Opgave 3

a) Laplacetransformer:

i)
$$\mathcal{L}\{t u(t-1)\} = \mathcal{L}\{(t-1+1)u(t-1)\}$$
$$= \mathcal{L}\{(t-1)u(t-1)\} + \mathcal{L}\{u(t-1)\}$$
$$= e^{-s}\left(\mathcal{L}\{t u(t)\} + \mathcal{L}\{u(t)\}\right)$$
$$= \underline{\underline{e^{-s}\left(\frac{1}{s^2} + \frac{1}{s}\right)}}$$

ii)
$$\mathcal{L}\{4e^{-2t}u(t)\}$$
$$= 4\,\mathcal{L}\{u(t)\}\Big|_{s=s+2}$$
$$= 4\,\frac{1}{s}\Big|_{s=s+2} = \underline{\underline{\frac{4}{s+2}}}$$

iii)
$$\mathcal{L}\{(t^2-2)u(t-2)\}$$
$$= \mathcal{L}\{((t-2)^2 - 2^2 + 4t - 2)u(t-2)\}$$
$$= \mathcal{L}\{((t-2)^2 + 4(t-2) + 8 - 2^2 - 2)u(t-2)\}$$

Opgave 3 (fortsat)

a)

iii) fortsat

$$\mathcal{L}\{((t-2)^2 + 4(t-2) + 2)\, u(t-2)\}$$

$$= e^{-2s}\left(\mathcal{L}\{(t^2 + 4t + 2)\, u(t)\}\right)$$

$$= \underline{\underline{e^{-2s}\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{2}{s}\right)}}$$

iv) $\mathcal{L}\{t\,u(t) + u(t-9)\}$

$$= \underline{\underline{\frac{1}{s^2} + e^{-9s}\frac{1}{s}}}$$

v) $\mathcal{L}\{(-t)(u(t) - u(t-1))\}$

$$= -\mathcal{L}\{t\,u(t)\} + \mathcal{L}\{t\,u(t-1)\}$$

$$= -\frac{1}{s^2} + \mathcal{L}\{(t-1+1)\,u(t-1)\}$$

$$= -\frac{1}{s^2} + e^{-s}\mathcal{L}\{(t+1)\,u(t)\}$$

$$= \underline{\underline{-\frac{1}{s^2} + e^{-s}\left(\frac{1}{s^2} + \frac{1}{s}\right)}}$$

Opgave 3 (fortsat)

a) fortsat

vi) $\mathcal{L}\{4e^{-2t}u(t-4)\}$

$= 4\,\mathcal{L}\{u(t-4)\}\Big|_{s=s+2}$

$= 4e^{-4s}\,\mathcal{L}\{u(t)\}\Big|_{s=s+2}$

$= 4e^{-4s}\,\frac{1}{s}\Big|_{s=s+2}$

$= \underline{\underline{4e^{-4(s+2)}\,\frac{1}{s+2}}}$

vii)

$\mathcal{L}\{t^3\,4e^{-2t}u(t)\}$

$= 4\,\mathcal{L}\{t^3u(t)\}\Big|_{s=s+2}$

$= 4\,\frac{3\cdot 2\cdot 1}{s^4}\Big|_{s=s+2}$

$= \underline{\underline{\frac{24}{(s+2)^4}}}$

Opgave 3

a)

viii)

$$\mathcal{L}\{t e^{3t} u(t)\} - \mathcal{L}\{t e^{3t} u(t-2)\}$$

$$= \mathcal{L}\{t u(t)\}\Big|_{s=s-3} - \mathcal{L}\{t u(t-2)\}\Big|_{s=s-3}$$

$$= \frac{1}{s^2}\Big|_{s=s-3} - \mathcal{L}\{(t-2+2)u(t-2)\}\Big|_{s=s-3}$$

$$= \frac{1}{(s-3)^2} - e^{-2s}\mathcal{L}\{(t+2)u(t)\}\Big|_{s=s-3}$$

$$= \frac{1}{(s-3)^2} - e^{-2s}\left(\frac{1}{s^2} + \frac{2}{s}\right)\Big|_{s=s-3}$$

$$= \underline{\underline{\frac{1}{(s-3)^2} - e^{-2(s-3)}\left(\frac{1}{(s-3)^2} + \frac{2}{s-3}\right)}}$$

ix)

$$\mathcal{L}\left\{\int_0^t u(t)\,dt\right\} = \frac{1}{s}\mathcal{L}\{u(t)\}$$

$$= \underline{\underline{\frac{1}{s^2}}}$$

x)

$$\mathcal{L}\left\{\int_0^t t u(t-1) + 8u(t)\,dt\right\}$$

$$= \frac{1}{s}\mathcal{L}\{t u(t-1) + 8u(t)\}$$

$$= \frac{1}{s}\left(e^{-s}\mathcal{L}\{(t+1)u(t)\} + \mathcal{L}\{8u(t)\}\right)$$

$$= \underline{\underline{\frac{1}{s}\left(e^{-s}\left(\frac{1}{s^2} + \frac{1}{s}\right) + 8\frac{1}{s}\right)}}$$

Opgave 3

xi/

$$\mathcal{L}\left\{\int_0^t -e^{-3t}u(t)\,dt\right\}$$

$$= -\frac{1}{s}\,\mathcal{L}\left\{e^{-3t}u(t)\right\}$$

$$= -\frac{1}{s}\left[\frac{1}{s}\Big|_{s=s+3}\right]$$

$$= -\frac{1}{s}\,\frac{1}{s+3}$$