

## Limits :

$$\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \quad \lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0 \quad \lim_{x \rightarrow \infty} \frac{\cos x}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$$

$$\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{k}{x}\right)^x = e^k$$

$$\lim_{x \rightarrow \infty} \frac{\log x}{x^m} = 0$$

$$\lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{x} = n$$

$$\lim_{x \rightarrow 0} \frac{(a+x)^n - a^n}{x} = na^{n-1}$$

$$\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = \frac{\tan^{-1} x}{x} = \cos x = 1$$

$$\lim_{n \rightarrow \infty} [h(x)]^{g(x)} = e^{\lim_{x \rightarrow \infty} (h(x)-1) g(x)} \quad \begin{array}{l} h(x) \rightarrow 0 \\ g(x) \rightarrow \infty \end{array}$$

Differential Calculus :  $\frac{d}{dx} f(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$$1. \frac{d}{dx}(x^n) = \underline{\underline{n x^{n-1}}}$$

$$18. (\tan^{-1} x) = \frac{1}{x^2+1}$$

$$2. d(c) = 0$$

$$19. (\cot^{-1} x) = \frac{-1}{x^2+1}$$

$$3. d(x) = 1$$

$$20. (\sec^{-1} x) = \frac{-1}{x\sqrt{1-x^2}} \parallel \frac{1}{x\sqrt{x^2-1}}$$

$$5. d(e^x) = e^x$$

$$21. (\csc^{-1} x) = \frac{1}{x\sqrt{1-x^2}} \parallel \frac{-1}{x\sqrt{x^2-1}}$$

$$6. d(a^x) = a^x \log a$$

$$7. d(\log x) = \frac{dx}{x}$$

$$22. d\left(\frac{1}{x}\right) = \frac{-1}{x^2}$$

$$8. d(\sin x) = \cos x$$

$$23. d(e^{ax}) = ae^{ax}$$

$$9. d(\cos x) = -\sin x$$

$$\Rightarrow e^{ax} \cdot \frac{d}{dx}(ax)$$

$$10. d(uv) = vu' + uv'$$

—

$$24. de^{ax+b} = ae^{ax+b}$$

$$25. \frac{d}{dx} f(x) e^x \Rightarrow e^x [1+f(x)]$$

$$12. d(\tan x) = \sec^2 x$$

$$13. d(\sec x) = \sec x \tan x$$

$$14. d(\cot x) = -\operatorname{cosec}^2 x$$

$$15. d(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$16. d(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$17. d(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

# Integration : Unit 9

exponent  
7  
Substitution: ILATE  
reversed log  
LT Trigon  
Algebra

$$\int f'(x) dx = f(x) + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int e^x dx = e^x + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos mx dx = \frac{\sin mx}{m} + c$$

$$\int \frac{dx}{x} = \log x + c$$

Sinx

$$1 x^n dx$$

$$2. 1 dx$$

$$3. \frac{1}{2\sqrt{x}} dx$$

$$4. \frac{-1}{x^2} dx$$

$$5. \cos x dx$$

$$6. \sin x dx$$

$$7. \sec^2 x dx$$

if	subs
$a^2 - x^2$	$x = a \sin \theta$
$x^2 + a^2$	$x = a \tan \theta$
$x^2 - a^2$	$x = a \sec \theta$

X

4.5

$\frac{dy}{dx}$  $\int$ 

8.  $\sec x \tan x \, dx$   $\sec x + c$
9.  $\operatorname{cosec}^2 x \, dx$   $-\cot x + c$
10.  $\operatorname{cosec} x \cot x \, dx$   $-\operatorname{cosec} x + c$
- 
11.  $\tan x \, dx$   $\log |\sec x| + c$
12.  $\sec x \, dx$   $\log |\sec x + \tan x| + c$
13.  $\cot x \, dx$   $\log |\sin x| + c$
14.  $\operatorname{cosec} x \, dx$   $\log (\operatorname{cosec} x - \cot x) + c$
- 
15.  $\frac{1}{x} \, dx$   $\log x + c$
16.  $\frac{1}{\sqrt{1-x^2}} \, dx$   $\sin^{-1} x + c$
17.  $\frac{-1}{\sqrt{1-x^2}} \, dx$   $\cos^{-1} x + c$
18.  $\frac{1}{x^2+1} \, dx$   $\tan^{-1} x + c$
19.  $\frac{-1}{x^2+1} \, dx$   $\cot^{-1} x + c$
20.  $\frac{1}{x\sqrt{1-x^2}} \, dx$   $\sec^{-1} x + c$
21.  $\frac{-1}{x\sqrt{1-x^2}} \, dx$   $\operatorname{cosec}^{-1} x + c$
-

