Vorticity

Vorticity:

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_0 \\ \omega_1 \\ \omega_2 \end{pmatrix}$$

Velocity:

$$\mathbf{v} = \begin{pmatrix} v_0 \\ v_1 \\ v_2 \end{pmatrix}$$

Velocity Gradient:

$$\begin{aligned} \nabla \mathbf{v} &= \begin{pmatrix} \frac{\partial v_0}{\partial x_0} & \frac{\partial v_1}{\partial x_0} & \frac{\partial v_2}{\partial x_0} \\ \frac{\partial v_0}{\partial x_1} & \frac{\partial v_1}{\partial x_1} & \frac{\partial v_2}{\partial x_1} \\ \frac{\partial v_0}{\partial x_2} & \frac{\partial v_1}{\partial x_2} & \frac{\partial v_2}{\partial x_2} \end{pmatrix} \\ &= \begin{pmatrix} \nabla v_0 & \nabla v_1 & \nabla v_2 \end{pmatrix} \end{aligned}$$

Berechnung von $\boldsymbol{\omega} \cdot \nabla \mathbf{v} = \mathbf{k}$:

$$\mathbf{k} = \begin{pmatrix} k_0 \\ k_1 \\ k_2 \end{pmatrix}$$

Berechnung der Komponenten mit $i = 0, 1, 2$:

$$\begin{aligned} k_i &= \sum_{j=0}^{2} \omega_j \frac{\partial v_i}{\partial x_j} \\ &= \boldsymbol{\omega} \cdot \nabla v_i \end{aligned}$$

In Matrix-Schreibweise:

$$\begin{aligned} \mathbf{k}^T &= \boldsymbol{\omega}^T \nabla \mathbf{v} \\ \mathbf{k} &= (\nabla \mathbf{v})^T \boldsymbol{\omega} \end{aligned}$$