Opgave b partialbrøksopløsning

i)
$$\frac{s}{(s+1)(s-1)} = \frac{A_1}{s+1} + \frac{A_2}{s-1}$$

$$A_1 = \left.\frac{(s+1)s}{(s+1)(s-1)}\right|_{s=-1} = \frac{-1}{-2} = \frac{1}{2}$$

$$A_2 = \left.\frac{(s-1)s}{(s+1)(s-1)}\right|_{s=1} = \frac{1}{2}$$

$$\frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{2}}{s-1}$$

ii)
$$\frac{s}{s^2+4s+5} = \frac{s}{(s+2-j)(s+2+j)} = \frac{A_1}{s+2-j} + \frac{A_2}{s+2+j}$$

$$A_1 = \left.\frac{s}{s+2+j}\right|_{s=-2+j} = \frac{-2+j}{-2+j+2+j} = \frac{-2+j}{+2j} = +\cancel{2}j + \frac{1}{2}$$

$$A_2 = \left.\frac{s}{s+2-j}\right|_{s=-2-j} = \frac{-2-j}{-2-j+2\cancel{+}j} = \frac{+2+j}{+2j} = -j + \frac{1}{2}$$

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iii)

$$\frac{2+s}{(s+1)(s-1)(s-2)} = \frac{A_1}{s+1} + \frac{A_2}{s-1} + \frac{A_3}{s-2}$$

$$A_1 = \left.\frac{2+s}{(s-1)(s-2)}\right|_{s=-1} = \frac{1}{(-2)(-3)} = \frac{1}{6}$$

$$A_2 = \left.\frac{2+s}{(s+1)(s-2)}\right|_{s=1} = \frac{3}{2(-1)} = -\frac{3}{2}$$

$$A_3 = \left.\frac{2+s}{(s-1)(s+1)}\right|_{s=2} = \frac{4}{1\cdot 3} = \frac{4}{3}$$

iv)

$$\frac{1}{(s+3+j)(s+3-j)} = \frac{A_1}{s+3+j} + \frac{A_2}{s+3-j}$$

$$A_1 = \left.\frac{1}{s+3-j}\right|_{s=-3-j} = \frac{1}{j^2} = -1$$

$$A_2 = \left.\frac{1}{s+3+j}\right|_{s=-3+j} = \frac{1}{j^2} = -1$$

$$-\frac{1}{s+3+j} - \frac{1}{s+3-j}$$

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v)

$$\frac{1}{(s+6-4j)(s+6+4j)} = \frac{A_1}{s+6-4j} + \frac{A_2}{s+6+4j}$$

$$A_1 = \left.\frac{1}{s+6+4j}\right|_{s=-6+4j} = \frac{1}{8j} = -\frac{1}{8}j$$

$$A_2 = \left.\frac{1}{s+6-4j}\right|_{s=-6-4j} = -\frac{1}{8j} = \frac{1}{8}j$$

$$\frac{-\frac{1}{8}j}{s+6-4j} + \frac{\frac{1}{8}j}{s+6+4j}$$

vi)

$$\frac{s}{(s+6)(s+1)} + e^{-3s}\frac{4}{(s+6)(s+1)}$$

$$= \frac{A_1}{s+6} + \frac{A_2}{s+1} + e^{-3s}\left(\frac{A_3}{s+6} + \frac{A_4}{s+1}\right)$$

$$A_1 = \left.\frac{s}{s+1}\right|_{s=-6} = \frac{6}{5}$$

$$A_2 = \left.\frac{s}{s+6}\right|_{s=-1} = -\frac{1}{5}$$

$$A_3 = \left.\frac{4}{s+1}\right|_{s=-6} = -\frac{4}{5}$$

$$A_4 = \left.\frac{4}{s+6}\right|_{s=-1} = \frac{4}{5}$$

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Opgave b (fortsat)

ix)

$$\frac{9+s}{s^2+6} = \frac{9+s}{(s+\sqrt{6})(s-\sqrt{6})} =$$

$$\frac{A_1}{s+\sqrt{6}} + \frac{A_2}{s-\sqrt{6}}$$

$$A_1 = \left.\frac{9+s}{s-\sqrt{6}}\right|_{s=-\sqrt{6}} = \frac{9-\sqrt{6}}{-2\sqrt{6}} = \frac{9\sqrt{6}-6}{-12}$$

$$= \frac{-3\sqrt{6}+2}{4}$$

$$A_2 = \left.\frac{9+s}{s+\sqrt{6}}\right|_{s=\sqrt{6}} = \frac{9+\sqrt{6}}{2\sqrt{6}} = \frac{3\sqrt{6}+2}{4}$$

