Opgave 1

Vi har en funktion $f(t)$ som på Figur 1.

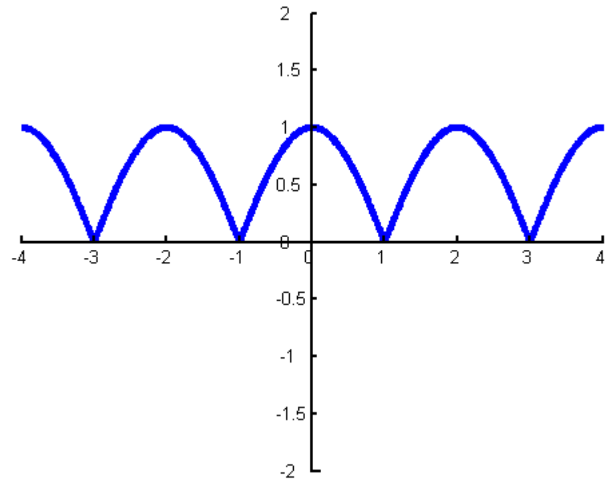

Figure 1: 1. periodisk funktion.

a) Periodetiden er $T = 2$. Funktionsforeskriften er givet som

$$f(t) = \cos\left(\frac{\pi}{2}t\right) \qquad -1 < t < 1$$

for $f(t)$.

b) Grundfrekvensen i rad/sec er:

$$\omega_0 = \frac{2\pi}{2} = \pi$$

c) Formlen for Fourierrækken for $f(t)$, hvor a_0, a_n og b_n er ukendte:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos(n\pi t) + b_n \sin(n\pi t))$$

d) Integralet for a_0 opskrives og beregnes

$$\begin{aligned} a_0 &= \int_{-1}^{1} \cos\left(\frac{\pi}{2}t\right) dt \\ &= \left[\frac{2}{\pi}\sin\left(\frac{\pi}{2}t\right)\right]_{-1}^{1} \\ &= \frac{2}{\pi} - (-\frac{2}{\pi}) \\ &= \frac{4}{\pi} \end{aligned}$$

Opgave 1 fortsat på næste side…

Den passer med det forventede da $\frac{a_0}{2} = 0,64$.

e) Vi opskriver integralet for a_n og beregner:

$$\begin{aligned}
a_n =& \frac{2}{2}\int_{-1}^{1} \cos\left(\frac{\pi}{2}t\right)\cos(\pi n t)\,dt \\
=& \left[\frac{\sin((\frac{\pi}{2}-\pi n)t)}{2(\frac{\pi}{2}-\pi n)} + \frac{\sin((\frac{\pi}{2}+\pi n)t)}{2(\frac{\pi}{2}+\pi n)}\right]_{-1}^{1} \\
=& \left(\frac{\sin(\frac{\pi}{2}-\pi n)}{2(\frac{\pi}{2}-\pi n)} + \frac{\sin(\frac{\pi}{2}+\pi n)}{2(\frac{\pi}{2}+\pi n)} - \frac{\sin(-(\frac{\pi}{2}-\pi n))}{2(\frac{\pi}{2}-\pi n)} - \frac{\sin(-(\frac{\pi}{2}+\pi n))}{2(\frac{\pi}{2}+\pi n)}\right) \\
=& (-1)^n\left(\frac{2}{2(\frac{\pi}{2}-\pi n)} + \frac{2}{2(\frac{\pi}{2}+\pi n)}\right) \\
=& (-1)^n\left(\frac{\frac{\pi}{2}+\pi n+\frac{\pi}{2}-\pi n}{(\frac{\pi}{2}-\pi n)(\frac{\pi}{2}+\pi n)}\right) \\
=& (-1)^n\left(\frac{1}{\frac{\pi}{4}-\pi n^2}\right)
\end{aligned}$$

f) Vi opskriver integralet for b_n og beregner:

$$\begin{aligned}
b_n =& \frac{2}{2}\int_{-1}^{1} \cos\left(\frac{\pi}{2}t\right)\sin(\pi n t)\,dt \\
=& -\left[\frac{\cos((\pi n-\frac{\pi}{2})t)}{2(\pi n-\frac{\pi}{2})} + \frac{\cos((\frac{\pi}{2}+\pi n)t)}{2(\frac{\pi}{2}+\pi n)}\right]_{-1}^{1} \\
=& -\left(\frac{\cos(\pi n-\frac{\pi}{2})}{2(\pi n-\frac{\pi}{2})} + \frac{\cos(\frac{\pi}{2}+\pi n)}{2(\frac{\pi}{2}+\pi n)} - \frac{\cos(-(\pi n-\frac{\pi}{2}))}{2(\pi n-\frac{\pi}{2})} + \frac{\cos(-(\frac{\pi}{2}+\pi n))}{2(\frac{\pi}{2}+\pi n)}\right) \\
=& 0
\end{aligned}$$

