



BHARATIYA VIDYA BHAVAN'S  
**SARDAR PATEL INSTITUTE OF TECHNOLOGY**

DEPARTMENT OF COMPUTER ENGINEERING

**CLASS: SE COMPS SEM: IV YEAR: SECOND COURSE CODE: CS205**

**SUBJECT NAME: DESIGN AND ANALYSIS OF ALGORITHM LAB NO: 604  
INDEX**

<b>Sr. No</b>	<b>TITLE</b>	<b>DATE OF EXPERIMENT</b>
1	Implementation of Insertion sort and Selection sort and also finding time complexity	09/02/2022
2	Implementation of Divide and Conquer method using Quick Sort algorithm.	17/02/2022
3	Bubble sort using Recursion	24/02/2022
4	Implementing of Dynamic Programming	17/03/2022
5	Implementing Greedy Algorithm using Huffman coding	17/03/2022
6	Implementing Djikstra's programming algorithm	31/03/2022
7	Backtracking approach for subset sum problem	07/04/2022
8	To solve 15 puzzle problem using Branch and Bound	14/04/2022
9	Experiment based on Approximation Algorithm (Set Cover Problem)	21/04/2022
10	Experiment of String Matching	28/04/2022

<b>Tools/software used:</b>	<b>Programming Language:</b> C, Java, Python <b>IDE:</b> Vs Code
<b>Name of the Faculty:</b>	Dr. Sudhir Dhage



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<b>Experiment No.</b>	1		

<b>AIM:</b>	To sort arrays using selection and insertion sort
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**EXPERIMENT 1**

<b>THEORY:</b>	<p><b>Sorting</b></p> <p>A Sorting Algorithm is used to rearrange a given array or list elements according to a comparison operator on the elements. The comparison operator is used to decide the new order of element in the respective data structure.</p> <p>In computer science, a sorting algorithm is an algorithm that puts elements of a list into an order. The most frequently used orders are numerical order and lexicographical order, and either ascending or descending. Efficient sorting is important for optimizing the efficiency of other algorithms (such as search and merge algorithms) that require input data to be in sorted lists. Sorting is also often useful for canonicalizing data and for producing human-readable output.</p> <p>Formally, the output of any sorting algorithm must satisfy two conditions:</p> <ol style="list-style-type: none"><li>1. The output is in monotonic order (each element is no smaller/larger than the previous element, according to the required order).</li><li>2. The output is a permutation (a reordering, yet retaining all of the original elements) of the input.</li></ol> <p><b>Selection sort</b></p>
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Selection sort is an in-place comparison sort. It has  $O(n^2)$  complexity, making it inefficient on large lists, and generally performs worse than the similar insertion sort. Selection sort is noted for its simplicity, and also has performance advantages over more complicated algorithms in certain situations.

The algorithm finds the minimum value, swaps it with the value in the first position, and repeats these steps for the remainder of the list. It does no more than  $n$  swaps, and thus is useful where swapping is very expensive.

**Time Complexity**

In computer science, selection sort is an in-place comparison sorting algorithm. It has an  $O(n^2)$  time complexity, which makes it inefficient on large lists, and generally performs worse than the similar insertion sort.

- Worst case time complexity:  $\Theta(N^2)$  comparisons and  $\Theta(N)$  swaps
- Average case time complexity:  $\Theta(N^2)$  comparisons and  $\Theta(N)$  swaps
- Best case time complexity:  $\Theta(N^2)$  comparisons and  $\Theta(N)$  swaps
- Space complexity:  $\Theta(1)$  auxillary space

**Worst Case Time Complexity**

The worst case is the case when the array is already sorted (with one swap) but the smallest element is the last element. For example, if the sorted number as  $a_1, a_2, \dots, a_N$ , then:  $a_2, a_3, \dots, a_N, a_1$  will be the worst case for our particular implementation of Selection Sort.

Worst Case:  $a_2, a_3, \dots, a_N, a_1$

The cost in this case is that at each step, a swap is done. This is because the smallest element will always be the last element and the swapped element which is kept at the end will be the second smallest element that is the smallest element of the new unsorted sub-array. Hence, the worst case has:



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- $N * (N+1) / 2$  comparisons
- N swaps

Hence, the time complexity is  $O(N^2)$ .

### **Best Case Time Complexity**

The best case is the case when the array is already sorted. For example, if the sorted number as  $a_1, a_2, \dots, a_N$ , then:  $a_1, a_2, a_3, \dots, a_N$  will be the best case for our particular implementation of Selection Sort.

This is the best case as we can avoid the swap at each step but the time spend to find the smallest element is still  $O(N)$ . Hence, the best case has:

- $N * (N+1) / 2$  comparisons
- 0 swaps

### **Average Case Time Complexity**

Based on the worst case and best case, we know that the number of comparisons will be the same for every case and hence, for average case as well, the number of comparisons will be constant.

Number of comparisons =  $N * (N+1) / 2$

Therefore, the time complexity will be  $O(N^2)$ .

To find the number of swaps,

- There are  $N!$  different combination of  $N$  elements
- Only for one combination (sorted order) there is 0 swaps.
- In the worst case, a combination will have  $N$  swaps. There are several such combinations.
- Number of ways to select 2 elements to swap =  $nC2 = N * (N-1) / 2$
- From sorted array, this will result in  $O(N^2)$  combinations which need 1 swap.

### **ALGO:**

1. Call insert to insert the element that starts at index 1 into the sorted subarray in index 0.
2. Call insert to insert the element that starts at index 2 into the sorted subarray in indices 0 through 1.



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- |  |  |
|--|--|
|  | <ol style="list-style-type: none"><li>3. Call insert to insert the element that starts at index 3 into the sorted subarray in indices 0 through 2.</li><li>4. ...</li><li>5. Finally, call insert to insert the element that starts at index <math>n-1n</math>, minus, 1 into the sorted subarray in indices 0 through <math>n-2n-2n</math>, minus, 2.</li></ol> |
|--|--|

**Working of Insertion Sort**

Suppose we need to sort the following array.

9	5	1	4	3
---	---	---	---	---

Initial array

1. The first element in the array is assumed to be sorted. Take the second element and store it separately in key.

Compare key with the first element. If the first element is greater than key, then key is placed in front of the first



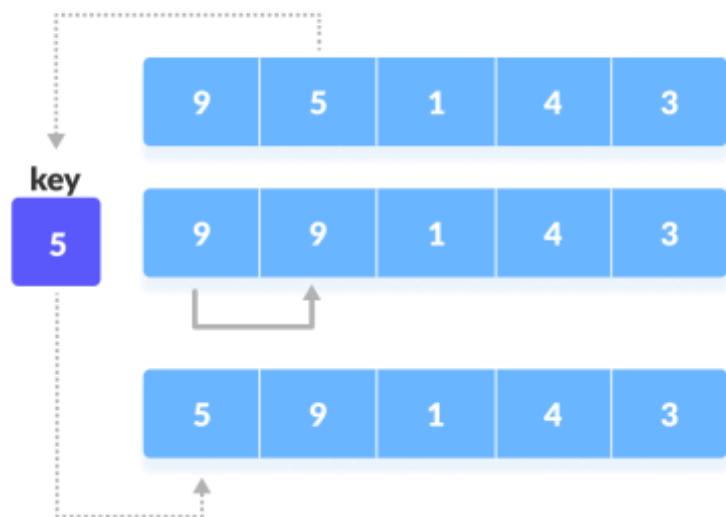
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element.

**step = 1**



If the first element is greater than key, then key is placed in front of the first element.

2. Now, the first two elements are sorted.

Take the third element and compare it with the elements on the left of it. Place it just behind the element smaller than it. If there is no element smaller than it, then place it at the beginning of the array.

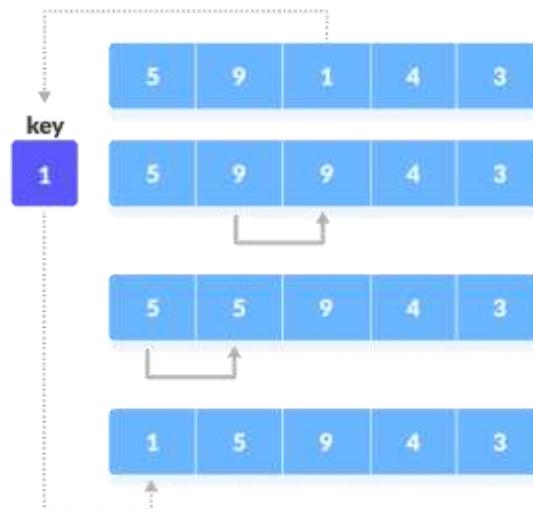


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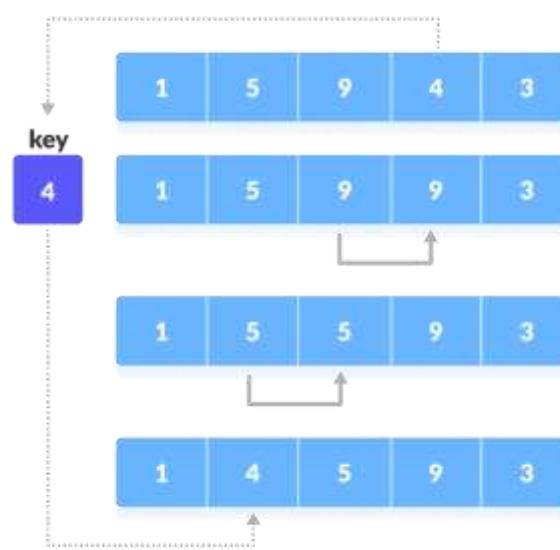
**step = 2**



Place 1 at the beginning

3. Similarly, place every unsorted element at its correct position.

**step = 3**



Place 4 behind 1

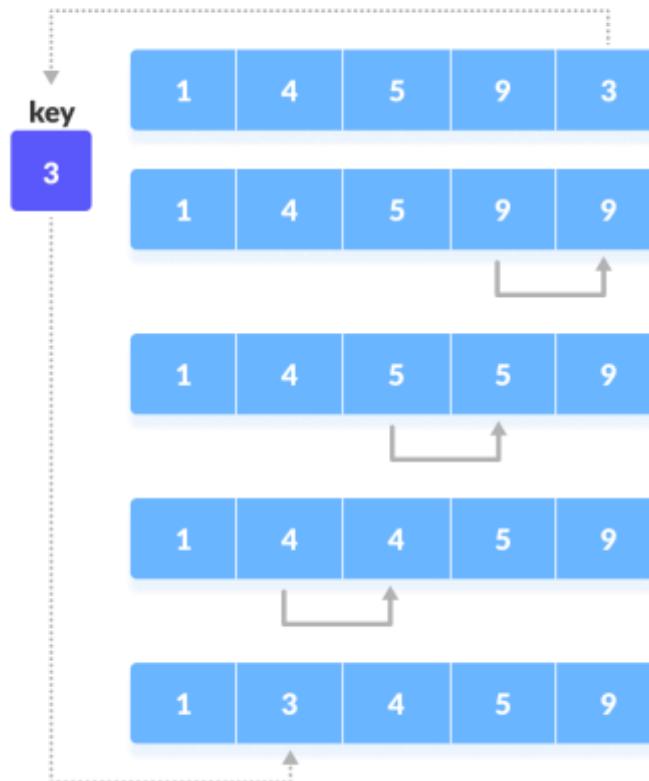


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**step = 4**



Place 3 behind 1 and the array is sorted

**Insertion Sort**

*Insertion sort* is a simple sorting algorithm that is relatively efficient for small lists and mostly sorted lists, and is often used as part of more sophisticated algorithms. It works by taking elements from the list one by one and inserting them in their correct position into a new sorted list similar to how we put money in our wallet. In arrays, the new list and the remaining elements can share the array's space, but insertion is expensive, requiring shifting all following elements over by



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one. Shellsort (see below) is a variant of insertion sort that is more efficient for larger lists.

### **Time Complexity**

The worst case time complexity of Insertion sort is  $O(N^2)$   
The average case time complexity of Insertion sort is  $O(N^2)$   
The time complexity of the best case is  $O(N)$ .  
The space complexity is  $O(1)$

### **Working Principle**

- Compare the element with its adjacent element.
- If at every comparison, we could find a position in sorted array where the element can be inserted, then create space by shifting the elements to right and insert the element at the appropriate position.
- Repeat the above steps until you place the last element of unsorted array to its correct position.

### **Best Case Analysis**

In Best Case i.e., when the array is already sorted,  $t_j = 1$   
Therefore,  $T(n) = C_1 * n + (C_2 + C_3) * (n - 1) + C_4 * (n - 1) + (C_5 + C_6) * (n - 2) + C_8 * (n - 1)$   
which when further simplified has dominating factor of  $n$  and gives  $T(n) = C * (n)$  or  $O(n)$

### **Worst Case Analysis**

In Worst Case i.e., when the array is reversly sorted (in descending order),  $t_j = j$   
Therefore,  $T(n) = C_1 * n + (C_2 + C_3) * (n - 1) + C_4 * (n - 1) * (n) / 2 + (C_5 + C_6) * ((n - 1) * (n) / 2 - 1) + C_8 * (n - 1)$   
which when further simplified has dominating factor of  $n^2$  and gives  $T(n) = C * (n^2)$  or  $O(n^2)$

### **Average Case Analysis**

Let's assume that  $t_j = (j-1)/2$  to calculate the average case  
Therefore,  $T(n) = C_1 * n + (C_2 + C_3) * (n - 1) + C_4/2 * (n - 1) * (n) / 2 + (C_5 + C_6)/2 * ((n - 1) * (n) / 2 - 1) + C_8 * (n - 1)$



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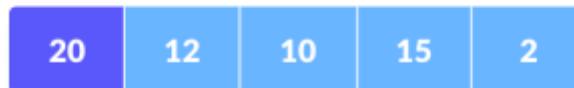
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$n - 1$ )  
which when further simplified has dominating factor of  $n^2$  and gives  $T(n) = C * (n^2)$  or  $O(n^2)$

**Working of Selection Sort:**

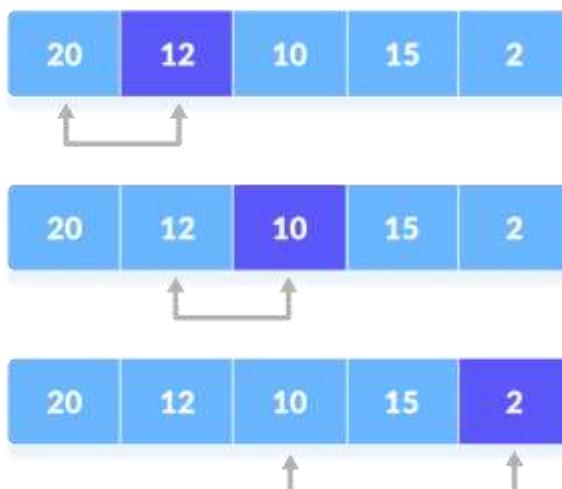
1. Set the first element as minimum.



Select first element as minimum

2. Compare minimum with the second element. If the second element is smaller than minimum, assign the second element as minimum.

Compare minimum with the third element. Again, if the third element is smaller, then assign minimum to the third element otherwise do nothing. The process goes on until the last element.



Compare minimum with the remaining elements



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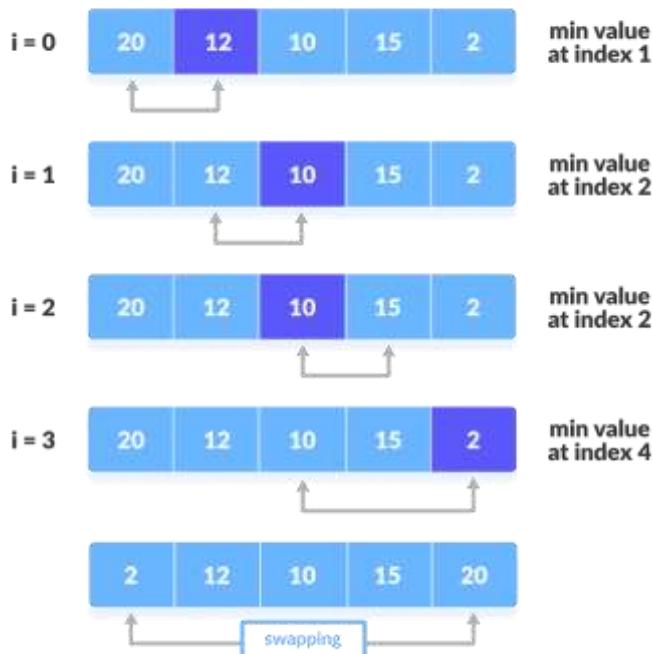
3. After each iteration, minimum is placed in the front of the unsorted list.



Swap the first with minimum

4. For each iteration, indexing starts from the first unsorted element. Step 1 to 3 are repeated until all the elements are placed at their correct positions.

step = 0



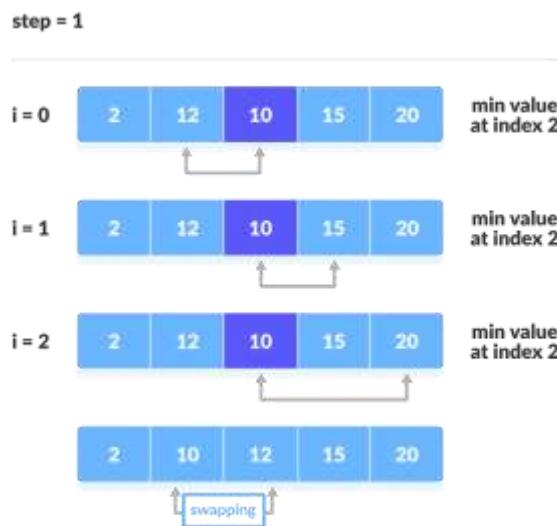
The first iteration



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The second iteration



The third iteration



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The fourth iteration

**ALGO:**

1. Find the smallest card. Swap it with the first card.
2. Find the second-smallest card. Swap it with the second card.
3. Find the third-smallest card. Swap it with the third card.
4. Repeat finding the next-smallest card, and swapping it into the correct position until the array is sorted.

**CODE:**

```
import java.io.BufferedReader;
import java.io.IOException;
import java.util.*;

public class Sort {
    public static void insertionSort(int array[]) {
        int arrayLen = array.length;
        for (int i = 1; i < arrayLen; ++i) {
            System.out.println("\n*****");
            int currentPos = array[i];
            System.out.print("\nIteration on Element " +
currentPos);
            int j = i - 1;
            while (j >= 0 && array[j] > currentPos) {

```



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```
        System.out.print("\nSwapping " + array[j] + " with
" + array[j + 1]);
        array[j + 1] = array[j];
        j--;
    }
    array[j + 1] = currentPos;
    System.out.print("\nArray : " + printArray(array));
}
}

public static void selectionSort(int array[]) {
    int arrayLen = array.length;

    for (int i = 0; i < arrayLen - 1; i++) {
        System.out.print("\n*****");
        ****;
        int minElement = i;
        for (int j = i + 1; j < arrayLen; j++) {
            if (array[j] < array[minElement]) {
                minElement = j;
            }
        }
        System.out.print("\nThe minimum Element is " +
array[minElement]);
        System.out.print("\nSwapping " + array[i] + " with " +
+ array[minElement]);
        int temp = array[minElement];
        array[minElement] = array[i];
        array[i] = temp;
        System.out.print("\nArray : " + printArray(array));
    }
}

public static String printArray(int array[]) {
    String result = "[ ";
    for (int i : array) {
        result += i + " ";
    }
}
```



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```
        result += "]";
        return result;
    }

    public static void main(String[] args) throws IOException {

        ArrayList<Integer> arrayList = new
        ArrayList<Integer>();

        int sortingArray[];
        BufferedReader br = new BufferedReader(new
        java.io.InputStreamReader(System.in));
        System.out.print("\n1.Manual Input\n2.Random
        Input\n3.Roll no Input : ");
        int choice = Integer.parseInt(br.readLine());

        if (choice == 1) {
            System.out.print("\nEnter the elements of the
            array(with space)\n-> ");
            String numbers[] = br.readLine().split(" ");
            for (String number : numbers)
                arrayList.add(Integer.parseInt(number));
        } else if (choice == 2) {
            System.out.print("\nEnter the size of the array : ");
            int size = Integer.parseInt(br.readLine());
            for (int i = 0; i < size; i++) {
                arrayList.add((int) (Math.random() * 100));
            }
        } else {
            System.out.print("\nEnter the roll no : ");
            int rollNo = Integer.parseInt(br.readLine());
            for (int i = 0; i < 10; i++)
                arrayList.add(rollNo + (rollNo + 1) * i);
            Collections.shuffle(arrayList);
        }

        sortingArray = new int[arrayList.size()];
        for (int i = 0; i < arrayList.size(); i++) {
```



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```
        sortingArray[i] = arrayList.get(i);
    }

    System.out.print("\nArray : " +
printArray(sortingArray));
    System.out.print("\n\n=====" +
"=====");
    System.out.print("\nWhich sorting algorithm do you
want to use?\n1. Insertion Sort\n2. Selection Sort\n-> ");
    int decision = Integer.parseInt(br.readLine());
    System.out.print("\n=====" +
"=====");
    System.out.print("\nBefore Sort: " +
printArray(sortingArray));
    switch (decision) {
        case 1:
            System.out.print("\nInsertion Sort");
            insertionSort(sortingArray);
            break;
        case 2:
            System.out.print("\nSelection Sort");
            selectionSort(sortingArray);
            break;
        default:
            System.out.print("\nInvalid Choice");
    }

    System.out.print("\n-----" +
-----");
    System.out.print("\nFinal After Sort: " +
printArray(sortingArray));

    System.out.print("\n\n1.Exit\n2.Continue\n-> ");
    int exit = Integer.parseInt(br.readLine());
    if (exit == 1)
        System.exit(0);
    else
        main(null);}}
```



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<b>OUTPUT:</b>	<p>Insertion Sort:</p> <pre>1.Manual Input 2.Random Input 3.Roll no Input : 3  Enter the roll no : 54</pre> <p>Best Case:</p> <pre>Enter the roll no : 54  1.Best Case 2.Average Case 3.Worst Case : 1  Array : [ 54 109 164 219 274 329 384 439 494 549 ]  ===== Which sorting algorithm do you want to use? 1. Insertion Sort 2. Selection Sort -&gt; 1  ===== Before Sort: [ 54 109 164 219 274 329 384 439 494 549 ] Insertion Sort *****</pre> <p>Iteration on Element 109</p> <pre>Array : [ 54 109 164 219 274 329 384 439 494 549 ] *****</pre> <p>Iteration on Element 164</p> <pre>Array : [ 54 109 164 219 274 329 384 439 494 549 ] *****</pre> <p>Iteration on Element 219</p> <pre>Array : [ 54 109 164 219 274 329 384 439 494 549 ] *****</pre> <p>Iteration on Element 274</p> <pre>Array : [ 54 109 164 219 274 329 384 439 494 549 ] *****</pre>
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```
Iteration on Element 329
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
```

```
Iteration on Element 384
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
```

```
Iteration on Element 439
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
```

```
Iteration on Element 494
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
```

```
Iteration on Element 549
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
-----
```

```
Final After Sort: [ 54 109 164 219 274 329 384 439 494 549 ]
```

Average Case:

- 1.Best Case
- 2.Average Case
- 3.Worst Case : 2

```
Array : [ 109 274 54 384 164 494 439 549 329 219 ]
```

```
=====
```

Which sorting algorithm do you want to use?

- 1. Insertion Sort
- 2. Selection Sort

-> 1

```
=====
Before Sort: [ 109 274 54 384 164 494 439 549 329 219 ]
Insertion Sort
```

```
*****
```

```
Iteration on Element 274
```

```
Array : [ 109 274 54 384 164 494 439 549 329 219 ]
```

```
*****
```



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```
Iteration on Element 54
Swapping 274 with 54
Swapping 109 with 274
Array : [ 54 109 274 384 164 494 439 549 329 219 ]
*****  
  
Iteration on Element 384
Array : [ 54 109 274 384 164 494 439 549 329 219 ]
*****  
  
Iteration on Element 164
Swapping 384 with 164
Swapping 274 with 384
Array : [ 54 109 164 274 384 494 439 549 329 219 ]
*****  
  
Iteration on Element 494
Array : [ 54 109 164 274 384 494 439 549 329 219 ]
*****  
  
Iteration on Element 439
Swapping 494 with 439
Array : [ 54 109 164 274 384 439 494 549 329 219 ]
*****  
  
Iteration on Element 549
Array : [ 54 109 164 274 384 439 494 549 329 219 ]
*****  
  
Iteration on Element 329
Swapping 549 with 329
Swapping 494 with 549
Swapping 439 with 494
Swapping 384 with 439
Array : [ 54 109 164 274 329 384 439 494 549 219 ]
*****  
  
Iteration on Element 219
Swapping 549 with 219
Swapping 494 with 549
Swapping 439 with 494
Swapping 384 with 439
Swapping 329 with 384
Swapping 274 with 329
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
-----  
Final After Sort: [ 54 109 164 219 274 329 384 439 494 549 ]
```



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Worth Case:

```
=====
Before Sort: [ 549 494 439 384 329 274 219 164 109 54 ]
Insertion Sort
*****  
  
Iteration on Element 494
Swapping 549 with 494
Array : [ 494 549 439 384 329 274 219 164 109 54 ]
*****  
  
Iteration on Element 439
Swapping 549 with 439
Swapping 494 with 549
Array : [ 439 494 549 384 329 274 219 164 109 54 ]
*****  
  
Iteration on Element 384
Swapping 549 with 384
Swapping 494 with 549
Swapping 439 with 494
Array : [ 384 439 494 549 329 274 219 164 109 54 ]
*****  
  
Iteration on Element 329
Swapping 549 with 329
Swapping 494 with 549
Swapping 439 with 494
Swapping 384 with 439
Array : [ 329 384 439 494 549 274 219 164 109 54 ]
*****  
  
Iteration on Element 274
Swapping 549 with 274
Swapping 494 with 549
Swapping 439 with 494
Swapping 384 with 439
Swapping 329 with 384
Array : [ 274 329 384 439 494 549 219 164 109 54 ]
*****
```



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```
Iteration on Element 219
Swapping 549 with 219
Swapping 494 with 549
Swapping 439 with 494
Swapping 384 with 439
Swapping 329 with 384
Swapping 274 with 329
Array : [ 219 274 329 384 439 494 549 164 109 54 ]
*****
```

```
Iteration on Element 164
Swapping 549 with 164
Swapping 494 with 549
Swapping 439 with 494
Swapping 384 with 439
Swapping 329 with 384
Swapping 274 with 329
Swapping 219 with 274
Array : [ 164 219 274 329 384 439 494 549 109 54 ]
*****
```

```
Iteration on Element 109
Swapping 549 with 109
Swapping 494 with 549
Swapping 439 with 494
Swapping 384 with 439
Swapping 329 with 384
Swapping 274 with 329
Swapping 219 with 274
Swapping 164 with 219
Array : [ 109 164 219 274 329 384 439 494 549 54 ]
*****
```

```
Iteration on Element 54
Swapping 549 with 54
Swapping 494 with 549
Swapping 439 with 494
Swapping 384 with 439
Swapping 329 with 384
Swapping 274 with 329
Swapping 219 with 274
Swapping 164 with 219
Swapping 109 with 164
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
```

---

Final After Sort: [ 54 109 164 219 274 329 384 439 494 549 ]



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Insertion Sort :-

Best Case:-  
Array :- [54, 109, 164, 219, 274, 329, 384, 439, 494, 549]  
for every iteration there would be any swap required since its already sorted

Worst Case:-  
Array :- [549, 494, 439, 384, 329, 274, 219, 164, 109, 54].