Opgave d

i) $(s + \frac{7}{2})^2 - \frac{7^2}{4} + 5 = \underline{\underline{(s + \frac{7}{2})^2 - 7{,}25}}$

ii) $s^2 + 10s = \underline{\underline{(s + 5)^2 - 100}}$

iii) $s^2 + s + 1 = (s + \frac{1}{2})^2 + 1 - \frac{1}{4} = \underline{\underline{(s + \frac{1}{2})^2 + \frac{3}{4}}}$

iv) $s^2 + 5 = \underline{\underline{s^2 + 5}}$

v) $s^2 + 2s - 15 = (s + 1)^2 - 1 - 15 = \underline{\underline{(s + 1)^2 - 16}}$

vi) $s^2 \cancel{+} 7s + 1 = \underline{\underline{(s + 3{,}5)^2 - 2{,}5}}$

vii) $s^2 + 4s^2 + 5 = \underline{\underline{5(s)^2 + 5}}$

viii) $s^2 - 10 = \underline{\underline{s^2 - 10}}$

ix) $(s + 1)^2 - 1 + 1 = \underline{\underline{(s + 1)^2}}$

Opgave e

i) $\mathcal{L}^{-1}\left\{\frac{s}{s^2+7s+5}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s+\frac{7}{2})^2 - 2,69^2}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s+3,5}{(s+3,5)^2-2,69^2} - 3,5\frac{1}{2,69}\frac{2,69}{(s+3,5)^2-2,69^2}\right\}$

$$= \underline{\underline{e^{-3,5t}\left(\cosh(2,69t) - 1,30\sinh(2,69t)\right)u(t)}}$$

ii) $\mathcal{L}^{-1}\left\{\frac{1}{(s+5)^2-10^2}\right\} = \underline{\underline{\frac{1}{10}\sinh(10t)\,u(t)\,e^{-5t}}}$

iii) $\mathcal{L}^{-1}\left\{\frac{s+6}{(s+\frac{1}{2})^2+0,87^2}\right\} = \underline{\underline{e^{-\frac{1}{2}t}u(t)\left(\cosh(0,87t) + \frac{5,5}{0,87}\sinh(0,87t)\right)}}$

$= e^{-\frac{1}{2}t}u(t)\left(\cosh(0,87t) + 6,32\sinh(0,87t)\right)$

iv) $\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+5}\right\} = \underline{\underline{\cos(\sqrt{5}t) + \frac{1}{\sqrt{5}}\sin(\sqrt{5}t)}}$

v) $\mathcal{L}^{-1}\left\{\frac{4}{(s+1)^2-4^2}\right\} = \underline{\underline{e^{-t}u(t)\,\sinh(4t)}}$

vi) $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{(s+3,5)^2-1,58^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+3,5)^2-1,58^2}\right\}\Bigg|_{t=t-2}$

Opgave e (fortsat)

vi) fortsat

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+3{,}5)^2-1{,}58^2}\right\}\Bigg|_{t=t-2} = \frac{1}{1{,}58}\sinh(1{,}58t)\,e^{-3{,}5t}u(t)\Bigg|_{t=t-2}$$

$$= \underline{\underline{\frac{1}{1{,}58}\sinh\big(1{,}58(t-2)\big)\,e^{-3{,}5(t-2)}\,u(t-2)}}$$

vii) $$\mathcal{L}^{-1}\left\{\frac{1-e^{5s}}{s^2+4s^2+5}\right\} = \mathcal{L}^{-1}\left\{\frac{1/5}{s^2+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1/5\,e^{5s}}{s^2+1}\right\}$$

$$= \underline{1/5\sin(t)u(t) - \frac{1}{5}\left[\sin(t)u(t)\right]}\Bigg|_{t=t+5}$$

$$= \underline{\underline{1/5\sin(t)u(t) - 1/5\sin(t+5)u(t+5)}}$$

viii) $$\mathcal{L}^{-1}\left\{\frac{3}{s^2-10}\right\} = \underline{\underline{\frac{3}{\sqrt{10}}\sinh(\sqrt{10}\,t)\,u(t)}}$$

ix) $$\mathcal{L}^{-1}\left\{\frac{s+10}{(s+1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{9}{(s+1)^2}\right\}$$

$$= \underline{\underline{e^{-t}u(t) + 9e^{-t}\,\frac{1}{\cancel{}}t}}$$