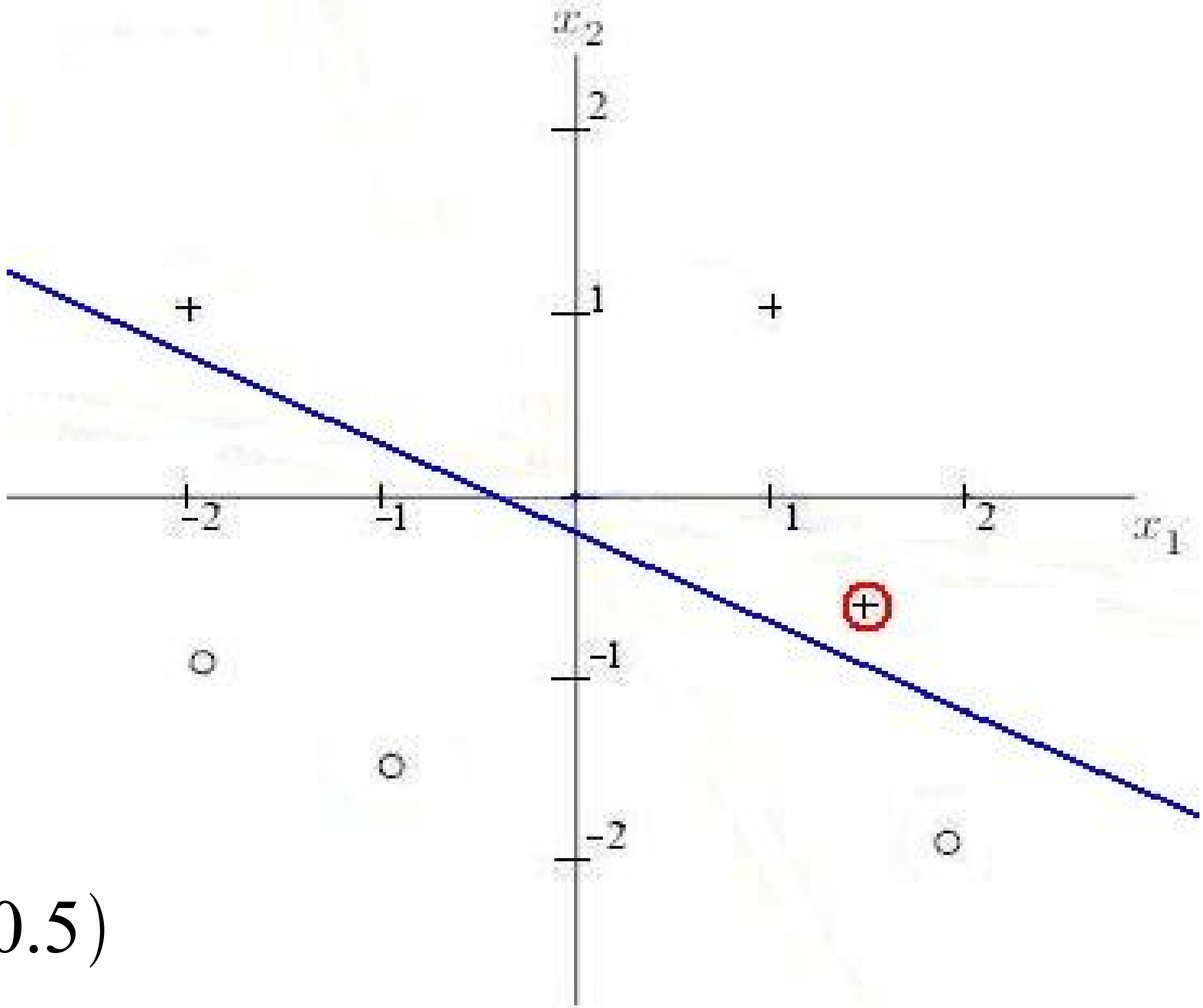
$$w = \begin{pmatrix} 0.2 \\ 0.5 \\ 1 \end{pmatrix}$$

$x_1 = 1.5,\ x_2 = -0.5$

$$w_0 = w_0 + 0.2 * 1$$

$$w_1 = w_1 + 0.2 * 1.5$$

$$w_2 = w_2 + 0.2 * (-0.5)$$



The End