1) Array :- [494, 549, 439, 384, 329, 274, 219, 164, 109, 54]  
2) Array :- [439, 109, 164, 219, 274, 329, 384, 439, 494, 549]  
; swapping to right until the min comes first

9) Array :- [54, 109, 164, 219, 274, 329, 384, 439, 494, 549]

For the Average Case:  
Every Iteration, the array element will float up using swaps

**Complexity of Insertion Sort:** The best case complexity of insertion sort is  $O(n)$  times, i.e. when the array is previously sorted. In the same way, when the array is sorted in reverse order, the first element of the unsorted array is to be compared with each element in the sorted set. So, in the worst case, running time of Insertion sort is quadratic, i.e.,  $O(n^2)$ .



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Time Complexity :-

- Insertion Sort

Algorithm	Cost of Step	Times
i) for $j=0$ to $n-1$	$C_1$	$n$
e) $key A[j]$	$C_2$	$n-1$
i) $i = j-1$	$C_3$	$n-1$
q) while $i \geq 0$ & $A[i] > key$	$C_4$	$\sum_{j=0}^{n-1} t_j$
5) $A[i+1] = A[i]$	$C_5$	$\sum_{j=1}^{n-1} t_j - 1$
6) $i = i+1$	$C_6$	$n-1$
e) $A[i+1] = key$	$C_7$	$n-1$

Total Time =  $C_1 n + C_2(n-1) + C_3(n-1) + C_4 \sum_{j=1}^{n-1} t_j$   
 $+ C_5 \sum_{j=1}^{n-1} t_{j+1} + C_6 \sum_{j=1}^{n-1} t_{j+1} - 1 + C_7(n-1)$

Best Case :-

As Array is already sorted, control wont enter while loop

$\therefore$  Line 5 and 6 will be  $\Theta$  times of time of 4 is  $(n-1)$  times

$$T(n) = C_1 n + C_2(n-1) + C_3(n-1) + C_4(n-1) + 0 + 0$$
$$C_7(n-1)$$
$$= C_1 + C_2 + C_3 + C_4(n-1) + (-1)(C_2 + C_3 + C_4 + C_7)$$
$$T(n) = O(n)$$

Worst Case :-

While loop will worth anytime as now are in descending order

$$Time \propto \frac{1}{2} (4, 5, 6) = \frac{n(n-1)}{2}$$



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**Selection Sort:**

**Best Case:**

```
=====
Before Sort: [ 54 109 164 219 274 329 384 439 494 549 ]
Selection Sort
*****
The minimum Element is 54
Swapping 54 with 54
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
The minimum Element is 109
Swapping 109 with 109
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
The minimum Element is 164
Swapping 164 with 164
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
The minimum Element is 219
Swapping 219 with 219
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
The minimum Element is 274
Swapping 274 with 274
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
The minimum Element is 329
Swapping 329 with 329
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
The minimum Element is 384
Swapping 384 with 384
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
The minimum Element is 439
Swapping 439 with 439
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
The minimum Element is 494
Swapping 494 with 494
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
-----
Final After Sort: [ 54 109 164 219 274 329 384 439 494 549 ]
```



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**Average Case:**

```
=====
Before Sort: [ 329 494 164 54 439 219 384 109 549 274 ]
Selection Sort
*****
The minimum Element is 54
Swapping 329 with 54
Array : [ 54 494 164 329 439 219 384 109 549 274 ]
*****
The minimum Element is 109
Swapping 494 with 109
Array : [ 54 109 164 329 439 219 384 494 549 274 ]
*****
The minimum Element is 164
Swapping 164 with 164
Array : [ 54 109 164 329 439 219 384 494 549 274 ]
*****
The minimum Element is 219
Swapping 329 with 219
Array : [ 54 109 164 219 439 329 384 494 549 274 ]
*****
The minimum Element is 274
Swapping 439 with 274
Array : [ 54 109 164 219 274 329 384 494 549 439 ]
*****
The minimum Element is 329
Swapping 329 with 329
Array : [ 54 109 164 219 274 329 384 494 549 439 ]
*****
The minimum Element is 384
Swapping 384 with 384
Array : [ 54 109 164 219 274 329 384 494 549 439 ]
*****
The minimum Element is 439
Swapping 494 with 439
Array : [ 54 109 164 219 274 329 384 439 549 494 ]
*****
The minimum Element is 494
Swapping 549 with 494
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
-----
Final After Sort: [ 54 109 164 219 274 329 384 439 494 549 ]
```



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**Worst Case:**

```
=====
Before Sort: [ 549 494 439 384 329 274 219 164 109 54 ]
Selection Sort
*****
The minimum Element is 54
Swapping 549 with 54
Array : [ 54 494 439 384 329 274 219 164 109 549 ]
*****
The minimum Element is 109
Swapping 494 with 109
Array : [ 54 109 439 384 329 274 219 164 494 549 ]
*****
The minimum Element is 164
Swapping 439 with 164
Array : [ 54 109 164 384 329 274 219 439 494 549 ]
*****
The minimum Element is 219
Swapping 384 with 219
Array : [ 54 109 164 219 329 274 384 439 494 549 ]
*****
The minimum Element is 274
Swapping 329 with 274
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
The minimum Element is 329
Swapping 329 with 329
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
The minimum Element is 384
Swapping 384 with 384
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
The minimum Element is 439
Swapping 439 with 439
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
*****
The minimum Element is 494
Swapping 494 with 494
Array : [ 54 109 164 219 274 329 384 439 494 549 ]
-----
Final After Sort: [ 54 109 164 219 274 329 384 439 494 549 ]
```



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2020300053

Selection Sort

Best Case :-  
Array :- [ 54, 109, 164, 219, 274, 329, 384, 439, 494, 549 ]

Since all the elements is already sorted, the minimum element will always be the current element, and therefore no comparisons done.

Average case :-  
Array :- [ 329, 494, 164, 54, 99, 219, 384, 109, 549, 274 ]

- i) Min Element = 54  
Array = [ 54, 329, 494, 164, 439, 219, 384, 109, 549, 274 ]
- ii) Min Element = 109  
Array = [ 54, 109, 164, 329, 439, 219, 384, 494, 549, 274 ]
- iii) Min Element = 164  $\rightarrow$  skip
- iv) Min Element = 219  
Array = [ 54, 109, 164, 219, 439, 329, 384, 494, 549, 274 ]
- v) Min Element = 274  
Array = [ 54, 109, 164, 219, 274, 439, 329, 384, 494, 549 ]
- vi) Min Elements = 329  
Array = [ 54, 109, 164, 219, 274, 329, 384, 439, 494, 549 ]
- vii) Min Element = 384  
Array = [ 54, 109, 164, 219, 274, 329, 384, 439, 494, 549 ]
- viii) Min Element = 439  
Array = [ 54, 109, 164, 219, 274, 329, 384, 439, 494, 549 ]
- ix) Min Element = 494  
Array = [ 54, 109, 164, 219, 274, 329, 384, 439, 494, 549 ]



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	<p>2020300056</p> <p>Page No. _____ Date _____</p> <p>Worst Case:-</p> <p>Array = [54, 494, 439, 384, 329, 274, 219, 164, 109, 54].</p> <p>i) Min. Element = 54 Array = [54, 494, 439, 384, 329, 274, 219, 164, 109, 54].</p> <p>ii) Min. Element = 109 Array = [54, 109, 439, 384, 329, 274, 219, 164, 494, 54].</p> <p>iii) Min. Element = 164 Array = [54, 109, 164, 384, 329, 274, 219, 439, 494, 54].</p> <p>iv) Min. Element = 219 Array = [54, 109, 164, 219, 329, 274, 384, 439, 494, 54].</p> <p>v) Min. Element = 274 Array = [54, 109, 164, 219, 274, 329, 384, 439, 494, 54].</p> <p>vi) Min. Element = 329 Array = [54, 109, 164, 219, 274, 329, 384, 439, 494, 54].</p> <p>vii) Min. Element = 384 Array = [54, 109, 164, 219, 274, 329, 384, 439, 494, 54].</p> <p>viii) Min. Element = 439 Array = [54, 109, 164, 219, 274, 329, 384, 439, 494, 54].</p> <p>ix) Min. Element = 494 Array = [54, 109, 164, 219, 274, 329, 384, 494, 494, 54].</p> <p>x) Min. Element = 54 Array = [54, 109, 164, 219, 274, 329, 384, 494, 494, 54].</p> <p>Comparision keep happening but it is already sorted</p> <p>xi) Min. Element = 54 Array = [54, 109, 164, 219, 274, 329, 384, 494, 494, 54].</p>
--	--



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2020300054  
Selection Sort

Algorithm	Cost of step	Times
i) for $i=0$ to $A.length$	$C_1$	$n$
a) $min = i$	$C_2$	$n-1$
b) for $j=i+1$ to $A.length$	$C_3$	$\frac{n(n+1)}{2}$
c) $temp = A[j]$	$C_4$	$n-1$
d) $A[i] = A[min]$	$C_5$	$n-1$
e) $A[min] = temp$	$C_6$	$n-1$

Total Time

$$T(n) = C_1 n + C_2(n-1) + C_3 \frac{n(n-1)}{2} + C_4(n-1) + C_5(n-1) + C_6(n-1)$$
$$= \frac{C_3 n^2}{2} + (C_1 + C_2 + C_3 + C_4 + C_5 + C_6) n$$
$$- (C_2 + C_4 + C_5 + C_6)$$
$$= an^2 + bn + c$$
$$= O(n^2)$$

Time Complexity -  $O(n^2)$  for all cases

**Complexity of Selection Sort:** As the working of selection, sort does not depend on the original order of the elements in the array, so there is not much difference between best case and worst case complexity of selection sort. Therefore, in both the cases, the complexity is  $O(n^2)$ .

**CONCLUSION:** Concepts learnt during procedural programming of the problem



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- Learnt about the time complexity and proved and perform complexity cases on selection and insertion sort
- Learnt how to write Insertion and Selection sort using the algorithm
- Learnt how to different cases differ in computation power as the number of elements increase in array
- I learnt about the advantages and disadvantages about selection and insertion sort and how to improve the overall time complexity
- Learnt how to derive and compute the time complexity of selection sort and insertion sort



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<b>Name</b>	<b>Pratik Pujari</b>		
<b>UID no.</b>	<b>2020300054</b>	<b>Class:</b>	<b>Comps C Batch</b>
<b>Experiment No.</b>	2		

<b>AIM:</b>	To implement divide and conquer sorting algorithms
<b>THEORY:</b>	<p><b>What is QuickSort ?</b></p> <p>Quick Sort is a divide and conquer algorithm. It creates two empty arrays to hold elements less than the pivot value and elements greater than the pivot value, and then recursively sort the sub arrays. There are two basic operations in the algorithm, swapping items in place and partitioning a section of the array.</p> <p><b>Important Characteristics of Quick Sort:</b></p> <ul style="list-style-type: none"><li>• Quick Sort is useful for sorting arrays.</li><li>• In efficient implementations Quick Sort is not a stable sort, meaning that the relative order of equal sort items is not preserved.</li><li>• Overall time complexity of Quick Sort is <math>O(n\log n)</math>. In the worst case, it makes <math>O(n^2)</math> comparisons, though this behavior is rare.</li><li>• The space complexity of Quick Sort is <math>O(n\log n)</math>. It is an in-place sort (i.e. it doesn't require any extra storage)</li></ul> <p><b>Problem Statement</b></p> <p>Consider the following array: 50, 23, 9, 18, 61, 32. We need to sort this array in the most efficient manner without using extra place (inplace sorting).</p>



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**Solution**

**Step 1:**

- **Make any element as pivot:** Decide any value to be the pivot from the list. For convenience of code, we often select the rightmost index as pivot or select any at random and swap with rightmost. Suppose for two values "Low" and "High" corresponding to the first index and last index respectively.
  - In our case low is 0 and high is 5.
  - Values at low and high are 50 and 32 and value at pivot is 32.
- **Partition the array on the basis of pivot:** Call for partitioning which rearranges the array in such a way that pivot (32) comes to its actual position (of the sorted array). And to the left of the pivot, the array has all the elements less than it, and to the right greater than it.
  - In the partition function, we start from the first element and compare it with the pivot. Since 50 is greater than 32, we don't make any change and move on to the next element 23.
  - Compare again with the pivot. Since 23 is less than 32, we swap 50 and 23. The array becomes 23, 50, 9, 18, 61, 32
  - We move on to the next element 9 which is again less than pivot (32) thus swapping it with 50 makes our array as 23, 9, 50, 18, 61, 32.
  - Similarly, for next element 18 which is less than 32, the array becomes 23, 9, 18, 50, 61, 32. Now 61 is greater than pivot (32), hence no changes.
  - Lastly, we swap our pivot with 50 so that it comes to the correct position.

Thus the pivot (32) comes at its actual position and all elements to its left are lesser, and all elements to the right are greater than itself.

**Step 2:** The main array after the first step becomes 23, 9, 18, 32, 61, 50



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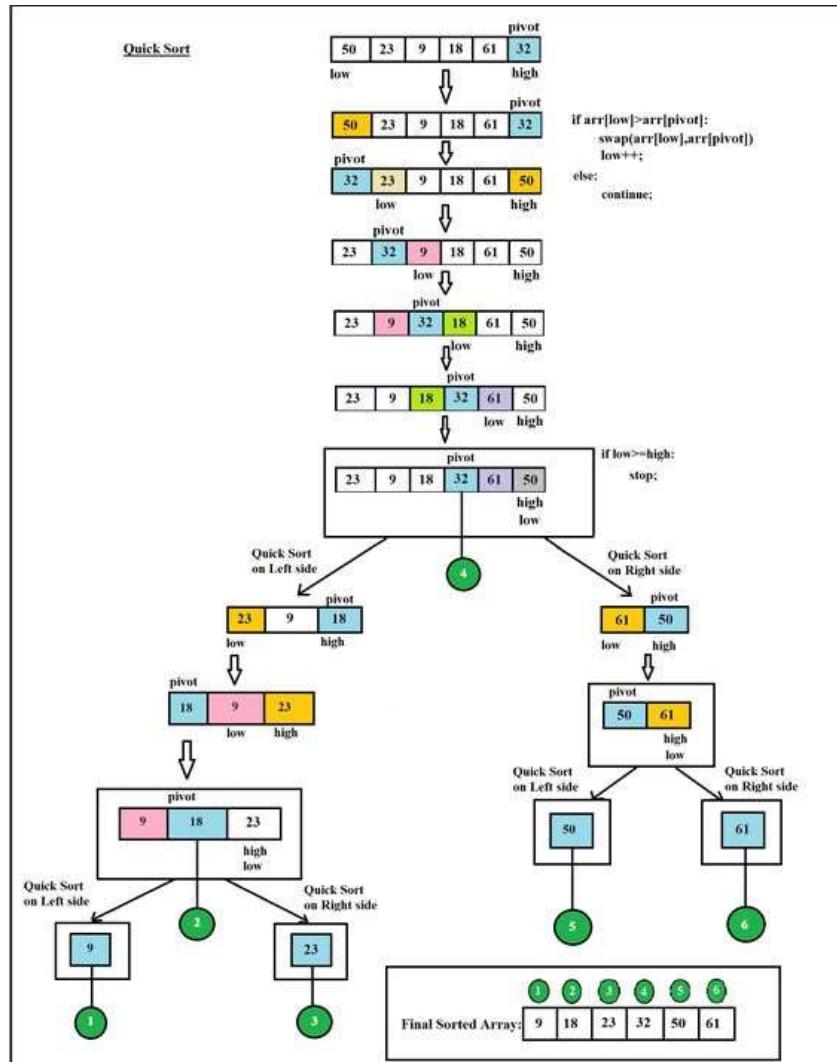
**Step 3:** Now the list is divided into two parts:

1. Sublist before pivot element
2. Sublist after pivot element

**Step 4:** Repeat the steps for the left and right sublists recursively. The final array thus becomes

9, 18, 23, 32, 50, 61.

The following diagram depicts the workflow of the Quick Sort algorithm which was described above.





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**ALGORITHM:**

1. Find a “pivot” item in the array. This item is the basis for comparison for a single round.
2. Start a pointer (the left pointer) at the first item in the array.
3. Start a pointer (the right pointer) at the last item in the array.
4. While the value at the left pointer in the array is less than the pivot value, move the left pointer to the right (add 1). Continue until the value at the left pointer is greater than or equal to the pivot value.
5. While the value at the right pointer in the array is greater than the pivot value, move the right pointer to the left (subtract 1). Continue until the value at the right pointer is less than or equal to the pivot value.
6. If the left pointer is less than or equal to the right pointer, then swap the values at these locations in the array.
7. Move the left pointer to the right by one and the right pointer to the left by one.
8. If the left pointer and right pointer don't meet, go to step 1

**Applications of quick sort**

- Commercial Computing is used in various government and private organizations for the purpose of sorting various data like sorting files by name/date/price, sorting of students by their roll no., sorting of account profile by given id, etc.
- The sorting algorithm is used for information searching and as Quicksort is the fastest algorithm so it is widely used as a better way of searching.
- It is used everywhere where a stable sort is not needed.
- It is tail -recursive and hence all the call optimization can be done.



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**EXPERIMENT 1**

<b>CODE:</b>	<pre>Program Code Quicksort: import java.util.Arrays;  public class quickSort {      int totalSwaps = 0;     int totalComparisons = 0;      // This function takes last element as pivot, places     public int partition(int arr[], int low, int high) {          int pivot = low;         low++;         int comparisons = 0;         int swaps = 0;         System.out.print("\n-----");         System.out.print("\nPivot: " + arr[high] + " Low: " +         low + " High: " + high);         System.out.print("\nBefore Swaps-&gt; Array: " +         printArray(arr));         do {             // increment low pointer until that element is larger             // than the pivot element             while (low &lt; high &amp;&amp; arr[low] &lt;= arr[pivot]) {                 low++;                 comparisons++;             }             // decrement high pointer until that element is             // smaller than the pivot element             while (high &gt; pivot &amp;&amp; arr[high] &gt; arr[pivot]) {                 high--;                 comparisons++;             }         }     } }</pre>
--------------	---



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```
// if the low and high pointers cross each other swap
the corresponding elements
if (low < high) {
    int temp = arr[low];
    arr[low] = arr[high];
    arr[high] = temp;
    swaps++;

}
} while (low < high);
// swap the pivot element and the element pointed by
high
System.out.print("\n\nSwapping pivot and high: " +
arr[pivot] + " with " + arr[high]);
int temp = arr[high];
arr[high] = arr[pivot];
arr[pivot] = temp;
totalComparisons += comparisons;
totalSwaps += swaps;
System.out.println("\n\nAfter Swaps -> Array: " +
printArray(arr));
System.out.print("\nSwaps: " + swaps + "
Comparisons: " + comparisons);
return high;
}

// swap function
public void swap(int arr[], int i, int j) {
    int temp = arr[i];
    arr[i] = arr[j];
    arr[j] = temp;
}

public void sort(int arr[], int low, int high) {
    if (low < high) {
        /*
         * pi is partitioning index, arr[pi] is
         * now at right place
         */
    }
}
```



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```
int pi = partition(arr, low, high);

// Recursively sort elements before
// partition and after partition
sort(arr, low, pi - 1);
sort(arr, pi + 1, high);
}

}

/* A utility function to print array of size n */
public String printArray(int arr[]) {
    return Arrays.toString(arr);
}

}

Main Class:
import java.util.ArrayList;
import java.util.Collections;
import java.util.Scanner;

public class Driver {
    public static void main(String[] args) {
        // User Input
        Scanner input = new Scanner(System.in);
        System.out.print("\n" + "QUICKSORT");
        System.out.print("\n-----");
        System.out.print("\n-----\n");
        System.out.print("Enter the roll no: ");
        int rollNo = input.nextInt();

        int array[];
        ArrayList<Integer> list = new ArrayList<Integer>();

        // Case input
        System.out.print("\n1.Random Case\n2.Worst
Case\n3.Manual Case :");
        int choice = input.nextInt();
        if (choice == 2) {
            // Worst Case
        }
    }
}
```



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```
for (int i = 9; i >= 0; i--)
    list.add(rollNo + (rollNo + 1) * i);

} else if (choice == 1) {
    // Random Case
    for (int i = 0; i < 10; i++)
        list.add(rollNo + (rollNo + 1) * i);

    Collections.shuffle(list);
} else {
    // Manual Case
    System.out.print("\nEnter the elements: ");
    for (int i = 0; i < 10; i++)
        list.add(input.nextInt());
}

array = new int[10];
for (int i = 0; i < 10; i++)
    array[i] = list.get(i);
quickSort qSort = new quickSort();
System.out.print("\nBefore Sorting: " +
qSort.printArray(array));
System.out.print("\n-----\n");
qSort.sort(array, 0, array.length - 1);
System.out.print("\n-----\n");
System.out.print("\nAfter Sorting: " +
qSort.printArray(array));
System.out.print("\n-----\n");
System.out.print("\nTotal swaps: " +
qSort.totalSwaps);
System.out.print("\nTotal comparisons: " +
qSort.totalComparisons);
input.close();
}
```



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**OUTPUT:**

**Best case**

```
        QUICKSORT
-----
Enter the roll no: 54
1.Random Case
2.Worst Case
3.Manual Case :3
Enter the elements: 274 54 109 219 164 439 329 384 549 494
Before Sorting: [274, 54, 109, 219, 164, 439, 329, 384, 549, 494]
-----
Pivot: 494 Low: 1 High: 9
Before Swaps-> Array: [274, 54, 109, 219, 164, 439, 329, 384, 549, 494]
Swapping pivot and high: 274 with 164
After Swaps -> Array: [164, 54, 109, 219, 274, 439, 329, 384, 549, 494]
Swaps: 0 Comparisons: 9
-----
Pivot: 219 Low: 1 High: 3
Before Swaps-> Array: [164, 54, 109, 219, 274, 439, 329, 384, 549, 494]
Swapping pivot and high: 164 with 109
After Swaps -> Array: [109, 54, 164, 219, 274, 439, 329, 384, 549, 494]
Swaps: 0 Comparisons: 3
-----
Pivot: 54 Low: 1 High: 1
Before Swaps-> Array: [109, 54, 164, 219, 274, 439, 329, 384, 549, 494]
Swapping pivot and high: 109 with 54
After Swaps -> Array: [54, 109, 164, 219, 274, 439, 329, 384, 549, 494]
Swaps: 0 Comparisons: 0
-----
Pivot: 494 Low: 6 High: 9
Before Swaps-> Array: [54, 109, 164, 219, 274, 439, 329, 384, 549, 494]
Swapping pivot and high: 439 with 384
After Swaps -> Array: [54, 109, 164, 219, 274, 384, 329, 439, 549, 494]
Swaps: 0 Comparisons: 4
```



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```
Pivot: 329 Low: 6 High: 6
Before Swaps-> Array: [54, 109, 164, 219, 274, 384, 329, 439, 549, 494]

Swapping pivot and high: 384 with 329

After Swaps -> Array: [54, 109, 164, 219, 274, 329, 384, 439, 549, 494]
Swaps: 0 Comparisons: 0

Pivot: 494 Low: 9 High: 9
Before Swaps-> Array: [54, 109, 164, 219, 274, 329, 384, 439, 549, 494]

Swapping pivot and high: 549 with 494

After Swaps -> Array: [54, 109, 164, 219, 274, 329, 384, 439, 494, 549]
Swaps: 0 Comparisons: 0

After Sorting: [54, 109, 164, 219, 274, 329, 384, 439, 494, 549]

Total swaps: 0
Total comparisons: 16
```

**Time Complexity of Quick sort**

- **Best case scenario:** The best case scenario occurs when the partitions are as evenly balanced as possible, i.e their sizes on either side of the pivot element are either equal or have size difference of 1 of each other.
  - Case 1: The case when sizes of sublist on either side of pivot becomes equal occurs when the subarray has an odd number of elements and the pivot is right in the middle after partitioning. Each partition will have  $(n-1)/2$  elements.
  - Case 2: The size difference of 1 between the two sublists on either side of pivot happens if the subarray has an even number,  $n$ , of elements. One partition will have  $n/2$  elements with the other having  $(n/2)-1$ .

