M4LP Assignment 2

Viggo de Breij Thijmen van der Meijden Lex Klaassen

Group $\boxed{42}$

Abstract

Roughly this was how we made the exercises. Viggo made the first and second exercise, Lex did the second and third, and Thijmen helped along the way and also did the fourth. The fifth was really a colaborative piece between the three of us.

1 Calculations over λ-terms, exercise 1.4 and 1.5

(4) $(\lambda xy.yxx)(\lambda x.xy)(\lambda x.xx)$
$(\lambda y.y(\lambda x.xy)(\lambda x.xy)(\lambda x.xx) \rightarrow_\beta$
$(\lambda x.xx)(\lambda x.xy)(\lambda xy.) \rightarrow_\beta$
$(\lambda x.xy)(\lambda x.xy)(\lambda x.xy) \rightarrow_\beta$
$(\lambda x.xy)y(\lambda x.xy) \rightarrow_\beta$
$yy(\lambda x.xy)$

Or the other conversion: $(\lambda xy.yxx)(\lambda x.xy)(\lambda x.xx) \rightarrow_\beta$
$(\lambda x.xx)(\lambda x.xy)(\lambda x.xy) \rightarrow_\beta$
$(\lambda x.xx)(\lambda x.xyy) \rightarrow_\alpha$
$(\lambda x.xx)(\lambda x.xyz) \rightarrow_\beta$
$(\lambda x.xyz)(\lambda x.xyz)$

(5) $\Big(\lambda y.\big(\lambda x.(\lambda x.yx)y\big)x\Big)(\lambda y.xy)$
$(\lambda x.(\lambda x.(\lambda y.xy)x)(\lambda y.xy))x \rightarrow_\alpha$
$(\lambda x.(\lambda x.(\lambda y.zy)x)(\lambda y.xy))x \rightarrow_\beta$
$(\lambda x.(\lambda y.xy)(\lambda y.xy))x \rightarrow_\beta$
$(\lambda y.xy)(\lambda y.xy) \rightarrow_\beta$
$x(\lambda y.xy)$

2 Typing λ-terms

(3) $\big(\exists@\text{PIZZA}\big) \rightarrow \big((\text{DELICIOUS}(x)) \wedge (\text{CROQUETTE}(x))\big)$

$$\dfrac{\dfrac{\rightarrow}{(e\to t)\to t}\quad \dfrac{\exists}{(e\to t)\to t}\quad \dfrac{\dfrac{\text{PIZZA}}{e\to t}\quad \dfrac{x}{e}}{t}{@} \quad \dfrac{\dfrac{\dfrac{\vee}{t\to t}\quad \dfrac{\dfrac{\text{DELICIOUS}}{e\to t}\quad \dfrac{x}{e}}{t}{@}}{t\to t}{@}\quad \dfrac{\dfrac{\text{CROQUETTE}}{e\to t}\quad \dfrac{x}{e}}{t}{@}}{t}{@}}{\text{Typing fails}}\times$$

(4) $((\lambda x.\forall @ x) @ \textsc{boy}) \vee (y = z)$

$$\cfrac{\cfrac{\vee}{t \to (t \to t)} \quad \cfrac{\forall}{(e \to t) \to t} \quad \cfrac{x}{e} \quad \cfrac{\text{BOY}}{e \to t} \quad \cfrac{\cfrac{\cfrac{=}{e \to (e \to t)} \quad \cfrac{y}{e}}{e \to t}{\scriptstyle @} \quad \cfrac{z}{e}}{t}{\scriptstyle @}}{\text{Typing fails}}{\scriptstyle \times}$$

(5) $((\textsc{in}(\texttt{utrecht})) \wedge (\textsc{with}(\texttt{ashley}))$

$$\cfrac{\cfrac{\wedge}{t \to (t \to t)} \quad \cfrac{\cfrac{\text{IN}}{e \to (e \to t)} \quad \cfrac{\texttt{utrecht}}{e}}{e \to t}{\scriptstyle @} \quad \cfrac{\cfrac{\text{WITH}}{e \to (e \to t)} \quad \cfrac{\texttt{ashley}}{e}}{e \to t}{\scriptstyle @}}{\text{Typing fails}}{\scriptstyle \times}$$

(6) $\lambda x.\lambda y.\big((\textsc{wine}(x)) \vee (y @ \textsc{dance})\big)$
interpreting y as being of type $(e \to t) \to t$.

$$\cfrac{\cfrac{x}{e} \quad \cfrac{\cfrac{y}{(e \to t) \to t} \quad \cfrac{\cfrac{\cfrac{\wedge}{t \to (t \to t)} \quad \cfrac{\cfrac{\text{WINE}}{e \to t} \quad \cfrac{x}{e}}{t}{\scriptstyle @}}{t \to t}{\scriptstyle @} \quad \cfrac{\cfrac{y}{(e \to t) \to t} \quad \cfrac{\text{DANCE}}{e \to t}}{t}{\scriptstyle @}}{t}{\scriptstyle @}}{((e \to t) \to t) \to t}{\scriptstyle \lambda.}}{e \to (((e \to t) \to t) \to t)}{\scriptstyle \lambda.}$$