$\frac{d}{dx}$  $\int$ 

$$22 \quad \frac{1}{a^2 + x^2} dx$$

$$\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$23 \quad \frac{1}{a^2 - x^2} dx$$

$$\frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

$$24 \quad \frac{1}{x^2 - a^2} dx$$

$$\frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$25 \cdot \frac{1}{\sqrt{a^2 - x^2}} dx$$

$$\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$26 \quad \frac{1}{\sqrt{x^2 + a^2}} dx$$

$$\log |x + \sqrt{x^2 + a^2}| + C$$

$$27 \quad \frac{1}{\sqrt{x^2 - a^2}} dx$$

$$\log |x + \sqrt{x^2 - a^2}| + C$$

$$28 \cdot \sqrt{a^2 - x^2} dx$$

$$\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$29 \quad \sqrt{x^2 + a^2} dx$$

$$\frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$30 \quad \sqrt{x^2 - a^2} dx$$

$$\frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$$

$$31 \cdot e^{-ax} x^n dx$$

$$\frac{n!}{a^{n+1}}$$

$$32. \quad e^x \left[ f(x) + f'(x) \right] dx$$

$$e^x f(x) + C$$

$$33. \quad \frac{f'(x)}{f(x)} dx$$

$$\log |f(x)| + C$$

$$34. \quad [f'(x)]^n f'(x) dx$$

$$\frac{[f(x)]^{n+1}}{n+1} + C$$

$\frac{dy}{dx}$

$\int$

34. for : turn,  $ax^2 + bx + c$

$$\int \frac{dx}{ax^2 + bx + c}, \quad = a \left[ x^2 + \frac{b}{a}x + \frac{c}{a} \right]$$

$$\int \frac{dx}{\sqrt{ax^2 + bx + c}} = a \left[ \left( x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a^2} \right]$$

using completion squares.

and then use

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} \sin^{-1} \left( \frac{x}{a} \right) + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} \log \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

35.

for :

$$\int \frac{px + q}{ax^2 + bx + c} dx, \quad \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx$$

$$\textcircled{I} \quad px + q = A \frac{d}{dx} (ax^2 + bx + c) + B$$

$$\hookrightarrow px + q = A (2ax + b) + B$$

$$\int \frac{px+q}{ax^2+bx+c} dx \rightarrow \int \frac{A(2ax+b)+B}{ax^2+bx+c} dx$$

$$\hookrightarrow = A \int \frac{2ax+b}{ax^2+bx+c} dx + B \int \frac{dx}{ax^2+bx+c}$$

$\hookrightarrow \int \frac{f'(x)dx}{f(x)} = \log |f(x)| \quad // \text{ completion of squares (34)}$

$$\Rightarrow A \log |ax^2+bx+c| + B \int \frac{dx}{ax^2+bx+c}$$


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$$\int \frac{px+q}{\sqrt{ax^2+bx+c}} dx \rightarrow \int \frac{A(2ax+b)+B}{\sqrt{ax^2+bx+c}} dx$$

$$\hookrightarrow = A \int \frac{2ax+b}{\sqrt{ax^2+bx+c}} dx + B \int \frac{1}{\sqrt{ax^2+bx+c}} dx$$

$\hookrightarrow$  use:

$$\int [f(\frac{x}{\sqrt{ax^2+bx+c}})]^n f'(x) dx \quad // \text{use completion of squares : } ax^2+bx+c \quad (34)$$

$$= \frac{[f(x)]^{n+1}}{n+1}$$

$$\Rightarrow A \left( 2 \sqrt{ax^2+bx+c} \right) + B \int \frac{1}{\sqrt{ax^2+bx+c}} dx$$

