* *Mobility Model* & Estimation Step

$$\begin{bmatrix} x_t \\ y_t \\ \Theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \Theta_{t-1} \end{bmatrix} + \mathrm{T} * \begin{bmatrix} v_t cos\Theta_{t-1} & 0 \\ v_t sin\Theta_{t-1} & 0 \\ 0 & 1 \end{bmatrix} + \mathrm{w}_k \Leftarrow \boldsymbol{q}_j^{(t)} = \boldsymbol{x}_j^{(t-1)} + \boldsymbol{v}_j(t)\Delta t,$$

jacobian

$$F_{t-1} = \begin{bmatrix} 1 & 0 & -v_t * t * sin\Theta_{t-1} \\ 0 & 1 & v_t * t * cos\Theta_{t-1} \\ 0 & 0 & 1 \end{bmatrix}$$

EKF Estimation

$$\boxed{\begin{aligned} & \hat{x}_t^{-} = f(\hat{x}_{t-1}) \\ & \boldsymbol{q}_j^{(t)} = \boldsymbol{x}_j^{(t-1)} + \boldsymbol{v}_j(t)\Delta t, \end{aligned}}$$

$$\boxed{\hat{P}_t^{-} = F P_{t-1} F^T + Q_t}$$

$$\hat{Q_t} = \begin{bmatrix} v_t cos\Theta_{t-1} & 0 \\ v_t sin\Theta_{t-1} & 0 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} v_t cos\Theta_{t-1} & 0 \\ v_t sin\Theta_{t-1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_w^2 \end{bmatrix}$$

* *Observation Model* & Kalman Gain, Correction Step

$$\begin{bmatrix} r_t^k \\ \phi_t^k \end{bmatrix} = \begin{bmatrix} d_{kj}^{(t)} = \|x_k^{(t)} - x_j^{(t)}\|, \\ \tan^{-1} \dfrac{x_{2k}^{(t)} - x_{2j}^{(t)}}{x_{1k}^{(t)} - x_{1j}^{(t)}}. \end{bmatrix} + n_t^l$$

$$r_t^k = \sqrt{\left(x_{k(t)} - x_{j(t)}\right)^2 + \left(y_{k(t)} - y_{j(t)}\right)^2}$$

$$\phi_t^l = \tan^{-1}((y_{k(t)} - y_{j(t)})/(x_{k(t)} - x_{j(t)}))$$

jacobian

$$H_t = \begin{bmatrix} \dfrac{\partial r_t^k}{\partial x_{k(t)}} & \dfrac{\partial r_t^k}{\partial y_{k(t)}} & \dfrac{\partial r_t^k}{\partial \theta_{k(t)}} \\ \dfrac{\partial \phi_t^k}{\partial x_{k(t)}} & \dfrac{\partial \phi_t^k}{\partial y_{k(t)}} & \dfrac{\partial \phi_t^k}{\partial \theta_{k(t)}} \end{bmatrix}$$

$$z(t) = \begin{matrix} \varphi_{1kj}^{w}(t) = \sigma_{d_{kj}}^2 \cos^2 \theta_{kj}^{(t)} + (d_{kj}^{(t)})^2 \sigma_{\theta_{kj}}^2 \sin^2 \theta_{kj}^{(t)} \\ \varphi_{2kj}^{w}(t) = \sigma_{d_{kj}}^2 \sin^2 \theta_{kj}^{(t)} + (d_{kj}^{(t)})^2 \sigma_{\theta_{kj}}^2 \cos^2 \theta_{kj}^{(t)}, \end{matrix}$$

EKF Correction

$$x_t^{\wedge} = x_t^{\wedge -} + K_t(z_t - h(x_t^{\wedge -}))$$

$$P_t = P_t^{-} - K_t H P_t^{-1}$$

Kalman Gain

$$K_t = P_t^{\wedge -} H_t^T (H_t P_{t-1} H_t^T + R)^\wedge -1)$$

$$R = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{bmatrix}$$

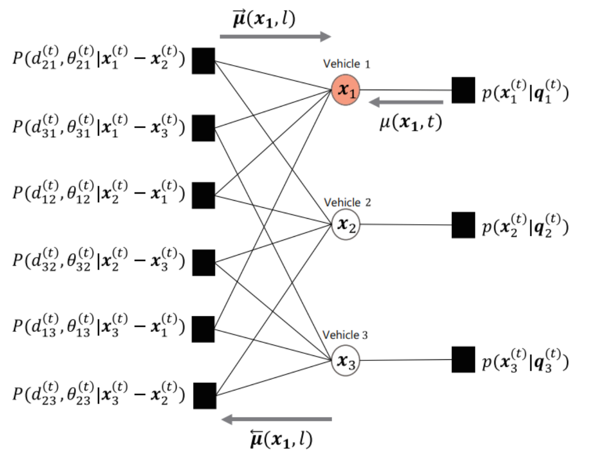
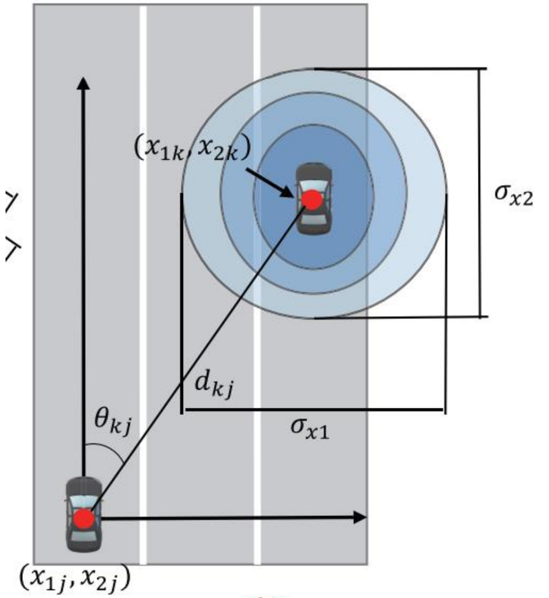


Figure 3. Graphical model associated with example in Figure 1.



(b)