Student Name: _____

TSSM
Creating VCE Success

GENERAL MATHEMATICS 2024

Unit 4
Key Topic Test 3 – Transition Matrices

Recommended writing time*: 45 minutes
Total number of marks available: 25 marks

QUESTION BOOK

* The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this test: pens, pencils, highlighters, erasers, sharpeners and rulers, approved CAS calculator and one bound reference book.
- Students are NOT permitted to bring into the room for this test: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 9 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.

SECTION A – Multiple-choice questions

Instructions for Section A
• All questions are worth one mark.
• Answer all questions by circling the correct response.
• Marks are not deducted for incorrect answers.
• No marks will be awarded if more than one answer is completed for any question

Use the following information to answer Questions 1 and 2

A railway knows that 200 carriages will be needed to carry passengers from point *A* to point *B*. At the end of each week, it finds that 10% of the carriages that started at *A*, ended at point *B*, and 15% of the carriages that started at point *B* ended at point *A*.

Question 1
A transition matrix, T that could represent this situation is

A.
$$\begin{array}{c} \\ to \end{array} \begin{array}{c} from \\ \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array}\begin{bmatrix} 0.1 & 0.85 \\ 0.9 & 0.15 \end{bmatrix} \end{array}$$

B.
$$\begin{array}{c} \\ to \end{array} \begin{array}{c} from \\ \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array}\begin{bmatrix} 0.9 & 0.85 \\ 0.1 & 0.15 \end{bmatrix} \end{array}$$

C.
$$\begin{array}{c} \\ to \end{array} \begin{array}{c} from \\ \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array}\begin{bmatrix} 0.9 & 0.15 \\ 0.1 & 0.85 \end{bmatrix} \end{array}$$

D.
$$\begin{array}{c} \\ to \end{array} \begin{array}{c} from \\ \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array}\begin{bmatrix} 0.1 & 0.9 \\ 0.85 & 0.15 \end{bmatrix} \end{array}$$

E.
$$\begin{array}{c} \\ to \end{array} \begin{array}{c} from \\ \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array}\begin{bmatrix} 1.1 & 0.15 \\ 0.1 & 1.15 \end{bmatrix} \end{array}$$
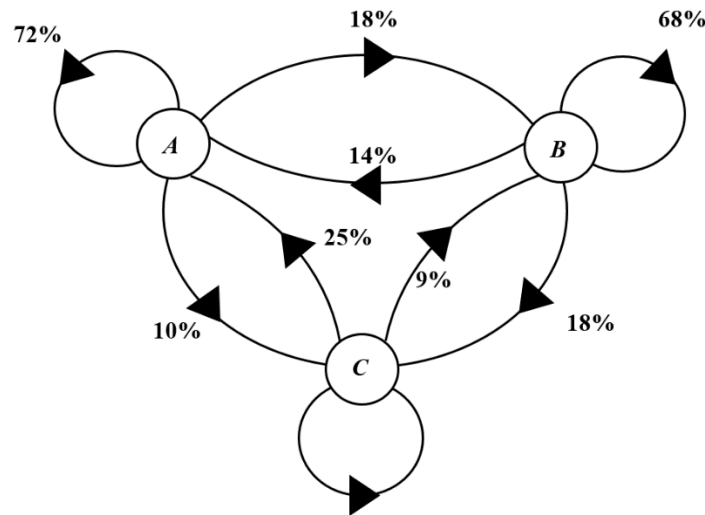
Question 2
If equal number of carriages started at both points, the number of carriages located at point B at the end of three weeks is:

A. 91

B. 88

C. 100

D. 95

E. 112

Use the following information to answer Questions 3, 4 and 5

An e-scooter hire business has three locations **A**, **B** and **C** from which clients can hire an e-scooter. Clients may deliver their e-scooter to any of the three locations. Over the period of a week, the number of e-scooters at each location changes according to the transition diagram below:

Question 3

The percentage of e-scooters that are hired from C and delivered to C each week is:

   A. 9%   **B.** 18%   **C.** 10%

   D. 25%   **E.** 66%

Question 4

If the business began the week with 100 e-scooters at each location. The number of e-scooters at location B at the end of 2 weeks is:

   A. 93   **B.** 92   **C.** 90

   D. 117   **E.** 95

Question 5

In the long run, the number of e-scooters located at location C at the end of each week is

   A. 122   **B.** 92   **C.** 93

   E. 85   **E.** 86

SECTION B - Short-answer questions

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Instructions for Section B
- Answer each question in the space provided.
- Please provide appropriate workings and use exact answers unless otherwise specified.
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Question 1   **(10 marks)**

For a certain football league, gambling agencies are able to predict whether a team is likely to win, lose or draw their next match, based on their previous results.

