Module 2 BOOLEAN ALGEBRA

Boolean postulates and laws: De-Morgan's Theorem, Principal of Duality, Boolean expression Boolean function Minimization of Boolean expressions – Sum of product (SOP) Product of sums (POS), Minterm, Maxterm, Canonical forms, Conversion between conical forms, Karnaugh map Minimization, Don't Care Conditions. Quine-McCluskey method.

Boolean Algebra

- Boolean algebra is a mathematical method of describing the operation of digital circuit inplace of the tabular form such as the truth table.
- It is a set of rules, ~~an~~ laws and theorems by which logical operations can be ~~carried~~ expressed.
- A digital circuit or a system can be expressed and analyzed in a systematic and convenient way.
- The boolean algebra is in the form of an equation The equation contains three elements: input variables, output variables and Boolean operators
- The variable can have a value of 0 or 1, which is 0 in case of LOW and 1 in case of high.
- The complement of A is represented by $\bar{A}$ or A'

Boolean Addition : The basic rules of Boolean addition are:

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 1$

- It is same as logic OR operation

Boolean Multiplication :- The basic rules of Boolean multiplication are:

$0 \cdot 0 = 0$

$0 \cdot 1 = 0$

$1 \cdot 0 = 0$

$1 \cdot 1 = 1$

- It is same as binary multiplication
- It " " " logical AND operation

Properties of Boolean algebra

There are three basic properties of Boolean algebra namely commutativity, associativity and distributativity

(i) Commutative property : Boolean addition & multiplication are commutative.

$A + B = B + A$ — The order in which variables are ORed does not make a differance

$A \cdot B = B \cdot A$ — The order in which variables are ANDed does not make a differance.

(ii) Associative property : Boolean addition & multiplication are associative.

$A + (B + C) = (A + B) + C$. — The ORing of variables is not effected by the way. they are grouped

$A \cdot (B \cdot C) = (A \cdot B) \cdot C$ — The ANDing of the variable is not effected by the way they are grouped

(iii) Distributive property

- The Boolean addition is distributive over Boolean multiplication

$$A + (B.C) \text{ or } A + BC = (A+B)(A+C)$$

- The Boolean multiplication is distributive over Boolean addition

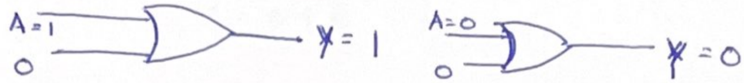
$$A.(B+C) = A.B + A.C$$

→ The other basic laws of Boolean algebra.

Sr NO	Law
1	$A+0 = A$
2	$A+1 = 1$
3	$A.0 = 0$
4	$A.1 = A$
5	$A+A = A$
6	$A+\bar{A} = 1$
7	$A.A = A$
8	$A.\bar{A} = 0$
9	$(\bar{A})'$ or $\bar{\bar{A}} = A$
10	$A + AB = A$
11	$A + \bar{A}B = A + B$
12	$AB + \bar{A}C + BC = AB + \bar{A}C$ → Consensus theorem

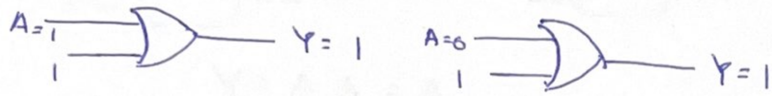
Rule 1 $A+0 = A$

use OR gate



$Y = A+0 = A$

Rule 2 $A+1 = 1$ use OR gate

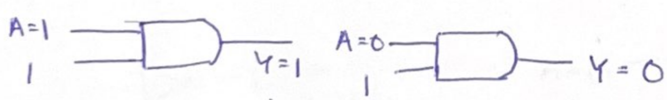


$Y = A+1 = 1$

Rule 3 $A \cdot 0 = 0$ use AND gate

$Y = A \cdot 0 = 0$

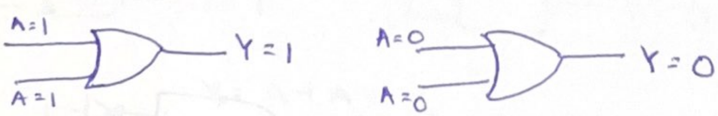
Rule 4 $A \cdot 1 = A$ use AND gate



$\therefore Y = A \cdot 1 = A$

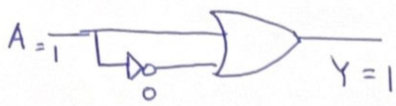
Rule 5

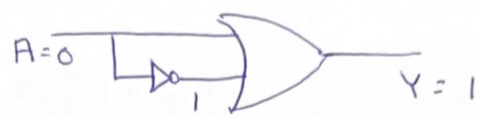
$A + A = A$ use OR ~~AND~~ gate



$\therefore Y = A + A = A$

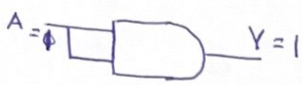
Rule 6 $A + \bar{A} = 1$ use OR gate

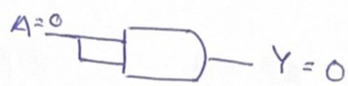




$Y = A + \bar{A} = 1$

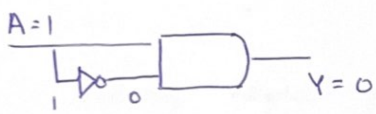
Rule 7 $A \cdot A = A$ use AND gate

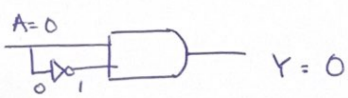




$Y = A \cdot A = A$

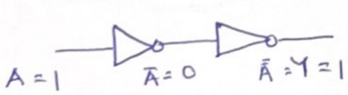
Rule 8 $A \cdot \bar{A} = 0$ use AND gate





$Y = A \cdot \bar{A} = 0$

Rule 9 $\bar{\bar{A}}$ or $\bar{A}'$



$\bar{\bar{A}} = A$

Rule 10 $A + AB = A$

use factoring

LHS = $A \cdot 1 + AB = A(1 + B)$

use rule 2 (ie $1 + B = 1$

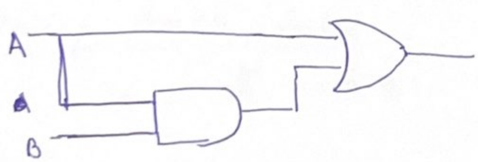
we get $= A \cdot 1$ use rule 4 $(A \cdot 1 = A)$

$= A = RHS$

using truth table.

A	B	AB	A + AB
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

using logic gates



Rule 11 $A + \bar{A}B = A + B$

LHS $= A + \bar{A}B$

use rule 10 $(A + AB = A)$, we get

$= A + AB + \bar{A}B$

use rule 7 (~~Area~~ $A \cdot A = A$), we get.

$= A \cdot A + AB + \bar{A}B$

use rule 8 $(A\bar{A} = 0)$

$= A \cdot A + A \cdot B + \bar{A}A + \bar{A}B$

$= (A + \bar{A})(A + B)$ factoring

use rule 6 $A + \bar{A} = 1$, we get

$= 1 \cdot (A + B)$

use Rule 4 $(A \cdot 1 = A)$ remove 1

$= A + B =$ RHS.

Using Truth Table

A	B	$\bar{A}$	$\bar{A}B$	$A + \bar{A}B$	$A + B$
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

using logic gate