# Introduction to Probability and Statistics

Session 2 Exercises

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**Exercise 1:** Poll results

On October 14, 2003, the New York Times reported that a recent poll indicated that $52\%$ of the population was in favor of the job performance of president Bush, with a margin of error (i.e., $95\%$ confidence interval (CI)) of $\pm 4\%$.

a) What does this mean?

b) How many people were questioned?

**Exercise 2:** Wireless communication with multiple antennas

A device transmits a signal towards a receiver. The receiver measures the signal strength, which is affected by Gaussian noise and, hence, is normally distributed with mean $\mu$ and variance $\sigma^2 = 4$. The receiver has 9 receiving antennas and each one records the value of the signal strength, which are

$$[5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5]$$

a) Calculate the $95\%$ CI using $Z_{\alpha/2}$

b) Calculate the $95\%$ CI using $T_{\alpha/2,n-1}$ and without knowing the variance

**Exercise 3:** Execution time of an algorithm.

We measured the execution time of an image processing algorithm and obtained an average execution time of $\mu = 12.995$ ms. Assume that our measurements are subject to Gaussian noise with zero mean and variance $\sigma^2 = 1.997$

a) Get the maximum likelihood estimates for $\mu$, denoted as $\hat{\mu}_n$ after taking a sample with $n = 10$ and $n = 1000$.

b) Plot the collected measurements along with the estimated $\hat{\mu}_n$.

c) Calculate the $95\%$ CI for both values of $n$ assuming that the variance $\sigma^2$ is known.

d) Calculate the $95\%$ CI for both values of $n$ assuming that the variance is **not known** using $Z_{\alpha/2}$

e) Calculate the $95\%$ CI for both values of $n$ assuming that the variance is **not known** using $T_{\alpha/2,n-1}$

f) Which of these CIs is wider?

**Exercise 4:** CIs with unknown parameters and small sample size.

Suppose the data 2.5, 5.5, 8.5, 11.5 was drawn from a $N(\mu, \sigma^2)$ distribution with unknown parameters. Give the $95\%$, $80\%$, and $50\%$ CIs for $\mu$.

**Exercise 5:** Chapter 9, problem 4 and 9

The following data indicate the relationship between $x$, the specific gravity of a wood sample, and $Y$, its maximum crushing strength in compression parallel to the grain

| $x_i$ | $y_i$(psi) | $x_i$ | $y_i$(psi) |
|---|---|---|---|
| .41 | 1850 | .39 | 1760 |
| .46 | 2620 | .41 | 2500 |
| .44 | 2340 | .44 | 2750 |
| .47 | 2690 | .43 | 2730 |
| .42 | 2160 | .44 | 3120 |

a) Plot a scatter diagram. Does a linear relationship seem reasonable?

b) Estimate the regression coefficients.

c) Predict the maximum crushing strength of a wood sample whose specific gravity is 0.43.

d) Estimate the variance of an individual response.