3 Recipes for semantic composition exercise 3.2

$$\cfrac{\cfrac{\cfrac{\text{No}}{\begin{gathered}\text{NP/N}\\ [\![\text{No}]\!]\end{gathered}} \quad \cfrac{\text{dog}}{\begin{gathered}\text{N}\\ [\![\text{dog}]\!]\end{gathered}}}{\begin{gathered}\text{NP}\\ [\![\text{No}]\!]@[\![\text{dog}]\!]\end{gathered}}{\scriptstyle >} \quad \cfrac{\cfrac{\text{is}}{\begin{gathered}\text{(S}\backslash\text{NP)/PP}\\ [\![\text{is}]\!]\end{gathered}} \quad \cfrac{\cfrac{\text{in}}{\begin{gathered}\text{PP/NP}\\ [\![\text{in}]\!]\end{gathered}} \quad \cfrac{\cfrac{\text{a}}{\begin{gathered}\text{NP/N}\\ [\![\text{a}]\!]\end{gathered}} \quad \cfrac{\text{room}}{\begin{gathered}\text{N}\\ [\![\text{room}]\!]\end{gathered}}}{\begin{gathered}\text{NP}\\ [\![\text{a}]\!]@[\![\text{room}]\!]\end{gathered}}{\scriptstyle >}}{\begin{gathered}\text{PP}\\ [\![\text{in}]\!]@([\![\text{a}]\!]@[\![\text{room}]\!])\end{gathered}}{\scriptstyle >}}{\begin{gathered}\text{S}\backslash\text{NP}\\ [\![\text{is}]\!]@([\![\text{in}]\!]@([\![\text{a}]\!]@[\![\text{room}]\!]))\end{gathered}}{\scriptstyle >}}{\begin{gathered}\text{S}\\ ([\![\text{No}]\!]@[\![\text{dog}]\!])@([\![\text{is}]\!]@([\![\text{in}]\!]@([\![\text{a}]\!]@[\![\text{room}]\!])))\end{gathered}}{\scriptstyle <}$$

4 Computing semantics exercise 4.2

Semantic lexicon
$[\![\text{no}]\!] = \lambda PQ.\forall x(P(x) \to \neg Q(x))$
$[\![\text{dog}]\!] = \textsc{dog}$
$[\![\text{is}]\!] = \lambda PQ.Q@P$
$[\![\text{in}]\!] = \lambda Px.P@(\lambda y.\textsc{in}(y,x))$ $\textsc{in}(x,y)$ is interpreted as "x is in y".
$[\![\text{a}]\!] = \lambda PQ.\exists x(Q(x) \wedge P(x))$
$[\![\text{room}]\!] = \textsc{room}$

We are inserting correct lexical terms in $([\![\text{No}]\!]@[\![\text{dog}]\!])@([\![\text{is}]\!]@([\![\text{in}]\!]@([\![\text{a}]\!]@[\![\text{room}]\!])))$.

$[\![\text{a}]\!]@[\![\text{room}]\!] = \lambda PQ.\exists x(Q(x) \wedge P(x))@\texttt{room} \to_\beta \lambda Q.\exists x(Q(x) \wedge \texttt{ROOM}(x)$

$[\![\text{in}]\!]([\![\text{a}]\!]@[\![\text{room}]\!]) = \lambda Px.P@(\lambda y.\texttt{IN}(x,y))@\lambda Q.\exists x(Q(x) \wedge \texttt{ROOM}(x) \to_\beta$

$= \lambda x.(\lambda Q.(\exists x(Q(x) \wedge \texttt{ROOM}(x)))@(\lambda y.\texttt{IN}(x,y))) \to_\alpha$

$= \lambda x.(\lambda Q.(\exists z(Q(z) \wedge \texttt{ROOM}(z)))@(\lambda y.\texttt{IN}(x,y))) \to_\beta$

$= \lambda x.(\exists z((\lambda y.\texttt{IN}(x,y))(z) \wedge \texttt{ROOM}(z))) \to_\beta$

$[\![\text{is}]\!]([\![\text{in}]\!]([\![\text{a}]\!]@[\![\text{room}]\!])) = (\lambda PQ.Q@P)@(\lambda x.\exists z(\texttt{IN}(x,z) \wedge \texttt{ROOM}(z))) \to_\beta$

$= (\lambda Q.Q@(\lambda x.\exists z(\texttt{IN}(x,z) \wedge \texttt{ROOM}(z)))) \to_\beta$

$[\![\text{No}]\!]@[\![\text{dog}]\!] = \lambda PQ.\forall x(P(x) \to \neg Q(x))@\texttt{DOG} \to_\beta$

$[\![\text{No}]\!]@[\![\text{dog}]\!] = \lambda Q.\forall x(\texttt{DOG}(x) \to \neg Q(x))$

$([\![\text{is}]\!]@([\![\text{in}]\!]@([\![\text{a}]\!]@[\![\text{room}]\!])))@([\![\text{No}]\!]@[\![\text{dog}]\!]) = (\lambda Q.Q@(\lambda x.\exists z(\texttt{IN}(x,z) \wedge \texttt{ROOM}(z))))@\lambda Q.\forall x(\texttt{DOG}(x) \to \neg Q(x)) \to_\beta$

$= \lambda Q.\forall x(\texttt{DOG}(x) \to \neg Q(x))@(\lambda x.\exists z(\texttt{IN}(x,z) \wedge \texttt{ROOM}(z))) \to_\beta$

$= \forall x(\texttt{DOG}(x) \to \neg(\lambda x.\exists z(\texttt{IN}(x,z) \wedge \texttt{ROOM}(z)))@x) \to_\beta$

$= \forall x(\texttt{DOG}(x) \to \neg\exists z(\texttt{IN}(x,z) \wedge \texttt{ROOM}(z))$

5 Natural tableau reasoning

We tried to find a conclusive tableau solution but we were unable to. Our hypothesis is that it is neutral.