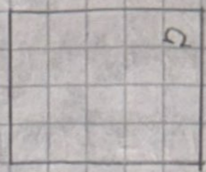
3

a) $P(A) = P(A)$
$P(A) = P(A \cup \emptyset)$ $\qquad (A \cup \emptyset = A)$

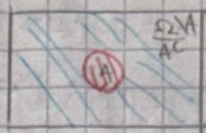
por el axioma 3

$P(A) = P(A) + P(\emptyset)$

$P(\emptyset) = 0$



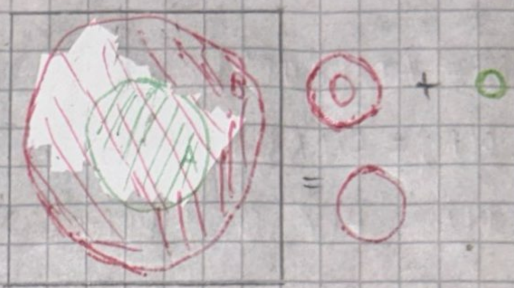
b) $P(A^c) = P(\Omega \setminus A)$
$P(A^c) = P(\Omega) - P(A)$
$P(A^c) = 1 - P(A)$



c) Sea $A \subset B$

$B = A \cup (B \setminus A)$

$P(B) = P(A \cup (B \setminus A))$
$P(B) = P(A) + P(B \setminus A)$



d) Sea A un evento, luego $A \subseteq \Omega$

$P(\Omega) = P(A) + P(\Omega \setminus A)$

$1 = P(A) + P(\Omega \setminus A)$

por el axioma 2 $P(\Omega \setminus A) \geq 0$ por lo que $P(A) \leq 1$

e) ya que $P(B) = P(A) + P(B \setminus A)$ y $P(B \setminus A) \geq 0$ (axioma 2)

$P(A) \leq P(A) + P(B \setminus A) \Leftrightarrow P(A) \leq P(B)$

f) si $A \cap B = \emptyset$ $\qquad P(A \cup B) = P(A) + P(B) - 0$
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

si $A \cap B \neq \emptyset$ $\qquad A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

$A = (A - B) \cup (A \cap B) \rightarrow P(A)$

g)

$A \cup B \cup C = A \cup (B \cup C)$

$P(A \cup (B \cup C)) = P(A) + P(B \cup C) - P(A \cap (B \cup C))$

$P(A \cup (B \cup C)) = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap (B \cup C))$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$P((A \cap B) \cup (A \cap C)) = P(A \cap B) + P(A \cap C) - P((A \cap B) \cap (A \cap C))$

$P((A \cap B) \cup (A \cap C)) = P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

h)

$A = A \cap (B \cup B^c) = (A \cap B) \cup (A \cap B^c)$

$P(A) = P((A \cap B) \cup (A \cap B^c))$

$P(A) = P(A \cap B) + P(A \cap B^c)$

$P(A) = P(A \cap B) + P(A - B)$

$P(A - B) = P(A) - P(A \cap B)$

i)

$P((A \cap B^c) \cup (B \cap A^c)) = P((A - B) \cup (B - A))$

$P((A \cap B^c) \cup (B \cap A^c)) = P(A - B) + P(B - A)$

$P((A \cap B^c) \cup (B \cap A^c)) = P(A) - P(A \cap B) + P(B) - P(B \cap A)$

$A \cap B = B \cap A \;\rightarrow\; P(A \cap B) = P(B \cap A)$

$P((A \cap B^c) \cup (B \cap A^c)) = P(A) + P(B) - 2\,P(A \cap B)$

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