

3

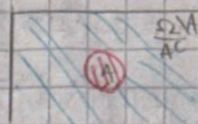
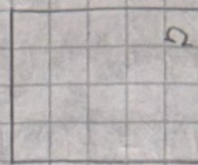
$$a) \begin{aligned} P(A) &= P(A) \\ P(A) &= P(A \cup \emptyset) \end{aligned} \quad (A \cup \emptyset = A)$$

por el axioma 3

$$P(A) = P(A) + P(\emptyset)$$

$$P(\emptyset) = 0$$

$$b) \begin{aligned} P(A^c) &= P(\Omega \setminus A) \\ P(A^c) &= P(\Omega) - P(A) \\ P(A^c) &= 1 - P(A) \end{aligned}$$



$$c) \text{ Sea } A \subset B$$

$$B = A \cup (B \setminus A)$$

$$P(B) = P(A \cup (B \setminus A))$$

$$P(B) = P(A) + P(B \setminus A)$$



$$\begin{aligned} & \text{Diagram showing a red circle } A \text{ and a green circle } B \setminus A \text{ separated by a plus sign, followed by an equals sign and a single red circle representing the union.} \\ & \text{= } \bigcirc \end{aligned}$$

$$d) \text{ Sea } A \text{ un evento, luego } A \in \mathcal{F}$$

$$P(\Omega) = P(A) + P(\Omega \setminus A)$$

$$1 = P(A) + P(\Omega \setminus A) =$$

por el axioma 2 $P(\Omega \setminus A) \geq 0$ por lo que $P(A) \leq 1$

$$e) \text{ ya que } P(B) = P(A) + P(B \setminus A) \text{ y } P(B \setminus A) \geq 0 \text{ (axioma 2)}$$

$$P(A) \leq P(A) + P(B \setminus A) \Leftrightarrow P(A) \leq P(B)$$

$$f) \text{ si } A \cap B = \emptyset \quad \begin{aligned} P(A \cup B) &= P(A) + P(B) - 0 \\ P(A \cup B) &= P(A) + P(B) - P(A \cap B) \end{aligned}$$

$$\text{si } A \cap B \neq \emptyset \quad A \cup B = (A - B) \cup (B - A) \cup (A \cap B) = C$$

$$A = (A - B) \cup (A \cap B) \rightarrow P(A)$$

g)

$$A \cup B \cup C = A \cup (B \cup C)$$

$$P(A \cup (B \cup C)) = P(A) + P(B \cup C) - P(A \cap (B \cup C))$$

$$P(A \cup (B \cup C)) = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap (B \cup C))$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$P((A \cap B) \cup (A \cap C)) = P(A \cap B) + P(A \cap C) - P((A \cap B) \cap (A \cap C))$$

$$P((A \cap B) \cup (A \cap C)) = P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

h)

$$A = A \cap (B \cup B^c) = (A \cap B) \cup (A \cap B^c)$$

$$P(A) = P((A \cap B) \cup (A \cap B^c))$$

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

$$P(A) = P(A \cap B) + P(A - B)$$

$$P(A - B) = P(A) - P(A \cap B)$$

i)

$$P((A \cap B^c) \cup (B \cap A^c)) = P((A - B) \cup (B - A))$$

$$P((A \cap B^c) \cup (B \cap A^c)) = P(A - B) + P(B - A)$$

$$P((A \cap B^c) \cup (B \cap A^c)) = P(A) - P(A \cap B) + P(B) - P(B \cap A)$$

$$A \cap B = B \cap A \rightarrow P(A \cap B) = P(B \cap A)$$

$$P((A \cap B^c) \cup (B \cap A^c)) = P(A) + P(B) - 2P(A \cap B)$$