Student Name: _____

GENERAL MATHEMATICS 2024

Unit 4
Key Topic Test 5 – Networks and Decision Mathematics: Introduction to graphs

Recommended writing time*: 45 minutes
Total number of marks available: 25 marks

QUESTION BOOK

* The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this test: pens, pencils, highlighters, erasers, sharpeners and rulers, approved CAS calculator and one bound reference book.
- Students are NOT permitted to bring into the room for this test: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 9 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.
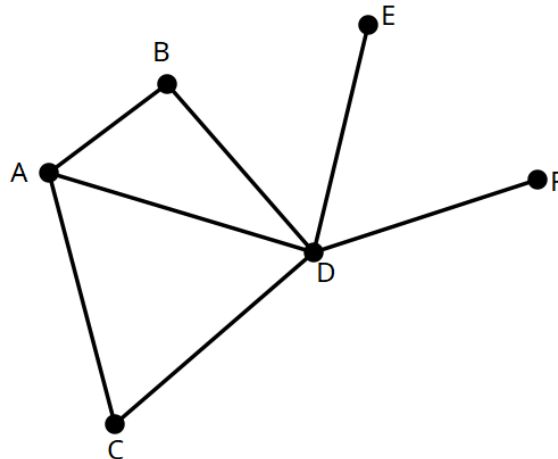
SECTION A – Multiple-choice questions

Instructions for Section A
• All questions are worth one mark.
• Answer all questions by circling the correct response.
• Marks are not deducted for incorrect answers.
• No marks will be awarded if more than one answer is completed for any question

The following information relates to Question 1 and 2

Consider the network

Question 1

The sum of the degrees of all vertices in this this network is:

    A. 12

    B. 13

    C. 14

    D. 15

    E. 16

Question 2

The number of edges that would need to be added such that a Eulerian trail exists is:

    A. 0   **B.** 1   **C.** 2   **D.** 3   **E.** 4

Question 3

A connected planar graph has 6 vertices and 10 edges.
The number of faces that this graph will have is:

    A. 3

    B. 4

    C. 5

    D. 6

    E. 7

Question 4

For the network shown, which of the following statements is false:

    A. The graph is connected

    B. There are 3 even degree vertices

    C. There are 2 faces

    D. The graph is simple

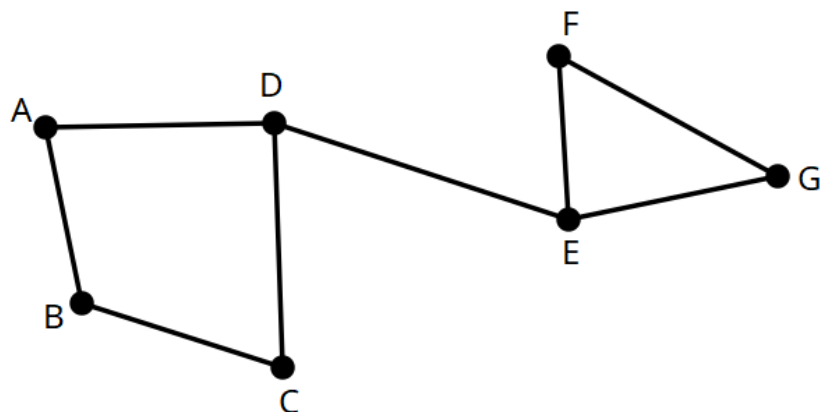   **E.** The graph contains a Hamiltonian circuit
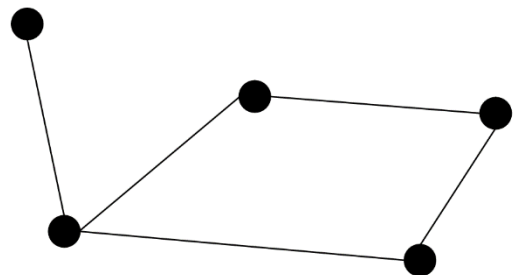
Question 5

The graph below has seven vertices labelled A, B, C, D, E, F and G, and eight edges.
The edge which can be described as a bridge is:

    A. *AD*

    B. *CD*

   **C.** *DE*

    D. *EF*

    E. *EG*

SECTION B - Short-answer questions

Instructions for Section B
• Answer each question in the space provided.
• Please provide appropriate workings and use exact answers unless otherwise specified.

Question 1 (5 marks)

Detective Holmes is on the trail of a criminal mastermind El Hapo. The edges on the following network show the flights that El Hapo took between countries over a 7 day period. Detective Holmes knows that El Hapo's first flight left **Mexico** and the last flight landed in **Peru**.
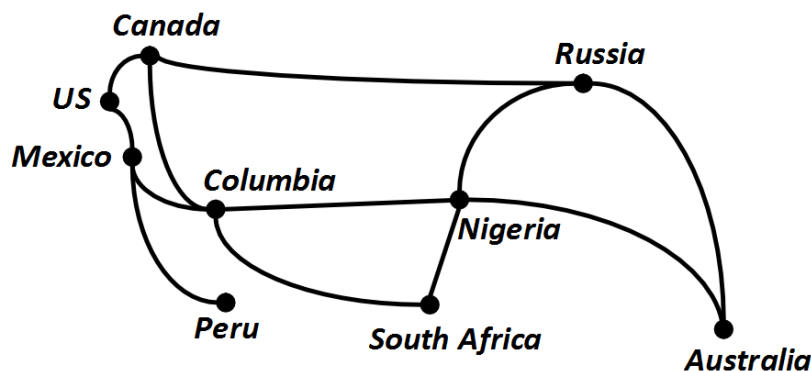
a. State the degree of the vertex Nigeria.

