Multiple Linear Regression.   In-class Exercise 3

EL-GY 6143 Intro Machine Learning. Prof. Sundeep Rangan

Question

We are given the following data

Sample number	Target y_i	Feature 1 x_{i1}	Feature 2 x_{i2}
1	3.0	0	1
2	5.0	2	3
3	9.0	4	8
4	10.0	6	10

- Q1. Write the equations to solve for the linear model using all four data points
    - Write the feature matrix and the equations for coefficients.
    - Do not solve them (you would need a computer)
- Q2. Can you find parameters that exactly fits the first three data points?
    - Just state if such parameters exist. You do not need to find them.

Solution

Q1. The feature matrix and target vector are:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 8 \\ 1 & 6 & 10 \end{bmatrix}, \quad y = \begin{bmatrix} 3 \\ 5 \\ 9 \\ 10 \end{bmatrix}$$

The coefficients in the model is:

$$\beta = (A^T A)^{-1} A^T y$$

Q2. If we look at only the first three data points, we have:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 8 \end{bmatrix}, \quad y = \begin{bmatrix} 3 \\ 5 \\ 9 \end{bmatrix}$$

To get an exact fit, we want $y = A\beta$. This is possible since, in this case, A is invertible (it is square and you can check that columns are linearly independent). So you can select $\beta = A^{-1}y$