**Problem 5.23**

**(a)**

$$P(X > 5) = 1 - \Phi\left(\frac{5-10}{6}\right) = 1 - \Phi\left(\frac{-5}{6}\right) =$$

$$1 - (1 - \Phi\left(\frac{5}{6}\right)) = \Phi\left(\frac{5}{6}\right) = \Phi\left(0.8333\right) = 0.7977$$

**(b)**

$$P(4 < X < 16) = \Phi\left(\frac{16-10}{6}\right) - \Phi\left(\frac{4-10}{6}\right) =$$

$$\Phi\left(1\right) - \Phi\left(-1\right) = 2\Phi\left(1\right) - 1 = 2 \cdot 0.8413 - 1 = 0.6826$$

**(c)**

$$P(X < 8) = \Phi\left(\frac{8-10}{6}\right) = \Phi\left(\frac{-1}{3}\right) =$$

$$1 - (1 - \Phi\left(\frac{1}{3}\right)) = 1 - 0.6306 = 0.3694$$

**(d)**

$$P(X < 20) = \Phi\left(\frac{20-10}{6}\right) = \Phi\left(\frac{5}{3}\right) =$$

$$\Phi\left(1.6667\right) = 0.9522$$

**(e)**

$$P(X > 16) = 1 - \Phi\left(\frac{16-10}{6}\right) = 1 - \Phi\left(1\right) = 0.1587$$

You can verify your calculation by using e.g. matlab function `normpdf`.

**Problem 5.25**

We will solve this problem in two steps.

**Step 1.** Let a $X$ be a r.v. representing the annual rainfall. As it is stated in the text, $X \sim N(\mu = 40, \sigma = 4)$. Probability that a rainfall exceeds 50 inches can be found as:

$$p = P(X > 50) = 1 - P(X \leq 50) =$$

$$1 - \Phi\left(\frac{50 - 40}{4}\right) = 1 - \Phi\left(25\right) = 0.0062$$

**Step 2.** Let an event $A$ be defined as a rainfall exceeds 50 inches. $p(A) = 0.0062$. Let a r.v. $Y$ be defined as a number of years out of 4 when a rainfall exceeds 50 inches. $Y$ is a binomial r.v. with parameters $n = 4$, $p = 0.0062$. The probability that we should find is given by

$$P(Y = 2) = \binom{4}{2}(0.0062)^2(1 - 0.0062)^2 = 0.00028$$

**Problem 5.28**

Let a r.v. $X$ be a diameter of a bolt. According to the text, $X \sim N(\mu = 1.20, \sigma = 0.005)$. A bolt will not meet the specification if it's diameter is larger than 1.21 or smaller than 1.21:

$$P(X \leq 1.19 \text{ } or \text{ } X \geq 1.21) = 1 - P(1.19 < X < 1.21) =$$

$$1 - \left(\Phi\left(\frac{1.21 - 1.20}{0.005}\right) - \Phi\left(\frac{1.19 - 1.20}{0.005}\right)\right) =$$

$$1 - (\Phi(2) - \Phi(-2)) = 2(1 - \Phi(2)) = 2(1 - 0.9772) = 0.0456$$

Answer: 4.56%

**Problem 5.37**

Let a r.v. $X$ be a time to repair a machine. As it is given in the text, $X$ is exponentially distributed with parameter $\lambda = 1$.

**(a)**

$$P(X > 2) = 1 - P(X \leq 2) = 1 - (1 - e^{-2}) = e^{-2} = 0.1353$$

**(b)**

Using memoryless property of $X$, we obtain

$$P(X > 3 \mid X > 2) = P(X > (3 - 2)) = P(X > 1) = e^{-1} = 0.3679$$

**Problem 5.5**

We first find $P(X > t)$:

$$P(X > t) = P(no \ \ arrival \ \ in \ \ [0, t]) = e^{-\lambda t}\frac{(\lambda t)^0}{0!} = e^{-\lambda t}$$

Cdf for $X$ for $x > 0$ is given by

$$F_X(x) = 1 - P(X > x) = 1 - e^{-\lambda t}$$

which is the cdf of exponential distribution with parameter $\lambda$.

**Problem 5.6**

(a) $X$ is normally distributed with $\mu_x = 2$ and $\sigma_x = 2$. $P(X > 1)$ is the same as $1 - F_X(1)$ where $F_X$ is a cdf for $X$. Cdf can be calculated e.g. by using matlab function `normcdf`. In this case it will be `normcdf(1,2,2)` $= 0.3085$ and thus, $P(X > 1) = 0.6915$.

Alternatively, one could use the standard normal distribution:

$$P(X > 1) = 1 - \Phi(\frac{1-2}{2}) = 1 - \Phi(-0.5) = \Phi(0.5) = 0.6915$$

(b) $Y$ is a normally distributed random variable with $\mu_y = -1$ and $\sigma_y = 4$.

$$P(-2 < Y < 1) = F_Y(1) - F_Y(-2) = 0.2902$$

For calculations matlab was used.

(c)

$$P(X > 2|Y < 1) = P(X > 2|3 - 2X < 1) =$$

$$P(X > 2|X > 1) = \frac{P(X > 2, X > 1)}{P(X > 1)} =$$

$$\frac{P(X > 2)}{P(X > 1)} = 0.72$$

(d)

$$E[X] = 2, \;\; Var(X) = 4$$

$$E[Y] = -1, \;\; Var(Y) = 16$$

(e)

$$Cov(X, 3-2X) = Cov(X, -2X) = -2Cov(X, X) = -2Var(X) = -8$$

(f) $X$ and $Y$ are not independent. This can be seen from the way we have introduced a r.v. $Y$. It is also confirmed by the fact that covariance of these two r.v. is not zero.

**Problem 5.7**

$$P(500 < X < 1000) = \frac{800 - 500}{800 - 400} = 3/4$$

**Problem 5.8**

(a)

$$1 - P(N = 0, t = 2) = 1 - e^{-2.5 \cdot 2} = 1 - e^{-5} = 0.99326$$

(b)

$$1 - P(N = 0, t = 2) - P(N = 1, t_2) = 1 - e^{-2.5 \cdot 2} - 5 \cdot e^{-5} = 1 - 6 \cdot e^{-5} = 0.95$$

(c)

$$1 - P(N = 0, t = 2) - P(N = 1, t_2) - P(N = 2, t = 2) =$$

$$1 - e^{-2.5 \cdot 2} - 5 \cdot e^{-5} - \frac{1}{2} \cdot 5^2 \cdot e^{-5} = 1 - 18.5 \cdot e^{-5}$$

In this calculation we have used a formula for probability for a certain number of events $N$ in an interval of duration $t$ for a Poisson distribution.

**Problem 5.9**

We are finding the cdf of $X$:

$$F_X(x) = P(X \leq x) = P(\ln(1-U) \leq x) = P(\frac{1}{1-U} \leq e^x) =$$

$$P(U \leq 1 - e^{-x}) = 1 - e{-}x$$

and this is a cdf for exponential random variable with parameter $\lambda = 1$.