Quicksort analysis

$$T(n) = E\left[\begin{array}{c}\text{\# of comperison} \\ \text{on array of size } n\end{array}\right]$$

$$T(n) = n + \frac{1}{n} \sum_{j=1}^{n} \left(T(j-1) + T(n-j) \right)$$

$$T(n) = n + \frac{1}{n} \left(2 \sum_{j=1}^{n-1} T(j) \right)$$

$$n\, T(n) = n^2 + 2 \sum_{j=1}^{n-1} T(j)$$

$$(n-1)\, T(n-1) = (n-1)^2 + 2 \sum_{j=1}^{n-2} T(j)$$

$$n\,T(n) - (n-1)\,T(n-1) = 2n - 1 + 2\,T(n-1)$$

$$n\,T(n) = 2n - 1 + (n+1)\,T(n-1)$$

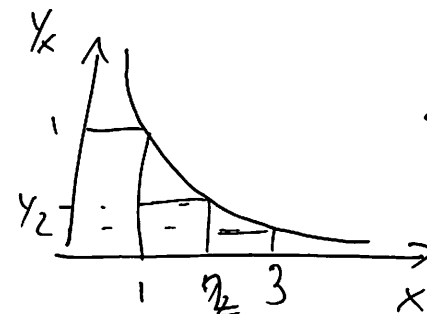$$\frac{T(n)}{n+1} = \frac{2n-1}{n(n+1)} + \frac{T(n-1)}{n}$$

$$\frac{T(n)}{(n+1)} \leq \frac{2}{(n+1)} + \frac{T(n-1)}{n}$$

$$\frac{T(n)}{(n+1)} \leq \frac{2}{(n+1)} + \frac{2}{n} + \cdots + 2$$

$$\Rightarrow T(n) \leq 2(n+1)\,H(n+1)$$

$$\Rightarrow T(n) = O(n \log n)$$

$$\ln n \leq H(n) \leq \ln n + 1$$

$$\int \frac{1}{x} = \ln n$$