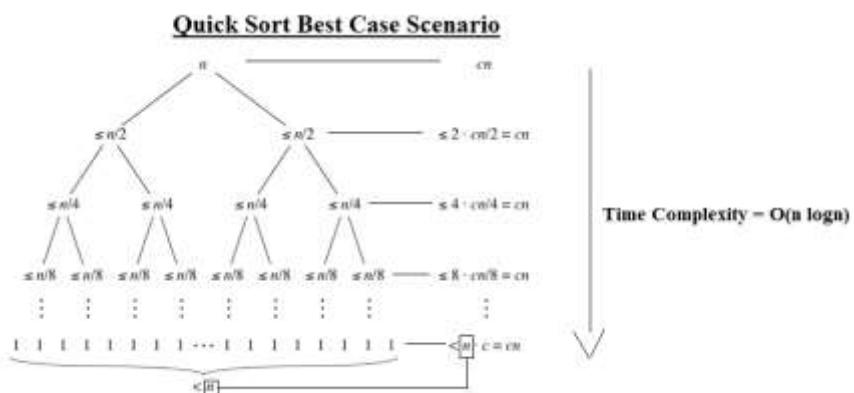
In either of these cases, each partition will have at most  $n/2$  elements, and the tree representation of the subproblem sizes will be as below:



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**Worst Case:**

```
Before Sorting: [549, 494, 439, 384, 329, 274, 219, 164, 109, 54]
-----
Pivot: 54 Low: 1 High: 9
Before Swaps-> Array: [549, 494, 439, 384, 329, 274, 219, 164, 109, 54]
Swapping pivot and high: 549 with 54
After Swaps -> Array: [54, 494, 439, 384, 329, 274, 219, 164, 109, 549]
Swaps: 0 Comparisons: 8
-----
Pivot: 109 Low: 1 High: 8
Before Swaps-> Array: [54, 494, 439, 384, 329, 274, 219, 164, 109, 549]
Swapping pivot and high: 54 with 54
After Swaps -> Array: [54, 494, 439, 384, 329, 274, 219, 164, 109, 549]
Swaps: 0 Comparisons: 8
-----
Pivot: 109 Low: 2 High: 8
Before Swaps-> Array: [54, 494, 439, 384, 329, 274, 219, 164, 109, 549]
Swapping pivot and high: 494 with 109
After Swaps -> Array: [54, 109, 439, 384, 329, 274, 219, 164, 494, 549]
Swaps: 0 Comparisons: 6
-----
Pivot: 164 Low: 2 High: 7
Before Swaps-> Array: [54, 109, 439, 384, 329, 274, 219, 164, 494, 549]
Swapping pivot and high: 109 with 164
After Swaps -> Array: [54, 109, 164, 384, 329, 274, 219, 439, 494, 549]
Swaps: 0 Comparisons: 4
-----
Pivot: 219 Low: 3 High: 6
Before Swaps-> Array: [54, 109, 164, 384, 329, 274, 219, 439, 494, 549]
Swapping pivot and high: 164 with 164
After Swaps -> Array: [54, 109, 164, 384, 329, 274, 219, 439, 494, 549]
Swaps: 0 Comparisons: 4
-----
Pivot: 219 Low: 4 High: 6
Before Swaps-> Array: [54, 109, 164, 384, 329, 274, 219, 439, 494, 549]
Swapping pivot and high: 384 with 219
After Swaps -> Array: [54, 109, 164, 219, 329, 274, 384, 439, 494, 549]
Swaps: 0 Comparisons: 2
```



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```
Pivot: 274 Low: 4 High: 5
Before Swaps-> Array: [54, 109, 164, 219, 329, 274, 384, 439, 494, 549]

Swapping pivot and high: 219 with 219

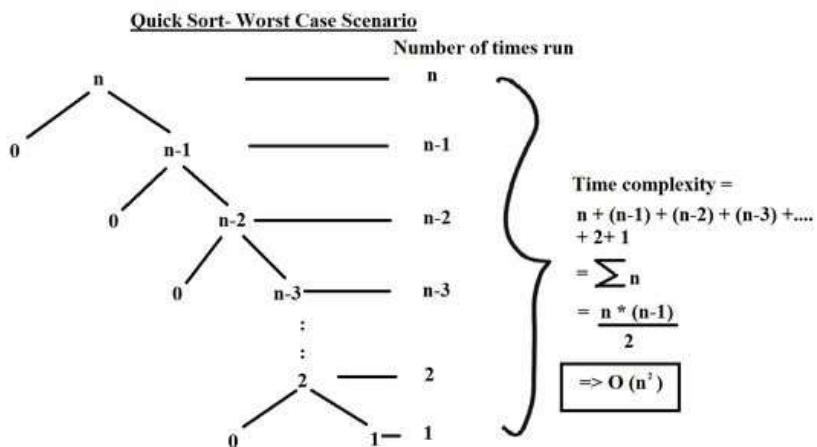
After Swaps -> Array: [54, 109, 164, 219, 329, 274, 384, 439, 494, 549]
Swaps: 0 Comparisons: 2
-----
Pivot: 274 Low: 5 High: 5
Before Swaps-> Array: [54, 109, 164, 219, 329, 274, 384, 439, 494, 549]

Swapping pivot and high: 329 with 274

After Swaps -> Array: [54, 109, 164, 219, 274, 329, 384, 439, 494, 549]
Swaps: 0 Comparisons: 0
-----
After Sorting: [54, 109, 164, 219, 274, 329, 384, 439, 494, 549]

Total swaps: 0
Total comparisons: 40
```

- **Worst case scenario:** This happens when we encounter the most unbalanced partitions possible, then the original call takes  $n$  time, the recursive call on  $n-1$  elements will take  $(n-1)$  time, the recursive call on  $(n-2)$  elements will take  $(n-2)$  time, and so on. The worst case time complexity of Quick Sort would be  $O(n^2)$ .





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Worst Case:-

54, 109, 164, 219, 274, 329, 384, 439, 494, 549

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑  
pivot j i j i j i j i j

No swap, comparison = 9

(54), 109, 164, 219, 274, 329, 384, 439, 494, 549

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑  
pivot j i j i j i j i j

No swap, comparison = 8 Total = 17

(54), (109), (164) 219 274, 329, 384, 439, 494, 549

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑  
pivot j i j i j i j i j

No swap, comparison = 7 Total = 24

.....

.....

.....

.....

(54), (109), (164), (219), (274), (329), (384), (439), 549

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑  
i, j, i, j, i, j, i, j, pivot

No swap, comparison = 1

Total Swap = 0, comparison = 45 =  $\left\lfloor \frac{n(n-1)}{2} \right\rfloor = \left\lfloor \frac{10(9)}{2} \right\rfloor = 45$



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**Average Case:**

```
Before Sorting: [54, 329, 274, 384, 494, 164, 439, 219, 549, 109]
-----
-----
Pivot: 109 Low: 1 High: 9
Before Swaps-> Array: [54, 329, 274, 384, 494, 164, 439, 219, 549, 109]

Swapping pivot and high: 54 with 54

After Swaps -> Array: [54, 329, 274, 384, 494, 164, 439, 219, 549, 109]
Swaps: 0 Comparisons: 9
-----
Pivot: 109 Low: 2 High: 9
Before Swaps-> Array: [54, 329, 274, 384, 494, 164, 439, 219, 549, 109]

Swapping pivot and high: 329 with 164

After Swaps -> Array: [54, 164, 274, 109, 219, 329, 439, 494, 549, 384]
Swaps: 2 Comparisons: 8
-----
Pivot: 219 Low: 2 High: 4
Before Swaps-> Array: [54, 164, 274, 109, 219, 329, 439, 494, 549, 384]

Swapping pivot and high: 164 with 109

After Swaps -> Array: [54, 109, 164, 274, 219, 329, 439, 494, 549, 384]
Swaps: 1 Comparisons: 3
-----
Pivot: 219 Low: 4 High: 4
Before Swaps-> Array: [54, 109, 164, 274, 219, 329, 439, 494, 549, 384]

Swapping pivot and high: 274 with 219

After Swaps -> Array: [54, 109, 164, 219, 274, 329, 439, 494, 549, 384]
Swaps: 0 Comparisons: 0
-----
Pivot: 384 Low: 7 High: 9
Before Swaps-> Array: [54, 109, 164, 219, 274, 329, 439, 494, 549, 384]

Swapping pivot and high: 439 with 384

After Swaps -> Array: [54, 109, 164, 219, 274, 329, 384, 439, 549, 494]
Swaps: 1 Comparisons: 3
```



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```
Pivot: 494 Low: 9 High: 9
Before Swaps-> Array: [54, 109, 164, 219, 274, 329, 384, 439, 549, 494]

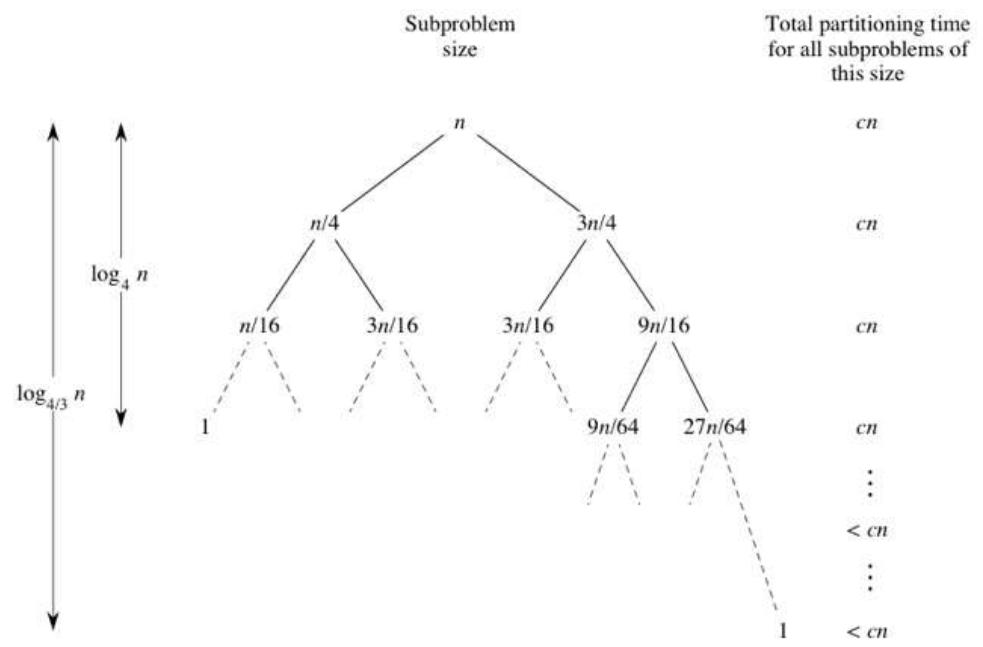
Swapping pivot and high: 549 with 494

After Swaps -> Array: [54, 109, 164, 219, 274, 329, 384, 439, 494, 549]
Swaps: 0 Comparisons: 0

-----
After Sorting: [54, 109, 164, 219, 274, 329, 384, 439, 494, 549]

-----
Total swaps: 4
Total comparisons: 23
```

**Average Case:** The average case run time of quick sort is  $O(n \log n)$ . This case happens when we don't exactly get evenly balanced partitions. We might get at worst a 3-to-1 split on either side of pivot element.





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Avg Case :-

Comparison - 9. swap = 0

54,329,274,384,494,164,439,219,549,100

54, 164, 244, 109, 219, 329, 439, 494, 549, 384  
Swaps = 2, Comparison = 8

54, 109, 164, 219, 889, 274, 32  
swap - 1 Comparison = 3

Pivot = .494. (last swap)

54, 109, 164, 219, 274, 329, 384, 439, 694, 51

Comparison = 23 , Swap = 4



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Best case and Average case Time complexity analysis

Time complexity analysis  
Best & Average case

$$T(0) = T(1) = 0 \quad (\text{base case})$$
$$T(N) = 2T(N/2) + N \quad [\text{Time to divide + combine}]$$
$$\frac{T(N)}{N} = 2\frac{T(N/2)}{N} + 1$$
$$\frac{T(N/2)}{N/2} = 1 + \frac{T(N/4)}{N/4}$$
$$\frac{T(N/4)}{N/4} = 1 + \frac{T(N/8)}{N/8} = 1 + \frac{T(1)}{1}$$

same as

$$\frac{T(2)}{2} = 1 + \frac{T(1)}{1}$$

Hence  $T(N) = 1 + 1 + 1 \dots$

$$\frac{T(N)}{N} = \log N \quad T(N) = N \log N$$

Hence time complexity of best case -  $O(n \log n)$   
its average case is slower to best, time complexity



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Worst Case Time Complexity Analysis:

Time complexity worst case.

Recurrence Relation

$$T(0) = T(1) = 0$$
$$T(N) = O(n) + T(n-1) + O(1) \quad [T(n) = T(\text{divide}) + T(\text{conquer}) + T(\text{combine})]$$
$$T(n) = n + T(n-1)$$
$$T(n-1) = n-1 + T(n-2)$$
$$\vdots$$
$$T(2) = 2 + T(1)$$
$$T(1) = 0 + T(0)$$

Hence  $T(N) = n + (n-1) + (n-2) + \dots + 2$   
 $= \frac{n^2}{2}$   
which is  $O(N^2)$

Time complexity of worst case is  $O(n^2)$

**RESULT:** Concepts gained from programming of problem:

- I have learnt about divide and conquer sorting algorithm like quicksort and mergesort
- Since the time complexity of following algo's are less than the  $O(n)$  which is  $O(n\log n)$  which makes them faster in case of large numbers of elements to sort as  $n\log n$  function has relatively constant slope at large values of  $N$
- In quicksort the time complexity of any worst case is  $O(n^2)$  while in mergesort its  $O(n\log n)$  for all cases
- I learnt how to use proper looping in order to avoid the `ArrayOutOfBoundsException`



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**EXPERIMENT (MergeSort)**

**CODE:**

```
import java.util.ArrayList;
import java.util.Arrays;
import java.util.Collections;
import java.util.Scanner;

public class mergeSortClass {
    // Merge two subarrays L and M into arr
    int shift = 0;

    void merge(int arr[], int p, int q, int r) {
        // Create temp array to store the merged subarrays
        int n1 = q - p + 1;
        int n2 = r - q;
        shift = 0;
        int L[] = new int[n1];
        int M[] = new int[n2];

        for (int i = 0; i < n1; i++)
            L[i] = arr[p + i];
        for (int j = 0; j < n2; j++)
            M[j] = arr[q + 1 + j];

        // Maintain current index of sub-arrays and main array
        int i, j, k;
        i = 0;
        j = 0;
        k = p;

        // Until we reach either end of either L or M, pick larger
        // among
        // elements L and M and place them in the correct
        // position at A[p..r]
        while (i < n1 && j < n2) {
            if (L[i] <= M[j]) {
                arr[k] = L[i];
                i++;
            } else {
                arr[k] = M[j];
                j++;
            }
            k++;
        }

        // Copy remaining elements of L, if there are any
        while (i < n1) {
            arr[k] = L[i];
            i++;
            k++;
        }

        // Copy remaining elements of M, if there are any
        while (j < n2) {
            arr[k] = M[j];
            j++;
            k++;
        }
    }
}
```



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```
i++;
} else {
    arr[k] = M[j];
    j++;
    shift++;
}
k++;
}

// When we run out of elements in either L or M,
// pick up the remaining elements and put in A[p..r]
while (i < n1) {
    arr[k] = L[i];
    i++;
    k++;
}

while (j < n2) {
    arr[k] = M[j];
    j++;
    k++;
}

// Divide the array into two subarrays, sort them and
// merge them
void mergeSort(int arr[], int l, int r) {
    if (l < r) {
        System.out.print("\n-----");
        System.out.print("-----");
        // m is the point where the array is divided into two
        // subarrays
        int m = (l + r) / 2;
        System.out.print("\nThe midposition is: " + m);

        System.out.print("\nLeft half: " +
Arrays.toString(Arrays.copyOfRange(arr, l, m)));
        mergeSort(arr, l, m);
    }
}
```



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```
System.out.print("\nRight half: " +
printArray(Arrays.copyOfRange(arr, m + 1, arr.length)));
mergeSort(arr, m + 1, r);
System.out.print("\nTotal Shift: " + shift);
merge(arr, l, m, r);
System.out.print("\nMerged: " + printArray(arr));
System.out.print("\n-----");
-----");
}
}

/* A utility function to print array of size n */
public String printArray(int arr[]) {
    return Arrays.toString(arr);
}

// Driver program
public static void main(String args[]) {
    Scanner input = new Scanner(System.in);
    System.out.print("\n" MERGESORT");
    System.out.print("\n-----\n");
    System.out.print("\nEnter the roll no: ");
    int rollNo = input.nextInt();

    int array[];
    ArrayList<Integer> list = new ArrayList<Integer>();

    // Case input
    System.out.print("\n1.Random Case\n2.Worst
Case\n3.Manual Case :");
    int choice = input.nextInt();
    if (choice == 2) {
        // Worst Case
        for (int i = 9; i >= 0; i--)
            list.add(rollNo + (rollNo + 1) * i);

    } else if (choice == 1) {
        // Random Case
```



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```
for (int i = 0; i < 10; i++)
    list.add(rollNo + (rollNo + 1) * i);
Collections.shuffle(list);
} else {
    // Manual Case
    System.out.print("\nEnter the elements: ");
    for (int i = 0; i < 10; i++)
        list.add(input.nextInt());
}
array = new int[10];
for (int i = 0; i < 10; i++)
    array[i] = list.get(i);
mergeSortClass ob = new mergeSortClass();
ob.mergeSort(array, 0, array.length - 1);
System.out.println("\nSorted array:" +
ob.printArray(array));
input.close();
}
}
```

**Pseudocode for MergeSort**

- Declare left and right var which will mark the extreme indices of the array
- Left will be assigned to 0 and right will be assigned to n-1
- Find mid = (left+right)/2
- Call mergeSort on (left,mid) and (mid+1,rear)
- Above will continue till left<right
- Then we will call merge on the 2 subproblems

**Merge sort Algorithm**

```
MergeSort(arr, left, right):
    if left > right
        return
    mid = (left+right)/2
    mergeSort(arr, left, mid)
    mergeSort(arr, mid+1, right)
    merge(arr, left, mid, right)
end
```



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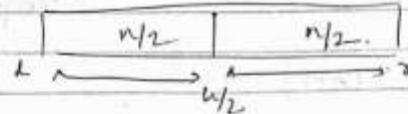
**OUTPUT:**

Pratik Pujari - 2020300054

Merge Sort Time Complexity

$$T(1) = T(0) = 1 \text{ (base case)}$$

$$T(n) = T(n/2) + T(n/2)n \quad \dots \textcircled{1}$$



Substitute  $n/2$  in above eqn

$$T(n/2) = 2T(n/4) + n/2 \quad \dots \textcircled{2}$$

Substituting  $\textcircled{2}$  in  $\textcircled{1}$

$$\begin{aligned} T(n) &= 2[2T(n/4) + n/2] + n \\ &= 2^2 T\left(\frac{n}{2^2}\right) + 2n \end{aligned}$$

$$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + 2n \quad \dots \textcircled{3}$$

Substitute  $n/4$  as  $n$  in eq 1

$$T(n/4) = 2T(n/8) + n/4 \quad \dots \textcircled{4}$$

Substitute eq  $\textcircled{4}$  in  $\textcircled{3}$

$$T(n) = 3 2^3 \left[ T\left(\frac{n}{2^3}\right) + 3n \right]$$



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	$\therefore T(n) = 2^4 T\left(\frac{n}{2^4}\right) + 40$ $T(n) = 2^i T\left(\frac{n}{2^i}\right) + i n$ $\text{Let } \frac{n}{2^i} = 1 \rightarrow \log n = i$ $T(n) = n T(1) + \log n$ $T(n) = n + n \log n$ $\therefore \text{Time complexity of merge sort} = O(n \log n)$
<b>OUTPUT:</b>	Random Case:  <pre>MERGESORT ----- Enter the roll no: 54 1.Random Case 2.Worst Case 3.Manual Case :1 ----- The midposition is: 4 Left half: [54, 109, 494, 384] ----- The midposition is: 2 Left half: [54, 109] ----- The midposition is: 1 Left half: [54]</pre>



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```
The midposition is: 0
Left half: []
Right half: [109, 494, 384, 164, 549, 439, 219, 274, 329]
=====
Total Shift: 0
Merged: [54, 109, 494, 384, 164, 549, 439, 219, 274, 329]
=====

Right half: [494, 384, 164, 549, 439, 219, 274, 329]
=====
Total Shift: 0
Merged: [54, 109, 494, 384, 164, 549, 439, 219, 274, 329]
=====

Right half: [384, 164, 549, 439, 219, 274, 329]
=====
The midposition is: 3
Left half: []
Right half: [164, 549, 439, 219, 274, 329]
=====
Total Shift: 0
Merged: [54, 109, 494, 164, 384, 549, 439, 219, 274, 329]
=====

=====
Total Shift: 1
Merged: [54, 109, 164, 384, 494, 549, 439, 219, 274, 329]
=====

Right half: [549, 439, 219, 274, 329]
=====
The midposition is: 7
Left half: [549, 439]
=====
The midposition is: 6
Left half: [549]
=====
The midposition is: 5
Left half: []
Right half: [439, 219, 274, 329]
=====
Total Shift: 2
Merged: [54, 109, 164, 384, 494, 439, 549, 219, 274, 329]
```



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Right half: [219, 274, 329]

Total Shift: 1

Merged: [54, 109, 164, 384, 494, 219, 439, 549, 274, 329]

Right half: [274, 329]

The midposition is: 8

Left half: []

Right half: [329]

Total Shift: 1

Merged: [54, 109, 164, 384, 494, 219, 439, 549, 274, 329]

Total Shift: 0

Merged: [54, 109, 164, 384, 494, 219, 274, 329, 439, 549]

Total Shift: 2

Merged: [54, 109, 164, 219, 274, 329, 384, 439, 494, 549]

Sorted array:[54, 109, 164, 219, 274, 329, 384, 439, 494, 549]



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<b>Name</b>	<b>Pratik Pujari</b>		
<b>UID no.</b>	<b>2020300054</b>	<b>Class:</b>	<b>Comps C Batch</b>
<b>Experiment No.</b>	3		

<b>AIM:</b>	To implement the Bubble sort algorithm using Recurrence Relations concepts.
<b>THEORY:</b>	<p><b>What is Recursion?</b> The process in which a function calls itself directly or indirectly is called recursion and the corresponding function is called as recursive function. Using recursive algorithm, certain problems can be solved quite easily. Examples of such problems are Towers of Hanoi (TOH), Inorder/Preorder/Postorder Tree Traversals, DFS of Graph, etc.</p> <p><b>What is base condition in recursion?</b> In the recursive program, the solution to the base case is provided and the solution of the bigger problem is expressed in terms of smaller problems.</p> <pre>int fact(int n) {     if (n &lt;= 1) // base case         return 1;     else         return n*fact(n-1); }</pre> <p>In the above example, base case for <math>n \leq 1</math> is defined and larger value of number can be solved by converting to smaller one till base case is reached.</p> <p><b>What is the difference between direct and indirect recursion?</b> A function fun is called direct recursive if it calls the same</p>



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function fun. A function fun is called indirect recursive if it calls another function say fun\_new and fun\_new calls fun directly or indirectly. Difference between direct and indirect recursion has been illustrated in Table 1.

**// An example of direct recursion**

```
void directRecFun()
{
    // Some code....
    directRecFun();
    // Some code...
}
```

**// An example of indirect recursion**

```
void indirectRecFun1()
{
    // Some code...
    indirectRecFun2();
    // Some code...
}
void indirectRecFun2()
{
    // Some code...
    indirectRecFun1();
    // Some code...
}
```

**Recurrence Relation**

A recurrence is an equation or inequality that describes a function in terms of its values on smaller inputs. To solve a Recurrence Relation means to obtain a function defined on the natural numbers that satisfy the recurrence.

**For Example**, the Worst Case Running Time  $T(n)$  of the MERGE SORT Procedures is described by the recurrence.

$$T(n) = \theta(1) \text{ if } n=1$$

$$2T\left(\frac{n}{2}\right) + \theta(n) \text{ if } n>1$$



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**What is Bubble Sort?**

Bubble sort is a simple sorting algorithm. This sorting algorithm is comparison-based algorithm in which each pair of adjacent elements is compared and the elements are swapped if they are not in order. This algorithm is not suitable for large data sets as its average and worst case complexity are of  $O(n^2)$  where  $n$  is the number of items.

**How Bubble Sort Works?**

We take an unsorted array for our example. Bubble sort takes  $O(n^2)$  time so we're keeping it short and precise.



Bubble sort starts with very first two elements, comparing them to check which one is greater.



In this case, value 33 is greater than 14, so it is already in sorted locations. Next, we compare 33 with 27.



We find that 27 is smaller than 33 and these two values must be swapped.



The new array should look like this –



Next we compare 33 and 35. We find that both are in already sorted positions.



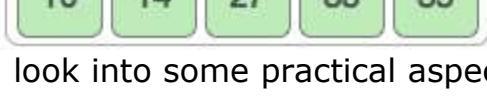
Then we move to the next two values, 35 and 10.