_____

1 mark

b. El Hapo took each flight exactly once. Detective Holmes knows that there is **one flight missing** from the network. Draw the missing flight on the network above.

1 mark

c. State the mathematical name given to this type of trip.

_____

1 mark

  d. <u>Including Mexico</u>, find the number of countries that El Hapo has to visit more than once on this trip.

_____

                                               1 mark
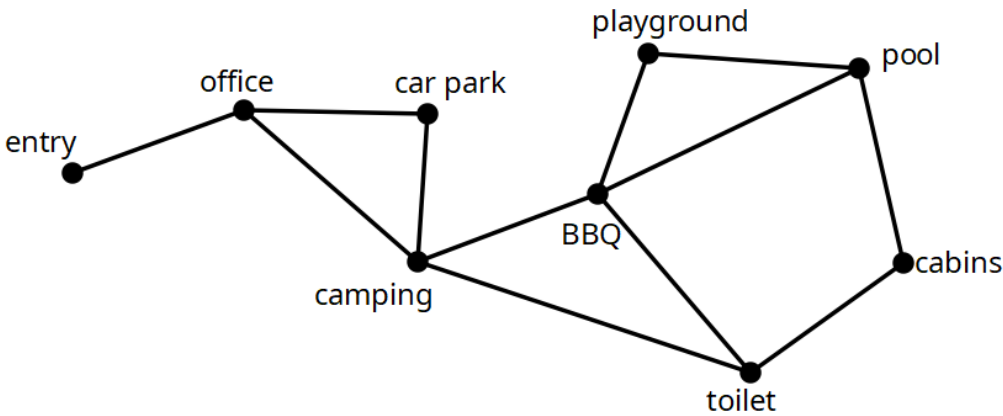
  e. List a possible travel route that El Hapo took.

_____

                                               1 mark

Question 2 (11 marks)

A caravan park that includes different facilities is shown in the network below. The key landmarks are represented by vertices and the edges show the roads connecting them.

  a. State the degree of the BBQ vertex

_____

                                               1 mark

  b. Complete the table below and hence use Euler's formula to show that this graph is planar.

Number of edges	
Number of vertices	
Number of faces	

$$\boxed{} \quad + \quad \boxed{} \quad - \quad \boxed{} \quad = \quad \boxed{}$$

                                               2 marks

The manager of the park begins at the entry and visits each key landmark once.

    c. State the mathematical term given to this route.
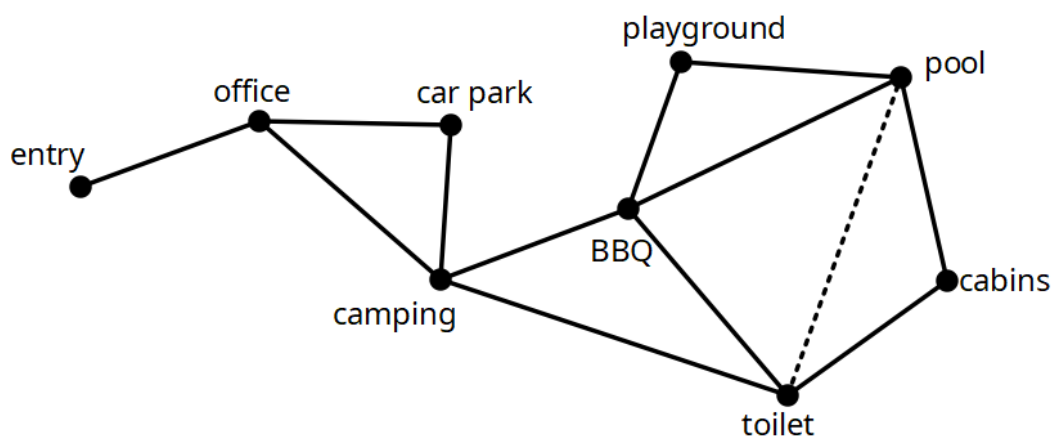
_____

                                1 mark

    d. State a possible route that the manager took.

_____

                                1 mark

    e. Find the number of odd degree vertices

_____

                                1 mark

There is also a walking track in the park that give direct access to the toilet from the pool. It is shown with a dotted line in the diagram below.

The cleaner wishes to travel along all roads and walking tracks in the caravan park.

    f. If the cleaner wishes to travel each road and walking track exactly once, state the possible key landmarks where they could start their journey.

_____

                                2 marks

g. State a possible route that the cleaner could take.

_____

1 mark

h. The cleaner wishes to start and finish this journey from the car park, still without repeating any road or walking trail. Between which key landmarks would a walking trail need to be added so that this is possible?
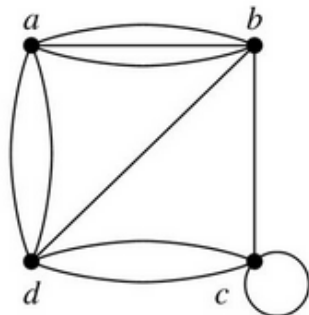
_____

1 mark

i. State the mathematical term for the route described in h.

_____

1 mark

Question 3   (4 marks)

Consider the network diagram below:

a. Complete the adjacency matrix below for this network.

$$\begin{array}{cccc} a & b & c & d \end{array}$$
$$\begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{array}{c} a \\ b \\ c \\ d \end{array}$$

2 marks

b. State the number of unique paths, from a to b with 2 edges or less.

_____

1 mark

c. State the degree of vertex c

_____

1 mark

END OF KEY TOPIC TEST