

$$T(n) = \begin{cases} 4T\left(\frac{n}{2}\right) + cn & n > 1 \\ c & n = 1 \end{cases}$$

level 0

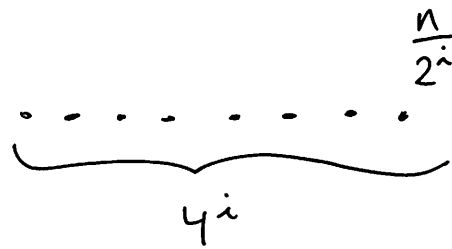


$$\longrightarrow cn$$

level 1

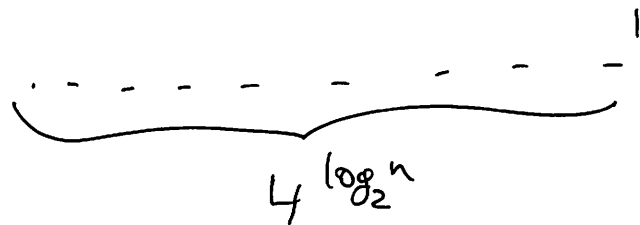
$$\longrightarrow 4c\frac{n}{2}$$

level i



$$\longrightarrow 4^i c \frac{n}{2^i} = 2^i cn$$

level  $\log_2 n$



$$2^{\log_2 n} cn = cn^2$$

Adding up:

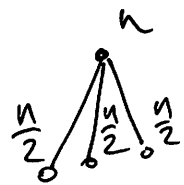
$$\sum_{i=0}^{\log_2 n} c 2^i n$$

$$= cn \sum_{i=0}^{\log_2 n} 2^i$$

$$\leq cn 2^{\log_2 n + 1}$$

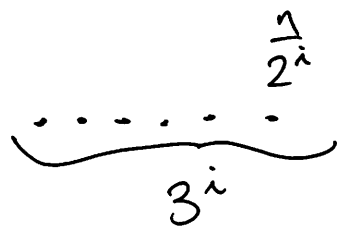
$$= O(n^2)$$

$$T(n) = \begin{cases} 3T\left(\frac{n}{2}\right) + cn & n > 1 \\ c & n = 1 \end{cases}$$

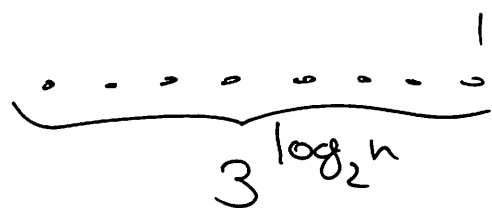


$$\rightarrow cn$$

$$\rightarrow 3c \frac{n}{2}$$



$$\left(\frac{3}{2}\right)^i cn$$



$$\left(\frac{3}{2}\right)^{\log_2 n} cn$$

Adding up

$$\sum_{i=0}^{\log_2 n} \left(\frac{3}{2}\right)^i cn$$

$$= O\left(\left(\frac{3}{2}\right)^{\log_2 n} cn\right)$$

$$= O\left(2^{\log_2 \frac{3}{2}} \cdot \log_2 n \cdot n\right)$$

$$= O\left(2^{\log_2 n \cdot \log_2 \frac{3}{2}} \cdot n\right)$$

$$= O\left(n^{\log_2 \frac{3}{2}} \cdot n\right)$$

$$= O\left(n^{\log_2 3} \cdot \frac{1}{n} \cdot n\right) = O\left(n^{\log_2 3}\right)$$

$$\frac{3}{2} = 2^{\log_2 \frac{3}{2}}$$

$$\begin{aligned} \log_2 \frac{3}{2} &= \log_2 3 - \log_2 2 \\ &= \log_2 3 - 1 \end{aligned}$$

$$T(n) = \begin{cases} \frac{3}{c} T\left(\frac{n}{2}\right) + cn & n > 1 \\ c & n = 1 \end{cases}$$

$$a = 3$$

$$b = 2$$

$$f(n) = cn$$

$$\log_2 3 \approx 1.6$$

$f(n)$  compare to  $n^{\log_2 3}$

$\Rightarrow$  Case 1 of MT

$$T(n) = O(n^{\log_2 3})$$

$$T(n) = \begin{cases} \frac{4}{c} T\left(\frac{n}{2}\right) + cn \end{cases}$$

$$a = 4 \quad b = 2 \quad f(n) = cn$$

compare  $f(n)$  to  $n^{\log_2 4} = n^2$

$\Rightarrow$  case 1 of MT

$$T(n) = O(n^{\log_2 4}) = O(n^2)$$

$$T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + cn \\ c \end{cases}$$

$$a = 2 \quad b = 2 \quad f(n) = cn$$

compare

$$cn \quad \text{with} \quad n^{\log_2 2} = n$$

$$f(n) = \Theta\left(n^{\log_2 2} \log^0 n\right)$$

$\Rightarrow$  Case 2 of  $T_n$  with  $k=0$

$$\begin{aligned} \Rightarrow T(n) &= O\left(n^{\log_2 2} \log n\right) \\ &= O(n \log n) \end{aligned}$$

$$T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + cn^2 \\ c \end{cases}$$

$$a = 2 \quad b = 2 \quad f(n) = cn^2$$

compare  $f(n)$  with  $n^{\log_2 2} = n$

$\Rightarrow$  case 3 of MT

and so

$$T(n) = O(n^2)$$