g) Vi opskriver Fourierrækken:

$$\begin{aligned}
f(t) =& \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos n\pi t + b_n \sin n\pi t) \\
=& \frac{\frac{4}{\pi}}{2} + \sum_{n=1}^{\infty}\left((-1)^n\left(\frac{1}{\frac{\pi}{4}-\pi n^2}\right)\cos n\pi t + 0\sin n\pi t\right) \\
=& \frac{2}{\pi} + \sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{\frac{\pi}{4}-\pi n^2}\right)\cos n\pi t
\end{aligned}$$

h) Vi opskriver Fourierrækken for de første 3 led:

$$f(t) = \frac{2}{\pi} - \left(\frac{1}{\frac{\pi}{4}-\pi}\right)\cos\pi t + \left(\frac{1}{\frac{\pi}{4}-\pi 4}\right)\cos 2\pi t - \left(\frac{1}{\frac{\pi}{4}-\pi 9}\right)\cos 3\pi t$$

$n = 1, 2, 3,$

i) Tegn løsningen fra h) er tegnet i Matlab:

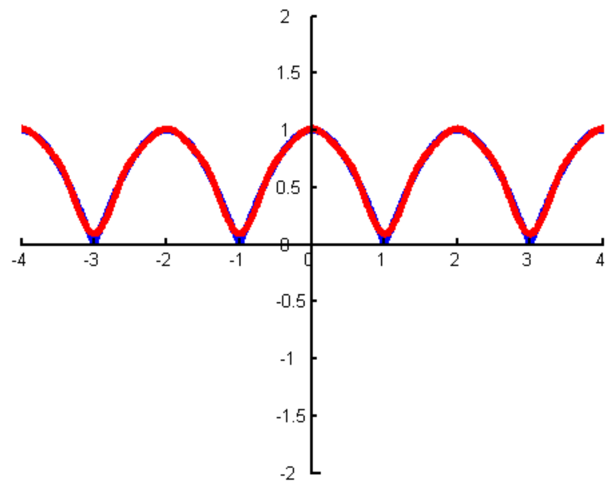

Figure 2: 1. Foruierrækken med de tre første led.

Opgave 2

Vi har en funktion $f(t)$ som på Figur 3.

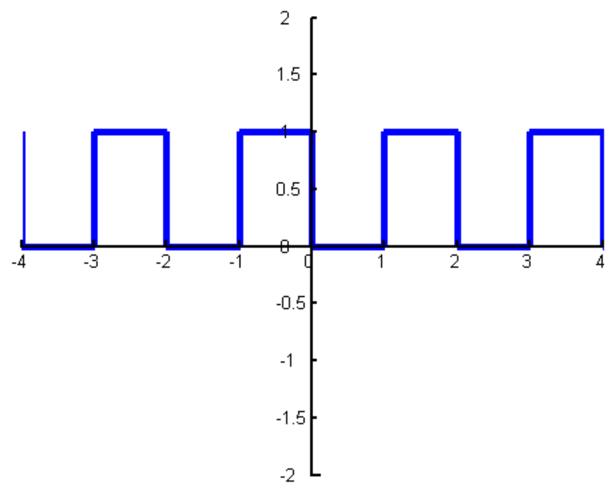

Figure 3: 2. periodisk funktion.

a) Periodetiden er $T = 2$. Funktionsforeskriften er givet som

$$f(t) = \begin{cases} 0 & 0 < t < 1 \\ 1 & 1 < t < 2 \end{cases}$$

for $f(t)$.

b) Grundfrekvensen i rad/sec er:

$$\omega_0 = \frac{2\pi}{2} = \pi$$

c) Formlen for Fourierrækken for $f(t)$, hvor a_0, a_n og b_n er ukendte:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos(n\pi t) + b_n \cos(n\pi t))$$

d) Integralet for a_0 opskrives og beregnes

$$\begin{aligned} a_0 =& \frac{2}{2}\int_0^1 0dt + \int_1^2 1dt \\ =& [t]_1^2 \\ =& 2 - 1 \\ =& 1 \end{aligned}$$

Den passer med det forventede da $\frac{a_0}{2} = 0,5$.

