SAAS CX Homework 5

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Spring 2022

Problem 0.1.

1. $5 \times 4 \times 3 = \boxed{60}$
2. $\binom{5}{3} = \boxed{10}$
3. a) $n(n-1)(n-2)\ldots(n-k+1) = \frac{n!}{(n-k)!} =_n P_k$

   b) $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Problem 0.2.

1. $\frac{1}{4}$
2. $\frac{1}{2}$
3. $\frac{15}{16}$
4. 1

Problem 0.3. Using PIE yields

$$\left(\frac{1}{10}+\frac{1}{10}+\frac{1}{10}\right)-\left(\frac{1}{100}+\frac{1}{100}+\frac{1}{100}\right)+\frac{1}{1000}=\boxed{\frac{271}{1000}}$$

Problem 0.4. Using conditional probability yields

$$\begin{aligned}\mathbb{P}[C] &= \mathbb{P}[C|S]\mathbb{P}[S]+\mathbb{P}[C|R]\mathbb{P}[R]\\ &= (0.9)(0.6)+(0.8)(0.4)=\boxed{0.86}\end{aligned}$$

Problem 0.5. Observing

$$X = X_p + 5X_n + 10X_d + 25X_q$$

for

$$\begin{aligned}X_p &\sim \text{Bin}(8, 0.5)\\ X_n &\sim \text{Bin}(4, 0.5)\\ X_d &\sim \text{Bin}(6, 0.5)\\ X_q &\sim \text{Bernoulli}(0.5)\end{aligned}$$

We have by linearity

$$\begin{aligned} \mathbb{E}[X] &= \mathbb{E}[X_p] + 5\mathbb{E}[X_n] + 10\mathbb{E}[X_d] + 25\mathbb{E}[X_q] \\ &= 4 + 5(2) + 10(3) + 25(0.5) \\ &= \boxed{56.5} \end{aligned}$$

and by independence

$$\begin{aligned} \mathrm{Var}(X) &= \mathrm{Var}(X_p) + \mathrm{Var}(5X_n) + \mathrm{Var}(10X_d) + \mathrm{Var}(25X_q) \\ &= \mathrm{Var}(X_p) + 25\,\mathrm{Var}(X_n) + 100\,\mathrm{Var}(X_d) + 625\,\mathrm{Var}(X_q) \\ &= 2 + 25(1) + 100(1.5) + 625(0.25) = 333.25 \\ \sigma &= \sqrt{\mathrm{Var}(X)} = \boxed{18.26} \end{aligned}$$

Thus, the 95%-confidence interval is

$$[\mu - 2\sigma, \mu + 2\sigma] = \boxed{[19.99, 93.01]}$$

That is, we expect that roughly 95% of the time, our experiment of flipping the aforementioned set of coins will yield between ~ 20 and ~ 93 cents landing face-up.