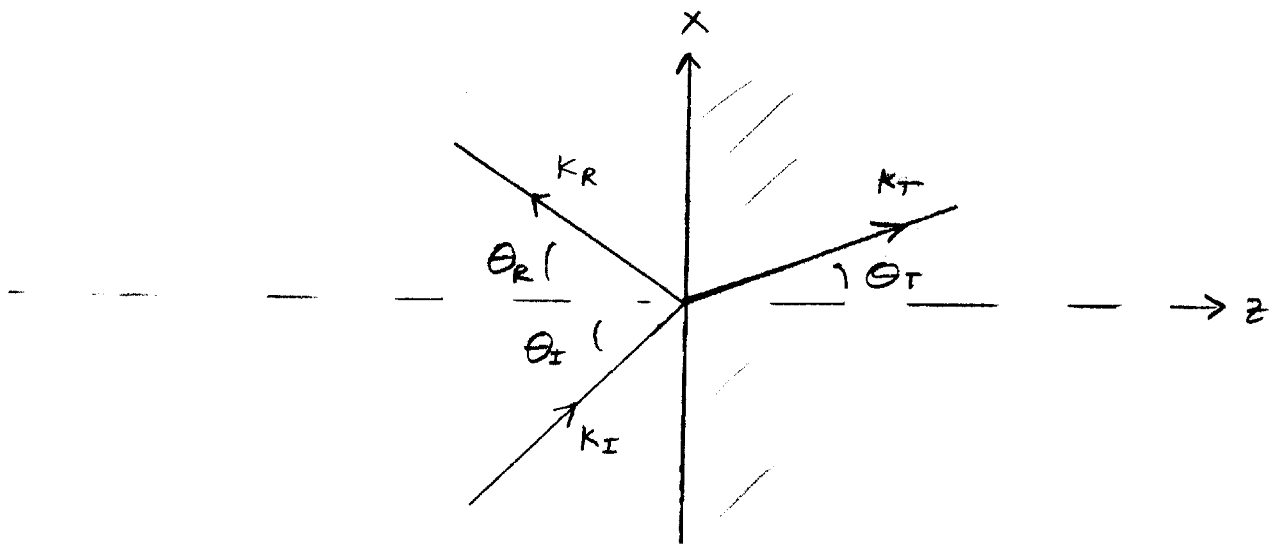
Chapter 9, lecture

Reflection and transmission of electromagnetic waves at a boundary.



We would like $\tilde{E}_{0R}$, $\tilde{E}_{0T}$ in terms of $\tilde{E}_{0I}$
To get these, along with other information, we will do the following:

1) Write expressions for single frequency incident, reflected and transmitted waves

2) The requirement that the oscillatory portion of each wave be the same at the boundary leads to $\theta_I = \theta_R$ and Snell's law.

3) Apply specifically electromagnetic boundary conditions gives the desired relations for the complex amplitudes of the waves.

First, write expressions for the waves:

$$E_I = \tilde{E}_{0I} e^{i(\vec{k}_I \cdot \vec{r} - \omega t)}$$

$$E_R = \tilde{E}_{0R} e^{i(\vec{k}_R \cdot \vec{r} - \omega t)}$$

$$E_T = \tilde{E}_{0T} e^{i(\vec{k}_T \cdot \vec{r} - \omega t)}$$

Matching at the boundary:

$$\tilde{E}_{0I} e^{i(\vec{k}_I \cdot \vec{r} - \omega t)} + \tilde{E}_{0R} e^{i(\vec{k}_R \cdot \vec{r} - \omega t)} = \tilde{E}_{0T} e^{i(\vec{k}_T \cdot \vec{r} - \omega t)}$$

The exponent of each term must be the same

$$k_{Ix} x + k_{Iy} y = k_{Rx} x + k_{Ry} y = k_{Tx} x + k_{Ty} y$$

since $z = 0$

$$k_{Ix} = k_{Rx} = k_{Tx}$$

$$k_{Iy} = k_{Ry} = k_{Ty}$$

We may choose the coordinate system so that the incident wave is in the xz plane. Then, all waves must be in the xz plane (the plane of incidence).

