0: Strong Induction (slide invariant)

Prove that every $\mathbf{n \geq 2}$ can be expressed as a product of itself.

Let $P(n)$ be "$n = p_0 p_1 p_2 \cdots p_j$ where $p_0, p_1, p_2, \cdots, p_j$ are prime." We go by strong induction on n

Base Case ($n = 2$): $2 = 2$ and 2 is a prime number. Thus, since it can be written as a product of itself, which is prime, $P(2)$ is true.

Inductive Hypothesis: Suppose that $P(l)$ is true for all $2 \leq l < k$ where $k \geq 2$.

Inductive Step: We go by cases:

- k is prime: If k is prime, then there is no number smaller than k divides it. Thus, k can be written as a product of itself which is prime and $P(k)$ holds if k is prime.
- k is composite: If k is composite, then it can be written as $k = m \cdot n$ where $1 < m, n < k$. Since $m < k$ and $n < k$, then

  $$k = m \cdot n$$

  $$k = (p_0^m p_1^m p_2^m \cdots p_i^m) \cdot (p_0^n p_1^n p_2^n \cdots p_j^n) \text{ (by Inductive Hypothesis)}$$

  A product of products is still a product, so k can be written as a product of primes, thus $P(k)$ is true even when k is composite

Since $P(k)$ is true for both cases of k being prime or composite, we have proved the statement for all $n \geq 2$ by induction. $\square$