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	<p></p> <p>We know then that 10 is smaller 35. Hence they are not sorted.</p> <p></p> <p>We swap these values. We find that we have reached the end of the array. After one iteration, the array should look like this –</p> <p></p> <p>To be precise, we are now showing how an array should look like after each iteration. After the second iteration, it should look like this –</p> <p></p> <p>Notice that after each iteration, at least one value moves at the end.</p> <p></p> <p>And when there's no swap required, bubble sorts learns that an array is completely sorted.</p> <p></p> <p>Now we should look into some practical aspects of bubble sort.</p>
<b>PSEUDOCODE:</b>	<pre>begin BubbleSort(int arr[],int n):     for i=0 to i&lt; (n-1)         if arr[i] is greater than arr[i+1]             swap arr[i] and arr[i+1]          if n-1 is greater than 1 return BubbleSort(arr,n-1)</pre>



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**EXPERIMENT 1**

<b>CODE:</b>	<pre>Bubble Sort code: import java.util.Arrays;  public class BubbleSort {      int counter = 0;     int swapNum = 0;     static int maxPos = 0;     String swaps = "Swaps:  ";      // swap function     public static void swap(int[] arr, int i, int j) {         int temp = arr[i];         arr[i] = arr[j];         arr[j] = temp;     }      // bubble sort function     public void bubbleSortIteration(int arr[]) {         int n = arr.length;         System.out.printf("Iteration\tSwap\t\tArray\n");         for (int i = 0; i &lt; n - 1; i++) {             for (int j = 0; j &lt; n - i - 1; j++) {                 ++counter;                 if (arr[j] &gt; arr[j + 1]) {                     // swap arr[j+1] and arr[j]                     System.out.printf("%d\t\t%d&lt;- &gt;%d\t\t%s\n", counter, arr[j], arr[j + 1], printArr(arr));                      // swapping the elements                     int temp = arr[j];                     arr[j] = arr[j + 1];                     arr[j + 1] = temp;                 } else {                     System.out.printf("%d\t\t----\t\t%s\n", counter, printArr(arr));                 }             }         }     }      // printing the array     public String printArr(int arr[]) {         String str = "[";         for (int i = 0; i &lt; arr.length; i++) {             str += arr[i] + ",";         }         str += "]";         return str;     } }</pre>
--------------	---



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```
        }
    }
}

public void bubbleSortRec(int[] arr, int n) {

    for (int i = 0; i < n - 1; i++) {
        if (arr[i] > arr[i + 1]) {
            ++swapNum;
            // swap arr[i+1] and arr[i]
            swaps += arr[i] + "<->" + arr[i + 1] + " | ";
            // swapping the elements
            swap(arr, i, i + 1);
        }
    }

    if (n - 1 > 1) {
        System.out.print("\n\n" + swaps);
        System.out.print(
            "\nUnsorted + Sorted => " + printArr(arr, 0,
n - 1) + " " + printArr(arr, n - 1, arr.length)
            + "\n");
        swaps = "Swaps: |";
        bubbleSortRec(arr, n - 1);
    }
}

public int getSwapCount() {
    return swapNum;
}

public int[] bubbleSort(int i, int[] arr) {
    if (arr[i] > arr[i + 1] && (i + 1) < arr.length - 1) {
        swap(arr, i, i + 1);
        maxPos++;
    }
    if (i < arr.length - 1) {
```



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```
System.out.print("\n-----\n-----");
System.out.println("\nMax Places " + arr[i] + " is
shifted " + maxPos + " times");
System.out.println("\nArray: " + printArr(arr));
maxPos = 0;
bubbleSort(i + 1, arr);
}
return arr;
}

public String printArr(int arr[]) {
// printing the array
if (arr.length > 10)
    return "...";
else
    return Arrays.toString(arr);
}

public String printArr(int[] arr, int start, int end) {
// printing the array
if (arr.length > 10)
    return "...";
else
    return Arrays.toString(Arrays.copyOfRange(arr,
start, end));
}
// They need the array size case

}

Driver Code:
import java.util.ArrayList;
import java.util.Arrays;
import java.util.Collections;
import java.util.Scanner;

public class Driver {
```



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```
public static void main(String[] args) throws Exception {  
  
    // ArrayList of Integers  
    ArrayList<Integer> list = new ArrayList<Integer>();  
    // array for sorting  
    int[] array;  
    // User input  
    Scanner input = new Scanner(System.in);  
    System.out.print("\n1.Random Array\n2.Roll.no  
Input : ");  
    int choice = input.nextInt();  
    if (choice == 1) {  
        System.out.print("\nEnter the size of the array : ");  
        int size = input.nextInt();  
        for (int i = 0; i < size; i++) {  
            list.add((int) (i + (Math.random() * 100)));  
        }  
        Collections.sort(list);  
    } else {  
        System.out.print("\nEnter the roll no: ");  
        int roll = input.nextInt();  
        for (int i = 0; i < 10; i++) {  
            list.add(roll + (roll + 1) * i);  
        }  
    }  
    System.out.print("\n1.Best Case\n2.Worst  
Case\n3.Average Case\n4.Manual Choice\nEnter your  
choice: ");  
    int newChoice = input.nextInt();  
    int listSize = list.size();  
    array = new int[listSize];  
    switch (newChoice) {  
        case 1:  
            for (int i = 0; i < listSize; i++) {  
                array[i] = list.get(i);  
            }  
            break;  
        case 2:  
            for (int i = listSize - 1; i >= 0; i--) {  
                array[i] = list.get(i);  
            }  
    }  
}
```



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```
        array[(listSize - 1) - i] = list.get(i);
    }
    break;
case 3:
    Collections.shuffle(list);
    for (int i = 0; i < listSize; i++) {
        array[i] = list.get(i);
    }
    break;
case 4:
    if (choice == 1) {
        System.out.print("\nAre u sure u want to
enter the array manually?(y/n): ");
        String choice1 = input.next().toLowerCase();
        if (choice1.equals("y")) {
            System.out.print("Enter the size of the
array: ");
            int size = input.nextInt();
            list.clear();
            System.out.print("Enter the elements of
the array(with space): ");
            for (int i = 0; i < size; i++) {
                list.add(input.nextInt());
            }
        }
        break;
    default:
        System.out.println("Invalid choice");
        break;
    }
    int arrLen = array.length;
    // Array segregation
    switch (arrLen) {
        case 0:
            input.close();
            throw new Exception("Array is empty");
        case 1:
            input.close();
```



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**OUTPUT:**

Best Case:

```
1. Random Array  
2. Roll.no Input : 1
```

```
Enter the size of the array : 6
```

```
1. Best Case  
2. Worst Case  
3. Average Case  
4. Manual Choice  
Enter your choice: 1
```

```
Array: [11, 16, 20, 37, 78, 90]  
*****
```

```
Bubble Sort
```

```
1. Iteration Bubble Sort  
2. Recursive Bubble Sort : 2
```

```
Before sorting: [11, 16, 20, 37, 78, 90]
```

```
-----  
Swaps: |  
Unsorted + Sorted => [11, 16, 20, 37, 78] [90]
```

```
-----  
Swaps: |  
Unsorted + Sorted => [11, 16, 20, 37] [78, 90]
```

```
-----  
Swaps: |  
Unsorted + Sorted => [11, 16, 20] [37, 78, 90]
```

```
-----  
Swaps: |  
Unsorted + Sorted => [11, 16] [20, 37, 78, 90]
```

```
Total Swaps: 0
```

```
-----  
Final Array: [11, 16, 20, 37, 78, 90]
```



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	<p>Best Case:-</p> <p>Array: [11, 16, 20, 37, 78, 80]</p> <p>for First Pass</p> <p>Array = [11, 16, 20, 37, 78, 80] <math>\Rightarrow</math> 5 comparison</p> <p>Swaps: 0</p> <p>for second Pass</p> <p>Array :- [11, 16, 20, 37, 78] + [80] <math>\Rightarrow</math> 4 comparisons</p> <p>Swaps = 0</p> <p>for Third Pass</p> <p>Array: [11, 16, 20, 37] + [78, 80] <math>\Rightarrow</math> 3 comparison</p> <p>Swap = 0</p> <p>for Fourth Pass</p> <p>Array = [11, 16, 20] + [37, 78, 80] <math>\Rightarrow</math> 2 comparison</p> <p>for Last Pass Array: [11, 16, 20, 37, 78, 80]</p> <p>Swaps = 0, Comparison = 15</p>
--	---

Average Case:

1. Best Case  
2. Worst Case  
3. Average Case  
4. Manual Choice  
Enter your choice: 3

Array: [99, 55, 5, 36, 6, 97, 98]

\*\*\*\*\*



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**Bubble Sort**

1. Iteration Bubble Sort
2. Recursive Bubble Sort : 3

Before sorting: [99, 55, 5, 36, 6, 97, 98]

Swaps: |99<->55 | 99<->5 | 99<->36 | 99<->6 | 99<->97 | 99<->98 |  
Unsorted + Sorted => [55, 5, 36, 6, 97, 98] [99]

Swaps: |55<->5 | 55<->36 | 55<->6 |  
Unsorted + Sorted => [5, 36, 6, 55, 97] [98, 99]

Swaps: |36<->6 |  
Unsorted + Sorted => [5, 6, 36, 55] [97, 98, 99]

Swaps: |  
Unsorted + Sorted => [5, 6] [36, 55, 97, 98, 99]

Total Swaps: 10

Final Array: [5, 6, 36, 55, 97, 98, 99]

Average Case -

Array: [99, 55, 5, 36, 6, 97, 98]

→ For First Pass:

Array: [99, 55, 5, 36, 6, 97, 98]

Since 99 is largest, it does 6 swaps and reaches last

Array: [55, 5, 36, 6, 97, 98, 99]



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→ for Second Pass  
Array: [55, 5, 36, 6, 97, 98, 99]  
55 reaches at 97 and stops; 3 comparison/swap  
Array: [5, 36, 6, 55, 97, 98, 99]  
→ for Third Pass  
Array: [5, 36, 6, 55, 97, 98, 99] (Already at top)  
→ for fourth Pass  
Array: [5, 36, 6, 55, 97, 98, 99]  
swap at B6, 6 = 1 swap  
Array: [5, 6, 36, 55, 97, 98, 99]  
∴ Array is sorted.

**Worst Case:**

```
Enter the size of the array : 5
1.Best Case
2.Worst Case
3.Average Case
4.Manual Choice
Enter your choice: 2

Array: [87, 82, 58, 24, 1]
*****
```



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**Bubble Sort**

1. Iteration Bubble Sort  
2. Recursive Bubble Sort : 2

Before sorting: [87, 82, 58, 24, 1]

-----  
Swaps: |87<->82 | 87<->58 | 87<->24 | 87<->1 |  
Unsorted + Sorted => [82, 58, 24, 1] [87]

-----  
Swaps: |82<->58 | 82<->24 | 82<->1 |  
Unsorted + Sorted => [58, 24, 1] [82, 87]

-----  
Swaps: |58<->24 | 58<->1 |  
Unsorted + Sorted => [24, 1] [58, 82, 87]

-----  
Total Swaps: 10

-----  
Final Array: [1, 24, 58, 82, 87]

**Time Complexity of Bubble Sort:**

**Worst Case Time Complexity**

$\Theta(N^2)$  is the Worst Case Time Complexity of Bubble Sort. This is the case when the array is reversely sorted. The number of swaps of two elements is equal to the number of comparisons in this case as every element is out of place.

$T(N) = C(N) = S(N) = N * (N-1) / 2$



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- Therefore, in the worst case:
- Number of Comparisons:  $O(N^2)$  time
  - Number of swaps:  $O(N^2)$  time
  -

**Best Case Time Complexity**

$\Theta(N)$  is the Best Case Time Complexity of Bubble Sort.

This case occurs when the given array is already sorted.

$$T(N) = C(N) = NT(N) = C(N) = N$$

$$S(N) = 0$$

Therefore, in the best case:

- Number of Comparisons:  $N = O(N)$  time
- Number of swaps:  $0 = O(1)$  time

**Average Case Time Complexity**

$\Theta(N^2)$  is the Average Case Time Complexity of Bubble Sort.

The number of comparisons is constant in Bubble Sort so in average case, there is  $O(N^2)$  comparisons. This is because irrespective of the arrangement of elements, the number of comparisons  $C(N)$  is same.

For the number of swaps, consider the following points:

- If an element is in index  $I_1$  but it should be in index  $I_2$ , then it will take a minimum of  $I_2 - I_1$  swaps to bring the element to the correct position.
- An element  $E$  will be at a distance of  $I_3$  from its position in sorted array
- The sum of maximum difference in position across all elements will be:

$$\begin{aligned} & (N-1) + (N-3) + (N-5) \dots + 0 + \dots + (N-3) + (N-1) \\ &= N \times N - 2 \times (1 + 3 + 5 + \dots + N/2) \\ &= N^2 - 2 \times N^2 / 4 \\ &= N^2 - N^2 / 2 \\ &= N^2 / 2 \end{aligned}$$

Therefore, in average, the number of swaps =  $O(N^2)$ .

Therefore, in the average case time complexity of Bubble sort:

- Number of Comparisons:  $O(N^2)$  time
- Number of swaps:  $O(N^2)$  time



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**TIME  
COMPLEXITY:**

Pratik Pujari 2020300054  
h=5

public void BubbleSort(int arr[], int n){

for (int i=0; i<(n-1); i++)

if (arr[i] > arr[i+1]) {

int temp = arr[i];

arr[i] = arr[i+1];

arr[i+1] = temp;

}

if (n-1 > 1) {

BubbleSort(arr, n-1);

}

}

Time complexity of Recursive Bubble Sort

$T(N) = T(N-1) + (N-1)$  for loop

$T(N-1) = T(N-2) + (N-2)$

$T(1) = 1$

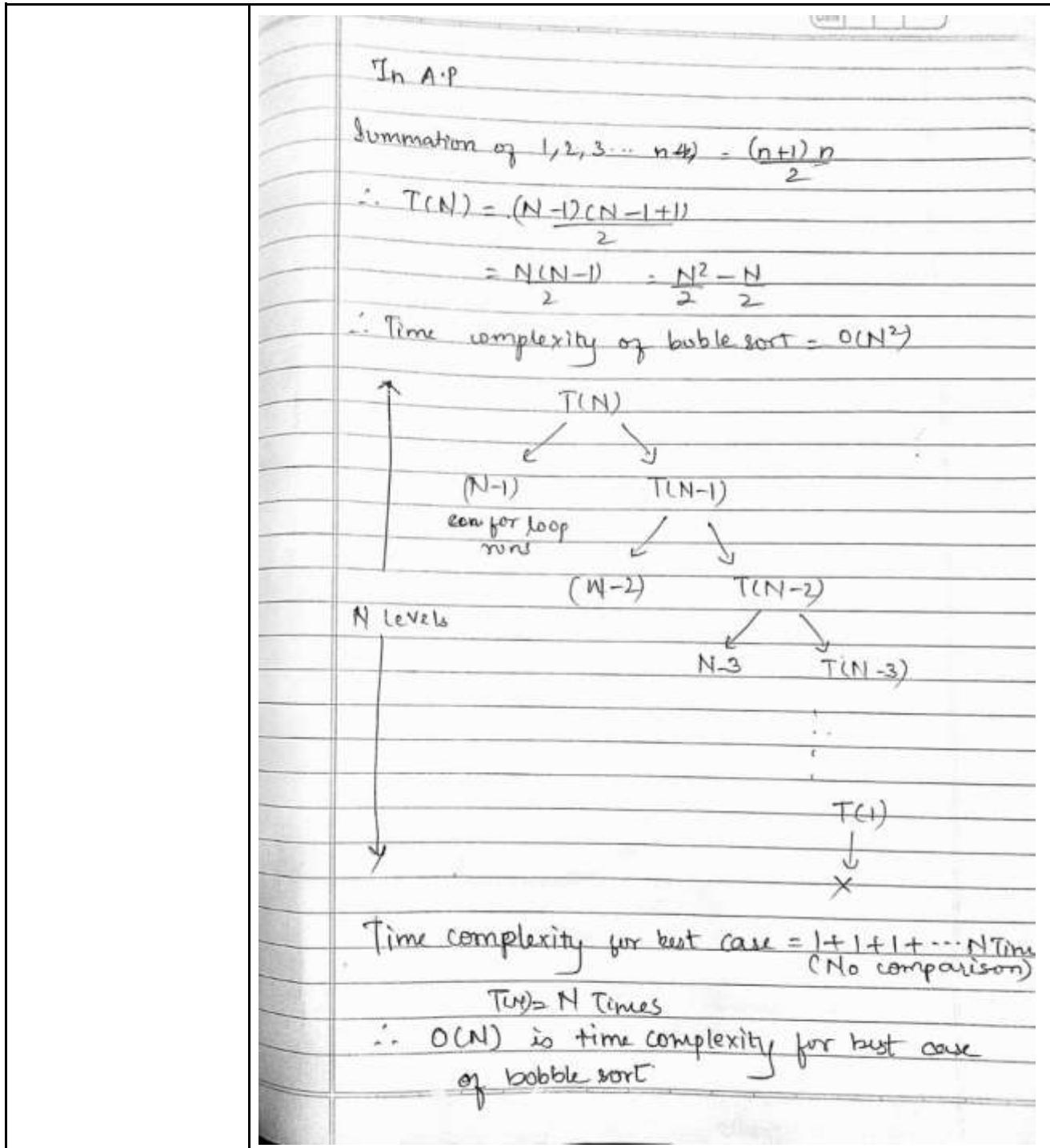
$\therefore T(N) = (N-1) + (N-2) \dots 1$



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**RESULT:** Things learnt during procedural programming during solving of the problem



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- Learnt how to implement bubble sort using iteration and recursion
- Learnt how to use exception in order to show that the array is of invalid length
- Learnt different time complexity cases of bubble sort and how it can be used.
- Learnt bubble sort required more time and space as it goes through the array  $N^2$  times

**Extra Outputs:**

```
Swaps: |67<->41 | 67<->16 | 67<->26 | 83<->34 | 83<->27 | 83<->71 | 83<->49 | 83<->15 | 83<->82 |  
83<->60 | 83<->15 |  
Unsorted + Sorted =>  ...  ...
```

```
Swaps: |41<->16 | 41<->26 | 67<->34 | 67<->27 | 71<->49 | 71<->15 | 82<->60 | 82<->15 |  
Unsorted + Sorted =>  ...  ...
```

```
Swaps: |41<->34 | 41<->27 | 67<->49 | 67<->15 | 71<->60 | 71<->15 |  
Unsorted + Sorted =>  ...  ...
```

```
Swaps: |34<->27 | 49<->15 | 67<->60 | 67<->15 |  
Unsorted + Sorted =>  ...  ...
```

```
Swaps: |41<->15 | 60<->15 |  
Unsorted + Sorted =>  ...  ...
```

```
Swaps: |34<->15 | 49<->15 |  
Unsorted + Sorted =>  ...  ...
```

```
Swaps: |27<->15 | 41<->15 |  
Unsorted + Sorted =>  ...  ...
```

```
Swaps: |26<->15 | 34<->15 |  
Unsorted + Sorted =>  ...  ...
```

```
Swaps: |16<->15 | 27<->15 |  
Unsorted + Sorted =>  ...  ...
```

```
Swaps: |26<->15 |  
Unsorted + Sorted =>  ...  ...
```

```
Swaps: |16<->15 |  
Unsorted + Sorted =>  ...  ...
```

Total Swaps: 53

```
Final Array: [15, 15, 16, 26, 27, 34, 41, 49, 60, 67, 71, 82, 83, 86]
```

(If the array is too big it shows the '....')



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(When the array size is 0/1 it shows error)

```
1.Random Array
2.Roll.no Input : 1

Enter the size of the array : 0

1.Best Case
2.Worst Case
3.Average Case
4.Manual Choice
Enter your choice: 1
Exception in thread "main" java.lang.Exception: Array is empty
at Driver.main(Driver.java:78)
```

(User gets a chance to reenter array)

```
1.Random Array
2.Roll.no Input : 1

Enter the size of the array : 5

1.Best Case
2.Worst Case
3.Average Case
4.Manual Choice
Enter your choice: 4

Are u sure u want to enter the array manually?(y/n): y
Enter the size of the array: 5
Enter the elements of the array(with space): 1 3 6 2 7

Array: [1, 3, 6, 2, 7]
*****
```



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(Iterative Bubble sort)

```
1.Random Array
2.Roll.no Input : 1
```

Enter the size of the array : 6

```
1.Best Case
2.Worst Case
3.Average Case
4.Manual Choice
Enter your choice: 1
```

```
Array: [20, 75, 91, 93, 96, 97]
*****
```

Bubble Sort

```
1.Iteration Buuble Sort
2.Recursive Buuble Sort : 1
```

Before sorting: [20, 75, 91, 93, 96, 97]

Iteration	Swap	Array
1	----	[20, 75, 91, 93, 96, 97]
2	----	[20, 75, 91, 93, 96, 97]
3	----	[20, 75, 91, 93, 96, 97]
4	----	[20, 75, 91, 93, 96, 97]
5	----	[20, 75, 91, 93, 96, 97]
6	----	[20, 75, 91, 93, 96, 97]
7	----	[20, 75, 91, 93, 96, 97]
8	----	[20, 75, 91, 93, 96, 97]
9	----	[20, 75, 91, 93, 96, 97]
10	----	[20, 75, 91, 93, 96, 97]
11	----	[20, 75, 91, 93, 96, 97]
12	----	[20, 75, 91, 93, 96, 97]
13	----	[20, 75, 91, 93, 96, 97]
14	----	[20, 75, 91, 93, 96, 97]
15	----	[20, 75, 91, 93, 96, 97]

Final Array: [20, 75, 91, 93, 96, 97]



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(Roll no input)

Bubble Sort

1. Iteration Bubble Sort  
2. Recursive Bubble Sort : 2

Before sorting: [439, 109, 164, 329, 384, 274, 219, 549, 54, 494]

-----  
Swaps: |439<->109 | 439<->164 | 439<->329 | 439<->384 | 439<->274 | 439<->219 | 549<->54 | 549<->494 |

Unsorted + Sorted => [109, 164, 329, 384, 274, 219, 439, 54, 494] [549]

Swaps: |384<->274 | 384<->219 | 439<->54 |

Unsorted + Sorted => [109, 164, 329, 274, 219, 384, 54, 439] [494, 549]

Swaps: |329<->274 | 329<->219 | 384<->54 |

Unsorted + Sorted => [109, 164, 274, 219, 329, 54, 384] [439, 494, 549]

Swaps: |274<->219 | 329<->54 |

Unsorted + Sorted => [109, 164, 219, 54, 274] [329, 384, 439, 494, 549]

Swaps: |219<->54 |

Unsorted + Sorted => [109, 164, 54, 219] [274, 329, 384, 439, 494, 549]

Swaps: |164<->54 |

Unsorted + Sorted => [109, 54, 164] [219, 274, 329, 384, 439, 494, 549]

Swaps: |109<->54 |

Unsorted + Sorted => [54, 109] [164, 219, 274, 329, 384, 439, 494, 549]

Total Swaps: 20

-----  
Final Array: [54, 109, 164, 219, 274, 329, 384, 439, 494, 549]



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<b>Name</b>	<b>Pratik Pujari</b>		
<b>UID no.</b>	<b>2020300054</b>	<b>Class:</b>	<b>Comps C Batch</b>
<b>Experiment No.</b>	4		

<b>AIM:</b>	To implement problems of Dynamic Programming
<b>THEORY:</b>	<p><b>What is Dynamic Programming?</b> Dynamic Programming (DP) is an algorithmic technique for solving an optimization problem by breaking it down into simpler subproblems and utilizing the fact that the optimal solution to the overall problem depends upon the optimal solution to its subproblems.</p> <p><b>Characteristics of Dynamic Programming</b> Before moving on to understand different methods of solving a DP problem, let's first take a look at what are the characteristics of a problem that tells us that we can apply DP to solve it.</p> <p><b>Top-down with Memoization</b> In this approach, we try to solve the bigger problem by recursively finding the solution to smaller sub-problems. Whenever we solve a sub-problem, we cache its result so that we don't end up solving it repeatedly if it's called multiple times. Instead, we can just return the saved result. This technique of storing the results of already solved subproblems is called Memoization.</p> <p><b>Bottom-up with Tabulation</b> Tabulation is the opposite of the top-down approach and avoids recursion. In this approach, we solve the problem "bottom-up" (i.e. by solving all the related sub-problems first). This is typically done by filling up an n-dimensional</p>



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table. Based on the results in the table, the solution to the top/original problem is then computed. Tabulation is the opposite of Memoization, as in Memoization we solve the problem and maintain a map of already solved sub-problems. In other words, in memoization, we do it top-down in the sense that we solve the top problem first (which typically recurses down to solve the sub-problems).

### **What is the Knapsack Problem?**

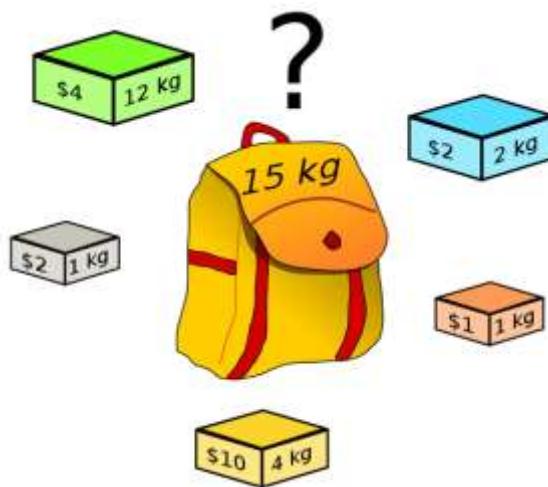
The Knapsack Problem is a famous *Dynamic Programming* Problem that falls in the optimization category.

It derives its name from a scenario where, given a set of items with specific weights and assigned values, the goal is to maximize the value in a knapsack while remaining within the weight constraint.

Each item can only be selected once, as we don't have multiple quantities of any item.

### **Problem:**

Given a Knapsack of a maximum capacity of  $W$  and  $N$  items each with its own value and weight, throw in items inside the Knapsack such that the final contents has the maximum value. Yikes !!





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Here's the general way the problem is explained – Consider a thief gets into a home to rob and he carries a knapsack. There are fixed number of items in the home – each with its own weight and value – Jewellery, with less weight and highest value vs tables, with less value but a lot heavy. To add fuel to the fire, the thief has an old knapsack which has limited capacity. Obviously, he can't split the table into half or jewellery into 3/4ths. He either takes it or leaves it

## Example:

Knapsack Max weight:  $W = 10$  (units)

Total items: N = 4

Values of items:  $v[] = \{10, 40, 30, 50\}$

Weight of items:  $w[] = \{5, 4, 6, 3\}$

A cursory look at the example data tells us that the max value that we could accommodate with the limit of max weight of 10 is  $50 + 40 = 90$  with a weight of 7.

## Approach:

The way this is optimally solved is using dynamic programming – solving for smaller sets of knapsack problems and then expanding them for the bigger problem. Let's build an Item x Weight array called V (Value array):  $V[N][W] = 4 \text{ rows} * 10 \text{ columns}$

Each of the values in this matrix represent a smaller Knapsack problem.

**Base case 1 :** Let's take the case of 0th column. It just means that the knapsack has 0 capacity. What can you hold in them? Nothing. So, let's fill them up all with 0s.

**Base case 2 :** Let's take the case of 0 row. It just means that there are no items in the house. What do you do hold in your knapsack if there are no items. Nothing again !!! All zeroes.

### Value Array - Item 0, Weight 0



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**Solution:**

1) Now, let's start filling in the array row-wise. What does row 1 and column 1 mean? That given the first item (row), can you accommodate it in the knapsack with capacity 1 (column). Nope. The weight of the first item is 5. So, let's fill in 0. In fact, we wouldn't be able to fill in anything until we reach the column 5 (weight 5).

2) Once we reach column 5 (which represents weight 5) on the first row, it means that we could accommodate item 1. Let's fill in 10 there (remember, this is a Value array):

Value Array - Item 1, Weight 5											
	Weight 0	Weight 1	Weight 2	Weight 3	Weight 4	Weight 5	Weight 6	Weight 7	Weight 8	Weight 9	Weight 10
Item 0	0	0	0	0	0	0	0	0	0	0	0
Item 1	0	0	0	0	0	10	0	0	0	0	0
Item 2	0	0	0	0	0	0	0	0	0	0	0
Item 3	0	0	0	0	0	0	0	0	0	0	0
Item 4	0	0	0	0	0	0	0	0	0	0	0

3) Moving on, for weight 6 (column 6), can we accommodate anything else with the remaining weight of 1 (weight – weight of this item => 6 – 5). Hey, remember, we are on the first item. So, it is kind of intuitive that the rest of the row will just be the same value too since we are unable to add in any other item for that extra weight that we have got.

