PROBABILITY THEORY

mm 4

Problem 4.1 (problem 5.2 from Sheldon Ross, 3rd ed.)

A communication channel transmits the digits 0 and 1. However, due to static, the digit transmitted is incorrectly received with probability 0.2. Suppose that we want to transmit an important message consisting of one binary digit. To reduce chances of error, we transmit 00000 instead of 0 and 11111 instead of 1. If the receiver of the message uses "majority" decoding, what is the probability that the message will be incorrectly decoded? What independence assumptions are you making?

Problem 4.2 (problem 5.4 from Sheldon Ross, 3rd ed.)

Suppose that a particular trait (such as eye color or left-handedness) of a person is classified on the basis of one pair of genes, and suppose that d represents a dominant gene and r represents a recessive gene. Thus, a person with dd genes is pure dominant, one with rr is pure recessive, and one with rd is hybrid. The pure dominance and the hybrid are alike in appearance. Children receive one gene from each parent. If, with respect to a Particular trait, 2 hybrid parents have a total of 4 children, what is the probability that 3 of the 4 children have the outward appearance of the dominant gene?

Problem 4.3 (problem 5.11 from Sheldon Ross, 3rd ed.)
If you buy a lottery ticket in 50 lotteries, in each of which your chance of winning a prize is 1/100, what is (approximate) probability that you will win a prize (a) at least once, (b) exactly once, (c) at least twice?

Problem 4.4 (problem 5.17 from Sheldon Ross, 3rd ed.)
If X is a Poisson r.v. with mean λ, show that $P\{X = i\}$ first increases and then decreases as i increases, reaching its maximum value when i is the largest integer less or equal to λ.

Problem 4.5 (problem from *ProbabilityCourse.com)*

Let X be a discrete random variable with the following pmf:

$$P(X = 2) = 0.1$$
$$P(X = 4) = 0.2$$
$$P(X = 5) = 0.2$$
$$P(X = 8) = 0.3$$
$$P(X = 10) = 0.2$$

(a) Draw pmf of X.

(b) Find $P(X < 6)$;

(c) Find $P(2.5 < X < 7.5)$;

(d) Find $P(X = 2|X < 6)$;

Problem 4.6 (problem from *ProbabilityCourse.com)*

Two dice are rolled and two numbers are observed, X and Y.

(a) Find pmfs of X and Y.

(b) Are X and Y independent? Find the joint pmf of X and Y.

(c) Find $P(X > 3|Y = 2)$.

(d) Let $Z = X + Y$. Find $E[Z]$.

(e) Find $P(X = 1|Z = 3)$.

(f) Find $Var(Z)$.

(g) Find $Cov(X, Y)$.

Problem 4.7 (problem from *ProbabilityCourse.com)*
I roll a fair die repeatedly until a number larger than 4 is observed. If N is the total number of times that I roll the die, find $P(N = k)$ where $k = 1, 2, 3, ...$ How many trials we will need on average?

Problem 4.8 (problem from *ProbabilityCourse.com)*
The number of customers arriving at a grocery store is a Poisson random variable. On average 10 customers arrive per hour. Let X be the number of customers arriving from 10:00 to 11:30. Find $P(10 < X \leq 15)$