# Qualitative Reasoning (QR)

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## 1 Introduction

Qualitative Reasoning (QR) is an area of Artificial Intelligence that automates reasoning about continuous aspects of the physical world, such as space, time, and quantity, for the purpose of problem solving and planning using qualitative rather than quantitative information [1].

It creates non-numerical descriptions of physical systems and their behavior, preserving important behavioral properties and qualitative distinctions [2]. The goal is to develop representation and reasoning methods that enable computer programs to reason about the behavior of physical systems, without precise quantitative information.

In this paper we will explore QR and use methods to represent a simple physical system from which the computer can reason.

## 2 System

Our experimental physical system is a simple kitchen sink which can be filled with water and also can be emptied.

### 2.1 Description

For simplicity we consider the sink as being a single entity with multiple quantities:

- **Inflow**(of water into the sink): [0,+]
- **Outflow**(of water out of the sink): [0,+,Max]
- **Volume**(of the water in the sink): [0,+,Max]
- **Height**(of the water in the sink): [0,+,Max]
- **Pressure**(of the water in the sink): [0,+,Max]

and the following dependencies:

- **I+**(Inflow, Volume)
- **I-**(Outflow, Volume)
- **P+**(Volume, Height)
- **P+**(Height, Pressure)
- **P+**(Pressure, Outflow)
- **VC**(Volume(Max), Height(Max), Pressure(Max), Outflow(Max))
- **VC**(Volume(0), Height(0), Pressure(0), Outflow(0)).

## 2.2 Assumptions

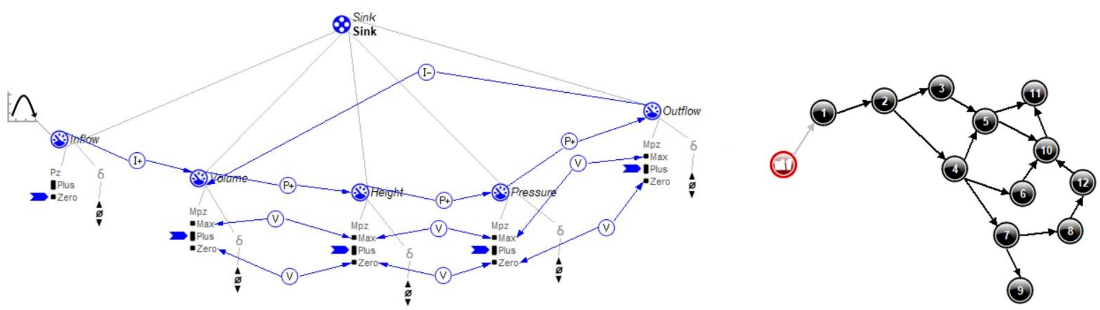
For this problem various solutions can be derives based on which assumptions we make about the system. For our experiment we made the following assumptions:

- The inflow follows a **parabola**(positive) shaped function
- The VC dependencies specified above are **bidirectional**
- In the initial state, the **inflow is 0**(there is no water coming from the tap)
- In the initial state, there is **some** water in the sink, but **not the maximum** amount possible.

According to these assumptions, the expected scenario is that we have a kitchen sink with some water in it(not the maximum amount) and someone is turning the tap on, in an increasing manner($\delta > 0$) till some "maximum" point($\delta = 0$), and then is turning the tap off, in a decreasing manner($\delta < 0$), till 0(a parabola shaped function). By this behaviour, we expect that the sink will be filled with water till some point(maximum or not) and then it will be drained.

## 2.3 Garp3

In order to build and visualize our system initial state(including the assumptions made)[1a] and the transition state graph[1b], we used Garp3, a workbench for building, simulating, and inspecting qualitative models. The system state descriptions are shown in Table 3 below.




(a) System's initial state (b) Transition state graph

Figure 1: Initial state model and transition state graph.

| | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 | S10 | S11 | S12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Inflow(v,$\delta$)** | *0,+* | *+,+* | *+,0* | *+,0* | *+,-* | *+,-* | *+,-* | *+,-* | *+,-* | *0,0* | *0,0* | *0,0* |
| **Volume(v,$\delta$)** | *+,-* | *+,-* | *+,-* | *+,0* | *+,-* | *+,0* | *+,+* | *Max,0* | *Max,+* | *+,-* | *0,0* | *Max,-* |
| **Height(v,$\delta$)** | *+,-* | *+,-* | *+,-* | *+,0* | *+,-* | *+,0* | *+,+* | *Max,0* | *Max,+* | *+,-* | *0,0* | *Max,-* |
| **Pressure(v,$\delta$)** | *+,-* | *+,-* | *+,-* | *+,0* | *+,-* | *+,0* | *+,+* | *Max,0* | *Max,+* | *+,-* | *0,0* | *Max,-* |
| **Outflow(v,$\delta$)** | *+,-* | *+,-* | *+,-* | *+,0* | *+,-* | *+,0* | *+,+* | *Max,0* | *Max,+* | *+,-* | *0,0* | *Max,-* |

Table 1: State descriptions.