e) Vi opskriver integralet for a_n og beregner:

$$\begin{aligned} a_n =& \frac{2}{2}\int_1^2 1 \cdot \cos(\pi n t)\, dt \\ =& \left[\frac{1}{\pi n}\sin(\pi n t)\right]_1^2 \\ =& 0 \end{aligned}$$

f) Vi opskriver integralet for b_n og beregner:

$$\begin{aligned} b_n =& \frac{2}{2}\int_1^2 1 \cdot \sin(\pi n t)\, dt \\ =& \left[-\frac{1}{\pi n}\cos(\pi n t)\right]_1^2 \\ =& -\frac{2}{\pi n} \; n\ er\ ulige \end{aligned}$$

g) vi opskriver Fourierrækken:

$$\begin{aligned} f(t) =& \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\pi t + b_n \sin n\pi t) \\ =& 0,5 + \sum_{n=1}^{\infty} 0 \cos n\pi t - \sum_{n=1,3,5\cdots}^{\infty} \frac{2}{\pi n}\sin n\pi t \\ =& 0,5 - \sum_{n=1,3,5\cdots}^{\infty} \frac{2}{\pi n}\sin n\pi t \end{aligned}$$

h) Vi opskriver Fourierrækken for de første 3 led:

$$f(t) = 0,5 - \frac{2}{\pi}\sin \pi t - \frac{2}{\pi 3}\sin 3\pi t$$

$n = 1, 2, 3,$

i) Løsningen fra h) er tegnet i Matlab:

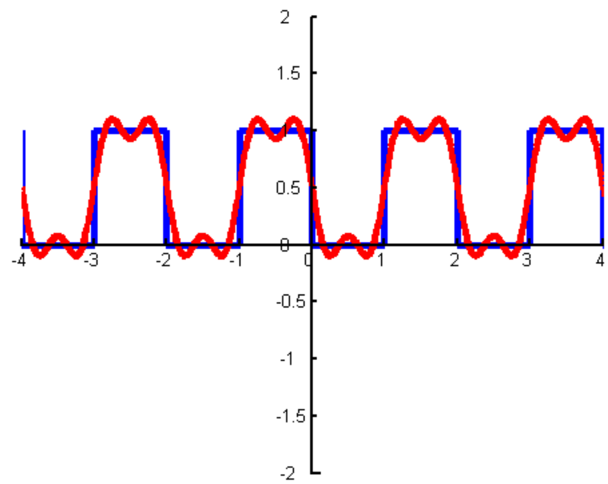

Figure 4: 1. Foruierrækken med de tre første led.

Opgave 3

Vi har en funktion $f(t)$ som på Figur 5.

a) Periodetiden er $T = 4$. Funktionsforeskriften er givet som

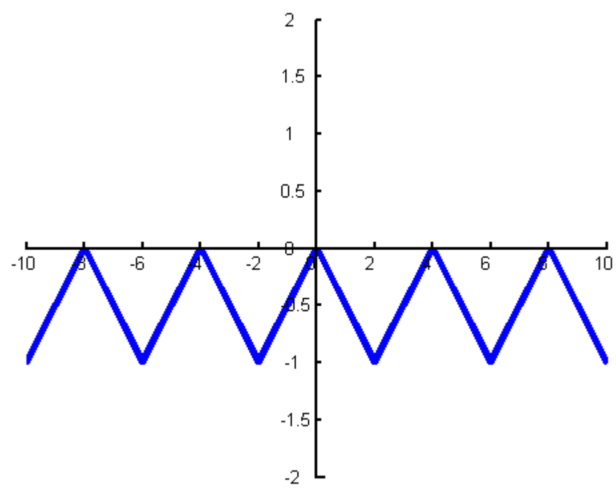

Figure 5: 3. periodisk funktion.

$$f(t) = \begin{cases} -\frac{1}{2}t & 0 < t < 2 \\ \frac{1}{2}t - 2 & 2 < t < 4 \end{cases}$$

for $f(t)$.

b) Grundfrekvensen i rad/sec er:

$$\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

c) Formlen for Fourierrækken for $f(t)$, hvor a_0, a_n og b_n er ukendte:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos(n\frac{\pi}{2}t) + b_n \cos(n\frac{\pi}{2}t))$$

d) Integralet for a_0 opskrives og beregnes

$$\begin{aligned} a_0 =& \frac{2}{4}\int_0^2 -\frac{1}{2}tdt + \int_2^4 \frac{1}{2}t - 2dt \\ =& \frac{1}{2}\left[-\frac{1}{4}t^2\right]_0^2 + \left[\frac{1}{4}t^2 - 2t\right]_2^4 \\ =& \frac{1}{2}\left(-\frac{1}{4}2^2 + \left(\frac{1}{4}4^2 - 2\cdot 4 - (\frac{1}{4}2^2 - 2\cdot 2)\right)\right) \\ =& -1 \end{aligned}$$

Den passer med det forventede da $\frac{a_0}{2} = -0,5$.

e) Vi opskriver integralet for a_n og beregner:

$$\begin{aligned} a_n =& \frac{2}{4}\int_0^2 -\frac{1}{2}t \cdot \cos\left(\frac{\pi}{2}nt\right)dt + \frac{2}{4}\int_2^4 (\frac{1}{2}t - 2)\cdot \cos\left(\frac{\pi}{2}nt\right)dt \\ =& -\frac{1}{4}\int_0^2 t \cdot \cos\left(\frac{\pi}{2}nt\right)dt + \frac{1}{4}\int_2^4 (t-1)\cdot \cos\left(\frac{\pi}{2}nt\right)dt \\ =& \frac{1}{4}\frac{8\cdot 2}{\pi^2 n^2} \\ =& \frac{4}{\pi^2 n^2} \end{aligned}$$