## First Fundamental Theorem:

$$F(x) = \int_a^b f(x) dx \quad \frac{d}{dx} F(x) = f(x)$$

$$F'(x) = f(x)$$

## Second Fundamental Theorem

$$\int_a^b f(x) dx = [F(x)]_a^b \\ = F(b) - F(a)$$

### Some important properties:

$$1. \int_a^b f(x) dx = \int_a^b f(y) dy$$

$$2. \int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$3. \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \quad || \quad a < c < b$$

$$4. \int_a^b [df(x) + \nu g(x)] dx = \alpha \int_a^b f(x) dx + \nu \int_a^b g(x) dx$$

$$5. \text{ If } x = g(u),$$

$$\int_a^b f(x) dx \Rightarrow \int_c^d f(g(u)) \frac{dg(u)}{du} du$$

$$\text{where, } g(c) = a ; \quad g(d) = b$$

# Integration Properties:

6.  $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

$$\hookrightarrow \int_0^a f(x) dx = \int_0^a f(a-x) dx$$


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7. If  $f(x)$  is even :  $f(-x) = f(x)$

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

8. If  $f(x)$  is odd :  $f(-x) = -f(x)$

$$\int_{-a}^a f(x) dx = \emptyset$$

9.  $\int_0^{2a} f(x) dx = \int_0^a [f(x) + f(2a-x)] dx$

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10. If  $f(2a-x) = f(x)$

$$\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$$

11. If  $f(2a-x) = -f(x)$

$$\int_0^{2a} f(x) dx = \emptyset$$

12.  $\int_0^a x f(x) dx = \frac{a}{2} \int_0^a f(x) dx \quad || \quad f(a-x) = f(x)$

# Integration Properties:

## Extra Properties:

$$\text{i) } \int_0^{\pi} g(\sin x) dx = 2 \int_0^{\pi/2} g(\sin x) dx$$

$$\text{ii) } \int_0^{2\pi} g(\cos x) dx = 2 \int_0^{\pi} g(\cos x) dx$$

$$\text{iii) If } f(a+x) = f(x),$$

$$\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$$

# Double derivative test for 2 var func

for  $f(x, y)$

$$\rightarrow \text{critical points} \rightarrow f_x = 0 \\ \rightarrow f_y = 0$$

and then if:

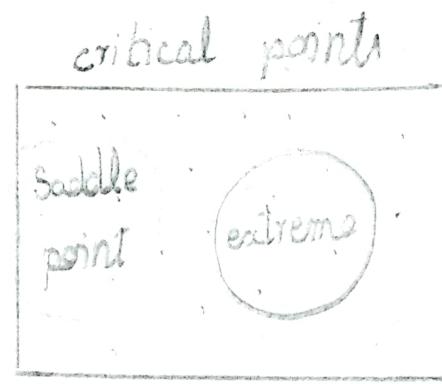
$$\rightarrow f_{xx} \cdot f_{yy} - (f_{xy})^2 = 0 \quad \text{inconsistent needs investigation}$$

$$\rightarrow f_{xx} \cdot f_{yy} - (f_{xy})^2 < 0 \quad \text{saddle point}$$

$$\rightarrow f_{xx} \cdot f_{yy} - (f_{xy})^2 > 0 \Rightarrow \text{has extrema}$$

dome  $\hookleftarrow$  maxima  $f_{xx} < 0 \quad \& \& \quad f_{xx} \cdot f_{yy} - (f_{xy})^2 > 0$

bowl  $\hookleftarrow$  minima  $f_{xx} > 0 \quad \& \& \quad f_{xx} \cdot f_{yy} - (f_{xy})^2 > 0$



# + Partial Derivatives :

for  $F(x_0, y_0)$ ,  $\frac{\partial F(x_0, y_0)}{\partial x}$

$$\frac{\partial F}{\partial x} = F_x = \left. \frac{\partial F}{\partial x} (x_0, y_0) \right|_{x_0, y_0}$$

$$\frac{\partial F}{\partial y} = F_y = \left. \frac{\partial F}{\partial y} (x_0, y_0) \right|_{x_0, y_0}$$


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# for  $U(x, y)$

\*  $U_x = \frac{\partial U}{\partial x}; U_y = \frac{\partial U}{\partial y}$

\*  $U_{xx} = \frac{\partial^2 U}{\partial x^2}; U_{yy} = \frac{\partial}{\partial y} \left( \frac{\partial U}{\partial y} \right)$

\*  $U_{xy} = \frac{\partial}{\partial x} \left( \frac{\partial U}{\partial y} \right); U_{yx} = \frac{\partial}{\partial y} \left( \frac{\partial U}{\partial x} \right)$

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\* for  $U(x, y) A \rightarrow \mathbb{R}^2$

