Bonus - 2.2

Sunday, November 12, 2023 7:43 PM

1)

$$\frac{\delta}{\delta D(x)} L^{(D)} = \frac{\delta}{\delta D(x)} \left[-\frac{1}{2} E_{x \sim P_{data}} [\log D(x)] - \frac{1}{2} E_{x \sim P_{model}} [\log(1 - D(x))] \right]$$

$$= \frac{\delta}{\delta D(x)} \left[-\frac{1}{2} \int \log D(x) \, P_{data}(x) - \frac{1}{2} \int \log(1 - D(x)) \, P_{model}(x) \right]$$

$$= -\frac{1}{2} \left[\int \frac{\delta}{\delta D(x)} \log D(x) \, P_{data}(x) + \int \frac{\delta}{\delta D(x)} \log(1 - D(x)) \, P_{model}(x) \right]$$

$$= -\frac{1}{2} \left[\int \frac{P_{data}(x)}{D(x)} - \int \frac{P_{model}(x)}{1 - D(x)} \right]$$

$$\boxed{\frac{\delta}{\delta D(x)} L^{(D)} = \frac{1}{2} \int \frac{P_{model}(x)}{1 - D(x)} - \frac{1}{2} \int \frac{P_{data}(x)}{D(x)}}$$

2)

$$\frac{\delta}{\delta D(x)} L^{(D)} = \frac{1}{2} \int \frac{P_{model}(x)}{1 - D(x)} - \frac{1}{2} \int \frac{P_{data}(x)}{D(x)} = 0$$

$$\frac{1}{2} \int \frac{P_{model}(x)}{1 - D(x)} = \frac{1}{2} \int \frac{P_{data}(x)}{D(x)}$$

$$\int \frac{P_{model}(x)}{1 - D(x)} = \int \frac{P_{data}(x)}{D(x)}$$

We note that the two integrals will be equal if the values under both integrals are the same i.e.,

$$\frac{P_{model}(x)}{1-D(x)} = \frac{P_{data}(x)}{D(x)}$$

$$\frac{P_{model}(x)}{P_{data}(x)} = \frac{1-D(x)}{D(x)}$$

$$\frac{P_{model}(x)}{P_{data}(x)} = \frac{1}{D(x)} - 1$$

$$\frac{1}{D(x)} = \frac{P_{model}(x) + P_{data}(x)}{P_{data}(x)}$$

$$\boxed{D(x) = \frac{P_{data}(x)}{P_{model}(x) + P_{data}(x)}}$$

3) This estimating fraction makes sense as it's essentially the probability of a point x being in the probability distribution of the data. The

total pixel space is comprised from pixels from the true distrubution p_{data} and the generator model p_{model}, so we sum them & divide by that quantity to get the probability that the pixel is in the true distribution.