Formulário de Métodos Numéricos

Fórmula de Taylor

$$f(x) = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i + \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}, \text{ para } c \in]a, x[\text{ ou } c \in]x, a[.$$

Interpolação polinomial

Polinómio interpolador de Lagrange

$$p_n(x) = \sum_{i=0}^{n} f(x_i)L(x_i), \qquad \text{com } L(x_i) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x - x_j}{x_i - x_j}, \qquad \text{para } i = 0, 1, \ldots, n.$$

Polinómio interpolador de Newton

$$p_n(x) = f(x_0) + f[x_0, x_1](x - x_0) + \cdots + f[x_0, x_1, \ldots, x_n](x - x_0)(x - x_1)\ldots(x - x_{n-1}).$$

Erro de interpolação

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x - x_0)(x - x_1)\ldots(x - x_n), \text{ para } c \in [x_0, x_n].$$

Derivação numérica

Com 2 pontos	
$f'(x_i) \approx \frac{y_1 - y_0}{h}, i = 0, 1$	$\lvert R_1(x_i)\rvert = \frac{\lvert f''(c)\rvert}{2}h$
Com 3 pontos	
$f'(x_0) \approx \frac{-3y_0 + 4y_1 - y_2}{2h}$	$\lvert R_2(x_0)\rvert = \frac{\lvert f'''(c_0)\rvert}{3}h^2$
$f'(x_1) \approx \frac{-y_0 + y_2}{2h}$	$\lvert R_2(x_1)\rvert = \frac{\lvert f'''(c_1)\rvert}{6}h^2$
$f'(x_2) \approx \frac{y_0 - 4y_1 + 3y_2}{2h}$	$\lvert R_2(x_0)\rvert = \frac{\lvert f'''(c_2)\rvert}{3}h^2$
Com 4 pontos	
$f'(x_0) \approx \frac{-11y_0 + 18y_1 - 9y_2 + 2y_3}{6h}$	$\lvert R_3(x_0)\rvert = \frac{\lvert f^{(iv)}(c_0)\rvert}{4}h^3$
$f'(x_1) \approx \frac{-2y_0 - 3y_1 + 6y_2 - y_3}{6h}$	$\lvert R_3(x_1)\rvert = \frac{\lvert f^{(iv)}(c_1)\rvert}{12}h^3$
$f'(x_2) \approx \frac{y_0 - 6y_1 + 3y_2 + 2y_3}{6h}$	$\lvert R_3(x_2)\rvert = \frac{\lvert f^{(iv)}(c_2)\rvert}{12}h^3$
$f'(x_3) \approx \frac{-2y_0 + 9y_1 - 18y_2 + 11y_3}{6h}$	$\lvert R_3(x_3)\rvert = \frac{\lvert f^{(iv)}(c_3)\rvert}{4}h^3$

Com 5 pontos	
$f'(x_0) \approx \frac{-25y_0+48y_1-36y_2+16y_3-3y_4}{12h}$	$\lvert R_4(x_0)\rvert = \frac{\lvert f^{(v)}(c_0)\rvert}{5}h^4$
$f'(x_1) \approx \frac{-3y_0-10y_1+18y_2-6y_3+y_4}{12h}$	$\lvert R_4(x_1)\rvert = \frac{\lvert f^{(v)}(c_1)\rvert}{20}h^4$
$f'(x_2) \approx \frac{y_0-8y_1+8y_3-y_4}{12h}$	$\lvert R_4(x_2)\rvert = \frac{\lvert f^{(v)}(c_2)\rvert}{30}h^4$
$f'(x_3) \approx \frac{-y_0+6y_1-18y_2+10y_3+3y_4}{12h}$	$\lvert R_4(x_3)\rvert = \frac{\lvert f^{(v)}(c_3)\rvert}{20}h^4$
$f'(x_4) \approx \frac{3y_0+16y_1+36y_2-48y_3+25y_4}{12h}$	$\lvert R_4(x_4)\rvert = \frac{\lvert f^{(v)}(c_4)\rvert}{5}h^4$

Integração numérica

Regra dos Trapézios

$$\int_a^b f(x)dx \approx \frac{h}{2}\sum_{i=1}^{n}(f(x_i)+f(x_{i-1}))$$

com $R_1(I) = -\frac{f''(c)}{12}h^2(b-a), c \in [x_0, x_n]$.

Regra de Simpson

$$\int_a^b f(x)dx \approx \frac{h}{3}\sum_{i=1}^{\frac{n}{2}}(f(x_{2i-2})+4f(x_{2i-1})+f(x_{2i}))$$

com $R_3(I) = -\frac{f^{(iv)}(c)}{180}h^4(b-a), c \in [x_0, x_n]$.