Introduction to Probability and Statistics

Session 1 Quiz

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1) Which of the following statements are true?
   a) A histogram is a discrete approximation of a continuous probability density function.
   b) A histogram is only valid to approximate discrete probability functions.
   c) The larger the number of bins, the larger the sample size needed to get a smooth approximation for the continuous probability density function (PDF).
2) Through inferential statistics, we aim to:
   a) Characterize the statistics of the sample.
   b) Characterize the parameters of the population by observing similar populations.
   c) Characterize the parameters of the population through the statistics of the sample.
3) Which of the following statements is false?
   a) Any sample (regardless its size or statistics) from a population is representative.
   b) Each value X_i from a sample is a random variable drawn from the population distribution F.
   c) The sample statistics may not be representative of the population parameters.
4) If X is a Gaussian random variable with mean μ_x and variance σ_x^2 and Y is another independent Gaussian random variable with mean μ_y and variance σ_y^2, which of the following statements are true?
   a) X and Y are completely characterized by their means μ_x and μ_y and by their standard deviations σ_x and σ_y.
   b) The probability of a realization from X to be below a value ϵ is given by
   $$P(X < \epsilon) = 1 - Q(\epsilon).$$
   c) The random variable $A = aX + Y$, with a being a positive constant, is distributed as $A \sim \mathcal{N}(a\mu_x + \mu_y, a\sigma_x^2 + \sigma_y^2)$.

5) If $X_1, \ldots, X_n$ for large n are independent random variables with mean μ and variance σ^2, then the distribution of $Y = \sum_{i=1}^{n} X_i$ is given by:
   a) A Gaussian random variable with mean $n\mu$ and variance σ^2/n.
   b) A Gaussian random variable with mean $n\mu$ and variance $n\sigma^2$.
   c) Will be Gaussian only if all X_i are Gaussian distributed.

6) What is the likelihood function?
   a) The joint density function (JDF) of the sample $f_{X_1,X_2,\ldots,X_n}(x_1, x_2, \ldots, x_n)$
   b) The JDF of the sample conditioned to parameter θ, given as $f_{X_1,X_2,\ldots,X_n}(x_1, x_2, \ldots, x_n; \theta)$
   c) The density function of an arbitrary random variable (RV) $f_{X_i}(x_i)$

7) The Maximum Likelihood Estimator (MLE) is formally defined as
   a) $\arg\max_{\theta} P_{X_1,X_2,\ldots,X_n}(x_1, x_2, \ldots, x_n; \theta)$
   b) $\arg\max_{\theta} \log\left(P_{X_1,X_2,\ldots,X_n}(x_1, x_2, \ldots, x_n; \theta)\right)$
   c) Both are correct

8) Which of the following statements are true?
   a) An estimator is less biased as the estimate is closer to the true value.
   b) The estimator of a constant (deterministic) value cannot be a random variable.
   c) The variance of the sample mean tends to zero as the sample size goes to infinity