
0: Inclusion-Exclusion

How many ways are there to pick three numbers from three sets A , B , C such that:

- $A = \{1, 2, 3, \dots, n\}$
- $B = \{1, 2, 3, \dots, m\}$
- $C = \{1, 2, 3, \dots, l\}$
- At least one of the numbers is a one

We use the inclusion Exclusion principle and calculate this by first looking at the total number of ways there are to pick three numbers from the three sets and then subtracting from that the number of ways to pick three numbers from the three sets where none of them is a one.

To find the total number of ways to pick three numbers from the three sets without any restriction, we note that there are n , m , and l ways to pick from A , B , C respectively. By the rule of product, we can say that there are $n * m * l$ ways to pick three numbers from the three sets without any restriction.

To calculate the total number of ways to pick three numbers from three sets where none of the numbers are 1, we note that we're essentially excluding 1 from each set. As such, there are $n - 1$, $m - 1$, and $l - 1$ ways to pick from A , B , C respectively. By the rule of product, we can say that there are $(n - 1)(m - 1)(l - 1)$ ways to pick three numbers from the three sets where none of the numbers are 1.

Now, to find the number of ways three numbers from the three sets where at least one of them is a one, we subtract the numbers of ways there are to pick no ones from the total number of ways to pick three numbers. As such, the total number of ways to pick three numbers from the three sets where at least one of them is a one is $\boxed{nml - (n - 1)(m - 1)(l - 1)}$.