Opgave 2

A]

i) $$\mathcal{L}\left\{\frac{dv(t)}{dt}\right\} = \mathcal{L}\{u(t)\}$$

$$\Downarrow$$

$$sV(s) - v(0) = \frac{1}{s}$$

da $v(0) = 0$ har vi:

$$\underline{\underline{sV(s) = \frac{1}{s}}}$$

ii) isolér $V(s)$:

$$V(s) = \frac{1}{s}\,\frac{1}{s}$$

$$\underline{\underline{V(s) = \frac{1}{s^2}}}$$

iii) $$\mathcal{L}^{-1}\{V(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}$$

$$\underline{\underline{v(t) = t\,u(t)}}$$

B) Lxs.

$$\frac{dh(t)}{dt} = 2t\,u(t)$$

$$\mathcal{L}\left\{\frac{dh(t)}{dt}\right\} = \mathcal{L}\left\{2t\,u(t)\right\}$$

$$sH(s) - h(0) = 2\,\mathcal{L}\left\{t\,u(t)\right\}$$

da $h(0) = 0$

$$sH(s) = 2\,\frac{1}{s^2}$$

$$\Updownarrow$$

$$H(s) = 2\,\frac{1}{s^3}$$

$$\mathcal{L}^{-1}\left\{H(s)\right\} = \mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\}$$

$$\underline{\underline{h(t) = t^2\,u(t)}}$$

c)

Laplace for $f(t)$:

$$\frac{d^2 f(t)}{dt} + f(t) = \delta(t)$$

where $f(0) = 0$, $f'(0) = 0$

$$\mathcal{L}\left\{ \frac{d^2 f(t)}{dt} \right\} + \mathcal{L}\{ f(t) \} = \mathcal{L}\{ \delta(t) \}$$

$$\Downarrow$$

$$s^2 F(s) + F(s) = 1$$

$$F(s)(s^2 + 1) = 1$$

$$F(s) = \frac{1}{s^2+1}$$

$$\mathcal{L}^{-1}\{ F(s) \} = \mathcal{L}^{-1}\left\{ \frac{1}{s^2+1} \right\}$$

$$\underline{\underline{f(t) = \sin(t)}}$$

D] Laplace for $y(t)$

$$\int_0^t y(t)\,dt = t\,u(t)$$

$$\mathcal{L}\left\{\int_0^t y(t)\,dt\right\} = \mathcal{L}\{t\,u(t)\}$$

$$\frac{1}{s}\,Y(s) = \frac{1}{s^2}$$

$$Y(s) = \frac{s}{s^2}$$

$$= \frac{1}{s}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$$

$$\underline{\underline{y(t) = u(t)}}$$