Value Array - Item 1, Weight 6											
	Weight 0	Weight 1	Weight 2	Weight 3	Weight 4	Weight 5	Weight 6	Weight 7	Weight 8	Weight 9	Weight 10
Item 0	0	0	0	0	0	0	0	0	0	0	0
Item 1	0	0	0	0	0	10	10	0	0	0	0
Item 2	0	0	0	0	0	0	0	0	0	0	0
Item 3	0	0	0	0	0	0	0	0	0	0	0
Item 4	0	0	0	0	0	0	0	0	0	0	0

4) So, the next interesting thing happens when we reach the column 4 in third row. The current running weight is 4. We should check for the following cases.

- 1) Can we accommodate Item 2 – Yes, we can. Item 2's weight is 4.
- 2) Is the value for the current weight is higher without Item 2? – Check the previous row for the same weight. Nope. the previous row\* has 0 in it, since we were not able to accommodate Item 1 in weight 4.
- 3) Can we accommodate two items in the same weight so that we could maximize the value? – Nope. The remaining weight after deducting the Item2's weight is 0.



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Value Array - Item 2, Weight 4											
	Weight 0	Weight 1	Weight 2	Weight 3	Weight 4	Weight 5	Weight 6	Weight 7	Weight 8	Weight 9	Weight 10
Item 0	0	0	0	0	0	0	0	0	0	0	0
Item 1	0	0	0	0	0	10	10	10	10	10	10
Item 2	0	0	0	0	40						
Item 3	0										
Item 4	0										

**Why previous row?**

Simply because the previous row at weight 4 itself is a smaller knapsack solution which gives the max value that could be accumulated for that weight until that point (traversing through the items).

Exemplifying,

- 1) The value of the current item = 40
- 2) The weight of the current item = 4
- 3) The weight that is left over =  $4 - 4 = 0$
- 4) Check the row above (the Item above in case of Item 1 or the cumulative Max value in case of the rest of the rows). For the remaining weight 0, are we able to accommodate Item 1? Simply put, is there any value at all in the row above for the given weight?

The calculation goes like so :

- 1) Take the max value for the same weight without this item:

previous row, same weight = 0

$$\Rightarrow V[\text{item-1}][\text{weight}]$$

- 2) Take the value of the current item + value that we could accommodate with the remaining weight:

Value of current item

+ value in previous row with weight 4 (total weight until now (4) - weight of the current item (4))

$$\Rightarrow \text{val}[\text{item-1}] + V[\text{item-1}][\text{weight-wt[item-1]}]$$

Max among the two is 40 (0 and 40).

- 3) The next and the most important event happens at column 9 and row 2. Meaning we have a weight of 9 and we have two items. Looking at the example data we could accommodate the first two items. Here, we consider few things:

1. The value of the current item = 40
2. The weight of the current item = 4
3. The weight that is left over =  $9 - 4 = 5$



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4. Check the row above. At the remaining weight 5, are we able to accommodate Item 1.

Value Array - Item 2, Weight 9 (accommodates 1 and 2)										
	Weight 0	Weight 1	Weight 2	Weight 3	Weight 4	Weight 5	Weight 6	Weight 7	Weight 8	Weight 9
Item 0	0	0	0	0	0	0	0	0	0	0
Item 1	0	0	0	0	0	10	10	10	10	10
Item 2	0	0	0	40	40	40	40	40	40	50
Item 3	0									
Item 4	0									

So, the calculation is :

1) Take the max value for the same weight without this item:

previous row, same weight = 10

2) Take the value of the current item + value that we could accumulate with the remaining weight:

Value of current item (40)

+ value in previous row with weight 5 (total weight until now (9) - weight of the current item (4))= 10

10 vs 50 = 50.

At the end of solving all these smaller problems, we just need to return the value at  $V[N][W]$  – Item 4 at Weight 10:

Value Array - Final set										
	Weight 0	Weight 1	Weight 2	Weight 3	Weight 4	Weight 5	Weight 6	Weight 7	Weight 8	Weight 9
Item 0	0	0	0	0	0	0	0	0	0	0
Item 1	0	0	0	0	0	10	10	10	10	10
Item 2	0	0	0	40	40	40	40	40	40	50
Item 3	0	0	0	40	40	40	40	40	40	70
Item 4	0	0	0	50	50	50	50	50	50	90

### Complexity

Analysing the complexity of the solution is pretty straightforward. We just have a loop for W within a loop of N =>  $O(N*W)$

### PSEUDOCODE:

```

array m[0..n, 0..W];
for j from 0 to W do:
    m[0, j] := 0
for i from 1 to n do:
    m[i, 0] := 0

for i from 1 to n do:
    for j from 0 to W do:
        if w[i] > j then:
            m[i, j] := m[i-1, j]
        else:
            m[i, j] := max(m[i-1, j], m[i-1, j-w[i]] + v[i])
    
```



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**EXPERIMENT 1**

<b>CODE:</b>	<pre>import java.util.Scanner; import java.io.BufferedReader; import java.util.ArrayList;  public class KnapSack {     public static void listToArray(int array[], ArrayList&lt;Integer&gt; list) {         for (int i = 0; i &lt; list.size(); i++) {             array[i] = list.get(i);         }     }      public static void main(String[] args) throws Exception {          Scanner input = new Scanner(System.in);          // ArrayList for storing temp values         ArrayList&lt;Integer&gt; _temp = new ArrayList&lt;Integer&gt;();          String numbers[];         int length = 0;         int values[], weight[], W;          System.out.print("\n\t\tKNAPSACK ALGORITHM");         BufferedReader br = new BufferedReader(new         java.io.InputStreamReader(System.in));         // Take values input         System.out.print("\nEnter the Values of the array(with space)\n-&gt; ");          String inputstr = br.readLine();         if (!inputstr.equals("")) {             numbers = inputstr.split(" ");             for (String number : numbers)                 _temp.add(Integer.parseInt(number));             length = _temp.size();         }     } }</pre>
--------------	---



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```
values = new int[length];
listToArray(values, _temp);

// weight input
weight = new int[length];
System.out.print("\n---- Weights of items ----\n");
for (int i = 0; i < length; i++) {
    System.out.print("\n-> Weight of item " + _temp.get(i) + " : ");
    weight[i] = input.nextInt();
}

// Max weight
System.out.print("\nEnter the Max Weight: ");
W = input.nextInt();

System.out.println("\nThe limit of max possible weight is " +
knapsack(values, weight, W));
input.close();

}

public static int knapsack(int val[], int wt[], int W) {

    System.out.print("\nFormulating the problem \n");

    // Get the total number of items.
    // Could be wt.length or val.length. Doesn't matter

    int N = wt.length;

    // Create a matrix.
    // Items are in rows and weight at in columns +1 on each side

    int[][] values = new int[N + 1][W + 1];

    // What if the knapsack's capacity is 0 - Set
    // all columns at row 0 to be 0

    for (int col = 0; col <= W; col++) {
```



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```
values[0][col] = 0;  
  
}  
  
// What if there are no items at home.  
// Fill the first row with 0  
for (int row = 0; row <= N; row++) {  
    values[row][0] = 0;  
}  
  
for (int item = 1; item <= N; item++) {  
  
// Let's fill the values row by row  
for (int weight = 1; weight <= W; weight++) {  
    // Is the current item's weight less  
    // than or equal to running weight  
    if (wt[item - 1] <= weight) {  
  
        // Given a weight, check if the value of the current  
        // item + value of the item that we could afford  
        // with the remaining weight is greater than the value  
        // without the current item itself  
  
        values[item][weight] = Math.max(val[item - 1] + values[item -  
1][weight - wt[item - 1]],  
            values[item - 1][weight]);  
    }  
  
    else {  
        // If the current item's weight is more than the  
        // running weight, just carry forward the value  
        // without the current item  
  
        values[item][weight] = values[item - 1][weight];  
    }  
}  
}
```



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	<pre>// Printing the matrix for (int[] rows : values) {     for (int col : rows) {         System.out.format("%5d", col);     }     System.out.println(); } return values[N][W]; }</pre>																																				
<b>OUTPUT:</b>	<pre>KNAPSACK ALGORITHM Enter the Values of the array (with space) -&gt; 2 5 8 1  ---- weights of items ----  -&gt; Weight of item '2' : 3 -&gt; Weight of item '5' : 6 -&gt; Weight of item '8' : 8 -&gt; Weight of item '1' : 1  Enter the Max weight: 15  Formulating the problem   0   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   0   0   0   2   2   2   5   5   5   7   7   7   7   7   7   7   7   7   7   7   0   0   0   2   2   2   5   5   8   8   8   10  10  10  10  13  13  13  13  15   0   0   0   2   2   2   5   5   8   8   8   10  10  10  10  13  13  13  13  15  15  The limit of max possible weight is 15</pre> <p>Weights :- {2, 3, 6, 4}</p> <p>Value :- {1, 5, 8, 3}</p> <p>Max weight = 5</p> <table border="1"><tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td></tr><tr><td>0</td><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td></tr><tr><td>0</td><td>2</td><td>2</td><td>2</td><td>2</td><td>0</td></tr><tr><td>0</td><td>2</td><td>2</td><td>2</td><td>2</td><td>3</td></tr><tr><td>0</td><td>2</td><td>2</td><td>4</td><td>6</td><td>6.</td></tr></table> <p>Max possible weight = 6</p>	0	0	0	0	0	0	0	2	2	2	2	2	0	2	2	2	2	2	0	2	2	2	2	0	0	2	2	2	2	3	0	2	2	4	6	6.
0	0	0	0	0	0																																
0	2	2	2	2	2																																
0	2	2	2	2	2																																
0	2	2	2	2	0																																
0	2	2	2	2	3																																
0	2	2	4	6	6.																																



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	<p><b>Input:</b> Weights: 0 Max Wgt: 1</p> <pre>KNAPSACK ALGORITHM Enter the values of the array(with space) -&gt;  ---- Weights of items ----  Enter the Max Weight: 1  Formulating the problem 0 0  The limit of max possible weight is 0</pre>
	<p>No of weights: 1, Max Weight: 5 Weight: 10 Weight val1: 5</p> <pre>KNAPSACK ALGORITHM Enter the Values of the array(with space) -&gt; 10  ---- Weights of items ----  -&gt; Weight of item '10' : 5  Enter the Max Weight: 5  Formulating the problem 0 0 0 0 0 0 0 0 0 0 0 10  The limit of max possible weight is 10</pre>



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```
KNAPSACK ALGORITHM
Enter the Values of the array(with space)
-> 10 20 30 40

---- Weights of items ----

-> Weight of item '10' : 12
-> Weight of item '20' : 13
-> Weight of item '30' : 15
-> Weight of item '40' : 19

Enter the Max Weight: 10

Formulating the problem
 0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0

The limit of max possible weight is 0
```

No of weights: 4, Max Weight: 7  
Weights: 80 60 40 20  
Values: 2 4 6 8

```
KNAPSACK ALGORITHM
Enter the Values of the array(with space)
-> 80 60 40 20

---- Weights of items ----

-> Weight of item '80' : 2
-> Weight of item '60' : 4
-> Weight of item '40' : 6
-> Weight of item '20' : 8

Enter the Max Weight: 7

Formulating the problem
 0  0  0  0  0  0  0  0
 0  0  80  80  80  80  80  80
 0  0  80  80  80  80  140 140
 0  0  80  80  80  80  140 140
 0  0  80  80  80  80  140 140

The limit of max possible weight is 140
```



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	<p>No of weights: 5, max Weight: 15 Weights: 4 2 1 10 2 Values: 12 2 1 4 1</p> <pre>KNAPSACK ALGORITHM Enter the Values of the array(with space) -&gt; 4 2 1 10 2  ----- Weights of items -----  -&gt; Weight of item '4' : 12 -&gt; Weight of item '2' : 2 -&gt; Weight of item '1' : 1 -&gt; Weight of item '10' : 4 -&gt; Weight of item '2' : 1  Enter the Max Weight: 15  Formulating the problem   0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0   0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0   0  0  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2   0  1  2  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3   0  1  2  3  10  11  12  13  13  13  13  13  13  13  13  13  13  13   0  2  3  4  10  12  13  14  15  15  15  15  15  15  15  15  15  15  The limit of max possible weight is 15</pre>
	<p>No of weights: 10, Max weight: 165 Weight: 92 57 49 68 60 43 67 84 87 72 Values: 23 31 29 44 53 38 63 85 89 82</p> <pre>KNAPSACK ALGORITHM Enter the Values of the array(with space) -&gt; 92 57 49 68 60 43 67 84 87 72  ----- Weights of items -----  -&gt; Weight of item '92' : 23 -&gt; Weight of item '57' : 31 -&gt; Weight of item '49' : 29 -&gt; Weight of item '68' : 44 -&gt; Weight of item '60' : 53 -&gt; Weight of item '43' : 38 -&gt; Weight of item '67' : 63 -&gt; Weight of item '84' : 85 -&gt; Weight of item '87' : 89 -&gt; Weight of item '72' : 82  Enter the Max Weight: 165</pre> <p>Output:</p> <p>The limit of max possible weight is 309</p>



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	<p>No of weight: 15, max Weight: 750      Weight:      135 139 149 150 156 163 173 184 192 201 210 214 221 229      240      Values: 70 73 77 80 82 87 90 94 98 106 110 113 115 118 120</p> <div style="background-color: black; color: white; padding: 5px; text-align: center;"> <b>The limit of max possible weight is 1458</b> </div> <p>No of weight: 24, Max Weight: 6404180      Weight: 825594 1677009 1676628 1523970 943972 97426 69666      1296457 1679693 1902996 1844992 1049289 1252836 1319836      953277 2067538 675367 853655 1826027 65731 901489 577243      466257 369261        Values: 382745 799601 909247 729069 467902 44328 34610 698150      823460 903959 853665 551830 610856 670702 488960 951111      323046 446298 931161 31385 496951 264724 224916 169684</p> <div style="background-color: black; color: white; padding: 5px; text-align: center;"> <b>Formulating the problem</b> </div> <div style="background-color: black; color: white; padding: 5px; text-align: center;"> <b>The limit of max possible weight is 13549094</b> </div>
<b>TIME COMPLEXITY:</b>	<p>Time complexity for knapsack problem:-</p> <pre> for i=0 to (n+1)           - - - - - n+1     for j=0 to w             - - - - - w+         if w[i] &gt; j          - - - - - w             m[i,j] = m[i-1,j] - - - - - w         else             m[i,j] = max[m[i-1,j], m[i-1,j-w[i]]+v[i]] - - - - - w </pre> $T(w) = (n+1)(w+1 + w + w + w)$ $= (4w+1)(n+1)$ $T(w) = \Theta(w \cdot n)$ <p>Time complexity of Knapsack Algo = <math>\Theta(w \cdot n)</math></p>



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	<p><b>Time Complexity: <math>O(N*W)</math>.</b> where 'N' is the number of weight elements and 'W' is the capacity of the knapsack.</p>
<p><b>CONCLUSION:</b> Learnt how to solve dynamic programming problems by dividing bigger problems into smaller problems. Learnt about the time complexity of the Knapsack problem. Learnt how and why is the 2d array is used in the Knapack problem. Dynamic problems like Fibonacci series nth number uses array to store reccuring solution which is implemented here too.</p>	



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<b>Name</b>	<b>Pratik Pujari</b>		
<b>UID no.</b>	<b>2020300054</b>	<b>Class:</b>	<b>Comps C Batch</b>
<b>Experiment No.</b>	5		

<b>AIM:</b>	To implement Huffman encoding technique of greedy approach
<b>THEORY</b>	<p><b>What is Greedy Algorithm?</b></p> <p>A greedy algorithm is a simple, intuitive algorithm that is used in optimization problems. The algorithm makes the optimal choice at each step as it attempts to find the overall optimal way to solve the entire problem. Greedy algorithms are quite successful in some problems, such as Huffman encoding which is used to compress data, or Dijkstra's algorithm, which is used to find the shortest path through a graph.</p> <p><b>Structure of a Greedy Algorithm</b></p> <p>Greedy algorithms take all of the data in a particular problem, and then set a rule for which elements to add to the solution at each step of the algorithm. In the animation above, the set of data is all of the numbers in the graph, and the rule was to select the largest number available at each level of the graph. The solution that the algorithm builds is the sum of all of those choices.</p> <p>If both of the properties below are true, a greedy algorithm can be used to solve the problem.</p> <ul style="list-style-type: none"><li>• <b>Greedy choice property:</b> A global (overall) optimal solution can be reached by choosing the optimal choice at each step.</li><li>• <b>Optimal substructure:</b> A problem has an optimal substructure if an optimal solution to the entire problem contains the optimal solutions to the sub-problems.</li></ul>



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In other words, greedy algorithms work on problems for which it is true that, at every step, there is a choice that is optimal for the problem up to that step, and after the last step, the algorithm produces the optimal solution of the complete problem.

To make a greedy algorithm, identify an optimal substructure or subproblem in the problem. Then, determine what the solution will include (for example, the largest sum, the shortest path, etc.). Create some sort of iterative way to go through all of the subproblems and build a solution.

### **Limitations of Greedy Algorithms**

Sometimes greedy algorithms fail to find the globally optimal solution because they do not consider all the data. The choice made by a greedy algorithm may depend on choices it has made so far, but it is not aware of future choices it could make.

### **Huffman Coding**

Huffman encoding is another example of an algorithm where a greedy approach is successful. The Huffman algorithm analyzes a message and depending on the frequencies of the characters used in the message, it assigns a variable-length encoding for each symbol. A more commonly used symbol will have a shorter encoding while a rare symbol will have a longer encoding.

The Huffman coding algorithm takes in information about the frequencies or values of a particular symbol occurring. It begins to build the prefix tree from the bottom up, starting with the two least probable symbols in the list. It takes those symbols and forms a subtree containing them, and then removes the individual symbols from the list.

The algorithm sums the values of elements in a subtree and adds the subtree and its probability to the list. Next, the algorithm searches the list and selects the two symbols or subtrees with the smallest values. It uses those to make a new subtree, removes the original subtrees/symbols from the list, and then adds the



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new subtree and its combined probability to the list. This repeats until there is one tree and all elements have been added. At each subtree, the optimal encoding for each symbol is created and together composes the overall optimal encoding.

Huffman Encoding Example:

Suppose the string below is to be sent over a network.

B C A A D D D C C C A C A C A C

Huffman coding is done with the help of the following steps.

1. Calculate the frequency of each character in the string

1	6	5	3
B	C	A	D

2. Sort the characters in increasing order of the frequency. These are stored in a priority queue Q.

1	3	5	6
B	D	A	C

3. Make each unique character as a leaf node.

4. Create an empty node z. Assign the minimum frequency to the left child of z and assign the second minimum frequency to the right child of z. Set the value of the z as the sum of the above two minimum frequencies.



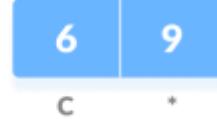


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5. Remove these two minimum frequencies from Q and add the sum into the list of frequencies (\* denote the internal nodes in the figure above).
6. Insert node z into the tree.
7. Repeat steps 3 to 5 for all the characters



15

8. For each non-leaf node, assign 0 to the left edge and 1 to the right edge.

For sending the above string over a network, we have to send the tree as well as the above compressed-code. The total size is given by the table below.

Character	Frequency	Code	Size
A	5	11	$5*2 = 10$
B	1	100	$1*3 = 3$
C	6	0	$6*1 = 6$
D	3	101	$3*3 = 9$
$4 * 8 = 32$ bits		15 bits	28 bits

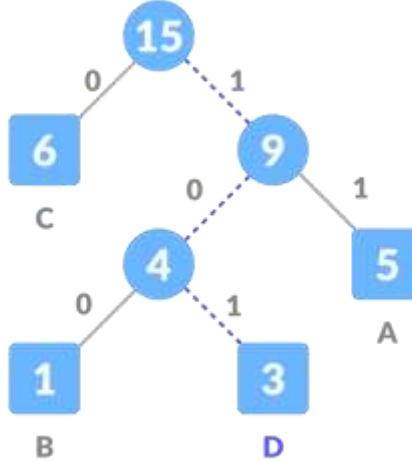
**Decoding the code**



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	<p>For decoding the code, we can take the code and traverse through the tree to find the character. Let 101 is to be decoded, we can traverse from the root as in the figure below.</p> 
<b>PSEUDOCODE:</b>	<h2><b>Huffman Algorithm</b></h2> <ol style="list-style-type: none"><li>1. Input:-Number of message with frequency count.</li><li>2. Output: - Huffman merge tree.</li><li>3. Begin</li><li>4. Let Q be the priority queue,</li><li>5. Q= {initialize priority queue with frequencies of all symbol or message}</li><li>6. Repeat n-1 times</li><li>7. Create a new node Z</li><li>8. X=extract_min(Q)</li><li>9. Y=extract_min(Q)</li><li>10. Frequency(Z) =Frequency(X) +Frequency(y);</li><li>11. Insert (Z, Q)</li><li>12. End repeat</li><li>13. Return (extract_min(Q))</li><li>14. End.</li></ol>



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**EXPERIMENT 1**

<b>CODE:</b>	<pre>Huffman Code import java.util.*;  class HuffmanNode {     // Huffman node class     int data;     char character;      // constructor     HuffmanNode left, right;      HuffmanNode() {         this.left = null;         this.right = null;     }      // constructor     HuffmanNode(char ch, int data) {         this.left = null;         this.right = null;         this.data = data;         this.character = ch;     }      class MyComparator implements     Comparator&lt;HuffmanNode&gt; {         // Comparator class         public int compare(HuffmanNode x, HuffmanNode y) {              return x.data - y.data;         }     }      public class Huffman {</pre>
--------------	---



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```
// Huffman class

char charArray[];
int charFreq[];
int characters;
// User input
Scanner input = new Scanner(System.in);
// Huffman tree
PriorityQueue<HuffmanNode> queue;
// hashmap to store the encodings
HashMap<Character, String> map = new
HashMap<Character, String>();

// User input method
public void userInput() {
    System.out.print("\nEnter the number of characters to
be read: ");
    characters = input.nextInt();

    // Priority queue to store the Huffman nodes
    queue = new
PriorityQueue<HuffmanNode>(characters, new
MyComparator());

    charArray = new char[characters];
    charFreq = new int[characters];

    System.out.print("\nEnter the characters below\n->");
    for (int i = 0; i < characters; i++) {
        charArray[i] = input.next().charAt(0);
    }

    System.out.print("\nEnter the Frequency of the
Characters\n");
    for (int i = 0; i < characters; i++) {
        System.out.print("-> " + charArray[i] + " : ");
        charFreq[i] = input.nextInt();
    }
}
```



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```
}

public void printArrays() {
    System.out.println();
    System.out.print("\n| Characters\t| + "
" Frequency\t|\n");
    System.out.print("-----"
\n");
    for (int i = 0; i < characters; i++) {
        System.out.print("\n|      " + charArray[i] + "\t| "
+ "      " + charFreq[i] + "\t|");
    }
}

public void setup(HashMap<Character, Integer> values)
{
    // setup method
    characters = values.size();
    charArray = new char[characters];
    charFreq = new int[characters];

    queue = new
PriorityQueue<HuffmanNode>(characters, new
MyComparator());

    for (int i = 0; i < values.size(); i++) {
        charArray[i] = (char) values.keySet().toArray()[i];
        charFreq[i] = (int) values.values().toArray()[i];
    }
    printArrays();
}

public HuffmanNode makeTree() {
    // makeTree method
    System.out.print("\nStarted Making the Huffman
Tree\n");
    for (int i = 0; i < characters; i++) {
```



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```
// Adding the nodes to the queue
HuffmanNode hNode = new
HuffmanNode(charArray[i], charFreq[i]);
queue.add(hNode);
}
HuffmanNode root = null;
int counter = 1;
// Making the tree
while (queue.size() > 1) {
    System.out.print("\n-----
-\n");
    System.out.print("\nStep " + counter);
    HuffmanNode x = queue.peek();
    queue.poll();

    HuffmanNode y = queue.peek();
    queue.poll();

    HuffmanNode f = new HuffmanNode();
    // Combining the nodes
    f.data = x.data + y.data;
    f.character = '+';
    // Adding the combined node to the queue
    f.left = x;
    f.right = y;
    root = f;
    printNode(root, x, y);
    queue.add(f);
    counter++;
}

return root;
}

public void printNode(HuffmanNode root, HuffmanNode
x, HuffmanNode y) {
    System.out.print("\n\nParent Node: " + " " +
root.data);
    System.out.print("\n| |");
}
```



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```
System.out.printf("\n| \\_Left Child:\t| " +
x.character + "\t| " + x.data + "\t|");
System.out.print("\n|");
System.out.printf("\n| \\_Right Child:\t| " +
y.character + "\t| " + y.data + "\t|");
}

public void printTree(HuffmanNode root, String
characters) {
    // System.out.println(root+" "+characters);
    if (root.left == null && root.right == null &&
Character.isLetter(root.character)) {
        map.put(root.character, characters);
        return;
    }
    // Printing the left child
    printTree(root.left, characters + "0");
    printTree(root.right, characters + "1");
}

public void displayMap() {
    // Displaying the map
    System.out.println("\n\nThe Encoding is: ");
    for (Map.Entry m : map.entrySet())
        System.out.println(m.getKey() + " " +
m.getValue());
}

Driver Code
import java.util.HashMap;

public class Driver {

    public static void main(String[] args) {
```



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```
Huffman HM = new Huffman();
// user INput
System.out.print("\n1.Character Input\n2.String
Input\nEnter your choice: ");
int choice = HM.input.nextInt();
switch (choice) {
    case 1:
        // character input
        HM.userInput();
        HM.printArrays();
        HuffmanNode root1 = HM.makeTree();
        HM.printTree(root1, "");
        HM.displayMap();
        break;
    case 2:
        // string input
        System.out.print("\nEnter the String to be
encoded(without space)");
        System.out.print("\n->");
        String str = HM.input.next();
        HashMap<Character, Integer> chars = new
HashMap<Character, Integer>();

        for (int i = 0; i < str.length(); i++) {
            // System.out.print("\n" + str.charAt(i));
            if
(Character.isLetter(str.toLowerCase().charAt(i)) &&
!chars.containsKey(str.charAt(i))) {
                chars.put(str.toLowerCase().charAt(i), 1);
            } else if (chars.containsKey(str.charAt(i))) {
                int value = chars.get(str.charAt(i));
                value++;
                chars.replace(str.charAt(i), value);
            }
        }
        System.out.print("\nAll the Characters in the
String are: ");
        HM.setup(chars);
```



