Opgaver repetion

Opg. a $\mathcal{L}\{f''(t) + f'(t) + f(t)\} = \mathcal{L}\{u(t)\}$

begyndelses værdier $f(0)=1$ $f'(0)=1$

$$\Rightarrow s^2F(s) - sf(0) - f'(0) + sF(s) - f(0) + F(s) = \frac{1}{s}$$

$$s^2F(s) - s - 1 + sF(s) - 1 + F(s) = \frac{1}{s}$$

$$F(s)(s^2 + s + 1) - s - 2 = \frac{1}{s}$$

$$F(s) = \frac{\frac{1}{s} + s + 2}{s^2 + s + 1}$$

$$= \frac{1 + s^2 + 2s}{s(s^2+s+1)} \Rightarrow \text{partialbrøksopløsning}$$

$$F \quad \frac{1}{s^2+s+1} + \frac{1}{s}$$

$$= \frac{1}{(s+\frac{1}{2})^2 + 1 - \frac{1}{4}} + \frac{1}{s}$$

$$= \frac{1}{(s+\frac{1}{2})^2 + \sqrt{\frac{3}{4}}^2} + \frac{1}{s}$$

$$\boxed{f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{\sqrt{\frac{3}{4}}} e^{-\frac{1}{2}t} \sin\left(\sqrt{\frac{2}{4}}t\right) u(t)}$$

OPG b

$$6\frac{d^2 f(t)}{dt^2} + 10\frac{df(t)}{dt} + 2f(t) = 0 \qquad f(0)=0 \qquad f'(0)=1$$

$$\mathcal{L}\{6f''(t) + 10f'(t) + 2f(t)\} = 0$$

$$6\left(s^2F(s) - sf(0) - f'(0)\right) + 10\left(sF(s) - f(0)\right) + 2F(s) = 0$$

$$6\left(s^2F(s) - s\cdot 0 - 1\right) + 10\left(sF(s) - 0\right) + 2F(s) = 0$$

$$6s^2F(s) - 6 + 10sF(s) + 2F(s) = 0$$

$$F(s)\left(6s^2 + 10s + 2\right) = 6$$

$$F(s) = \frac{6}{6s^2+10s+2} = \frac{1}{s^2 + \frac{10}{6}s + \frac{1}{3}} = \frac{1}{\left(s+\frac{10}{12}\right)^2 - \frac{10^2}{12^2} + \frac{1}{3}}$$

$$= \frac{1}{\left(s+\frac{4}{3}\right)^2 - \left(\frac{\sqrt{13}}{6}\right)^2}$$

$$\boxed{f(t) = e^{-\frac{4}{3}t}\, u(t) \cdot \frac{1}{\frac{\sqrt{13}}{6}} \sinh\left(\frac{\sqrt{13}}{6}t\right)}$$

Opg c

$$\frac{df(t)}{dt} + 2f(t) = u(t)$$

$$\mathcal{L}\{ f'(t) + 2f(t) \} = \mathcal{L}[u(t)] \qquad f(0) = 5$$

$$sF(s) - f(0) + 2F(s) = \frac{1}{s}$$

$$F(s)(s+2) - 5 = \frac{1}{s} \Rightarrow F(s)(s+2) = \frac{1}{s} + 5 \Rightarrow$$

$$F(s) = \frac{\frac{1}{s} + 5}{s+2} \Rightarrow F(s) = \frac{1 + 5s}{s^2 + 2s} \Rightarrow F(s) = \frac{5(s+1) - 5 + 1}{(s+1)^2 - 1^2}$$

$$\boxed{f(t) = 5e^{-t}u(t)\cosh(t) - 4e^{-t}u(t)\sinh(t)}$$

Opg d $\quad f(0)=1 \quad f'(0)=1$

$$\mathcal{L}\{f''(t)+2f'(t)+8f(t)\}=\mathcal{L}\{\delta(t)\} \Rightarrow$$

$$s^2F(s)-sf(0)-f'(0)+2(sF(s)-f(0))+8F(s)=1$$

$$F(s)(s^2+2s+8)-s-1-2\cdot 1=1$$

$$F(s)=\frac{s+4}{(s+1)^2-1^2+8}=\frac{s+1+3}{(s+1)^2+\sqrt{7}^2}$$

$$\boxed{f(t)=e^{-t}u(t)\left(\cos\sqrt{7}t+\frac{3}{\sqrt{7}}\sin\sqrt{7}t\right)}$$

Opg e

$$\mathcal{L}\{ f''(t) + 10 f'(t) + 2 f(t) \} = 0 \qquad f(0) = 0 \quad f'(0) = 1$$

$$s^2 F(s) - s \cdot f(0) - f'(0) + 10 (sF(s) - f(0)) + 2 F(s) = 0$$

$$F(s) (s^2 + 10s + 2) - 1 = 0$$

$$F(s) = \frac{1}{(s+5)^2 - 5^2 + 2} = \frac{1}{(s+5)^2 - \sqrt{23}^2}$$

$$\boxed{f(t) = e^{-5t} u(t) \frac{1}{\sqrt{23}} \sinh \sqrt{23}\, t}$$

OPG f

$$\mathcal{L}\{\int_0^t f(t)\,dt + 4f'(t) + f(t)\} = 0 \qquad f(0) = 3$$

$$\frac{1}{s}F(s) + 4(sF(s) - f(0)) + F(s) = 0$$

$$F(s)\left(\frac{1}{s} + 4s + 1\right) - 3 = 0$$

$$F(s) = \frac{3s}{1 + 4s^2 + s} = \frac{\frac{3}{4}s}{(s + \frac{1}{4\cdot 2})^2 - \frac{1}{8^2} + 1}$$

$$= \frac{\frac{3}{4}s}{(s + \frac{1}{8})^2 + \sqrt{\frac{63}{64}}^{\,2}}$$

$$\boxed{f(t) = \frac{3}{4}e^{-\frac{1}{8}t}u(t)\left(\cos\sqrt{\frac{63}{64}}\,t - \frac{1}{8}\cdot\sqrt{\frac{64}{63}}\sin\sqrt{\frac{63}{64}}\,t\right.}$$

Opg 5

$$\mathcal{L}\left[\int f(t)\,dt + f(t)\right] = \mathcal{L}[u(t)]$$

$$\frac{1}{s}F(s) + F(s) = \frac{1}{s}$$

$$F(s)(s+1) = 1$$

$$F(s) = \frac{1}{s+1}$$

$$\boxed{f(t) = e^{-t}u(t)}$$