

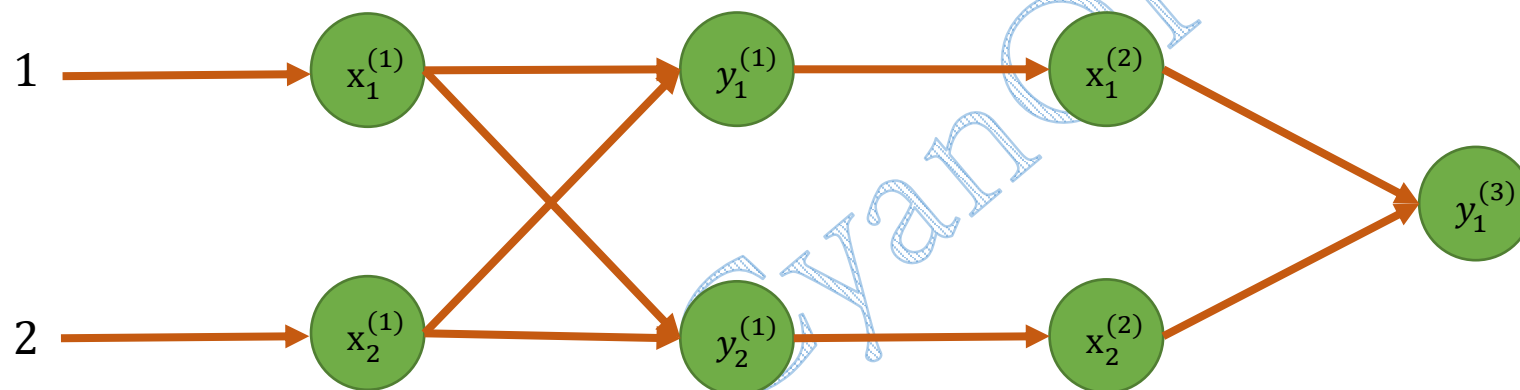
Input: $x_1^{(1)} = 1$ $x_2^{(1)} = 2$

Label: $A = 2$

激活函数 $y = \text{ReLU}(x) = \begin{cases} 0, & x < 0 \\ x, & x \geq 0 \end{cases}$

Learning rate(Lr) = 0.01

损失函数 最小均方误差-Minimum Mean Squared Error (MMSE) $J(w, b) = \text{Loss} = \frac{1}{2} (A - \hat{A})^2$