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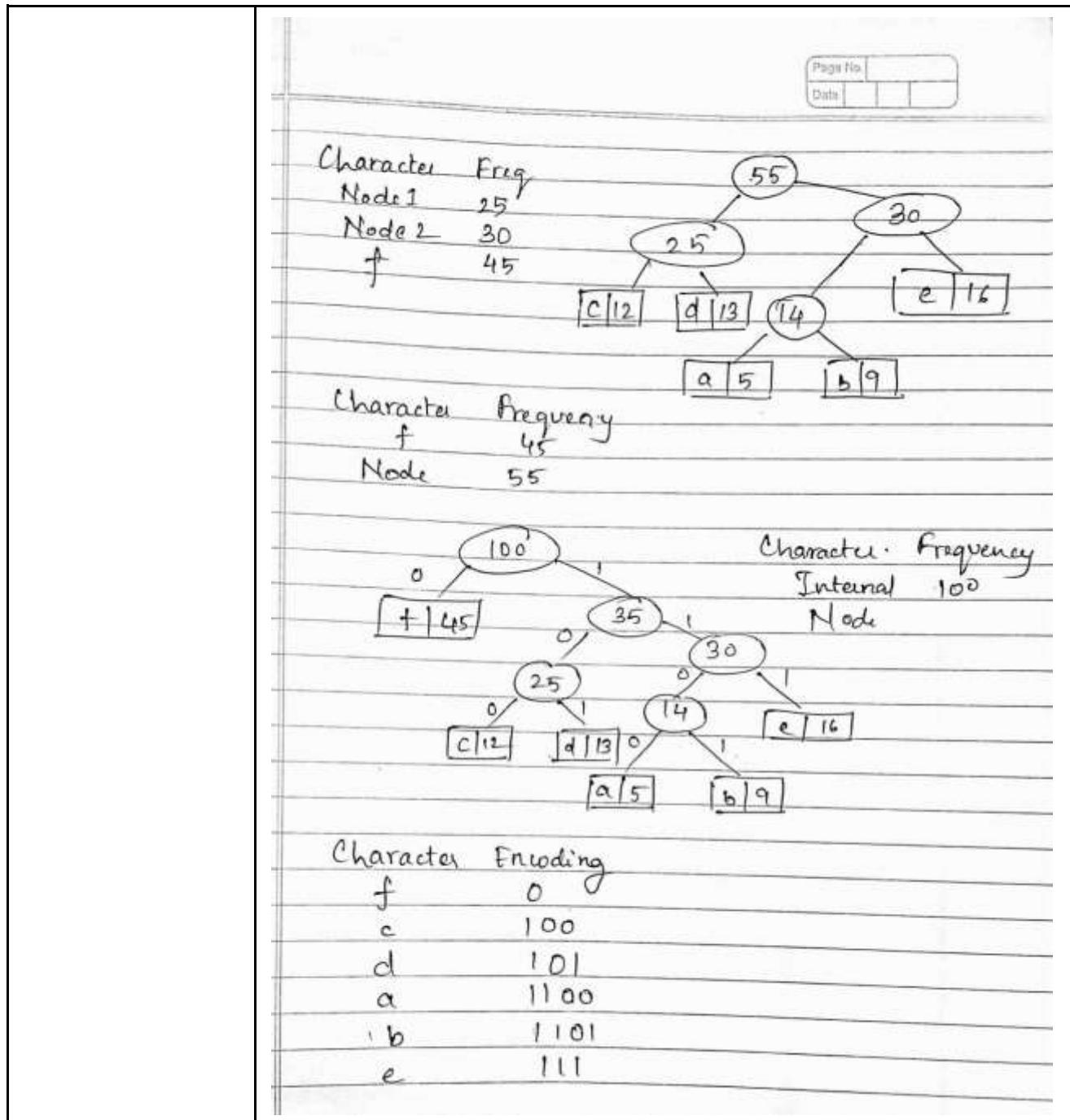
	<pre>HuffmanNode root2 = HM.makeTree(); HM.printTree(root2, ""); HM.displayMap(); break; default: break; } // HM.printQueue(); } }</pre>																						
<b>OUTPUT:</b>	<p>Character Frequency</p> <table><tbody><tr><td>a</td><td>5</td></tr><tr><td>b</td><td>9</td></tr><tr><td>c</td><td>12</td></tr><tr><td>d</td><td>13</td></tr><tr><td>e</td><td>16</td></tr><tr><td>f</td><td>45</td></tr></tbody></table> <p>Extract minimum nodes</p> <table><tbody><tr><td>Character</td><td>Frequency</td></tr><tr><td>Node 1</td><td>14</td></tr><tr><td>Node 2</td><td>25</td></tr><tr><td>e</td><td>16</td></tr><tr><td>f</td><td>45</td></tr></tbody></table>	a	5	b	9	c	12	d	13	e	16	f	45	Character	Frequency	Node 1	14	Node 2	25	e	16	f	45
a	5																						
b	9																						
c	12																						
d	13																						
e	16																						
f	45																						
Character	Frequency																						
Node 1	14																						
Node 2	25																						
e	16																						
f	45																						



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```
1.Character Input
2.String Input
Enter your choice: 2

Enter the String to be encoded(without space)
->dawdawdnkladawdlawdkl

All the Characters in the String are:
+-----+
|   Characters   |   Frequency   |
+-----+
|       a        |       5        |
|       d        |       6        |
|       w        |       4        |
|       k        |       2        |
|       l        |       3        |
|       n        |       1        |
Started Making the Huffman Tree
+-----+
Step 1
Parent Node: 3
| |
| \_Left Child: |   n   |   1   |
| \_Right Child: |   k   |   2   |
+-----+
Step 2
Parent Node: 6
| |
| \_Left Child: |   1   |   3   |
| \_Right Child: |   +   |   3   |
+-----+
Step 3
Parent Node: 9
| |
| \_Left Child: |   w   |   4   |
| \_Right Child: |   a   |   5   |
+-----+
```



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**Step 4**

Parent Node: 12

```
  | |  
  | \_Left Child: | + | 6 |  
  | \_Right Child: | d | 6 |
```

**Step 5**

Parent Node: 21

```
  | |  
  | \_Left Child: | + | 9 |  
  | \_Right Child: | + | 12 |
```

The Encoding is:

a 01  
d 11  
w 00  
k 1011  
l 100  
n 1010

From the above example

- String is read and character is categorized according to the frequency
- Priority queue (min heap) is used to store the Huffman node
- Huffman tree is created using the queue
- Every node is printed with its child data
- In-order transversal through the tree goes through each node and assigns the encoding to it.



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<b>TIME COMPLEXITY:</b>	<p>Operation of the Huffman algorithm.</p> <p>The time complexity of the Huffman algorithm is <b><math>O(n \log n)</math></b>. Using a heap to store the weight of each tree, each iteration requires <b><math>O(\log n)</math></b> time to determine the cheapest weight and insert the new weight. There are <b><math>O(n)</math></b> iterations, one for each item.</p>
<p><b>RESULT:</b> Things learnt during the procedural programming of the question</p> <ul style="list-style-type: none"><li>• Learnt about the Huffman encoding and decoding</li><li>• Learnt on how to store Huffman node priority queue (min heap)</li><li>• Learnt about the comparator class and used it in priority queue to find the min element</li><li>• Used priority queue which acts as a min heap to find the minimum most element in the heap</li><li>• Used HashMap to store the characters and frequency present in the string.</li></ul>	



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<b>Name</b>	<b>Pratik Pujari</b>		
<b>UID no.</b>	<b>2020300054</b>	<b>Class:</b>	<b>Comps C Batch</b>
<b>Experiment No.</b>	6		

<b>AIM:</b>	To implement Dijkstra algorithm
<b>THEORY:</b>	<p><b>What is BackTracking?</b></p> <p>Backtracking is an algorithmic technique where the goal is to get all solutions to a problem using the brute force approach. It consists of building a set of all the solutions incrementally. Since a problem would have constraints, the solutions that fail to satisfy them will be removed.</p> <p>It uses recursive calling to find a solution set by building a solution step by step, increasing levels with time. In order to find these solutions, a search tree named state-space tree is used. In a state-space tree, each branch is a variable, and each level represents a solution.</p> <p>A backtracking algorithm uses the depth-first search method. When it starts exploring the solutions, a bounding function is applied so that the algorithm can check if the so-far built solution satisfies the constraints. If it does, it continues searching. If it doesn't, the branch would be eliminated, and the algorithm goes back to the level before.</p> <p><b>When to Use a Backtracking Algorithm</b></p> <p>The backtracking algorithm is applied to some specific types of problems. For instance, we can use it to find a feasible solution to a decision problem. It was also found to be very effective for optimization problems.</p> <p>For some cases, a backtracking algorithm is used for the enumeration problem in order to find the set of all feasible solutions for the problem.</p>



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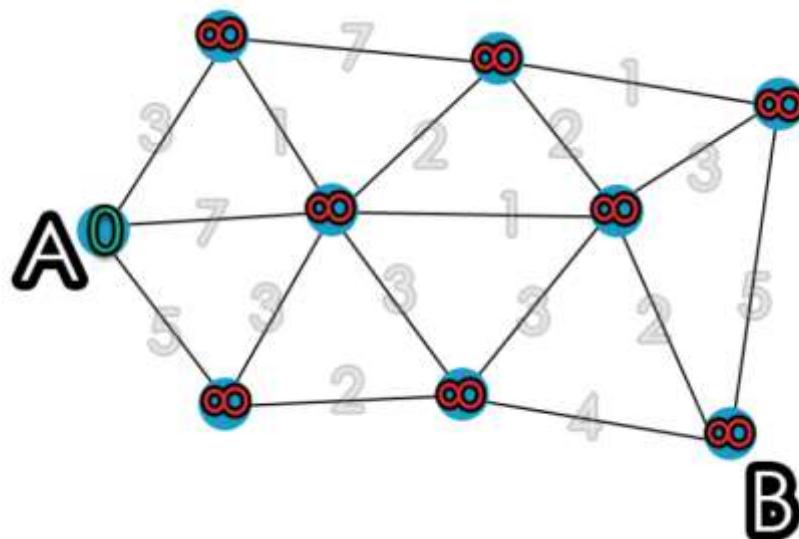
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On the other hand, backtracking is not considered an optimized technique to solve a problem. It finds its application when the solution needed for a problem is not time-bounded.

### **Dijkstra's Algorithm**

Dijkstra's algorithm finds a shortest path tree from a single source node, by building a set of nodes that have minimum distance from the source.



The graph has the following:

- vertices, or nodes, denoted in the algorithm by  $vv$  or  $uu$ ;
- weighted edges that connect two nodes:  $(u, vu, v)$  denotes an edge, and  $w(u, v)w(u, v)$  denotes its weight. In the diagram on the right, the weight for each edge is written in gray.

This is done by initializing three values:

- $dist[dist]$ , an array of distances from the source node  $ss$  to each node in the graph, initialized the following way:  $dist[dist](ss) = 0$ ; and for all other nodes  $vv$ ,  $dist[dist](vv) = \infty$ . This is done at the beginning because as the algorithm proceeds, the  $dist[dist]$  from the source to each node  $vv$  in the graph will be recalculated and finalized when the shortest distance to  $vv$  is found



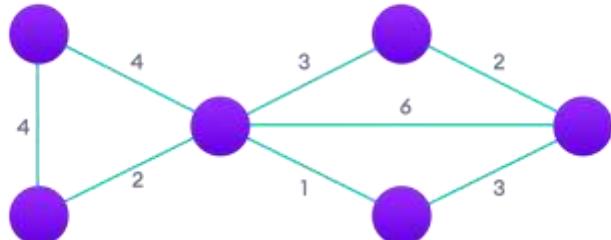
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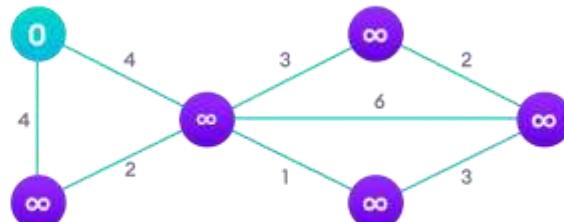
- |  |  |
|--|--|
|  | <ul style="list-style-type: none"><li>• <math>QQ</math>, a queue of all nodes in the graph. At the end of the algorithm's progress, <math>QQ</math> will be empty.</li><li>• <math>SS</math>, an empty set, to indicate which nodes the algorithm has visited. At the end of the algorithm's run, <math>SS</math> will contain all the nodes of the graph.</li></ul> |
|--|--|

It is easier to start with an example and then think about the algorithm.



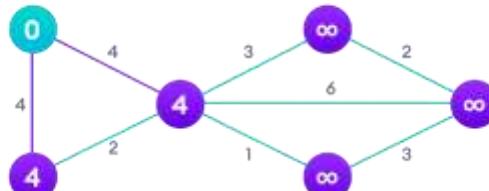
Step: 1

Start with a weighted graph



Step: 2

Choose a starting vertex and assign infinity path values to all other devices



Step: 3

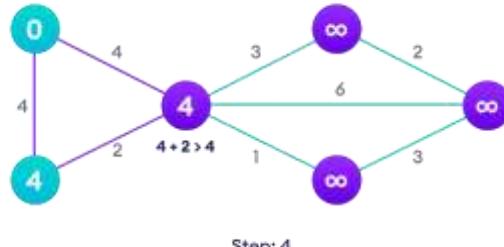


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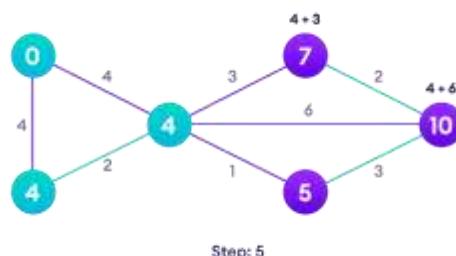
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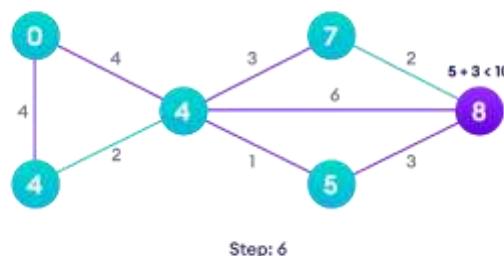
Go to each vertex and update its path length



If the path length of the adjacent vertex is lesser than new path length, don't update it



Avoid updating path lengths of already visited vertices





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	<p>After each iteration, we pick the unvisited vertex with the least path length. So we choose 5 before 7</p> <p>Step: 7</p> <p>Notice how the rightmost vertex has its path length updated twice</p> <p>Step: 8</p> <p>Repeat until all the vertices have been visited</p>
<b>PSEUDOCODE:</b>	<pre>1: function Dijkstra(Graph, source): 2:   for each vertex v in Graph:    // Initialization 3:     dist[v] := infinity // initial distance from source to vertex v is set to infinite 4:     previous[v] := undefined    // Previous node in optimal path from source 5:     dist[source] := 0 // Distance from source to source 6:     Q := the set of all nodes in Graph // all nodes in the graph are unoptimized - thus are in Q 7:     while Q is not empty:    // main loop 8:       u := node in Q with smallest dist[ ] 9:       remove u from Q 10:      for each neighbor v of u:    // where v has not yet been removed from Q. 11:        alt := dist[u] + dist_between(u, v) 12:        if alt &lt; dist[v]    // Relax (u,v)</pre>



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	13: dist[v] := alt 14: previous[v] := u 15: return previous[ ]
--	--

**EXPERIMENT 1**

<b>CODE:</b>	import java.util.*;  public class dijkstra {  int source;  // The main method is where the program starts. void dijkstra_solve(int[][] graph) { int count = graph.length; boolean[] visited = new boolean[count]; int[] distance = new int[count];  for (int i = 0; i < distance.length; i++) { distance[i] = Integer.MAX_VALUE; } distance[source] = 0; for (int k = 0; k < distance.length - 1; k++) {  int minVertex = findMin(distance, visited); visited[minVertex] = true; // explore the neighbours for (int i = 0; i < distance.length; i++) { if (graph[minVertex][i] != 0 && distance[minVertex] != Integer.MAX_VALUE) { // checking if the there exists an edge // between the two vertices, the neighbour // should not be visited // adding the weight of the edge to the // distance of the min vertex int newDistance = distance[minVertex] + graph[minVertex][i]; // Relaxation } } } } }
--------------	--



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```
        if (newDistance < distance[i]) {  
            // updating the distance of the vertex if the  
            // value is lesser than the  
            // previous value of the same vertex  
            distance[i] = newDistance;  
        }  
    }  
}  
}  
System.out.println("\nOutput\n(Vertex-> Distance):");  
for (int i = 0; i < distance.length; i++) {  
    if (distance[i] == Integer.MAX_VALUE || distance[i]  
== 0) {  
        continue;  
    }  
    System.out.println(i + "\t" + distance[i]);  
}  
  
// finding the minimum distance vertex  
static int findMin(int[] distance, boolean[] visited) {  
    int minVertex = -1; // initializing the minimum vertex  
    to -1  
    for (int i = 1; i < distance.length; i++) {  
        // if the vertex is not visited and the distance is  
        // lesser than the min vertex  
        if ((minVertex == -1 || distance[i] <  
distance[minVertex]) && !visited[i]) {  
            minVertex = i;  
        }  
    }  
    return minVertex; // returning the minimum vertex  
}  
  
public static void main(String[] args) throws Exception {  
    try // Driver code  
        Scanner sc = new Scanner(System.in));  
        System.out.println("-----Dijkstra's  
Algorithm-----");  
    }  
}
```



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```
System.out.println("\nInput(TestCases-> Vertices->
Edges-> Each edge with weights)\n");
int testCases = sc.nextInt();
dijkstra T = new dijkstra();
int negativeChecker = 0;
for (int i = 0; i < testCases; i++) {
    int v = sc.nextInt(); // vertices
    int e = sc.nextInt(); // edges
    int[][] graph = new int[1024][1024]; // adjacency matrix
    int src = sc.nextInt();
    T.source = src;
    int dest = sc.nextInt();
    int cost = sc.nextInt(); // cost of the edge // set
    to store the vertices
    // if cost is negative, then the edge is not added
    graph[src][dest] = cost;
    if (cost < 0) {
        negativeChecker = 1;
        System.out.print("\nNegative edge not
Added");
        continue;
    }
    for (int j = 0; j < e - 1; j++) {
        int p = sc.nextInt(); // source
        int q = sc.nextInt(); // destination
        cost = sc.nextInt(); // cost of the edge
        if (cost < 0) {
            negativeChecker = 1;
            System.out.print("\nNegative edge not
allowed");
            break;
        }
        graph[p][q] = cost;
    }
    if (negativeChecker != 1)
        T.dijkstra_solve(graph);
}
```



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	<pre>        sc.close();     } } }</pre>	
<b>OUTPUT:</b>	<pre>Input(TestCases-&gt; Vertices-&gt; Edges-&gt; Each e 1 3 3 5 7 1 6 5 2 7 6 3  Output (Vertex-&gt; Distance): 6      4 7      1</pre>	<pre>1 6 5 1 2 15 1 3 5 3 2 6 2 4 2 5 6 7</pre>
	<pre>1 6 7 10 20 3 20 40 -1 40 4 -2 4 80 1 4 2 -2 80 2 -4  Negative edge not allowed</pre>	<pre>Output (Vertex-&gt; Distance): 2      11 3      5 4      13</pre>
	<pre>1 7 7 15 3 2 15 19 5 3 14 3 19 8 2 19 10 4 14 8 1 8 16 5  Output (Vertex-&gt; Distance): 3      2 8      6 10     9 14     5 16     11 19     5</pre>	<pre>1 8 13 8 7 4 8 6 4 5 8 3 5 6 2 6 3 -2 6 4 4 3 7 3 3 4 -3 4 5 1 4 2 -2 1 4 -2 2 1 2  Negative edge not allowed</pre>



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		35	399	75	126	118	515	160	381
		36	425	76	499	119	494	161	412
		37	474	77	354	120	483	162	438
1	461	38	384	78	342	121	408	163	453
2	417	39	444	79	476	122	465	164	508
3	443	40	470	80	607	123	391	165	465
4	522	41	442	82	456	124	461	166	392
5	498	43	391	83	461	125	610	167	492
6	427	44	443	84	519	126	450	168	492
7	441	45	412	86	394	127	457	169	456
8	207	46	487	87	396	128	399	170	416
9	476	47	413	88	507	129	473	171	525
10	437	48	429	89	477	130	473	172	459
11	549	49	435	90	467	131	460	173	429
12	481	50	411	91	382	132	414	174	452
13	448	51	506	92	460	133	418	175	441
14	348	52	434	93	414	134	408	176	409
15	397	53	489	94	432	135	607	177	479
16	469	54	493	95	201	136	429	178	476
17	409	55	474	96	345	137	442	179	399
18	438	56	491	97	496	138	424	180	502
19	473	57	403	98	424	139	461	181	400
20	437	58	462	99	420	140	624	183	545
21	515	60	434	100	446	141	492	184	557
22	430	61	490	101	524	142	355	185	390
23	518	62	471	102	390	144	436	186	536
24	461	64	477	103	418	145	343	187	467
25	533	66	457	104	459	146	412	188	465
26	397	67	96	105	499	147	433	189	438
27	68	68	452	106	466	148	445	190	495
28	392	69	453	107	482	149	389	191	404
29	481	70	431	108	460	150	393	192	400
30	462	71	388	109	472	152	448	193	469
31	462	72	418	110	449	153	576	195	459
32	386	73	446	111	428	154	447	196	369
33	349	74	494	112	516	155	474	197	447
34				113	421	157	448	198	480
				114	288	158	408	199	524
				115	457	159	478	200	413
				116	494				
				117	555				



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201	477	246	436	288	444
202	623	247	380	289	463
203	468	249	532	291	394
204	312	250	356	292	452
205	384	251	39	293	514
206	531	252	443	294	431
207	515	253	491	295	552
209	468	254	456	296	432
210	371	255	513	297	401
211	420	256	441		
212	415	257	409		
213	355	258	497		
214	485	259	449		
217	509	260	440		
218	511	261	447		
219	438	262	498		
220	464	263	451		
221	385	264	456		
222	546	265	472		
223	461	266	361		
224	527	267	383		
226	477	268	476		
227	517	269	395		
228	432	270	530		
229	435	271	427		
230	454	272	528		
232	443	274	393		
233	442	275	402		
234	471	277	490		
235	483	278	415		
236	468	279	513		
237	490	280	446		
238	475	281	522		
240	386	282	493		
241	394	283	444		
242	498	284	426		
244	483	285	451		
245	464	286	412		
		287	503		



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Written Sum for the Dijkstra

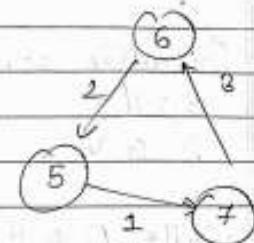
Pratik Rujau 2020300054

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No. of vertices: 3

No. of edges: 4

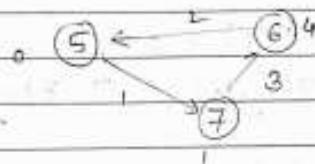
Edge 1	Edge 2	Distance
5	4	1
6	5	2
7	6	3



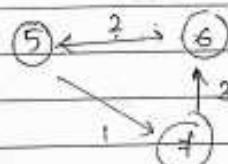
Source Vertex = 5

Initial dist [5] = 0

Path chosen: 5 7 (wt: 1)



Path chosen: 7, 6 (cost wt: 3)



Distance
5 → 0
6 → 4
7 → 1



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**TIME  
COMPLEXITY:**

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Dijkstra time complexity (Graph, Weight, Source)

Initialize source ( $G, s$ )  $\rightarrow O(V)$

$S = \emptyset$   $\rightarrow O(1)$

$P = G \cdot V$   $\rightarrow O(V)$  min heap

while  $Q \neq \emptyset$

$u = \text{Extract min}(Q)$   $O(\log V)$

$S = S \cup \{u\}$

for each vertex  $v \in G \cdot \text{Adj}[u]$

Relax ( $u, v, w$ )

$O(V \log V)$

$$\text{Overall } T_C = O(V) + O(1) + O(V) \\ + O(V \log V) + O(E \log V)$$

$$= O((V+E) \log V)$$

Matrix	Cost	Time
for $i = 1$	$C_1$	$V$
int $n = \min \text{dist}[l]$	$C_2$	$V$
visited [ $v$ ] = true	$C_3$	$V$
for $v = 1 \rightarrow V$	$C_4$	$V^2$
if ( $\text{distance}[v] +$ $\text{graph}[u][v] < \text{dist}[v]$ )	$C_5$	$V^2$

$$T_n = C_1 V + C_2 V + C_3 V + C_4 V^2 + C_5 V^2 \\ = O(V^2)$$

Both the time and space complexity obtained for the Dijkstra's algorithm is  $O(V^2)$  where  $V$  is the number of vertices. While if an adjacency list is used the time complexity will come out to be  $O(E \cdot \log V)$  where  $E$  is the number of edges. Since the Dijkstra's algorithm works on



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Greedy approach, it has a limitation that it doesn't work for the negative values of the distances because negative value and since Dijkstra works on the principle that once a vertex is discovered, it can't go back. Thus, it can't be used for the negative values

**CONCLUSION:** Learnt during the procedural programming of solving the problem

- Learnt about BackTracking and Dijkstra Algorithm
- Learnt how to use adjacency matrix in order to store the graph
- Learnt about time and space complexity of the shortest path algorithm
- Learnt about the advantages and disadvantages of dijkstra algorithm
- Learnt about the applications of a Dijkstra/ shortest path finder algorithm



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<b>UID no.</b>	<b>2020300054</b>	<b>Class:</b>	<b>Comps C Batch</b>
<b>Experiment No.</b>	7		

<b>AIM:</b>	To implement the concept of backtracking in subset sum problem
<b>THEORY:</b>	<p><b>What is Backtracking?</b></p> <p>Backtracking is an algorithmic-technique for solving problems recursively by trying to build a solution incrementally, one piece at a time, removing those solutions that fail to satisfy the constraints of the problem at any point of time (by time, here, is referred to the time elapsed till reaching any level of the search tree).</p> <p>There are three types of problems in backtracking –</p> <ol style="list-style-type: none"><li>1. Decision Problem – In this, we search for a feasible solution.</li><li>2. Optimization Problem – In this, we search for the best solution.</li><li>3. Enumeration Problem – In this, we find all feasible solutions.</li></ol> <p><b>When to Use a Backtracking Algorithm</b></p> <p>The backtracking algorithm is applied to some specific types of problems. For instance, we can use it to find a feasible solution to a decision problem. It was also found to be very effective for optimization problems.</p> <p>For some cases, a backtracking algorithm is used for the enumeration problem in order to find the set of all feasible solutions for the problem.</p> <p>On the other hand, backtracking is not considered an optimized technique to solve a problem. It finds its</p>



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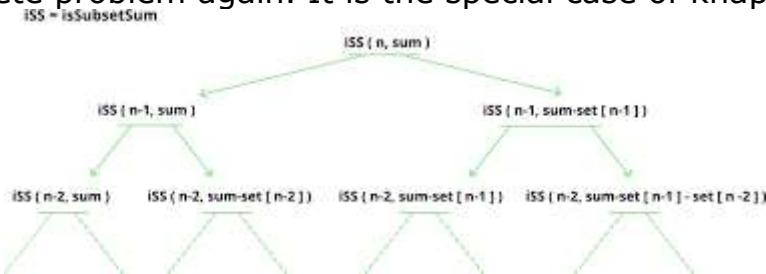
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application when the solution needed for a problem is not time-bounded.

### **Subset Sum Problem**

It is one of the most important problems in complexity theory. The problem is given an A set of integers  $a_1, a_2, \dots, a_n$  upto  $n$  integers. The question arises that is there a non-empty subset such that the sum of the subset is given as  $M$  integer?. For example, the set is given as  $[5, 2, 1, 3, 9]$ , and the sum of the subset is 9; the answer is YES as the sum of the subset  $[5, 3, 1]$  is equal to 9. This is an NP-complete problem again. It is the special case of knapsack



### **Example**

Consider the following array/ list of integers:

$\{1, 3, 2\}$

We want to find if there is a subset with sum 3.

Note that there are two such subsets  $\{1, 2\}$  and  $\{3\}$ . We will follow our backtracking approach.

Consider our empty set  $\{\}$

We add 1 to it  $\{1\}$  ( $\text{sum} = 1, 1 < 3$ )

We add 2 to it  $\{1, 3\}$  ( $\text{sum} = 3, 3 == 3$ , found)

We remove 3 from it  $\{1\}$  ( $\text{sum} = 1, 1 < 3$ )

We add 2 to it  $\{1, 2\}$  ( $\text{sum} = 3, 3 == 3$ , found)

We remove 2 and see that all elements have been considered.

Following diagram captures the idea:

$\{\} \rightarrow \{1\} \rightarrow \{1, 3\} \text{ (found)} \rightarrow \{1\} \rightarrow \{1, 2\} \text{ (found)} \rightarrow \{1\} \text{ (end)}$



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<b>PSEUDOCODE:</b>	subset_sum() if(subset is satisfying the constraint) print the subset exclude the current element and consider next element else generate the nodes of present level along breadth of tree and recur for next levels
--------------------	---

**EXPERIMENT 1**

<b>CODE:</b>	#include <stdio.h> #include <stdlib.h> static int total_nodes; void printValues(int A[], int size){ //prints the array printf("\n\n"); for (int i = 0; i < size; i++) { printf("%*d", 5, A[i]); } } void subset_sum(int s[], int t[], int s_size, int t_size, int sum, int ite, int const target_sum){ //increments the total node count total_nodes++; if (target_sum == sum) { // target sum found and printing the list. printValues(t, t_size); //finding all other valid pairs subset_sum(s, t, s_size, t_size - 1, sum - s[ite], ite + 1, target_sum); return; } else { //checking for all possible combinations for (int i = ite; i < s_size; i++) { t[t_size] = s[i];
--------------	--



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```
subset_sum(s, t, s_size, t_size + 1, sum
+ s[i], i + 1, target_sum);
}
}
}
void generateSubsets(int s[], int size, int target_sum){
//generating subsets of the initial array
int* tuplet_vector = (int*)malloc(size * sizeof(int));
subset_sum(s, tuplet_vector, size, 0, 0, 0, target_sum);
free(tuplet_vector);
}
int main(){
    int size;
    int target_sum;
//user input like array size and array and target sum
printf("\n-----");
printf("\nEnter the size of the set: ");
scanf("%d", &size);
int set[size];
printf("\n-----");
printf("\nEnter the elements of the set: ");
for (int i = 0; i < size; i++) {
    scanf("%d", &set[i]);
}
printf("The set is ");
printValues(set, size);
printf("\n-----");
printf("\nEnter the target sum: ");
scanf("%d", &target_sum);

//calling the functions and getting the result
generateSubsets(set, size, target_sum);
//printing the total nodes.
printf("\n\nTotal Nodes generated %d\n", total_nodes);
return 0;
}
```



