0: Rolling Dice

For both parts, we use the definition that two events A and B are independent if and only if $\Pr(A|B) = \Pr(A)$ and $\Pr(B|A) = \Pr(B)$.

(a) By the construction of `RollDie(6)`, we can say that $\Pr(D_1)$ $=\Pr(D_2) = \frac{1}{6}$. By the law of product, $\Pr(D_1 \cap D_2) = \frac{1}{6} * \frac{1}{6} = \frac{1}{36}$. We now perform the following calculations.

$$\begin{aligned}
\Pr(D_1|D_2) &= \frac{\Pr(D_1 \cap D_2)}{\Pr(D_2)} && \text{[definition of } \Pr(A|B)\text{]} \\
&= \frac{1/36}{1/6} \\
&= \frac{1}{6} \\
&= \Pr(D_2)
\end{aligned}$$

Using the commutativity $\cap$, a similar calculation is performed to show that $\Pr(D_2|D_1) = \Pr(D_1)$. Therefore, by the definition of independence, $\boxed{D_1 \text{ and } D_2 \text{ are independent.}}$

(b) There are 36 possible outcomes for `die1 + die2` (by law of product), and only 4 of those outcomes result in 5, i.e. $|S_5| = |\{(1,4),(2,3),(3,2),(4,1)\}| = 4$, so $\Pr(S_5) = \frac{4}{36} = \frac{1}{9}$. By looking at all the possibilities of S_5, we see that there is only one outcome where `die1` $= 1$, so $\Pr(D_1 \cap S_5) = \frac{1}{36}$.

$$\begin{aligned}
\Pr(D_1|S_5) &= \frac{\Pr(D_1 \cap S_5)}{\Pr(S_5)} && \text{[definition of } \Pr(A|B)\text{]} \\
&= \frac{1/36}{1/9} \\
&= \frac{1}{4} \\
&\neq \Pr(S_5)
\end{aligned}$$

Since $\Pr(D_1|S_5) \neq \Pr(S_5)$, by the definition of independence,
$\boxed{D_1 \text{ and } S_5 \text{ are not independent and therefore dependent.}}$