GENERAL MATHEMATICS 2024

Unit 4
Key Topic Test 1 – Matrix Arithmetic

Recommended writing time*: 45 minutes
Total number of marks available: 25 marks

SOLUTIONS

SECTION A – Multiple Choice (1 mark per question)

Question 1
Answer: A

Permutation matrix
- Contains only 1s and 0s
- Each row and column has exactly one 1

Question 2
Answer: D

$a_{21} = 1, a_{23} = -6$
$1 - -6 = 7$

Question 3
Answer: B

$p_{11} = -5a + 2c = -18$
$p_{21} = 2a + 1c = 9$

$a = 4, c = 1$
$a + c = 5$

Question 4
Answer: E

$2A + B$ is not possible as A is order 2×2 which doesn't match B of order 3×2

Question 5
Answer: D

$(A + C)$ order 2×3
B^{-1} order 3×2

Columns of $(A + C)$ match rows of B^{-1}

SECTION B – Short Answer

Question 1

a. The columns of A do not match the rows of C

1 mark

b. $b_{22} = 1$

1 mark

c. $A^{-1} = \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$ $or \frac{1}{3}\begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$

1 mark

d. $\begin{bmatrix} -12 & -2 & 14 \\ 12 & 10 & 20 \end{bmatrix}$

1 mark

e. $A^T = \begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}$

1 mark

f. $AB =$ order 2×3

(AB)C = order 2×2

1 mark

g. $P = \begin{bmatrix} -31 & 60 \\ 35 & 102 \end{bmatrix}$

$P^{-1} = \begin{bmatrix} -\frac{17}{877} & \frac{10}{877} \\ \frac{35}{5262} & \frac{31}{5262} \end{bmatrix}$

2 marks

h. $G = 3C \times 2B = \begin{bmatrix} 96 & 72 & 126 \\ 48 & 0 & -90 \\ -120 & 60 & 480 \end{bmatrix}$

G is a 3×3 matrix
$g_{21} = 48$

2 marks

Question 2

a. $\begin{bmatrix} 2 & 1 & 7 \\ 3 & 1 & 4 \\ 5 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9556 \\ 5899 \\ 3155 \end{bmatrix}$

1 mark

b. $A^{-1} = \begin{bmatrix} 2 & 1 & 7 \\ 3 & 1 & 4 \\ 5 & 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{7}{10} & \frac{13}{10} & -\frac{3}{10} \\ \frac{17}{10} & -\frac{33}{10} & \frac{13}{10} \\ \frac{1}{10} & \frac{1}{10} & -\frac{1}{10} \end{bmatrix}$

1 mark

c.

$$\begin{bmatrix} -\frac{7}{10} & \frac{13}{10} & -\frac{3}{10} \\ \frac{17}{10} & -\frac{33}{10} & \frac{13}{10} \\ \frac{1}{10} & \frac{1}{10} & -\frac{1}{10} \end{bmatrix} \times \begin{bmatrix} 9556 \\ 5899 \\ 3155 \end{bmatrix}$$

$x = 33, y = 880, z = 1230$ 2 marks

Question 3

a. $T^T = [1 \quad 8 \quad 20 \quad 16]$

1 mark

b. $B = \begin{bmatrix} 1.1 & 0 & 0 & 0 \\ 0 & 1.1 & 0 & 0 \\ 0 & 0 & 1.1 & 0 \\ 0 & 0 & 0 & 1.1 \end{bmatrix}$

1 mark

c. $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} t \\ f \\ u \\ r \\ i \end{bmatrix} = \begin{bmatrix} f \\ r \\ u \\ i \\ t \end{bmatrix}$

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2 marks

d. $P =$ order 5×5
P is a permutation matrix

1 mark