Transition to Mathematical Proofs
Chapter 1 - Logic Assignment Solutions

Instructions: For the below questions, show all of your work. For the proofs, be sure that you

(i) include a Discussion section;
(ii) write a complete proof in full English sentences;
(iii) if hand-writing, write legibly and clearly.

Question 1. Let $m \neq 0$ and b be real numbers. Show that there exists a unique x such that $mx + b = 0$.

Question 2. Prove the following biconditional statement.

> Let x be a real number. $-1 \leq x \leq 1$ if and only if $x^2 \leq 1$.

In proving this, it may be helpful to note that $-1 \leq x \leq 1$ is equivalent to $-1 \leq x$ and $x \leq 1$.

Question 3. Two whole numbers are said to *have the same parity* if they are both even or both odd. Prove the following biconditional statement:

> Let m and n be whole numbers. m and n have the same parity if and only if $m + n$ is even.

Question 4. Use *proof by contrapositive* to prove the following conditional statement.

> Let m and n be whole number. If $m \cdot n$ is odd, then m and n are both odd.

Question 5. We will investigate the following statement:

> Every odd whole number can be written as the difference of two perfect squares.

(a) For the odd whole numbers $n = -3, -1, 1, 3, 5, 7, 9$, write n as the difference of two perfect squares.

(b) Use any pattern that you found in (a) to help you write a proof of our statement.