PROBABILITY THEORY

Session 2

Warm up questions

(a) If we know marginal pdfs of random variables X and Y, are we able to find their joint pdf?

(b) Let X be a continuous r.v. Calculate $P(X = 100)$.

(c) Is it true that $E[g(X)] = g(E[X])$?

(d) Can variance of a random variable be negative?

(e) How varince of X will change, if we add a constant value c to X?

(f) What is the first and second moments of X?

(g) Does independence imply zero covariance?

Problem 1

The distribution function of the random variable X is given

$$F(x) = \begin{cases} 0 & x < 0 \\ x/2 & 0 \leq x < 1 \\ 2/3 & 1 \leq x < 2 \\ 11/12 & 2 \leq x < 3 \\ 1 & 3 \leq x \end{cases}$$

(a) Plot this distribution function.

(b) what is $P\{X > 1/2\}$?

(c) what is $P\{2 < X \leq 4\}$?

(d) what is $P\{X < 3\}$?

(e) what is $P\{X = 1\}$?

Problem 2

The amount of time, in hours, that a computer functions before breaking down is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \lambda e^{-x/100} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

What is the probability that a computer will function between 50 and 150 hours before breaking down? What is the probability that it will function less than 100 hours?

Problem 3
Let X be the number of the cars being repaired at a repair shop. We have the following information:

(a) At any time, there are at most 3 cars being repaired.

(b) The probability of having 2 cars at the shop is the same as the probability of having one car.

(c) The probability of having no car at the shop is the same as the probability of having 3 cars.

(d) The probability of having 1 or 2 cars is half of the probability of having 0 or 3 cars.

Find the PMF of X.

Problem 4

The joint density of X and Y is

$$f(x) = \begin{cases} xe^{-(x+y)} & x > 0, y > 0 \\ 0 & otherwise \end{cases}$$

(a) Compute the density of X.

(b) Compute the density of Y.

(c) Are X and Y independent?

Problem 5
If the density function of X equals

$$f(x) = \begin{cases} ce^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Find c. What is $P\{X > 2\}$?

Problem 6

If $E[X] = 2$ and $E[X^2] = 8$, calculate

(a) $E[(2 + 4X)^2]$

(b) $E[X^2 + (X + 1)^2]$

Problem 7

The density function of X is given by

$$f(x) = \begin{cases} a + bx^2 & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

If $E[X] = 3/5$, find a and b.

Problem 8

A product is classified according to the number of defects it contains and the factory that produces it. Let X_1 and X_2 be the random variables that represent the number of defects per unit (taking the possible values of 0, 1, 2, or 3) and the factory number (taking on possible values 1 or 2), respectively. The entries in the table represent the joint probability mass function of a random chosen product:

$$P(0,1) = 1/8; P(0,2) = 1/16; P(1,1) = 1/16; P(1,2) = 1/16;$$
$$P(2,1) = 3/16; P(2,2) = 1/8; P(3,1) = 1/8; P(3,2) = 1/4$$

(a) Find the marginal probability distributions of $X_1$and X_2.

(b) Find $E[X_1]$, $E[X_2]$, $Var[X_1]$, $Var[X_2]$, and $Cov(X_1, X_2)$.

Problem 9

A cdf of a r.v. X is

$$F(x) = \begin{cases} 0 & x \le 0 \\ x/4 & 0 < x \le 4 \\ 1 & x > 4 \end{cases}$$

Find $E[X]$.

Problem 10

Suppose that X has density function

$$f(x) = e^{-x}, x > 0$$

Calculate its means and variance.

Problem 11

Suppose you are given the distribution function F of a random variable X. Explain how you could determine $P\{X = 1\}$. (Hint. You will need to use the concept of a limit).