Student Name: _____

GENERAL MATHEMATICS 2024

Unit 4
Key Topic Test 4 – Leslie Matrices

Recommended writing time*: 45 minutes
Total number of marks available: 25 marks

QUESTION BOOK

* The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this test: pens, pencils, highlighters, erasers, sharpeners and rulers, approved CAS calculator and one bound reference book.
- Students are NOT permitted to bring into the room for this test: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 9 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.

SECTION A – Multiple-choice questions

Use the following information to answer Questions 1 and 2

Consider the Leslie matrix L and initial state matrix S_0 below.

From age group

$$L = \begin{bmatrix} 0 & 1.4 & 2.0 \\ 0.8 & 0 & 0 \\ 0 & 0.6 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2, \\ 3 \end{matrix} \quad \text{To age group} \qquad S_0 = \begin{bmatrix} 100 \\ 50 \\ 10 \end{bmatrix}$$

Question 1

The survival rate of age group 2 is:

A. 1.4

B. 2.0

C. 0.8

D. 0.6

E. 0

Question 2

The population of age group 1 after 2 cycles is:

A. 172

B. 48

C. 72

D. 90

E. 80

Use the following information to answer Questions 3 and 4

Details of the female population of a snake species in the Australian outback are shown below:

Age	0-1	1-2	2-3
Initial Population	100	100	100
Birth rate	0.6	0.3	0.1
Survival rate	0.5	0.8	0

Question 3

The Leslie matrix that describes this information is:

$$L = \begin{array}{c} \text{From age group} \\ \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.6 & a & b \\ c & 0 & 0 \\ 0 & d & 0 \end{bmatrix} \begin{array}{c} 1 \\ 2, \\ 3 \end{array} \end{array} \quad \text{To age group}$$

The values of a, b, c and d respectively are:

A. $0, 0, 0, 0$

B. $0.5, 0.8, 0.3, 0.1$

C. $0.3, 0.1, 0, 0.5$

D. $0.3, 0.1, 0.5, 0.8$

E. $0.5, 0.3, 0.8, 0.1$

Question 4
The population of age group 1 after 3 cycles is closest to:

A. 100

B. 68

C. 42

D. 40

E. 69

Question 5

Consider the following Leslie matrix L and population state matrix S_1.

$$L = \begin{bmatrix} 0 & 0.5 & 0.4 \\ 0.7 & 0 & 0 \\ 0 & 0.8 & 0 \end{bmatrix} \quad S_1 = \begin{bmatrix} 14 \\ 35 \\ 16 \end{bmatrix}$$

If $S_{n+1} = LS_n$, S_0 is equal to:

A. $\begin{bmatrix} 23.9 \\ 9.8 \\ 28 \end{bmatrix}$

B. $\begin{bmatrix} 50 \\ 50 \\ 50 \end{bmatrix}$

C. $\begin{bmatrix} 50 \\ 20 \\ 10 \end{bmatrix}$

D. $\begin{bmatrix} 10 \\ 40 \\ 50 \end{bmatrix}$

E. $\begin{bmatrix} 25 \\ 50 \\ 30 \end{bmatrix}$

SECTION B - Short-answer questions

Instructions for Section B
• Answer each question in the space provided.
• Please provide appropriate workings and use exact answers unless otherwise specified.

Question 1   (9 marks)

Details of the female population of an endangered bird species with a maximum life span of 6 years are shown below:

Age	0-2 years	2-4 years	4-6 years
Initial Population	50	220	30
Birth rate	0	0.6	0.4
Survival rate	0.3	0.7	0

    a. Present this information as a transition diagram by filling in the boxes below:
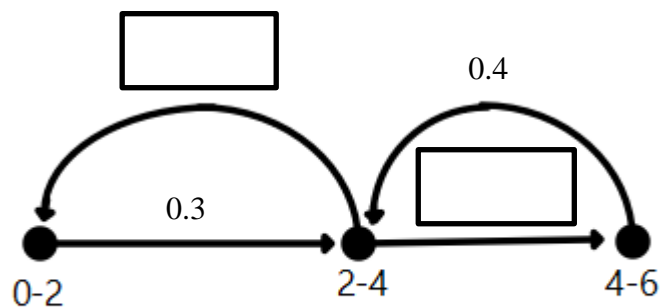
2 marks

    b. State the initial state matrix, S_0

1 mark

The breeding pattern of the female bird population can be modelled by the recurrence relation
$$S_{n+1} = LS_n$$

From Age Group

Where $L = \begin{bmatrix} 0 & 0.6 & 0.4 \\ 0.3 & 0 & 0 \\ 0 & 0.7 & 0 \end{bmatrix}$ To Age Group , and n is the number of 2 year periods since population was tracked.

c. Find the number of birds aged 2-4 are there after 3 2-year periods.

_____

_____

_____

2 marks

d. Find the percentage of birds in the 0-2 age category after 3 2-year periods. Round your answer to the nearest percent.

_____

_____

_____

2 marks

e. Using populations rounded to the nearest whole number, find the number of years until the female population of this bird species will be extinct.

_____

_____

_____

2 marks

Question 2 **(7 marks)**

A population of female Rabbits is modelled by a Leslie matrix L. Where S_n is the population of female rabbits after n cycles. The population follows the model:
$$S_{n+1} = L \times S_n$$

From Age Group

Where $L = \begin{bmatrix} 0 & 0.5 & 0.4 \\ 0.8 & 0 & 0 \\ 0 & 0.6 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$ To Age Group

with column headers $\begin{matrix} 1 & 2 & 3 \end{matrix}$

a. State the birth rate of age group 2.

_____

1 mark

b. State the survival rate of rabbits in age group 1.

_____

1 mark

S_0 shows the population of female rabbits that was initially recorded.

$$S_0 = \begin{bmatrix} 250 \\ 208 \\ 110 \end{bmatrix}$$

c. Use a matrix multiplication method to show that the initial population of female rabbits is 568.

_____

_____

_____

_____

1 mark

d. Find the percentage increase or decrease in the population of female rabbits after 1 cycle.

_____

_____

_____

2 marks

e. Find the population of rabbits in age group 2 after 5 cycles.

_____

_____

_____

2 marks

Question 3

The Leslie matrix, L and initial state matrix S_0 for the population of a species on the planet Zorg is given by:

From Age Group

$$\text{Where } L = \begin{matrix} 1 & 2 & 3 \\ \begin{bmatrix} 0 & 3 & 3 \\ 0.4 & 0 & 0 \\ 0 & 0.6 & 0 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{matrix} \text{ To Age Group }, \quad S_0 = \begin{bmatrix} 80 \\ 40 \\ 50 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

The population follows the model $S_{n+1} = L \times S_n$.

a. Find S_{54} and S_{55}, rounding values to the nearest whole number.

2 marks

b. If $S_{55} = k \times S_{54}$, find the value of k, rounding your answer to 1 decimal place.

_____

_____

1 mark

c. By referring to long term population, comment on the findings from **b.**

_____

_____

_____

1 mark

END OF KEY TOPIC TEST