They have found that:
- If a team wins a round, it has a 75% chance of winning the next round and a 5% chance of drawing the next round
- If a team loses a round, it has a 67% chance of losing the next round, and a 5% chance of drawing the next round
- If a team has a draw in a round, it then has an even chance of winning or losing the next round and no chance of drawing the next round

a. Complete the transition diagram to show this information

2 marks

b. Using the information above, complete this transition matrix:

This round

$$T = \begin{bmatrix} 0.75 & & 0.5 \\ & 0.67 & 0.5 \\ 0.05 & & 0 \end{bmatrix} \begin{matrix} W \\ L \\ D \end{matrix}$$

W   L   D (column headers)

Next round (right label)

2 marks

c. Complete this initial state matrix, showing how one team, the Falcons, performed in the final round of 2023, given that they won the game.

$$S_0 = \begin{matrix} W \\ L \\ D \end{matrix} \begin{bmatrix} \\ \\ \end{bmatrix}$$

1 mark

d. Given the transition matrix T predicts performance in the next round, state the probabilities (as percentages) that Falcons will win, lose or draw the next game they play. Note: this game will be the first round of the 2024 season. Assume the probabilities discussed above continue across seasons.

_____

_____

_____

2 marks

e. Find the probability that the Falcons will win in round 3 of 2024. Round your answer to the nearest whole percent.

_____

_____

_____

2 marks

f. In 2024, there will be 10 teams in the competition. In round one, 4 of the teams win, 4 lose and 2 draw. Of the 4 teams that win, how many are predicted to go on to be winners the next week?

_____

_____

_____

1 mark

Question 2 **(10 marks)**

Throughout winter at the local gym, members are able to choose from a Monday night group class of Boxing (B), Yoga (Y) or Cardio (C). The diagram below shows the change in class preferences each week on the Monday.
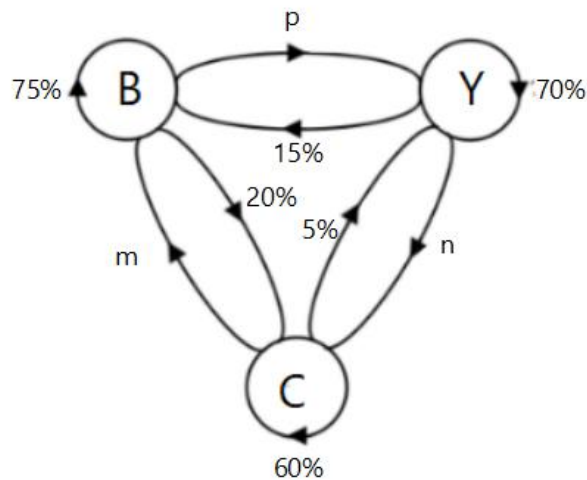
a. Find the values of m, n and p.

_____

_____

_____

2 marks

Last Monday, there were 40 members in the Boxing class, 15 in Yoga and 25 in Cardio. Represented in the initial state matrix S_0.

$$S_0 = \begin{bmatrix} 40 \\ 15 \\ 25 \end{bmatrix} \begin{matrix} B \\ Y \\ C \end{matrix}$$

b. Find the expected number of members in Yoga for the following Monday.

_____

_____

_____

_____

2 marks

c. Find the number of members that did the same class the following Monday.

_____

_____

_____

2 marks

d. Assuming the pattern continues, find the equilibrium state for members in each class.

_____

_____

_____

2 marks

During summer the matrix relation $S_{n+1} = T \times S_n + C$ describes the movement of members from week to week through the three different classes.

$$\text{This week}$$

Where: $T = \begin{bmatrix} 0.8 & 0.2 & 0.05 \\ 0.1 & 0.5 & 0.3 \\ 0.1 & 0.3 & 0.65 \end{bmatrix} \begin{matrix} B \\ Y \\ C \end{matrix} \text{ Next week}$ and $C = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} \begin{matrix} B \\ Y \\ C \end{matrix}$

e. Explain the net effect on member numbers from week to week under this model.

_____

_____

1 mark

f. Find S_2 given $S_1 = \begin{bmatrix} 50 \\ 30 \\ 20 \end{bmatrix} \begin{matrix} B \\ Y \\ C \end{matrix}$

_____

_____

1 mark

END OF KEY TOPIC TEST