Vi har at

$$\begin{aligned} \int_0^2 t\cdot\cos\left(\frac{\pi}{2}nt\right)dt =& [t\frac{2}{\pi n}\sin\left(\frac{\pi}{2}nt\right)]_0^2 - \int_0^2 \frac{2}{\pi n}\sin\left(\frac{\pi}{2}nt\right)dt \\ =& \left[\frac{2}{\pi n}\sin\left(\frac{\pi}{2}nt\right) + \frac{4}{\pi^2 n^2}\cos\left(\frac{\pi}{2}nt\right)\right]_0^2 \\ =& \frac{4}{\pi^2 n^2}\cos(\pi n) - \frac{4}{\pi^2 n^2} \\ =& -\frac{8}{\pi^2 n^2}\ \textit{n er ulige} \end{aligned}$$

Vi har at

$$
\begin{aligned}
\int_2^4 t\cdot\cos\left(\frac{\pi}{2}nt\right)dt =& [t\frac{2}{\pi n}\sin\left(\frac{\pi}{2}nt\right)]_2^4 - \int_2^4 \frac{2}{\pi n}\sin\left(\frac{\pi}{2}nt\right)dt \\
=& \left[\frac{2}{\pi n}\sin\left(\frac{\pi}{2}nt\right) + \frac{4}{\pi^2 n^2}\cos\left(\frac{\pi}{2}nt\right)\right]_2^4 \\
=& \frac{4}{\pi^2 n^2}\cos(2\pi n) - \frac{4}{\pi^2 n^2}\cos(\pi n) \\
=& \frac{8}{\pi^2 n^2}\ n\ er\ ulige
\end{aligned}
$$

Og vi har at

$$
\begin{aligned}
\int_2^4 \cos\left(\frac{\pi}{2}nt\right)dt =& \left[\frac{2}{\pi n}\sin\left(\frac{\pi}{2}nt\right)\right]_2^4 \\
=& 0
\end{aligned}
$$

f) Vi opskriver integralet for b_n og beregner med mathcad:

$$
\begin{aligned}
b_n =& \frac{2}{4}\int_0^2 -\frac{1}{2}t\cdot\sin\left(\frac{\pi}{2}nt\right)dt + \frac{2}{4}\int_2^4 (\frac{1}{2}t-2)\cdot\sin\left(\frac{\pi}{2}nt\right)dt \\
=& 0
\end{aligned}
$$

g) Vi opskriver Fourierrækken:

$$
\begin{aligned}
f(t) =& \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos n\omega t + b_n \sin n\omega t) \\
=& -0,5 + \sum_{n=1,3,5\cdots}^{\infty}\frac{8}{\pi^2 n^2}\cos n\frac{\pi}{2}t + \sum_{n=1}^{\infty} 0 \sin n\frac{\pi}{2}t \\
=& -0,5 + \sum_{n=1,3,5\cdots}^{\infty}\frac{8}{\pi^2 n^2}\cos n\frac{\pi}{2}t
\end{aligned}
$$

h) Vi opskriver Fourierrækken for de første 3 led:

$$
f(t) = -0,5 + \frac{4}{\pi^2}\cos\frac{\pi}{2}t + \frac{4}{\pi^2 9}\cos 3\frac{\pi}{2}t
$$

$n = 1, 2, 3,$

i) Løsningen fra h) er tegnet i Matlab:

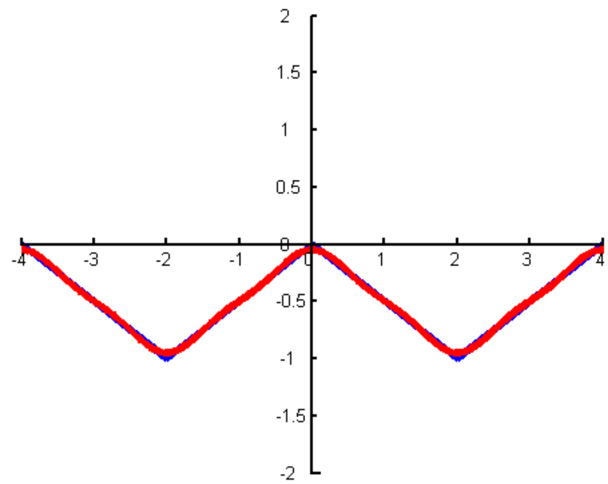

Figure 6: 3. Fourierrækken med de tre første led.