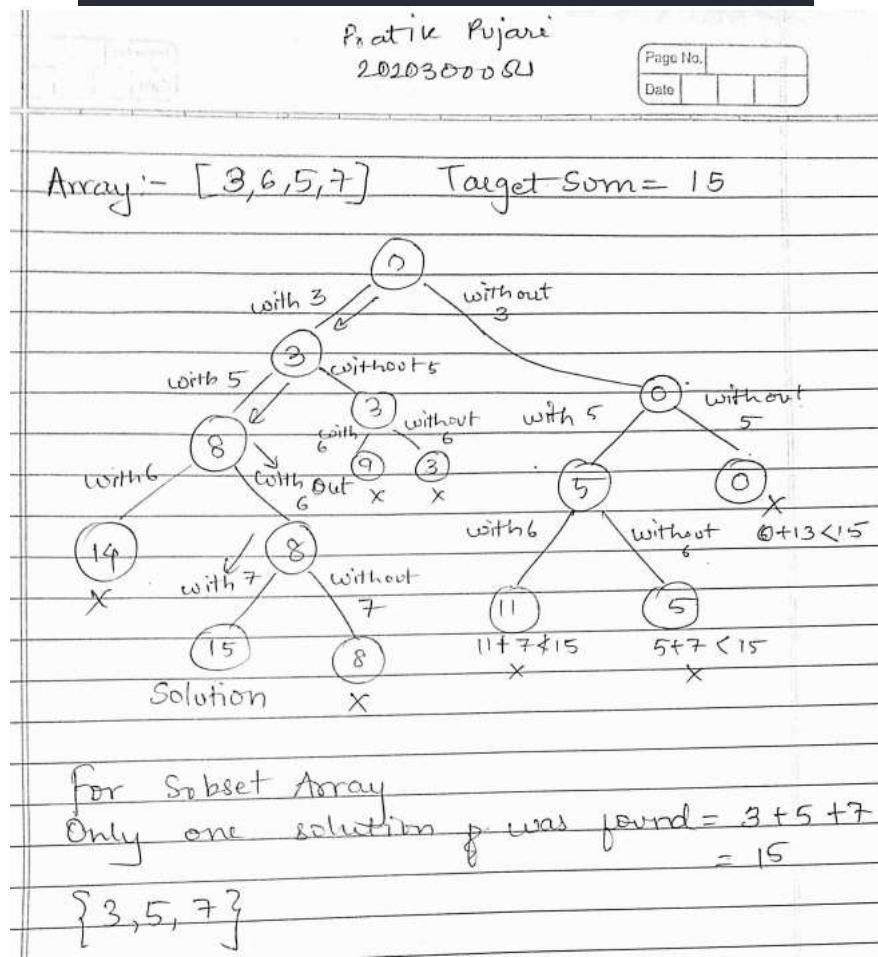
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**OUTPUT:**

```
-----  
Enter the size of the set: 6  
  
-----  
Enter the elements of the set: 12 1 61 5 9 2  
The set is  
  
12 1 61 5 9 2  
  
-----  
Enter the target sum: 24  
  
12 1 9 2  
  
-----  
Total Nodes generated 65
```





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<b>TIME COMPLEXITY:</b>	<p>The time complexity of the sum of subset problem is <math>O(2^n)</math> using backtracking approach ,where n is the number of elements in the array</p> <p>Since there will be two branches of each node and every element is atleast traversed once making it a binary tree.</p> <p>The total number of nodes can be calculated by the formula <math>2^{n-1}</math> where is the number of elements in the array.</p>
<p><b>CONCLUSION:</b> Things learnt during the procedural solving of the program.</p> <ul style="list-style-type: none"><li>• Learnt how to use backtracking for solving sums that use a binary tree structure.</li><li>• Learnt how to analyse the time complexity to find the number of nodes in diagram</li><li>• Learnt to implement backtracking solution to the subset problem.</li><li>• Learnt how to solve subset problem using the backtracking method using the tree way.</li></ul>	

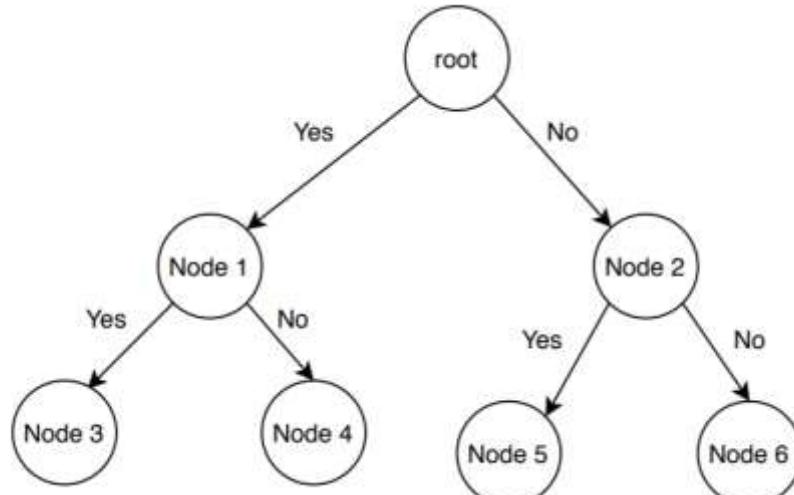


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<b>UID no.</b>	<b>2020300054</b>	<b>Class:</b>	<b>Comps C Batch</b>
<b>Experiment No.</b>	8		

<b>AIM:</b>	To implement 15 puzzle sum using the branch and bound
<b>THEORY:</b>	<p><b>What is Branch and Bound?</b></p> <p>Branch and bound algorithms are used to find the optimal solution for combinatorial, discrete, and general mathematical optimization problems. In general, given an NP-Hard problem, a branch and bound algorithm explores the entire search space of possible solutions and provides an optimal solution.</p> <p>A branch and bound algorithm consists of stepwise enumeration of possible candidate solutions by exploring the entire search space. With all the possible solutions, we first build a rooted decision tree. The root node represents the entire search space:</p>  <p><b>Advantages</b></p>



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In a branch and bound algorithm, we don't explore all the nodes in the tree. That's why the time complexity of the branch and bound algorithm is less when compared with other algorithms.  
If the problem is not large and if we can do the branching in a reasonable amount of time, it finds an optimal solution for a given problem.  
The branch and bound algorithm find a minimal path to reach the optimal solution for a given problem. It doesn't repeat nodes while exploring the tree.

### **Disadvantages**

The branch and bound algorithm are time-consuming.  
Depending on the size of the given problem, the number of nodes in the tree can be too large in the worst case.  
Also, parallelization is extremely difficult in the branch and bound algorithm.

Lets solve an example for branch and bound for 15 puzzle problem

1	2	3	4
5	6		8
9	10	7	11
13	14	15	12

Initial arrangement

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

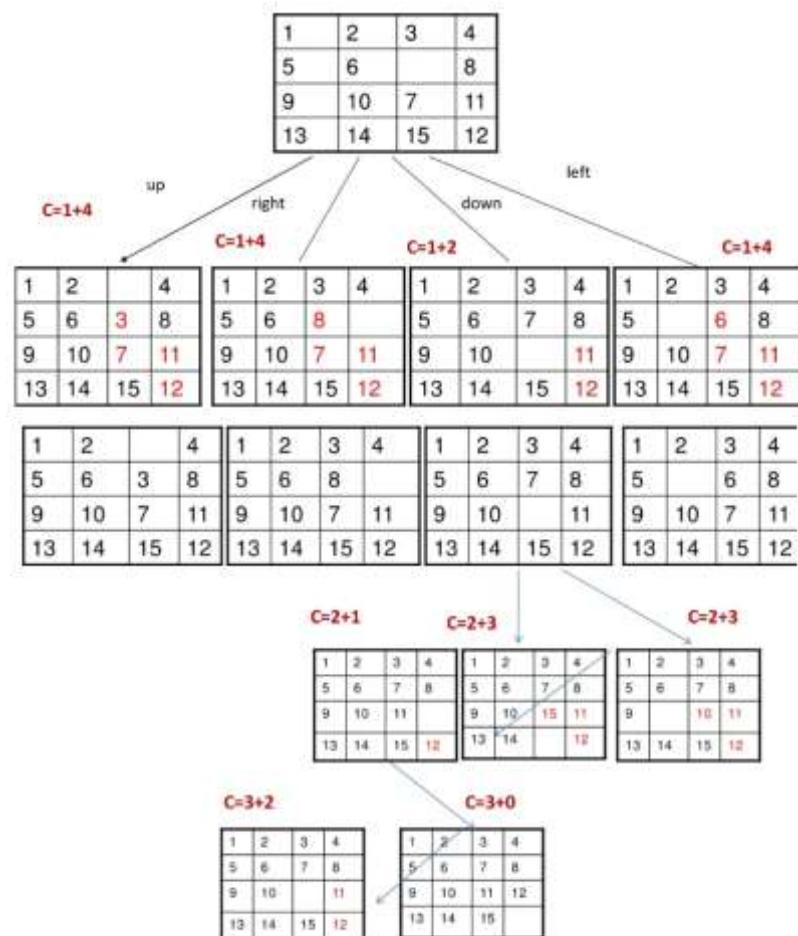
Goal arrangement



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**PSEUDOCODE:**

```
struct list_node
{
list_node *next;
list_node *parent;
float cost;
};
algorithm LCSearch(list_node *t)
{
    if (*t is an answer node)
    {
        print(*t);
        return;
    }
    E=t;
    Initialize the list of live nodes to be empty;
}

while (true) {
    for each child x of E
    {
        if x is an answer node {
            print the path from x to t;
            return; }
        Add (x);
        x->parent = E; }
        if there are no more live nodes
        {
            print ("No answer node");
            return;
        }
        E = Least();
    }
}
```



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**EXPERIMENT 1**

<b>CODE:</b>	<pre>import java.util.*;  class Node {     int[][] array;     int misplaced;     int recent;     // 1 up  2 right  3 down  4 left  }  class branchBounds {     int blankRow;     int blankRowIndex;// row index     int blackColIndex;// column index     int totalCost;     String isChoosen = "None";     int[][] targetMatrix = { { 1, 2, 3, 4 }, { 5, 6, 7, 8 }, { 9,         10, 11, 12 }, {             13, 14, 15, 0 } };      boolean isSolvable(int[][] arr, int row_len) {         int inv = inversions(arr); // method that counts the         // number of inversions (i &lt; j, arr[i] &gt; arr[j])         if (arr.length % 2 != 0) { // checks if n is odd             if (inv % 2 == 0) { // if the length is odd, and             // inversions are even the puzzle is solvable                 return true;             }             // row: even AND inversion: odd =&gt; solvable             // row: odd AND inversion: even =&gt; solvable         } else { // if n is even             if (this.blankRow % 2 == 0 &amp;&amp; inv % 2 != 0                    this.blankRow % 2 != 0 &amp;&amp; inv % 2 == 0) {                 return true;             }         }         return false;     } }</pre>
--------------	---



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```
}

int inversions(int[][] arr) {
    // count the number of inversions
    int no_inversions = 0;
    int[] arr2 = new int[arr.length * arr.length]; // 1d
array
    int k = 0;
    for (int i = 0; i < arr.length; i++) {
        for (int j = 0; j < arr.length; j++) {
            arr2[k] = arr[i][j]; // converting 2d array into 1d
array
            if (arr[i][j] == 0) { // blank tile(finding the index
of the blank tile)
                this.blankRow = arr.length - i; // finding the
index according to the convention (bottom->top):
                // 1..2 3.
                this.blankRowIndex = i;
                this.blackColIndex = j;
            }
            k++;
        }
    }
    printArray(arr);
    System.out.println();
    System.out.println("-----");
    System.out.println("X Mark is at -> " +
(blankRowIndex + 1) + ", " + (blackColIndex + 1));
    System.out.println("-----");

    System.out.println();
    for (int i = 0; i < arr2.length; i++) {
        for (int j = i + 1; j < arr2.length; j++) {
            if (arr2[i] > arr2[j] && arr2[j] != 0) { // not
considering the blank tile while finding out the
// inversions
            no_inversions++;
        }
    }
}
```



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```
        }
        System.out.println("Total inversions: " +
no_inversions);
        return no_inversions;
    }

    boolean isMatched(int[][] arr, int[][] sel) {
        // check if the selected array is the target array
        for (int i = 0; i < arr.length; i++) {
            for (int j = 0; j < arr.length; j++) {
                if (arr[i][j] != sel[i][j]) { // checks the array with
                    // the target array, as soon as it matches the while
                    // loop exits
                    return false;
                }
            }
        }
        return true;
    }

    int mismatch(int[][] arr) {
        // misplaced tiles
        int mislocations = 0;
        for (int i = 0; i < arr.length; i++) {
            for (int j = 0; j < arr.length; j++) {
                if (arr[i][j] != this.targetMatrix[i][j] && arr[i][j]
                != 0) { // checks the number of elements that dont
                    // match the target array
                    mislocations++;
                }
            }
        }
        return mislocations;
    }

    void solve(int[][] arr) {
        // Solving the puzzle
        int cost = Integer.MAX_VALUE;
```



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```
int level = 0;
int[][] temp_array = new int[arr.length][arr.length];

while (!isMatched(arr, targetMatrix)) {
    level++;
    for (int i = 0; i < arr.length; i++) {
        for (int j = 0; j < arr.length; j++) {
            // blank tile index
            // checking where the x mark is
            if (arr[i][j] == 0) {
                this.blankRow = arr.length - i;
                this.blankRowIndex = i;
                this.blackColIndex = j;
            }
        }
    }
    System.out.print("\nCosts->\n");
    int left[][] = leftShift(arr, temp_array,
    blankRowIndex, blackColIndex, level, cost);
    System.out.println("Left shift: " + ((int)
mismatch(left) + (int) level));

    int up[][] = upShift(arr, left, temp_array,
    blankRowIndex, blackColIndex, level, cost);
    System.out.println("Up shift: " + ((int)
mismatch(up) + (int) level));

    int right[][] = rightShift(arr, up, temp_array,
    blankRowIndex, blackColIndex, level, cost);
    System.out.println("Right shift: " + ((int)
mismatch(right) + (int) level));

    int[][] down = downShift(arr, right, temp_array,
    blankRowIndex, blackColIndex, level, cost);
    System.out.println("Down shift: " + ((int)
mismatch(down) + (int) level));
```



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```
for (int i = 0; i < down.length; i++) { // storing the
array for down shift
    for (int j = 0; j < down.length; j++) {
        down[i][j] = arr[i][j];
    }
}

if (blankRowIndex != arr.length - 1) {// checks if
the down shift is possible and doesnt go out of bounds
    int temp = down[blankRowIndex +
1][blackColIndex];
    down[blankRowIndex + 1][blackColIndex] =
down[blankRowIndex][blackColIndex];
    down[blankRowIndex][blackColIndex] = temp;
}

if (mismatch(down) + level <= cost) {// checking if
the cost is lower
    cost = mismatch(down) + level;
    for (int i = 0; i < left.length; i++) {
        for (int j = 0; j < left.length; j++) {
            temp_array[i][j] = down[i][j];
        }
    }
}

System.out.print("\nMinimum possible cost: " +
((int) mismatch(down) + (int) level) + "\n");

System.out.print("\nOperation performed: " +
isChoosen + "\n\n");

// after filtering through the whole level printing the
current
for (int i = 0; i < down.length; i++) {
    for (int j = 0; j < down.length; j++) {
        arr[i][j] = temp_array[i][j];
        // status of the matrix
    }
}
```



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```
        }
        printArray(arr);

        totalCost = totalCost + cost;

    }

    System.out.println("Total cost: " + totalCost);
}

public void printArray(int arr[][]) {
    for (int i = 0; i < arr.length; i++) {
        System.out.println("-----");
        for (int j = 0; j < arr.length; j++) {
            System.out.print(String.format("| %3d ", arr[i][j]));
        }
        System.out.println("|");
    }
    System.out.println("-----");
}

public int[][] leftShift(int[][] arr, int[][] temp_array, int blankRowIndex, int blackColIndex, int level, int cost) {
    // left shift
    int[][] left = new int[arr.length][arr.length];
    // storing the array for left shift
    for (int i = 0; i < left.length; i++) {
        for (int j = 0; j < left.length; j++) {
            left[i][j] = arr[i][j];
        }
    }
    // checks if the left shift is possible and doesnt go out of bounds
    if (blackColIndex != 0) {
        int temp = left[blankRowIndex][blackColIndex];
        left[blankRowIndex][blackColIndex] =
        left[blankRowIndex][blackColIndex - 1];
        left[blankRowIndex][blackColIndex - 1] = temp;
    }
}
```



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```
        }
        // checking if the cost is minimum
        if (mismatch(left) + level <= cost) {
            isChoosen = "Shifting left";
            cost = mismatch(left) + level; // assigning lower
cost
            for (int i = 0; i < left.length; i++) {
                for (int j = 0; j < left.length; j++) {
                    temp_array[i][j] = left[i][j]; // potential
candidate
                }
            }
        }
        return temp_array;
    }

    public int[][][] rightShift(int[][][] arr, int[][][] up, int[][][]
temp_array, int blankRowIndex, int blackColIndex,
        int level,
        int cost) {
        // right shift
        int[][] right = new int[arr.length][arr.length];
        // storing the array for right shift
        for (int i = 0; i < right.length; i++) {
            for (int j = 0; j < right.length; j++) {
                right[i][j] = arr[i][j];
            }
        }
        // checks if the right shift is possible and doesnt go
out of bounds
        if (blackColIndex != arr.length - 1) {
            int temp = right[blankRowIndex][blackColIndex];
            right[blankRowIndex][blackColIndex] =
right[blankRowIndex][blackColIndex + 1];
            right[blankRowIndex][blackColIndex + 1] = temp;
        }
        // checking if the cost is minimum
        if (mismatch(right) + level <= cost) {
            isChoosen = "Shifting right";
        }
    }
}
```



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```
cost = mismatch(right) + level; // assigning lower
cost
for (int i = 0; i < right.length; i++) {
    for (int j = 0; j < right.length; j++) {
        temp_array[i][j] = right[i][j]; // potential
candidate
    }
}
return temp_array;
}

public int[][] upShift(int arr[][], int left[][], int
temp_array[][], int blankRowIndex, int blankColIndex,
int level,
int cost) {
int[][] up = new int[arr.length][arr.length];

for (int i = 0; i < up.length; i++) {
    for (int j = 0; j < up.length; j++) {
        up[i][j] = arr[i][j]; // storing the array for up
shift
    }
}
if (blankRowIndex != 0) { // checks if the up shift is
possible and doesn't go out of bounds

    int temp = up[blankRowIndex - 1][blankColIndex];
    up[blankRowIndex - 1][blankColIndex] =
up[blankRowIndex][blankColIndex];
    up[blankRowIndex][blankColIndex] = temp;
}

if (mismatch(up) + level <= cost) { // checking if the
cost is lower
    isChoosen = "Shifting Up";
    cost = mismatch(up) + level;
    for (int i = 0; i < left.length; i++) {
        for (int j = 0; j < left.length; j++) {
```



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```
        temp_array[i][j] = up[i][j];
    }
}
return temp_array;
}

public int[][][] downShift(int arr[][][], int left[][][], int
temp_array[][][], int blankRowIndex, int blankColIndex,
int level,
int cost) {
int[][] down = new int[arr.length][arr.length];

for (int i = 0; i < down.length; i++) {
    for (int j = 0; j < down.length; j++) {
        down[i][j] = arr[i][j]; // storing the array for
down shift
    }
}
if (blankRowIndex != arr.length - 1) { // checks if the
down shift is possible and doesn't go out of bounds
    int temp = down[blankRowIndex +
1][blankColIndex];
    down[blankRowIndex + 1][blankColIndex] =
down[blankRowIndex][blankColIndex];
    down[blankRowIndex][blankColIndex] = temp;
}

if (mismatch(down) + level <= cost) { // checking if
the cost is lower
    isChoosen = "Shifting Down";
    cost = mismatch(down) + level;
    for (int i = 0; i < left.length; i++) {
        for (int j = 0; j < left.length; j++) {
            temp_array[i][j] = down[i][j];
        }
    }
}
return temp_array;
```



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```
}

}

public class puzzleSolver {
    public static void main(String[] args) {
        Scanner sc = new Scanner(System.in);

        branchBounds obj = new branchBounds();
        System.out.println("-----15 puzzle solve-----");
        System.out.println("Input Matrix: ");
        System.out.print("\nEnter the size of the matrix: ");
        int size = sc.nextInt();
        int[][] table = new int[size][size];
        System.out.print("\nEnter the elements of the matrix:");
    }

    for (int i = 0; i < size; i++) {
        for (int j = 0; j < size; j++) {
            table[i][j] = sc.nextInt();
        }
    }

    System.out.println("The Length of the puzzle is: " + table.length);
    if (obj.isSolvable(table, table[0].length)) {
        System.out.println("\nPuzzle is solvable");
        obj.solve(table); // solving the matrix
    } else {
        System.out.println("\n Puzzle is not solvable");
    }
    sc.close();
}
```



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**SCREENSHOT:**

**Not Solvable**

```
puzzlesolver
-----15 puzzle solve-----
Input Matrix:

Enter the size of the matrix: 4
The Length of the puzzle is: 4
-----
| 3 | 9 | 1 | 15 |
-----
| 14 | 11 | 4 | 6 |
-----
| 13 | 0 | 10 | 12 |
-----
| 2 | 7 | 8 | 5 |
-----

X Mark is at -> 3, 2
-----
Total inversions: 56

Puzzle is not solvable
```

**Solvable**

```
puzzlesolver
-----15 puzzle solve-----
Input Matrix:

Enter the size of the matrix: 4

Enter the elements of the matrix: 1 2 3 0 5 6 7 4 9 10 11 8 13 14 15 12
The Length of the puzzle is: 4
-----
| 1 | 2 | 3 | 0 |
-----
| 5 | 6 | 7 | 4 |
-----
| 9 | 10 | 11 | 8 |
-----
| 13 | 14 | 15 | 12 |
-----