$u \in U$  is harmonic in  $A$  if

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad || \quad U_{xx} + U_{yy} = 0$$

Laplace Eqn

\* Clairaut Theorem : If  $U_{xy}, U_{yx}$  are continuous

$$U_{xy} = U_{yx}$$

$$\star \sin^{-1}(x) = x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{x^5}{5} + \dots$$

↳ provided  $|x| \leq 1$

$$\star \tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

$$\star (1+x^n) = 1 + nx + nC_2 x^2 + nC_3 x^3 + \dots$$

$$\star \frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \dots \infty$$

Taylor Series:

$$\Rightarrow f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \quad (\text{for } x-a=0)$$

$$\Rightarrow f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots + \frac{(x-a)^n}{n!} f^{(n)}(a)$$

Derivatives:

$$\star \text{Linearization: } L(x) = f(a) + f'(a)(x-a)$$

$$\text{as } \frac{dy}{dx} = f'(a) = \frac{f(x) - f(a)}{x-a}$$

$$\star \boxed{\frac{\partial U}{\partial x}} = \lim_{h \rightarrow 0} \frac{F(x_0+h, y_0) - F(x_0, y_0)}{h}$$

$$\star \frac{\partial U}{\partial y} = \lim_{k \rightarrow 0} \frac{F(x_0, y_0+k) - F(x_0, y_0)}{k}$$

For  $U(x, y)$  :  $x = h(t)$   
 $y = g(t)$

$$\frac{dU}{dt} = \frac{\partial U}{\partial x} \times \frac{\partial x}{\partial t} + \frac{\partial U}{\partial y} \frac{\partial y}{\partial t}$$

For  $U(x, y)$

$$\therefore x = h(s, t)$$

$$y = g(s, t)$$

For  $U(x, y)$

~~$\frac{\partial U}{\partial x}$~~ 

$$\frac{dU}{ds} = \frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} \frac{\partial y}{\partial s}$$

$$\frac{\partial U}{\partial s} = \frac{\partial U}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial U}{\partial y} \frac{\partial y}{\partial s}$$

$$\therefore \frac{\partial U}{\partial t} = \frac{\partial U}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial U}{\partial y} \frac{\partial y}{\partial t}$$

important :

for  $U(x, y) : \frac{dy}{dx} = -\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}}$

Homogeneous  $\neq U(x, y)$  if , for some const  $t$

$$U(tx, ty) = t^p U(x, y)$$

$t$  - const

$p$  - degree

### Maclaurian series:

$$\Rightarrow f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^n}{n!} f^{(n)}(0)$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{x^n}{n!} f^{(n)}(0)$$


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Notes:

$$* \int_0^x f(x) dx = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} f\left(\frac{r}{n}\right)$$

$k = \binom{2n+1}{1}$

$$* \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$\sum_{\substack{k= \text{odd} \\ i=0}}^n \frac{x^k}{k!}$

$$* \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$\sum_{\substack{k=\text{even} \\ i=0}}^n \frac{x^k}{k!} \quad (k=2n)$

$$* \tan x = x + \frac{x^3}{3} + \frac{2}{15} x^5 + \dots$$

$\sum_{\substack{k=\text{odd} \\ i=0}}^n \frac{x^k}{k!}$

~~$$* \ln x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$~~

$\sum_{r=0}^n \frac{x^r}{r!}$

$$* |\ln(1+x)| = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \dots$$

$\sum_{r=0}^n (-1)^{r+1} \frac{x^{(r+1)}}{(r+1)}$

provided  $\underline{-1 \leq |x| \leq 1}$

If Homogeneous of degree  $P$ ,

Euler's Theorem,

$$x \frac{\partial F}{\partial x} + y \frac{\partial F}{\partial y} = P [F(x, y)]$$

so ...