则：

初始化权重：

$$w_{1,1}^{(1)} = w_{1,2}^{(1)} = w_{2,1}^{(1)} = w_{1,1}^{(2)} = 1$$

$$w_{2,2}^{(1)} = w_{2,1}^{(2)} = -1$$

$$\theta_{1,1}^{(1)} = \theta_{1,2}^{(1)} = \theta_{1,1}^{(2)} = 1$$

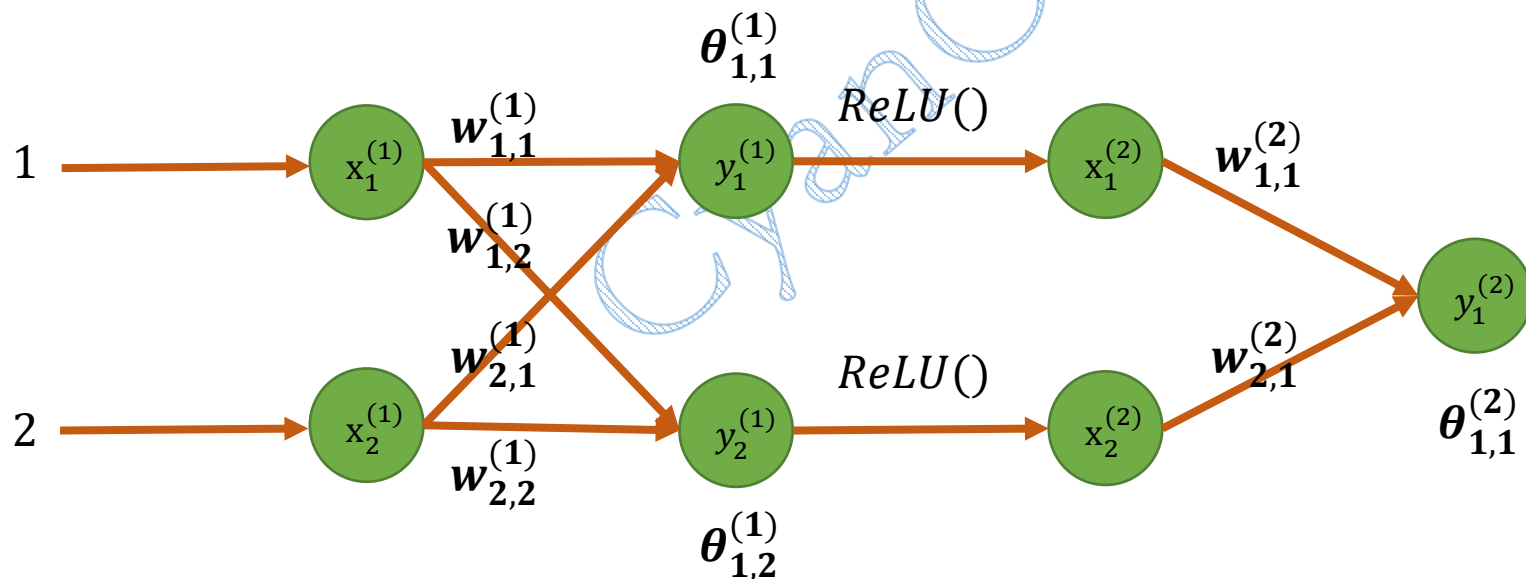
$$y_1^{(1)} = x_1^{(1)} * w_{1,1}^{(1)} + x_2^{(1)} * w_{2,1}^{(1)} + \theta_{1,1}^{(1)} = 4$$

$$x_1^{(2)} = \text{ReLU}(y_1^{(1)}) = 4$$

$$y_2^{(1)} = x_1^{(1)} * w_{1,2}^{(1)} + x_2^{(1)} * w_{2,2}^{(1)} + \theta_{1,2}^{(1)} = 0$$

$$x_2^{(2)} = \text{ReLU}(y_2^{(1)}) = 0$$

$$A = y_1^{(2)} = x_1^{(2)} * w_{1,1}^{(2)} + x_2^{(2)} * w_{2,1}^{(2)} + \theta_{1,1}^{(2)} = 5$$



$w_{1,1_grad}^{(1)}$ 表示参数 $w_{1,1}^{(1)}$ 的梯度

$$J(w, b) = \text{Loss} = \frac{1}{2} (A - \hat{A})^2 = 4.5$$

$$\tilde{w}_{1,1}^{(1)} = w_{1,1}^{(1)} - \text{Lr} * w_{1,1_grad}^{(1)}$$

$$w_{1,1_grad}^{(1)} = \frac{\partial J}{\partial w_{1,1}^{(1)}} = \frac{\partial J}{\partial A} * \frac{\partial A}{\partial x_1^{(2)}} * \frac{\partial x_1^{(2)}}{\partial y_1^{(1)}} * \frac{\partial y_1^{(1)}}{\partial w_{1,1}^{(1)}} = (A - \hat{A}) * w_{1,1}^{(2)} * 1 * x_1^{(1)} = 3$$

$$\tilde{w}_{1,1}^{(1)} = w_{1,1}^{(1)} - \text{Lr} * w_{1,1_grad}^{(1)} = 1 - 0.01 * 3 = 0.97$$

同理可求得 $\frac{\partial J}{\partial w_{1,2}^{(1)}}$, $\frac{\partial J}{\partial w_{2,1}^{(1)}}$, $\frac{\partial J}{\partial w_{2,2}^{(1)}}$, $\frac{\partial J}{\partial w_{1,1}^{(2)}}$, $\frac{\partial J}{\partial w_{2,1}^{(2)}}$, $\frac{\partial J}{\partial \theta_{1,1}^{(1)}}$, $\frac{\partial J}{\partial \theta_{1,2}^{(1)}}$, $\frac{\partial J}{\partial \theta_{1,1}^{(2)}}$