# 3 Algorithm

In order to get the insight on how the model generated the state graph with the state descriptions we implemented the algorithm for generating this data from the model, conditions and our assumptions.

### 3.1 Data encoding

We have encoded the **quantities, quantity spaces** and the **derivatives** as lists. We used **0** for zero, **1** for plus and **2** for max magnitudes, and **-1** for negative, **0** for zero and **1** for positive derivatives.

We used square matrices of 5x5 to encode the **proportional, influence, maxim** and **zero dependencies**. Every element of the matrices at position (i,j) is **-1** if there is a **I-** or **P-** condition between quantities i and j, **1** if there is a **I+, P+, Max** or **Zero** condition between quantities i and j and **0** for no condition.

Now that we have the input information encoded we can build the algorithm for generating the state graph with transitions and state descriptions. We encode the states as arrays with 10 elements: Inflow magnitude(mI), Inflow derivative($\delta I$), Volume magnitude(mV), Volume derivative($\delta V$), Height magnitude(mH), Height derivative($\delta H$), Pressure magnitude(mP), Pressure derivative($\delta P$), Outflow magnitude(mO) and Outflow derivative($\delta O$).

### 3.2 State generation

The first step is to generate all possible states within the quantity spaces. This can be easily done as generating all possible combinations of all possible values for our 10 variables.

We end up with $2 * 3 * 3 * 3 * 3 * 3 * 3 * 3 * 3 * 3 = 39366$ possible states.

### 3.3 State removal

The next step is to remove all the states that violate our conditions and assumptions. For this we take each state in part and we verify it:

*Assumptions*: because of the parabola shaped Inflow we remove the state if mI = 0 and $\delta I$ = -1.

*Max and Zero*: for every **Max** dependency between quantity i and quantity j, if magnitude(i) = Max and magnitude(j) != Max or magnitude(j) = Max and magnitude(i) != Max then the state is not valid. Same reasoning for **Zero** dependency.

*Influence*: for each quantity i we get the list of all quantities j that have an influence dependency with i, **I+**(j,i) or **I-**(j,i). If the product between the **magnitude** of j and the **sign of the influence**(j,i) has the same sign(+1 or -1) for all quantities in that list and the derivative of i it's not of that sign, then the state is not valid.

*Proportionality*: for each quantity i we get the list of all quantities j that have a proportional dependency with i, **P+**(j,i) or **P-**(j,i). If the product between the **derivative** of j and the **sign of the proportionality**(j,i) has the same sign(+1 or -1) for all quantities in that list and the derivative of i it's not of that sign, then the state is not valid.

*Influence and Proportionality of 0*: if a quantity i has at least an influence or a proportional dependency with quantity j, **I+**(j,i), **I-**(j,i), **P+**(j,i) or **P-**(j,i), but all the effects are 0(due to 0 magnitude for **I** and/or 0 derivative for **P**) and the derivative of i is not 0, then the state is not valid.

After applying the above mentioned state removal rules we end up with only **27 valid states**.

### 3.4 State transition

Now that we have all the valid states, for each state i we can compute the list of states j to which we can transition from state i in one step. In order to do this, we take all state pairs (i,j) and we find out if we can transition from state i to state j in one step:

*Same state*: if state i is the same as state j, then the transition is not valid because this transition is basically represented just as the state.

*Assumptions*: because the inflow follows the parabola function we have only a few valid transitions between consecutive states. From mI = 0 and $\delta I = 0$ we can transition only to mI = 0 and $\delta I = 0$; from mI = 0 and $\delta I = 1$ we can transition only to mI = 1 and $\delta I = 1$; from mI = 1 and $\delta I = 0$ we can transition only to mI = 1 and $\delta I$ = -1; from mI = 1 and $\delta I = 1$ we can transition only to mI = 1 and $\delta I = 1$ or mI = 1 and $\delta I = 0$; from mI = 1 and $\delta I$ = -1 we can transition only to mI = 1 and $\delta I$ = -1 or mI = 0 and $\delta I$ = 0. For any other combination the transition is not valid.

