Mathematical Model on Climate Change

Asiya Anas, Ashwin Nair, Faheena Thesni, Farsana, Govindram Niware

1. School of Mathematics, Indian Institute of Science Education and Research, Thiruvananthapuram
2. School of Data Science, Indian Institute of Science Education and Research, Thiruvananthapuram

* E-mail of the corresponding authors: asiyaanas20@iisertvm.ac.in ashwinnair20@iisertvm.ac.in faheenathes20@iisertvm.ac.in farsana20@iisertvm.ac.in govindram20@iisertvm.ac.in

Abstract

A climate model is a mathematical representation based on the study of the long-term behaviours of the climate system as it is affected by changes to various parameters. A differential equation model was developed and verified using the mean global temperature annually, forest area, and the daily amounts of precipitation. We mostly use the model used in [1] and check to see how it fits on real world data to check the effectiveness of the model described. One of our main contributions involve realising the effect of temperature on forest area and modelling the differential equation model described in [1] with our modifications. This research looks at the long-term behaviour of rainfall as it is affected by changes in forest area and global warming. To best capture the effects of severe weather hazards such as drought , global temperature and forest cover are considered annually, while rainfall is considered seasonally. A differential equation model was developed and validated using the annual mean global temperature, forest area, and daily rainfall amounts.

1. Introduction

Climate is an inevitable part of Earth's atmosphere. Climate change is an irreversible consequence of the global warming phenomenon.

Climate change models are essential tools for understanding and predicting the complex interactions between Earth's atmosphere, oceans, land surface, and biosphere. These models are designed to study the impacts of climate change on the environment, economy, and society.

In this model, we will explore the impact of climate change on forest areas, rainfall patterns, and temperature trends. The model will use historical data to establish a baseline and then project future scenarios based on various climate change scenarios. This model will provide valuable insights for policymakers and conservationists who are working to protect our ecosystem.

1.1 Statement of the Problem

Changes in climate have the potential to impact the economy and society. In the past few years, climate change and its impact on daily life have been a hot topic. It has grabbed the worldwide attention of scientists and leaders, obliging them to take action against climate change. This model is intended to answer the following questions:

1. Can we formulate a mathematical model that establishes the relationship between global temperature, seasonal precipitation and forest coverage?
2. What is the impact of rising rainfall trends on declining forest areas and global temperature?
3. How does global temperature change with forest area?
4. Is our model capable of predicting future climate changes?

2. The model

2.1 Assumptions:

In order to build the model, the following assumptions are made:

(i) Global temperature appears to be increasing exponentially. However, the global temperature cannot continue to rise indefinitely. It should be constrained at some temperature.

(ii) Today's forest area has decreased exponentially in comparison to the past. Because the forest area cannot fall below zero and cannot exceed the maximum area of the studied region, it is an example of logistic decay.

(iii) The amount of rainfall appears to be seasonal and thus periodic. To capture seasonal rainfall, a second order differential equation with a periodic solution should be considered.

(iv) The decrease in forest area is inversely proportional to the rise in the temperature and it is important to not overlook the effect of temperature on forest areas.

(v) Sources of change in a particular variable is considered to be zero from the other two variables.

2.2 Variables

Temperature: T

Rainfall: R

Forest Area: F

Time: t

2.3 Constants and their Dimensions

1. a_T : Rate of change of Temperature — $year^{-1}$
2. a_F : Rate of change of Forest area — $year^{-1}$
3. α : Rate of change of Rainfall — $year^{-1}$
4. b_T : Difference between two equilibrium points (T) — °C
5. b_f ; Difference between two equilibrium points (F) — km^2
6. m_T : Minimum global temperature — °C
7. m_f : Minimum global Forest area — km^2

2.4 General Model As Proposed in [1]:

1) $\frac{dT}{dt} = a_T\left(1 - \frac{T - m_T}{b_T}\right)(T - m_T)$

2) $\frac{dF}{dt} = -a_F\left(1 - \frac{F - m_f}{b_f}\right)(F - m_f)$

3) $\frac{d^2R}{dt^2} = -\alpha^2 R$

2.5 Explicit Solution:

$$T(t) = m_T + \frac{b_T A}{A + b_T e^{-a_T t}}$$

Where $A = (b_T(T_0 - m_T))/(b_T - T_0 + m_T$ and $T(0) = T_0$.

$F(t) = m_f + \frac{b_f B}{B + b_f exp(a_f t)}$

where $B = \frac{b_f(F_0 - m_f)}{b_f - F_0 + m_f}$ and $F(0) = F_0$

$R(t) = c_1 cos(\alpha(t)) + c_2 sin(\alpha(t))$

2.6 Stability Analysis

1.