X Mark is at -> 1, 4
-----
Total inversions: 9

Puzzle is solvable
```



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Costs->  
Left shift: 5  
Up shift: 4  
Right shift: 4  
Down shift: 3

Minimum possible cost: 3

Operation performed: Shifting Down

1	2	3	4	
5	6	7	0	
9	10	11	8	
13	14	15	12	

Costs->  
Left shift: 4  
Up shift: 4  
Right shift: 4  
Down shift: 3

Minimum possible cost: 3

Operation performed: Shifting Down

1	2	3	4	
5	6	7	8	
9	10	11	0	
13	14	15	12	



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	<p>Costs-&gt; Left shift: 4 Up shift: 4 Right shift: 4 Down shift: 3</p> <p>Minimum possible cost: 3</p> <p>Operation performed: Shifting Down</p> <table border="1"><tr><td>1</td><td>2</td><td>3</td><td>4</td><td></td></tr><tr><td>5</td><td>6</td><td>7</td><td>8</td><td></td></tr><tr><td>9</td><td>10</td><td>11</td><td>12</td><td></td></tr><tr><td>13</td><td>14</td><td>15</td><td>0</td><td></td></tr></table> <p>Total cost: 9</p>	1	2	3	4		5	6	7	8		9	10	11	12		13	14	15	0	
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5	6	7	8																		
9	10	11	12																		
13	14	15	0																		
<b>TIME COMPLEXITY</b>	<p>The Time complexity of the branch and Bound algorithm is given below: <b>Time Complexity: <math>O(n^2)</math></b></p> <p>The Space Complexity of the branch and Bound Algorithm is given below: <b>Space Complexity: <math>O(n)</math></b></p>																				



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## OUTPUT:

	<table border="1"> <tr><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr><td>5</td><td>6</td><td>X</td><td>8</td></tr> <tr><td>9</td><td>10</td><td>7</td><td>11</td></tr> <tr><td>13</td><td>14</td><td>15</td><td>12</td></tr> </table>	1	2	3	4	5	6	X	8	9	10	7	11	13	14	15	12
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**CONCLUSION:** Things learnt during the procedural programming of the problem of branch and bound

- Learnt one of the most popular algorithms used in the optimization problem is the branch and bound algorithm.
  - Learnt about some advantages and disadvantages of the branch and bound algorithm
  - Also noted how and when a branch and bound algorithm would be the right choice for a user to use.
  - Used branch and bound based algorithm for solving the 15 puzzle problem.



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<b>Name</b>	<b>Pratik Pujari</b>		
<b>UID no.</b>	<b>2020300054</b>	<b>Class:</b>	<b>Comps C Batch</b>
<b>Experiment No.</b>	9		

<b>AIM:</b>	To implement approximation algorithms
<b>THEORY:</b>	<p><b>What is the Approximation algorithm?</b> An approximation algorithm is a way of dealing with NP-completeness for an optimization problem. This technique does not guarantee the best solution. The goal of the approximation algorithm is to come close as much as possible to the optimal solution in polynomial time. Such algorithms are called approximation algorithms or heuristic algorithms.</p> <p><b>Features of Approximation Algorithm :</b> Here, we will discuss the features of the Approximation Algorithm as follows.</p> <ul style="list-style-type: none"><li>• An approximation algorithm guarantees to run in polynomial time though it does not guarantee the most effective solution.</li><li>• An approximation algorithm guarantees to seek out high accuracy and top quality solution(say within 1% of optimum)</li><li>• Approximation algorithms are used to get an answer near the (optimal) solution of an optimization problem in polynomial time</li></ul> <p><b>Performance Ratios</b> Suppose we work on an optimization problem where every solution carries a cost. An Approximate Algorithm returns a legal solution, but the cost of that legal solution may not be optimal.</p>



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For Example, suppose we are considering for a minimum size vertex-cover (VC). An approximate algorithm returns a VC for us, but the size (cost) may not be minimized.

Another Example is we are considering for a maximum size Independent set (IS). An approximate Algorithm returns an IS for us, but the size (cost) may not be maximum. Let  $C$  be the cost of the solution returned by an approximate algorithm, and  $C^*$  is the cost of the optimal solution.

We say the approximate algorithm has an approximate ratio  $P(n)$  for an input size  $n$ , where

$$\max \left( \frac{C}{C^*}, \frac{C^*}{C} \right) \leq P(n)$$

Intuitively, the approximation ratio measures how bad the approximate solution is distinguished with the optimal solution. A large (small) approximation ratio measures the solution is much worse than (more or less the same as) an optimal solution.

**Applications of Approximation algorithm :**

**1. The Vertex Cover Problem**

In the vertex cover problem, the optimization problem is to find the vertex cover with fewest vertices, and the approximation problem is to find the vertex cover with few vertices.

**2. Travelling Salesman Problem**

In the traveling salesperson problem, the optimization problem is to find the shortest cycle, and the approximation problem is to find a short cycle.

**3. The Set Covering Problem**



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	<p>This is an optimization problem that models many problems that require resources to be allocated. Here, a logarithmic approximation ratio is used.</p> <p><b>4. The Subset Sum Problem</b></p> <p>In the Subset sum problem, the optimization problem is to find a subset of <math>\{x_1, x_2, x_3, \dots, x_n\}</math> whose sum is as large as possible but not larger than target value <math>t</math>.</p> <p>Set Covering Problem</p> $U = \{1, 2, 3, 4, 5\}$ $S = \{S_1, S_2, S_3\}$ $S_1 = \{4, 1, 3\}, \quad \text{Cost}(S_1) = 5$ $S_2 = \{2, 5\}, \quad \text{Cost}(S_2) = 10$ $S_3 = \{1, 4, 3, 2\}, \quad \text{Cost}(S_3) = 3$ <p><b>Output:</b> Minimum cost of set cover is 13 and set cover is <math>\{S_2, S_3\}</math></p> <p>There are two possible set covers <math>\{S_1, S_2\}</math> with cost 15 and <math>\{S_2, S_3\}</math> with cost 13.</p>
<b>PSEUDOCODE:</b>	<p><u>Algorithm</u></p> <ol style="list-style-type: none"><li>1. <math>C \leftarrow \emptyset</math></li><li>2. While <math>C \neq U</math> do<ol style="list-style-type: none"><li>Find the set whose cost effectiveness is smallest, say <math>S</math></li><li>Let <math>\alpha = \frac{c(S)}{ S - C }</math></li><li>For each <math>e \in S - C</math>, set <math>\text{price}(e) = \alpha</math></li><li><math>C \leftarrow C \cup S</math></li></ol></li><li>3. Output picked sets</li></ol>



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**EXPERIMENT 1**

<b>CODE:</b>	<pre>import java.util.HashSet; import java.util.Scanner; import java.util.Set;  public class set_covering {     // calculate minimum cost of set cover     public static void minCost(Set&lt;Integer&gt; Univ, Set&lt;Integer&gt; S[], int cost[], int m) {         // Set I is used to get minimum cost         Set&lt;Integer&gt; I = new HashSet&lt;Integer&gt;();         int minCost = 0;         System.out.print("Best possible Solution sets\n");         // Loop continues till the time I contains all the elements of U         while (!I.equals(Univ)) {              // calculate the min cost             double min = Double.MAX_VALUE;             // loop to find the minimum cost             int index = -1;             double effCost[] = new double[m];             for (int i = 0; i &lt; m; i++) {                 // diff is difference between S[i] and I                 Set&lt;Integer&gt; diff = new HashSet&lt;Integer&gt;(S[i]);                 diff.removeAll(I);                 // if size of diff != 0                 if (diff.size() != 0) {                     effCost[i] = cost[i] / diff.size();                 }                 // if size of diff = 0                 else {                     effCost[i] = Double.MAX_VALUE;                 }                 // change min if effCost[i] &lt; min                 if (min &gt; effCost[i]) {</pre>
--------------	---



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```
        min = effCost[i];
        index = i;
    }
}
// Printing the S[i]
System.out.print("S[" + (index + 1) + "] ");
// Used to get union of I and S[index]
Set<Integer> union = new HashSet<Integer>(I);
union.addAll(S[index]);
// I = union of I and S[index]
I = union;
// Min cost
minCost = minCost + cost[index];
}
System.out.println(" Minimum cost: " + minCost);
}

public static void main(String args[]) {
    Scanner sc = new Scanner(System.in);
    // no of elements in U

    System.out.println("-----SET COVERING
PROBLEM-----\n");
    System.out.print("Enter the elements in Universal Set
: ");
    int n = sc.nextInt();
    // for storing the values in Universal set
    System.out.print("Enter the elements -> ");
    Set<Integer> Univ = new HashSet<Integer>();
    for (int i = 0; i < n; i++) {
        int temp = sc.nextInt();
        Univ.add(temp);
    }
    // no of sets in S
    System.out.print("Enter the number of sets : ");
    int m = sc.nextInt();
    // for storing the values in sets
    Set<Integer>[] S = new HashSet[m];
    int cost[] = new int[m];
```



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	<pre>for (int i = 0; i &lt; m; i++) {     S[i] = new HashSet&lt;Integer&gt;();     System.out.println("\nSet [" + (i + 1) + "] Details: ");     // no of elements in S[i]     System.out.print("Enter the elements : ");     String input = sc.next();     String[] elements = input.split(" ");     for (int j = 0; j &lt; elements.length; j++) {         S[i].add(Integer.parseInt(elements[j]));     }     // Cost of S[i]     System.out.print("Enter the cost(S[" + (i + 1) + "]) : ");     cost[i] = sc.nextInt(); } // method to get min cost minCost(Univ, S, cost, m); sc.close(); }</pre>
<b>OUTPUT:</b>	<pre>----- Subset Covering Prob -----  Enter the elements in Universal Set : 5 Enter the elements -&gt; 1 2 3 4 5 Enter the number of sets : 4  -----S[1]-----  Enter the elements of S[1]: 2 Enter the elements : 5 4 Enter the cost(S[1]) : 3  -----S[2]-----  Enter the elements of S[2]: 3 Enter the elements : 1 2 3 Enter the cost(S[2]) : 5</pre>



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```
-----S[3]-----  
Enter the elements of S[3]: 3  
Enter the elements : 1 2 3  
Enter the cost(S[3]) : 1
```

```
-----S[4]-----  
Enter the elements of S[4]: 5  
Enter the elements : 1 2 3 4 5  
Enter the cost(S[4]) : 5  
Best possible Solution sets is:  
->S[3] S[1] Minimum cost: 4
```

$U = \{5, 4, 3, 2, 1\}$

$S_1 = \{5, 4\}$   $c(S_1) = 3$        $S_2 = \{3, 2, 1\}$   $c(S_2) = 5$        $S_3 = \{3, 2, 1\}$   $c(S_3) = 1$

$S_4 = \{5, 4, 3, 2, 1\}$   $c(S_4) = 5$

Possible set  $\{S_1, S_2\}$  effective cost = 8

$\{S_1, S_3\}$  effective cost = 4

$\{S_4\}$  effective cost = 5

$\therefore$  Best possible combination =  $\{S_1, S_3\}$   
as the lowest cost is '4'



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<b>TIME COMPLEXITY</b>	<p>Time complexity</p> <p>let const be <math>\alpha</math></p> <p>Assume</p> <p><math>k-1</math> elements covered</p> <p>cost of <math>k/n &lt; \frac{n}{n-k+1}</math></p> <p><math>\therefore k^{\text{th}}</math> not covered</p> <p><math>\therefore</math> Per element cost is picked in set <math>\leq \alpha</math></p> <p>as greedy approx makes most cost off</p> <p>cost = sum of cost of all elements</p> <p>But <math>k = 1, 2, \dots, n</math></p> <p>cost <math>\leq \alpha(1 + \frac{1}{2} + \dots + \frac{1}{n})</math></p> <p><math>\therefore</math> <math>\leq \alpha \log n</math></p>
------------------------	---

**TIME & SPACE COMPLEXITY**

The time complexity for approximation problem using greedy approximation approach is found out to be  $O(\log n)$ .

**CONCLUSION:** After the procedural programming through this experiment:

- Learnt about NP complete problem and how approximation algorithm is used to solve it.
- Implemented Set covering problem where used sets for storing the subsets and calculated the minimum cost for covering the sets.
- Derived the Time Complexity for the Set Covering Problem using Approximation algorithm and found it to be  $O(\log n)$ .
- Further for more explanation, I solved the Subset Covering Problem and for presented its working and commented on the time complexity which was derived and justified the derivation.



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<b>UID no.</b>	<b>2020300054</b>	<b>Class:</b>	<b>Comps C Batch</b>
<b>Experiment No.</b>	10		

<b>AIM:</b>	Given a universe $U$ of $n$ elements, a collection of subsets of $U$ say $S = \{S_1, S_2, \dots, S_m\}$ where every subset $S_i$ has an associated cost. Find a minimum cost subcollection of $S$ that covers all elements of $U$ .
<b>THEORY:</b>	<p><b>NAÏVE STRING MATCHING</b></p> <p>The naïve approach tests all the possible placement of Pattern <math>P</math> [1.....m] relative to text <math>T</math> [1.....n]. We try shift <math>s = 0, 1, \dots, n-m</math>, successively and for each shift <math>s</math>. Compare <math>T</math> [<math>s+1, \dots, s+m</math>] to <math>P</math> [1.....m].</p> <p>The naïve algorithm finds all valid shifts using a loop that checks the condition <math>P</math> [1.....m] = <math>T</math> [<math>s+1, \dots, s+m</math>] for each of the <math>n - m + 1</math> possible value of <math>s</math>.</p> <p><b>NAIVE-STRING-MATCHER (<math>T, P</math>)</b></p> <ol style="list-style-type: none"><li>1. <math>n \leftarrow \text{length } [T]</math></li><li>2. <math>m \leftarrow \text{length } [P]</math></li><li>3. for <math>s \leftarrow 0</math> to <math>n - m</math></li><li>4. do if <math>P</math> [1.....m] = <math>T</math> [<math>s + 1, \dots, s + m</math>]</li><li>5. then print "Pattern occurs with shift" <math>s</math></li></ol> <p><b>Comparisons made with naïve algorithm</b></p> <ul style="list-style-type: none"><li>• <b>Worst case running time:</b><ul style="list-style-type: none"><li>– Test every position</li><li>– <math>P=aaaa, T=aaaaaaaaaaa</math></li></ul></li><li>• <b>Best case running time:</b><ul style="list-style-type: none"><li>– Test only first position</li><li>– <math>P=bbbb, T=aaaaaaaaaaa</math></li></ul></li></ul> <p>Can we do better?</p>



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**CALCULATION OF VALUE OF H IN TC O(LOG(M)).**  
**HINT MODULAR EXPONENTIATION**

Modular exponentiation is exponentiation performed over a modulus.

Modular exponentiation is the remainder when an integer  $b$  (the base) is raised to the power  $e$  (the exponent), and divided by a positive integer  $m$  (the modulus); i.e.,  $c = b^e \text{ mod } m$ .

It is  $O(\log(m))$  because the value is calculated as the sum of geometric progression depending on the value of  $d$  and  $m$  along with using the modulus with  $q$ .

**ROLLING HASH:**

Consider this example. Let's say you have a string "abcdeacdoe" & you want to find the pattern "bcd" in this string.

Now with the naive way, you will try to compare each character of pattern ("bcd") with the string characters ("abcdabc") forming three-character strings. This might be really inefficient especially as the input gets larger. If you use rolling hash, you can do this intelligently.

First you calculate the hash of first three letter substring (abc) in the string. To keep matters simple, let's say we use base 7 for this calculation (in the actual scenario we should mod it with a prime to avoid overflow):

**substring1** =  $a*7^0 +$

$b*7^1 + c*7^2 = 97*1 + 98*7 + 99*49 = 5634$  **pattern** =

$b*7^0 + c*7^1 + d*7^2 = 98*1 + 99*7 + 100*49 = 5691$

So you compare two hash values and since they are different, you move forward. Now you reach to second substring "bcd" in string. Here is where the fun begins. We can calculate the hash of this string without rehashing everything. As you can see, the window has moved forward only by dropping one character and adding another: a<-bc<-d

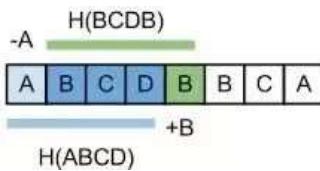
Below diagrams explains it better (though it's working on a four-letter string rather than three):



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\* credits: quora

From hash value we drop the first character value, divide the left out value with the base number and add the new char to the rightmost position with appropriate power of base: Here are the steps:

drop a =>  $(a*7^0 + b*7^1 + c*7^2) - a*7^0$

divide everything by 7 =>  $(b*7^1 + c*7^2)/7 \Rightarrow b*7^0 + c*7^1$

add d =>  $b*7^0 + c*7^1 + d*7^2$  [the power of base for d is (pattern-length-1)]

Thus new hash for "bcd" in input string would be =>  $(5634 - 97)/7 + 100*49 = 5691$  which matches pattern hash value. Now we can go ahead and compare the strings to verify if they are in fact same.

Visualize doing this for a large string which comprising of lets say, 30 lines. You will be able to get new hash value in constant time without much effort by just dropping and adding new character(s).

In real use cases, to avoid overflow because of large power, we will have to do modulo with large prime. This technique is often used for multi-pattern string search and forms the core of algorithms like the Karp-Rabin algorithm. This algorithm is an excellent choice to detect plagiarism.

**WHY ARE PRIME NUMBERS USED FOR CONSTRUCTING HASH FUNCTIONS?**

The reason why prime numbers are used is to minimize collisions when the data exhibits some particular patterns. First things first: If the data is random then there's no need for a prime number, you can do a mod operation against any number and you will have the same number of collisions for each possible value of the modulus.

But when data is not random then strange things happen. For example, consider numeric data that is always a multiple of 10.



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If we use mod 4 we find:

$$10 \bmod 4 = 2$$

$$20 \bmod 4 = 0$$

$$30 \bmod 4 = 2$$

$$40 \bmod 4 = 0 \quad 50 \bmod 4 = 2$$

*So from the 3 possible values of the modulus (0, 1, 2, 3), only 0 and 2 will have collisions, that is bad.*

If we use a prime number like 7:

$$10 \bmod 7 = 3$$

$$20 \bmod 7 = 6$$

$$30 \bmod 7 = 2$$

$$40 \bmod 7 = 4$$

$$50 \bmod 7 = 1$$

We also note that 5 is not a good choice but 5 is prime the reason is that all our keys are a multiple of 5. This means we have to choose a prime number that doesn't divide our keys, choosing a large prime number is usually enough.

So erring on the side of being repetitive the reason prime numbers are used is to neutralize the effect of patterns in the keys in the distribution of collisions of a hash function.

### **THE RABIN-KARP ALGORITHM**

Like the **Naive Algorithm**, the Rabin-Karp algorithm also slides the pattern one by one. But unlike the Naive algorithm, the Rabin Karp algorithm **matches the hash value** of the pattern with the hash value of the current substring of text, and if the hash values match then only it starts matching individual characters. So Rabin Karp algorithm needs to calculate hash values for the following strings.

1. Pattern itself.
2. All the substrings of the text of length m.

Since we need to efficiently calculate hash values for all the substrings of size m of text, we must have a hash function that has the following property.

Hash at the next shift must be efficiently computable from the current hash value and next character in text or we can say  $\text{hash}(\text{txt}[s+1 .. s+m])$  must be efficiently computable from  $\text{hash}(\text{txt}[s .. s+m-1])$  and  $\text{txt}[s+m]$  i.e.,  $\text{hash}(\text{txt}[s+1 .. s+m]) = \text{hash}(\text{txt}[s .. s+m-1]) \cdot \text{hash}(\text{txt}[s+m])$



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.. s+m]) = rehash(txt[s+m], hash(txt[s .. s+m-1])) and  
rehash must be O(1) operation.  
The hash function suggested by Rabin and Karp calculates  
an integer value. The integer value for a string is the  
numeric value of a string.

```
Input: txt[] = "THIS IS A TEST TEXT"
       pat[] = "TEST"
Output: Pattern found at index 10
```

```
Input: txt[] = "AABAACAAADAABAABA"
       pat[] = "AABA"
Output: Pattern found at index 0
        Pattern found at index 9
        Pattern found at index 12
```

Text : **A A B A A C A A D A A B A A B A**

Pattern : **A A B A**

<b>A A B A</b>	<b>A A B A</b>
<b>A A B A A C A A D A A B A A B A</b>	
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	
	<b>A A B A</b>

**Pattern Found at 0, 9 and 12**

*\*credits: documentation*

**Limitations of the Rabin-Karp Algorithm**

***Spurious Hit***

When the hash value of the pattern matches with the hash value of a window of the text but the window is not the actual pattern then it is called a spurious hit.

Spurious hit increases the time complexity of the algorithm. In order to minimize spurious hits, we use modulus. It greatly reduces the spurious hit.



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<b>PSEUDOCODE:</b>	<pre>n = t.length m = p.length h = dm-1 mod q p = 0 t0 = 0 for i = 1 to m     p = (dp + p[i]) mod q     t0 = (dt0 + t[i]) mod q for s = 0 to n - m     if p = ts if p[1.....m] = t[s + 1..... s + m]         print "pattern found at position" s     If s &lt; n-m         ts + 1 = (d (ts - t[s + 1]h) + t[s + m + 1]) mod q</pre>
--------------------	---

**EXPERIMENT 10**

<b>CODE:</b>	<pre>import java.util.ArrayList; import java.util.Scanner;  public class rabinKarp {     static ArrayList&lt;Integer&gt; locations = new ArrayList&lt;Integer&gt;();      // d is the number of characters in the input alphabet     public final static int         static void search(String pat, String txt, int q) {             int d = 256;             int M = pat.length();             int N = txt.length();             int i, j;             int p = 0; // hash value for pattern             int t = 0; // hash value for txt             int h = 1;             // The value of h would be "pow(d, M-1)%q"             for (i = 0; i &lt; M - 1; i++)                 h = (h * d) % q;             // Calculate the hash value of pattern and first</pre>
--------------	--



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```
// window of text
for (i = 0; i < M; i++) {
    p = (d * p + pat.charAt(i)) % q;
    t = (d * t + txt.charAt(i)) % q;
}
// Slide the pattern over text one by one
for (i = 0; i <= N - M; i++) {
    // Check the hash values of current window of text
    // and pattern. If the hash values match then only
    // check for characters one by one if (p == t) {
    /* Check for characters one by one */
    for (j = 0; j < M; j++) {
        if (txt.charAt(i + j) != pat.charAt(j))
            break;
    }
    // if p == t and pat[0...M-1] = txt[i, i+1,...i+M-1]
    if (j == M) {
        locations.add(i);
        System.out.println("Pattern found at index " +
i);
    }
}
// Calculate hash value for next window of text:
Remove
// leading digit, add trailing digit
if (i < N - M) {
    t = (d * (t - txt.charAt(i) * h) + txt.charAt(i + M)) %
q;
    // We might get negative value of t, converting it
    // to positive
    if (t < 0)
        t = (t + q);
}
/*
/* Driver program to test above function */
public static void main(String[] args) {
    Scanner sc = new Scanner(System.in);
    System.out.print("\nEnter the text: ");
```



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	<pre>String txt = sc.next(); System.out.print("\nEnter the pattern: "); String pat = sc.next(); int q = 101; // A prime number search(pat, txt, q); System.out.print("\nPattern found at following locations: \n"); for (int i = 0; i &lt; txt.length(); i++) {     System.out.print(txt.charAt(i)); } System.out.print("\n"); for (int i = 0; i &lt; txt.length() &amp;&amp; locations.size() != 0; i++) {     if (i == locations.get(0)) {         System.out.print("^");         locations.remove(0);     } else         System.out.print(" "); } sc.close(); }</pre>
<b>OUTPUT:</b>	<pre>Enter the text: aabadjwdaaab  Enter the pattern: aab Pattern found at index 0 Pattern found at index 9  Pattern found at following locations: aabadjwdaaab ^          ^</pre>



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Enter the text: helloworldworlelloworld

Enter the pattern: elow

Pattern found at index 1

Pattern found at index 15

Pattern found at following locations:

helloworldworlelloworld

^

^

Rabin-Karp Algorithm

Pratik Pujari

Text : 1 2 3 4 5 6 7 8 9 10 11  
c c a c c a a d b a

Pattern : 1 2 3  
d b a

d b a

$$4 \times 10^2 + 3 \times 10^1 + 10^0 \\ = 421$$

cc a

$$3 \times 10^2 + 3 \times 10^1 + 10^0$$

$$= 321$$

base c

$$= [(3 \times 10^2 + 1 \times 10^1 + 3 \times 10^0) \\ - 3 \times 10^2] \times 10$$

$$= 313$$

c c a

$$= 313 - 300 = 12 \times 10 + 3$$

$$= 133$$

Continuing with further pairs matching at  
index 9 & 10

No spurious hits

$O(mn-m)$  → Average

In case of spurious hit -  $O(nm)$

If value is exceeding the datatype using  
modulo.



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**TIME COMPLEXITY:**

Time complexity Analysis Pratik Pujari

Hence call hash( $s[1..m]$ )  
for Index from 1 to  $n-m+1$ .  
|| code to check if substring  
|| matches  
call hash( $s[index+1..index+m]$ )

Compared to  $\Theta(mn)$ , additive  $\Theta(m)$   
is largely ignored  
giving  $\Theta(m)+\Theta(n)+\Theta(mn)=\Theta(mn)$

Time complexity is  $\Theta(m+n)$  for  
cases when-

hash( $s[1..m]$ )  
index from 1 to  $n-m+1$   
|| code to check if substring matches  
call hash( $s[index+1..index+m]$ )  
|| which takes  $\Theta(1)$ , applies rolling hash

Giving  $\Theta(m)+\Theta(n)+\Theta(1)$   
 $= \Theta(m)+\Theta(n)$   
 $= \Theta(m+n)$

Assume the text is length  $n$  and the length of the word is  $m$ .  
The **best- and average-case** running time of Rabin-Karp  
is  **$\Theta(m+n)$**  because the rolling hash step takes  $\Theta(n)$  time  
and once the algorithm finds a potential match, it must  
verify each letter to make sure that the match is true



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	<p>The <b>worst-case</b> running time of Rabin-Karp is <b><math>O(nm)</math></b>. This would occur with an extremely awful hash function that resulted in a false positive at each step.</p>
<b>CONCLUSION:</b>	Things learnt during the procedural programming of the problem <ul style="list-style-type: none"><li>• Learnt the concept related to most of the String matching algorithms.</li><li>• Implemented Rabin Karp algorithm where the hash value of the text and the pattern is calculated and found out all the answers to additional questions assigned.</li><li>• Learnt how to find out the best and worst case time complexity.</li><li>• Also learnt how to solve Rabin Karp problems on paper.</li></ul>