*Derivative changes*: if a quantity q1 has the derivative **-1** in state i and **+1** in state j or **+1** in state i and **-1** in state j, then the transition is not valid because you can't go from -1 to +1 or from +1 to -1 without passing through **0** first.

*Magnitude changes*: if a quantity q1 has the magnitude **0** in state i and **Max** in state j or **Max** in state i and **0** in state j, then the transition is not valid because you can't go from 0 to Max or from Max to 0 without passing through **Plus** first.

*Magnitude by derivative changes*: if a quantity q1 has the derivative **-1** in state i and the magnitude in state j is **higher than** the magnitude in state i, then the transition in not valid because a negative derivative can't give an increase in the magnitude. Same logic for a derivative of **+1**. If the derivative is **0** in state i, then the magnitude has to **be the same** for state i and j because there is no change due to the 0 derivative.

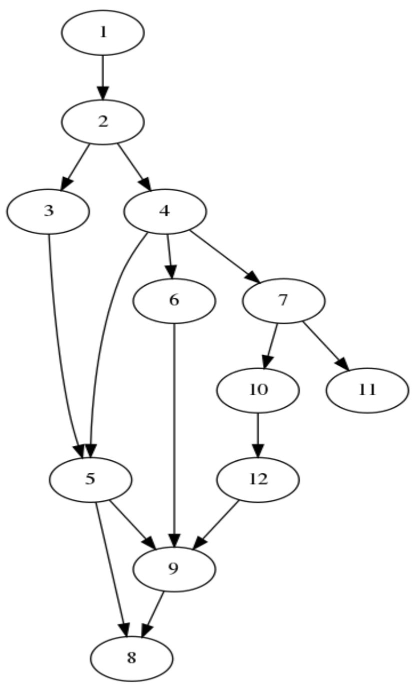
*Exogenous changes*: if the Inflow does not change at all from state i to state j(same magnitude and same derivative), then for a quantity q1(other than Inflow), if the magnitude in state j is the same as in state i but the derivative is different, then the transition in not valid because no change in the Inflow combined with no change in the magnitude can't generate change in the derivative.

*Fast Inflow changes*: if mI = 0 and $\delta I = 1$ or mI = 1 and $\delta I = 0$ in state i, then for a quantity q1(other than Inflow), if the derivative in state i is not 0 then the derivative of q1 in state j has to be the same as in state i because the change in Inflow in this case happens faster than the change in the derivative for other quantities. Otherwise, this transition is not valid.

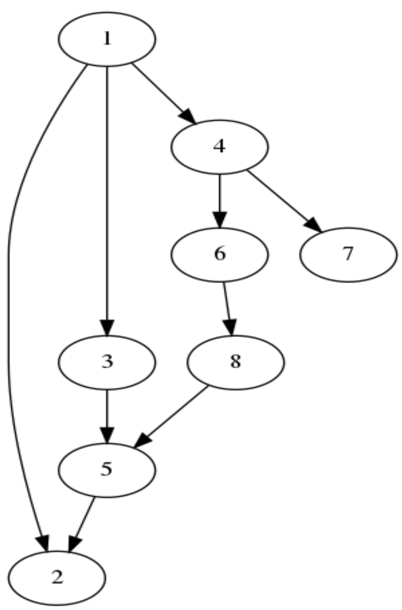
Any transition which is not considered invalid due to the above mentioned rules is a valid transition.

# 4 Results

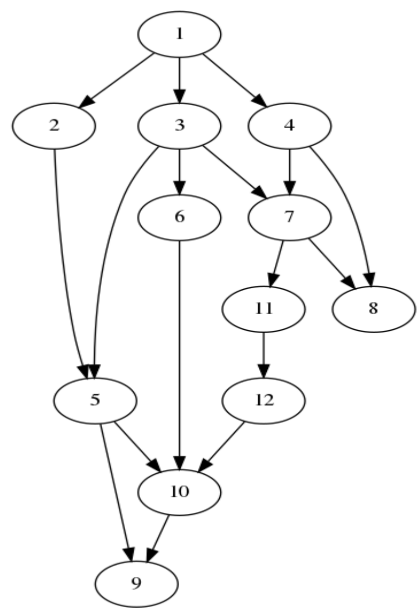
From all the valid states that we have we can choose an initial state and generate the state transition graph. In Figure 2 we have generated the state graph using different initial states. Because of the system description and assumptions all graphs should be acyclic, which is respected by our implementation.