$a_T(1 - \frac{T - m_T}{b_T})(T - m_T) = 0$

$T = m_T, m_T + b_T$ are the critical points

$\lim_{t \to \infty} T(t) = m_T + \frac{b_T A}{A} = m_T + b_T$

Therefore both critical solutions are stable

2.

$F = m_f, b_f + m_f$ are the critical points

$\lim_{t \to \infty} F(t) = m_f + \frac{0}{1} = m_f$ m_f is the minimum forest area and it is the stable point

$b_f + m_f \to$ global maximum of F(t)

Therefore both equilibrium points are stable

3.

$\frac{dR}{dt} = S$
$\frac{dS}{dt} = -\alpha^2 R$

Jacobian matrix:

$$\begin{bmatrix} 0 & 1 \\ -\alpha^2 & 0 \end{bmatrix}$$

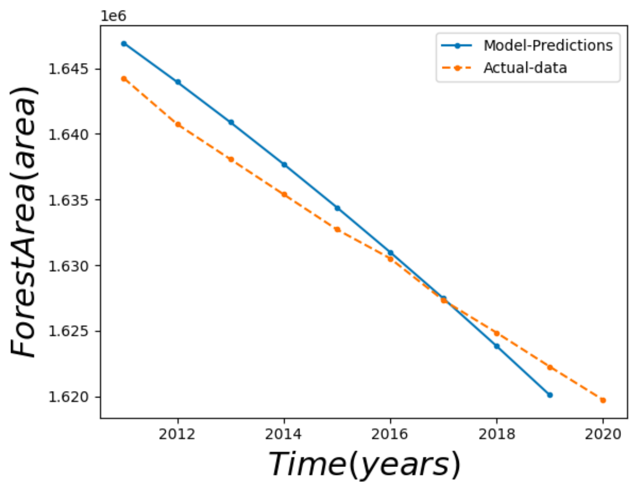
$\lambda^2 + \alpha^2 = 0$
$\lambda = \pm i\alpha$

(0,0) is a critical point (0,0) is the centre. Therefore, the solution is stable

2.7 Results

For testing of the model form [1] we have used the dataset [4] for the global annual forest area data, [2] for the global temperature of the earth and [3] for the seasonal rainfall data. We have the testing the model above using python using the SciPy library, specifically we have used the odeint() solver function in Scipy to solve the differential equations discussed above

The forest area model results:



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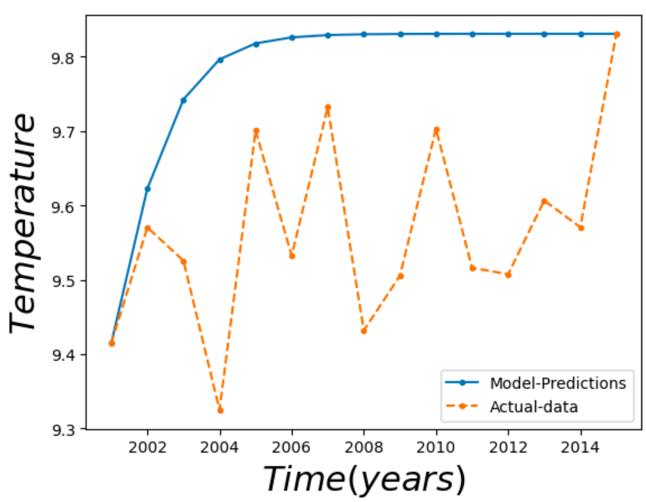
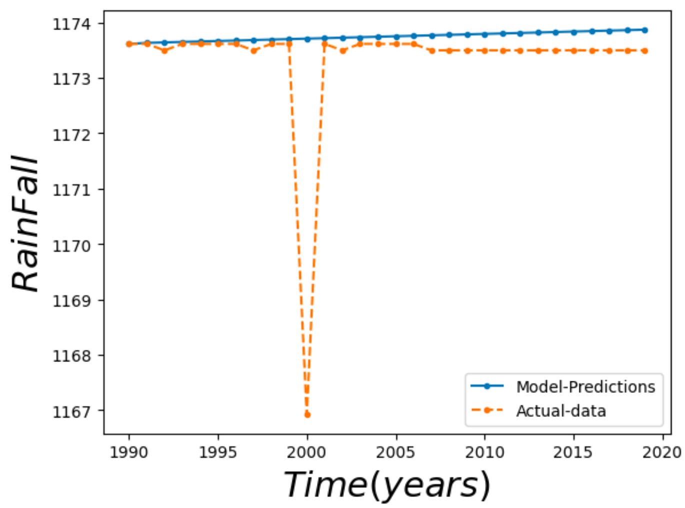
The temperature model results:



The rainfall model results:



2.8 Modified General Model

$$1)\ \frac{dT}{dt} = a_T\left(1 - \frac{T - m_T}{b_T}\right)(T - m_T)$$

$$2)\ \frac{dF}{dt} = -a_F\left(1 - \frac{F - m_f}{b_f}\right)(F - m_f) - c_f T$$

$$3)\ \frac{d^2R}{dt^2} = -\alpha^2(R - p)$$

2.9 Variables

1. Temperature: T
2. Rainfall: R
3. Forest Area: F
4. Time: t

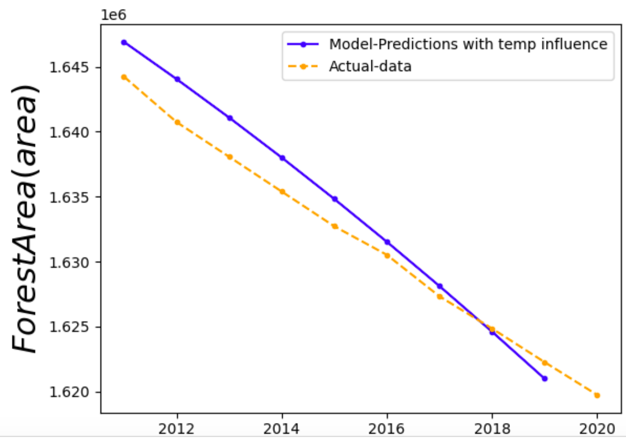
2.10 Constants and their dimensions

1.	a_T : Rate of change of Temperature	$year^{-1}$
2.	a_F : Rate of change of Forest area	$year^{-1}$
3.	α : Rate of change of Rainfall	$year^{-1}$
4.	b_T : Difference between two equilibrium points (T)	°C
5.	b_f : Difference between two equilibrium points (F)	km^2
6.	m_T : Minimum global temperature	°C
7.	m_f : Minimum global Forest area	km^2
8.	c_f : Constant depicting the interaction between forest area and temperature.	$year^{-1} km^2\, °C^{-1}$
9.	p : minimum rainfall	mm

2.11 Results for the Modified Model:

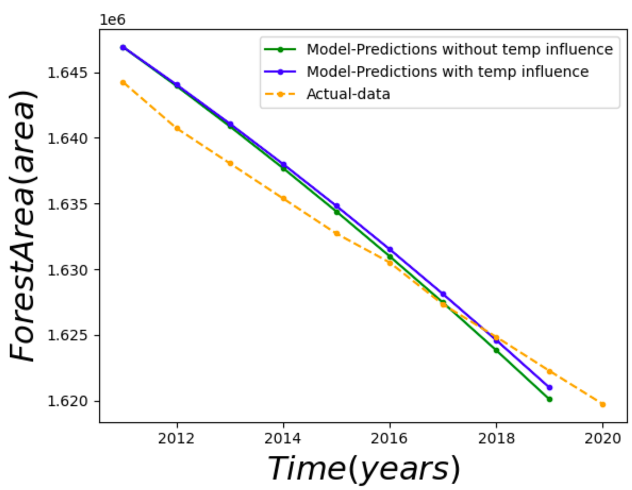
For testing of the modified model we have used the dataset [4] for the global annual forest area data, [2] for the global temperature of the earth and [3] for the seasonal rainfall data. We are testing the model above using python SciPy library, specifically we have used the odeint() solver function in Scipy to solve the differential equations discussed above

The forest area model results:



For better comparison with the original model from [1] and our new model we also plotting the two of them together.

The comparison results are:



We can observe that for the most part our modified model is close to the model defined by [1], but our model predictions are much better for prediction for a long time as it is more general and takes the changing temperature into account.

3. Conclusion

The relationship between global temperature dynamics, forest area, and amounts of rainfall has been mathematically formulated. As mean temperatures have risen, mean precipitation also has increased. This is expected because evaporation increases with increasing temperature, and there must be an increase in precipitation to balance the enhanced evaporation

There is a decrease in forest area due to deforestation to inhabit the increasing population.The modified model includes the interaction between temperature and forest area taking into consideration the increasing risk of forest fire due to increase in temperature.

References

[1] Genet Mekonnen Assefa : Mathematical Model on the Effects of Global Climate Change and Decreasing Forest Cover on Seasonal Rainfall DOI: 10.7176/MTM/9-1-03

[2] Climate Change: Earth Surface Temperature Data:
https://www.kaggle.com/datasets/berkeleyearth/climate-change-earth-surface-temperature-data?resource=download

[3] Precipitation Dataset:
https://data.worldbank.org/indicator/AG.LND.PRCP.MM?end=2019&start=2019&view=bar

[4] Forest Area Dataset:
https://data.worldbank.org/indicator/AG.LND.FRST.K2?end=2020&start=1990&view=chart