Expected Value:

Finite case [edit]

Let X be a random variable with a finite number of finite outcomes x_1, x_2, ..., x_k occurring with probabilities p_1 , p_2 , ..., p_k , respectively. The **expectation** of X is defined as

$$\operatorname{E}[X] = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k.$$

Since all probabilities p_i add up to 1 ($p_1 + p_2 + \ldots + p_k = 1$), the expected value is the weighted average, with p_i 's being the weights.

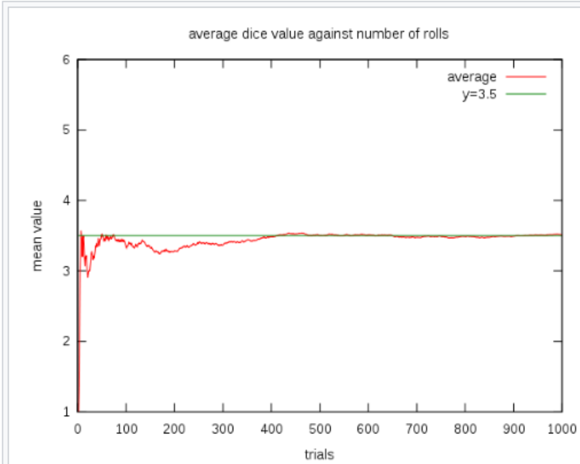
If all outcomes x_i are equiprobable (that is, $p_1 = p_2 = \ldots = p_k$), then the weighted average turns into the simple average. This is intuitive: the expected value of a random variable is the average of all values it can take; thus the expected value is what one expects to happen *on average*. If the outcomes x_i are not equiprobable, then the simple average must be replaced with the weighted average, which takes into account the fact that some outcomes are more likely than the others. The intuition however remains the same: the expected value of X is what one expects to happen *on average*.

X1p1+x2p2+…+xkpk

P1(x1+x2+…xk) = 1/k*(x1+x2+…xk) = average.

Examples [edit]

- Let X represent the outcome of a roll of a fair six-sided die. More specifically, X will be the number of pips showing on the top face of the die after the toss. The possible values for X are 1, 2, 3, 4, 5, and 6, all equally likely (each having the probability of $\frac{1}{6}$). The expectation of X is



An illustration of the convergence of sequence averages of rolls of a die to the expected value of 3.5 as the number of rolls (trials) grows.

$$\operatorname{E}[X] = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = 3.5.$$