Using $k_{Ix} = k_{Rx}$

$$k_I \sin\theta_I = k_R \sin\theta_R$$

$$\frac{\omega}{v_1} \sin\theta_I = \omega/v_1 \sin\theta_R$$

$$\boxed{\theta_I = \theta_R}$$

The angle of incidence is equal to the angle of reflection

Also $k_{Ix} = k_{Tx}$

$$k_I \sin\theta_I = k_T \sin\theta_T$$

$$\frac{k_I}{k_T} = \frac{\sin\theta_T}{\sin\theta_I}$$

$$\boxed{\frac{v_2}{v_1} = \frac{n_1}{n_2} = \frac{\sin\theta_T}{\sin\theta_I}}$$

since $v = c/n$

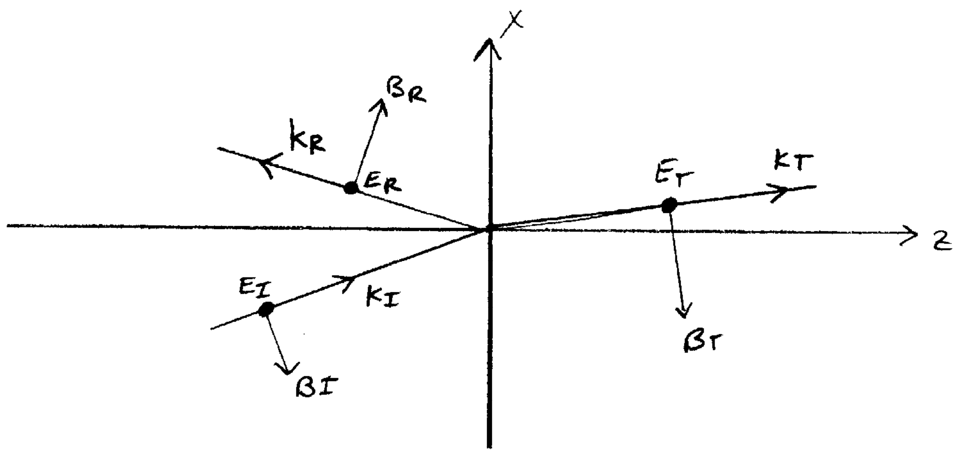
Snell's Law

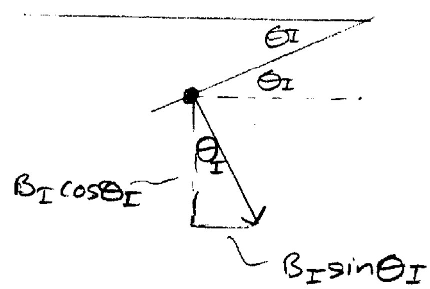
Thats as far as we can go (pretty far!) without the specific electromagnetic boundary conditions.

$$\epsilon_1 E^{\perp}_{Left} = \epsilon_2 E^{\perp}_{Right} \quad , \quad E^{\parallel}_{Left} = E^{\parallel}_{Right}$$

$$B^{\perp}_{Left} = B^{\perp}_{Right} \quad , \quad \frac{1}{\mu_1} B^{\parallel}_{Left} = \frac{1}{\mu_2} B^{\parallel}_{Right}$$

Now, to go further, pick a specific polarization. Let's choose perpendicular, since Griffiths does the parallel case.