Taylor Series

\* for  $U(x, y)$

$$\frac{dy}{dx} = - \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}}$$

\* for  $U(x, y)$ , Linearization at  $(a, b)$

$$\Rightarrow L(x, y) = [U(a, b) + (x-a) U_x(a, b)]$$

1° - linear +  $U(\cancel{x}, b) + (y-b) U_y(a, b) + \dots$

$$\Rightarrow f(x, y) = [f(a, b) + (x-a) f_x(a, b) + (y-b) f_y(a, b)]$$

$$2^{\circ} \text{ quadratic } + \frac{(x-a)^2}{2} f_{xx}(a, b)$$

$$+ (x-a)(y-b) f_{xy}(a, b)$$

$$* (x-a)(y-b) \cancel{f_{yx}}(a, b)$$

$$+ \frac{(y-b)^2}{2} f_{yy}(a, b)$$

# + Taylor Series

→ for  $f(x)$  type at  $(a)$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \quad \text{for } x-a = 0$$

$$\Rightarrow f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots + \frac{(x-a)^n}{n!} f^{(n)}(a)$$

→ Taylor series at origin  $a=0$

↳ MacLaurian series

$$\hookrightarrow f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} (x)^n$$

$$\hookrightarrow f(x) = f(0) + \frac{x}{1!} f'(0) + \cancel{\frac{x^2}{2!}} f''(0) + \dots + \frac{x^n}{n!} f^{(n)}(0)$$

→ for  $f(x, y)$  type at  $(a, b)$

$$\cancel{f(x,y)} = \sum_{n=0}^{\infty} \frac{1}{n!} \left( (x-a) \frac{\partial}{\partial x} + (y-b) \frac{\partial}{\partial y} \right)^n f(a, b)$$

$$\hookrightarrow f(a, b) + (x-a) f_x(a, b) + (y-b) f_y(a, b)$$

$$+ \frac{1}{2!} \left[ (x-a)^2 f_{xx} + 2(x-a)(y-b) f_{xy} + (y-b)^2 f_{yy} \right] \Big|_{(a,b)}$$

+ . . . .

→ Taylor series at origin  $(0, 0) \Rightarrow$  MacLaurian series

$$\hookrightarrow \left[ f(x, y) = \sum_{n=0}^{\infty} \frac{1}{n!} \left( x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \right)^n f(0, 0) \right]$$

## Notes :

\* Taylor series comes from fine tuning Linearization.

$$L(x) = f(a) + \cancel{f'(a)} (x-a) f'(a)$$

$$L(x, y) = f(a, b) + (x-a) f_x(a, b) + (y-b) f_y(a, b)$$

### \* Alternate notation

$$f_x(a, b) = f_x \Big|_{(a, b)}$$

$$\begin{aligned} x &= a+h \\ y &= b+k \end{aligned} \quad \text{so here so}$$

$$\begin{aligned} f(a+h, b+k) &= f(a, b) + (h f_x + k f_y) \Big|_{(a, b)} \\ &\quad + \frac{1}{2!} (h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy}) \Big|_{(a, b)} \\ &\quad + \dots \frac{1}{n!} \left( h \frac{\partial}{\partial x} + k \frac{\partial}{\partial y} \right)^n f \Big|_{(a, b)} \end{aligned}$$

### \* Linear = $1^\circ$

$$f(x, y) = [f + (x-a) f_x + (y-b) f_y] \Big|_{(a, b)} + \dots$$

$$\Rightarrow f(a, b) + (x-a) f_x(a, b) + (y-b) f_y(a, b) + \dots$$

$\hookrightarrow 3 \text{ terms}$

### \* Quadratic = $2^\circ$

$$\begin{aligned} f(x, y) &= \left[ f + (x-a) f_x + (y-b) f_y + \frac{(x-a)^2}{2} f_{xx} \right. \\ &\quad \left. + (x-a)(y-b) f_{xy} + \frac{(y-b)^2}{2} f_{yy} \right] \Big|_{(a, b)} \end{aligned}$$

6 terms  $\hookleftarrow$

+ ...

## Hyperbolic funcs:

some identities:

$$1. \cosh^2 x - \sinh^2 x = 1$$

$$2. \tanh^2 x - 1 = \operatorname{sech}^2 x$$

$$3. \coth^2 x - 1 = -\operatorname{cosech}^2 x$$

$$4. \sinh 2x = 2 \sinh x \cosh x$$

$$5. \cosh 2x = 1 + \sinh^2 x \\ = 2 \cosh^2 x - 1$$

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$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

derivatives

$$1. \frac{d}{dx} \sinh x = \cosh x$$

$$2. \frac{d}{dx} \cosh x = +\sinh x$$

$$3. \frac{d}{dx} \tanh x = \operatorname{sech}^2 x$$

$$4. \frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$$

$$5. \frac{d}{dx} \coth x = -\operatorname{cosech}^2 x$$

$$6. \frac{d}{dx} \operatorname{cosech} x = -\operatorname{cot} x \operatorname{cosech} x$$