(a) Initial state as described in assumptions




(b) Random initial state 1




(c) Random initial state 2

Figure 2: Transition state graph for different initial states.

| | Im | Id | Vm | Vd | Hm | Hd | Pm | Pd | Om | Od |
|---|---|---|---|---|---|---|---|---|---|---|
| State 1: | 0 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| State 2: | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| State 3: | 1 | 0 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| State 4: | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| State 5: | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| State 6: | 1 | -1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| State 7: | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| State 8: | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| State 9: | 0 | 0 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| State 10: | 1 | -1 | 2 | 0 | 2 | 0 | 2 | 0 | 2 | 0 |
| State 11: | 1 | -1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 |
| State 12: | 0 | 0 | 2 | -1 | 2 | -1 | 2 | -1 | 2 | -1 |

(a) States of the graph in Figure2a

| | Im | Id | Vm | Vd | Hm | Hd | Pm | Pd | Om | Od |
|---|---|---|---|---|---|---|---|---|---|---|
| State 1: | 1 | -1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| State 2: | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| State 3: | 1 | -1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| State 4: | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| State 5: | 0 | 0 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| State 6: | 1 | -1 | 2 | 0 | 2 | 0 | 2 | 0 | 2 | 0 |
| State 7: | 1 | -1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 |
| State 8: | 0 | 0 | 2 | -1 | 2 | -1 | 2 | -1 | 2 | -1 |

(b) States of the graph in Figure2b

| | Im | Id | Vm | Vd | Hm | Hd | Pm | Pd | Om | Od |
|---|---|---|---|---|---|---|---|---|---|---|
| State 1: | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| State 2: | 1 | 0 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| State 3: | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| State 4: | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| State 5: | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| State 6: | 1 | -1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| State 7: | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| State 8: | 1 | -1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 |
| State 9: | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| State 10: | 0 | 0 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| State 11: | 1 | -1 | 2 | 0 | 2 | 0 | 2 | 0 | 2 | 0 |
| State 12: | 0 | 0 | 2 | -1 | 2 | -1 | 2 | -1 | 2 | -1 |

(c) States of the graph in Figure2c

Figure 3: State descriptions of the Graphs

As we see our system generated the same state-graph as Garp3 (Fig. 2a and Fig. 1b). We have got the same behaviour when we start from the same assumptions. As we see from the graph (Fig. 2a) we have some ambiguity states where multiple outcomes are possible, for example state 2, 4, 5, 7, and 9. All these states can have multiple outcomes depending on the physical magnitude of all the entities, in particular when we have a proportionality dependence on the same entity but with the different sign. So these kind of situations create the ambiguity.

For example in State 4 we have the maximum *inflow* (because the magnitude is 1 and derivative is zero, it is the tip of the parabola) and all the other entities have a *plus* magnitude and no change (zero derivative). This means that the *Intra-state* of state 4 is described as all entities having no change and all of them have some magnitude which is neither zero nor max (except *Inflow*). It can be described as this: "The tap is open, we have some water in the sink and the outflow is the same as inflow so all entities have no change because there is no change in volume of water in the sink."

For this state we can have 3 different *Inter-State* transitions: 1) The *Inflow* decreases and all the others decreases(State 5), this is the case the *Outflow* is larger than *Inflow* so everything goes down. 2) The *inflow* decreases and all the others does not change(State 6), this is the case the *Outflow* equals to *Inflow* so nothing changes. 3) The *inflow* decreases and all the others decreases(State 7), this is the case the *Outflow* is smaller than *Inflow* so everything goes up.

# 5 Conclusion

In conclusion we can say that different problems require different approaches to solve them. The solution to a given problem depends very much on the assumptions made about the entities, environment behaviour and initial state. In our case the assumption of the parabolic inflow influenced how many states we have as well the initial state, as seen in Figure 2.

As well we have noticed that the complexity of the problem solving depends on the number of entities used. The more entities we use the more ambiguity we can have, especially if there is no proportionality or influence relation in-between entities.

# References


[1] *Qualitative Reasoning: Reaching Good Conclusions without Being Precise.* Association for the Advancement of Artificial Intelligence (AAAI).

[2] Bert Bredeweg and Peter Struss. *Current Topics in Qualitative Reasoning.* American Association for Artificial Intelligence, 2003.