

CS5691: Assignment 1

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1. Motivation

Many images have unnecessary details stored in them which are indistinguishable to the naked eye. Using EVD/SVD we can get the those eigenvalues/singular values that contain those details. So we can construct a new image by removing those values and lower the size of the image with very less improvisation. In this experiment we will look into the effect of EVD/SVD on the quality of an grayscale image.

2. EVD/SVD

EVD - Eigenvalue decomposition is the factorization of a matrix into a canonical form, whereby the matrix is represented in terms of its eigenvalues and eigenvectors. The decomposition can be derived from the fundamental property of eigenvectors:

$$Av = \lambda v$$

Reconstruction of the matrix from EVD:

$$A = P.D.P^{-1}$$

Where D is the diagonal matrix of all the eigenvalues and P is the column matrix of eigenvectors.

SVD - the singular value decomposition (SVD) is a factorization of a real or complex matrix. It generalizes the eigendecomposition of a square normal matrix with an orthonormal eigenbasis to any $m \times n$ matrix.

Reconstruction of the matrix from SVD:

$$A = U.\Sigma.V$$

Where Σ is the diagonal matrix for singular values. U is the column matrix of eigenvectors of $A.A^T$ and V is the transpose of column matrix of eigenvectors of $A^T.A$

3. Experimental Results

Below are the results for some values of k

- $k = 25$

Image of top 25 eigenvalues



Error image of top 25 eigenvalues

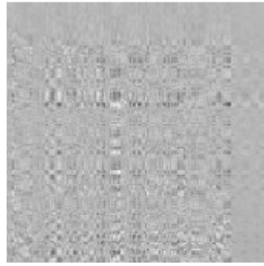
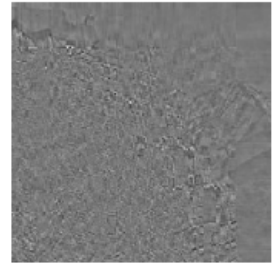


Image of top 25 singular values



Error image of top 25 singular values



- $k = 50$

Image of top 50 eigenvalues



Error image of top 50 eigenvalues

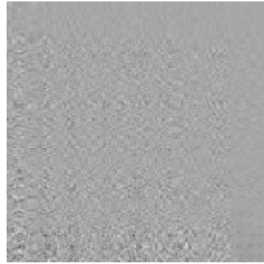
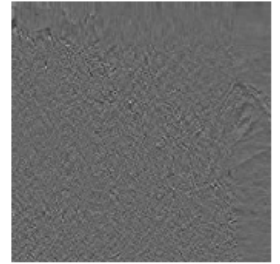


Image of top 50 singular values



Error image of top 50 singular values



- $k = 200$

Image of top 200 eigenvalues



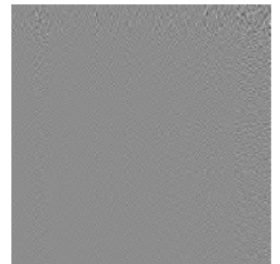
Error image of top 200 eigenvalues



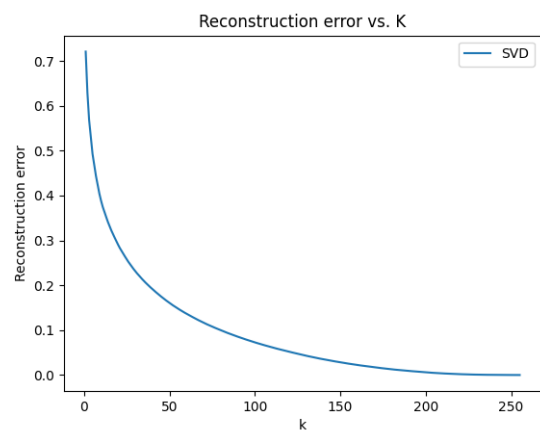
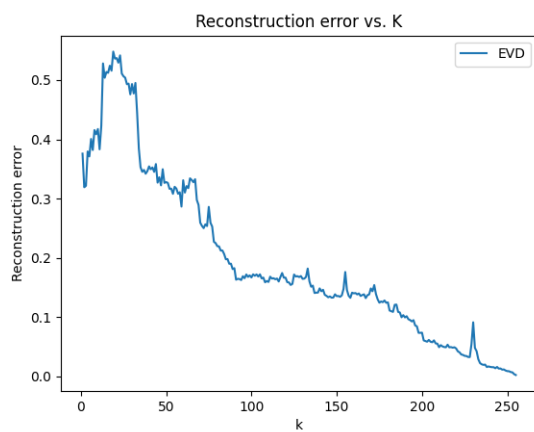
Image of top 200 singular values

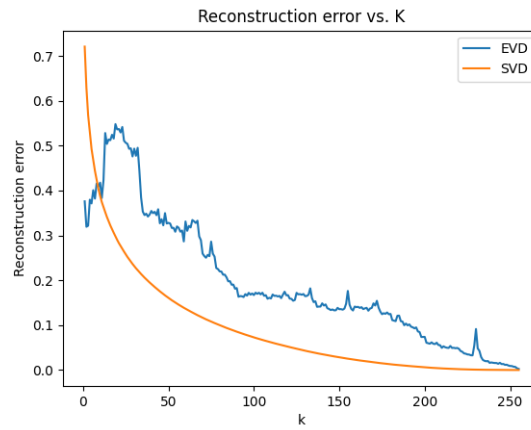


Error image of top 200 singular values



Below are the reconstruction error vs k graph for each EVD and SVD and their comparison





4. Inferences

- From the comparison of the error vs k graph of EVD and SVD we can infer that SVD is more reliable for compressing an image than EVD
- We can get most part of image with very less values in SVD
- For some values of k in EVD we get spike in error in reconstruction