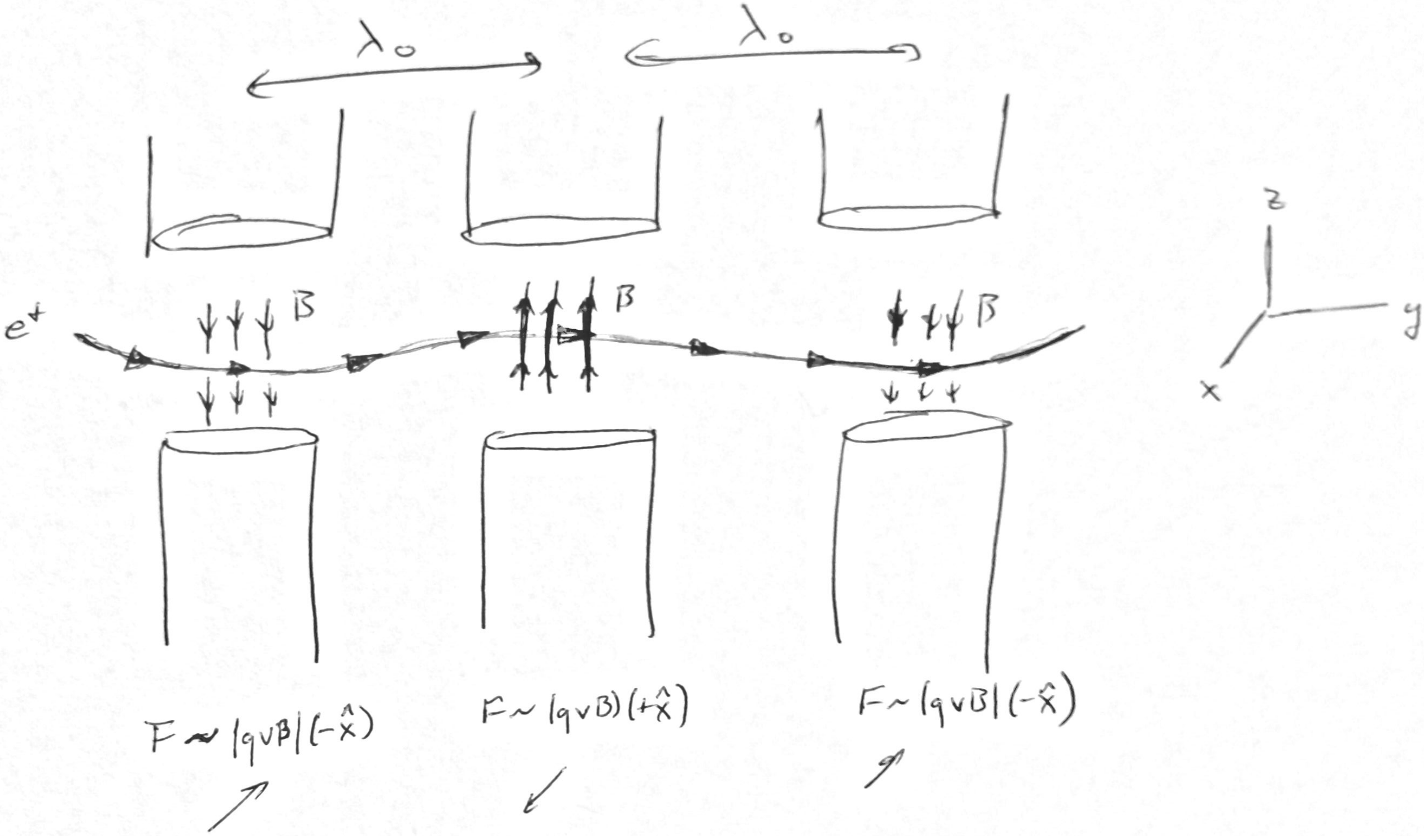
Undulators & Wigglers



Magnetic array causes relativistic charged particle to curve a bit. Say that without $\vec{B}$, the particle moves along the y axis with constant speed. With B, it oscillates in the xy plane:

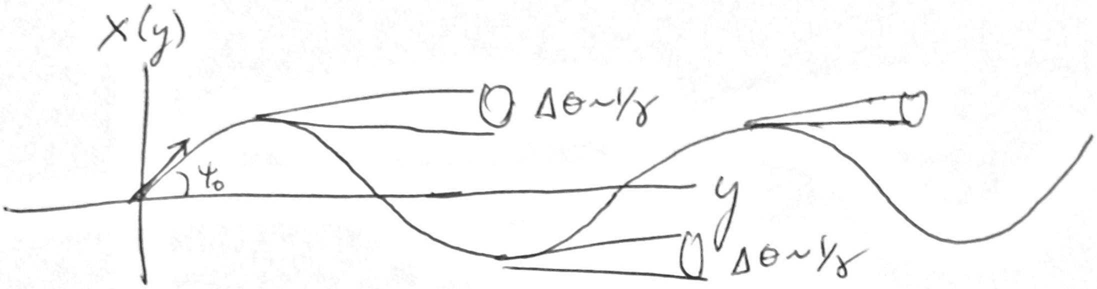
$$x \approx a \sin(2\pi y/\lambda_0)$$

a will depend on something like qB/E

(stronger field: more deflection

more energetic particle: less deflection.)

Define $\psi_0 \equiv \left.\frac{dx}{dy}\right|_{y=0} = \frac{2\pi a}{\lambda_0} \equiv k_0 a \quad (k_0 \equiv 2\pi/\lambda_0)$

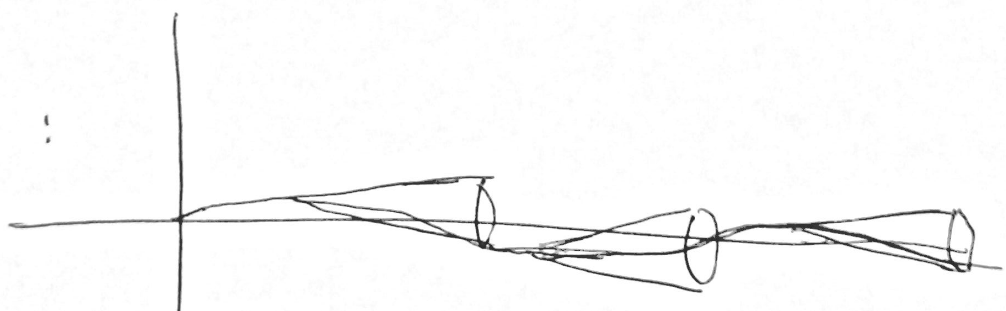


ψ_0 is the max angular deflection.

For $\gamma \gg 1$ (very relativistic particle) we get a searchlight beam of radiation flicking back and forth over the forward direction. Above is a wiggler, where $\Delta\theta \gg \psi_0$.

Undulator:

$\psi_0 \ll \Delta\theta$



Beam is "always on", each pulse adds coherently.

For a wiggler, the repetition rate of "on/off" seen by a fixed observer downstream is

$\nu_0 = \omega_0 / 2\pi$. If $\lambda_0 \sim$ cm, $\nu_0 \sim 10$ GHz.

Similar to a sychrotron with e^- spaced in bunches a few cm apart.

The frequency of the <u>radiation</u> peaks at

$$\omega_c \sim \underset{\substack{\uparrow \\ \text{the } \gamma^3 \\ \text{we saw before}}}{\gamma^3} \times \underbrace{c/R}_{\substack{\text{natural frequency of} \\ \text{the oscillatory motion} - R \text{ is} \\ \text{radius of curvature}}}$$

The minimum R for sinusoidal motion is

$$R = \frac{\lambda_0^2}{(2\pi)^2 a} = \frac{\lambda_0}{2\pi \psi_0}$$

Define $K \equiv \gamma \overset{\sim \psi_0/\theta_{max}}{\downarrow} \psi_0$. wigglers have $K \gg 1$, emit at

$\omega_c \sim \gamma^2 K \frac{2\pi c}{\lambda_0}$. K related to magnet by

$$K = \frac{e B_0 \lambda_0}{2\pi m c^2}$$

cf cyclotron frequency

Undulators have $\psi_0 \ll \Delta\theta$. In the particle's rest frame, the particle "sees" a magnet every $\frac{\lambda_0}{\gamma c}$ time units. γ accounts for Lorentz contraction in this frame. So the frequency of emission in the lab frame is (after accounting for relativistic doppler)

$$\omega \sim 2\gamma^2 \left(\frac{2\pi c}{\lambda_0}\right)$$

Same γ dep as the wiggler. for fixed K.

ptcl rest frame, lab frame

$$\omega' = \gamma\omega(1-\beta) \simeq \omega/2\gamma$$

$$\omega' = \frac{2\pi\gamma c}{\lambda_0} \Rightarrow \omega = 2\gamma^2 \frac{2\pi c}{\lambda_0}$$

The frequency distribution is sharply peaked here.

- A free electron laser operates by using the <u>emitted</u> radiation as a driving field to accelerate the same electrons <u>more</u> → "stimulated emission"
- undulators can be <u>brighter</u>, wigglers can be <u>higher energy</u>