PROBABILITY THEORY

mm 5

Problem 5.1 (problem 5.23 from Sheldon Ross, 3rd ed.)

If X is a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$, compute

(a) $P(X > 5)$;

(b) $P(4 < X < 16)$;

(c) $P(X < 8)$;

(d) $P(X < 20)$;

(e) $P(X > 16)$;

Problem 5.2 (problem 5.25 from Sheldon Ross, 3rd ed.)

The annual rainfall (in inches) in a certain region is normally distributed with $\mu = 40$ and $\sigma = 4$. What is the probability that in 2 out of the next 4 years the rainfall will exceed 50 inches? Assume that the rainfalls in different years are independent.

Problem 5.3 (problem 5.28 from Sheldon Ross, 3rd ed.)
A manufacturer produces bolts that are specified to be between 1.19 and 1.21 inches in diameter. If its production process results in a bolts diameter being normally distributed with mean 1.20 inches and standard deviation 0.005, what percentage of bolts will not meet specifications?

Problem 5.4 (problem 4.8 from Sheldon Ross, 3rd ed.)
The time (in hours) required to repair a machine in an exponentially distributed random variable with parameter $\lambda = 1$.

(a) what is the probability that a repair time exceeds 2 hours?

(b) what is the conditional probability that a repair takes at least 3 hours, given that its duration exceeds 2 hours?

Problem 5.5 (problem from *ProbabilityCourse.com)*

Suppose the number of customers arriving at a store obeys a Poisson distribution with an average of λ customers per unit time. That is, if Y is the number of customers arriving in an interval of length t, then $Y \sim Poisson(\lambda t)$. Suppose that the store opens at time $t = 0$. Let X be the arrival time of the first customer. Show that $X \sim Exponential(\lambda)$.

Problem 5.6 (problem from *ProbabilityCourse.com)*

Let $X \sim Norm(2, 4)$ and $Y = 3 - 2X$.

(a) Find $P(X > 1)$.

(b) Find $P(-2 < Y < 1)$.

(c) Find $P(X > 2|Y < 1)$.

(d) What is the mean and variance of X and Y?

(e Find $Cov(X, Y)$.

(f) Are X and Y independent random variables?

Problem 5.7
Let $X \sim Uniform(400, 800)$. Find $P(500 < X < 1000)$.

Problem 5.8
Customers arriving at a store according to a Poisson process with an average rate of 2.5 per hours. The store opens its door at 9 AM.

(a) What is the probability that the first customer arrives at the store before 11 AM?

(b) What is the probability that the first two customers arrive at the store before 11 AM?

(c) What is the probability that the first three customers arrive at the store before 11 AM?

Problem 5.9 (problem from *ProbabilityCourse.com)*
Let $U \sim Uniform(0, 1)$ and $X = -\ln(1-U)$. Show that $X \sim Exponential(1)$.