① $E^{\perp}_{Left} = E^{\perp}_{Right} = 0$

② $B^{\perp}_{Left} = B^{\perp}_{Right}$

$$\tilde{B}_{0I} \sin\theta_I + \tilde{B}_{0R} \sin\theta_R = \tilde{B}_{0T} \sin\theta_T$$

or, $$\frac{\tilde{E}_{0I}}{v_1} \sin\theta_I + \frac{\tilde{E}_{0R}}{v_1} \sin\theta_R = \frac{\tilde{E}_{0T}}{v_2} \sin\theta_T$$

This is

$$\tilde{E}_{0I} + \tilde{E}_{0R} = \frac{v_1}{v_2}\frac{\sin\theta_T}{\sin\theta_I}\tilde{E}_{0T}$$

$$n_1/n_2 = \frac{c/v_1}{c/v_2}$$

$$\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}$$

(3) $E^{\parallel}_{\text{Left}} = E^{\parallel}_{\text{Right}}$

$\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}$ same as above

(4) $\frac{1}{\mu_1} B^{\parallel}_{\text{Left}} = \frac{1}{\mu_2} B^{\parallel}_{\text{Right}}$

$$-\frac{1}{\mu_1}\tilde{B}_{0I}\cos\theta_I + \frac{1}{\mu_1}\tilde{B}_{0R}\cos\theta_I = \frac{-1}{\mu_2}\tilde{B}_{0T}\cos\theta_T$$

$$-\tilde{E}_{0I} + \tilde{E}_{0R} = -\frac{\mu_1 v_1}{\mu_2 v_2}\frac{\cos\theta_T}{\cos\theta_I}\tilde{E}_{0T}$$

$$= -\beta\alpha\,\tilde{E}_{0T}$$

using the book definitions $\alpha \doteq \frac{\cos\theta_T}{\cos\theta_I}$, $\beta = \frac{\mu_1 v_1}{\mu_2 v_2}$

combining 3 & 4

$$-\tilde{E}_{0I} + \tilde{E}_{0R} = -\beta\alpha E_{0I} - \beta\alpha E_{0R}$$

$$\tilde{E}_{0R} = \left(\frac{1-\beta\alpha}{1+\beta\alpha}\right)\tilde{E}_{0I}$$

$$\tilde{E}_{0T} = \left(\frac{2}{1+\beta\alpha}\right)\tilde{E}_{0I}$$

Reflection coefficient $\equiv$ $\frac{\text{Reflected intensity}}{\text{Incident intensity}}$

$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{1-\beta\alpha}{1+\beta\alpha}\right)^2$$

Since $I_I = \langle S \rangle \cdot \hat{z} = \frac{1}{2}\epsilon_1 v_1 E_{0I}^2 \cos\theta_I$

$I_R = \langle S \rangle \cdot \hat{z} = \frac{1}{2}\epsilon_1 v_1 E_{0R}^2 \cos\theta_R$

and $\theta_R = \theta_I$

Similarly,

$$T \equiv \frac{I_T}{I_I} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{E_{0T}}{E_{0I}}\right)^2 \frac{\cos\theta_T}{\cos\theta_I}$$

This is:

$$\frac{\epsilon_2 \mu_2 v_2}{\epsilon_1 \mu_1 v_1} \frac{\mu_1}{\mu_2} \left(\frac{2}{1+\beta\alpha}\right)^2 \alpha$$

$$= \frac{v_1^{\cancel{2}}}{v_2^{\cancel{2}}} \frac{\cancel{v_2}}{\cancel{v_1}} \frac{\mu_1}{\mu_2} \alpha \left(\frac{2}{1+\beta\alpha}\right)^2$$

$$T = \beta\alpha \left(\frac{2}{1+\beta\alpha}\right)^2$$

using definitions of α, β and

$$I_T = \langle S \rangle \cdot \hat{z} = \tfrac{1}{2} \epsilon_2 v_2 E_{0T}^2 \cos\theta_T$$

$$R + T = \frac{4\beta\alpha}{(1+\beta\alpha)^2} + \frac{(1-\beta\alpha)^2}{(1+\beta\alpha)^2} = \frac{1 - 2\beta\alpha + (\beta\alpha)^2 + 4\beta\alpha}{(1+\beta\alpha)^2}$$

$$= \frac{1 + 2\beta\alpha + (\beta\alpha)^2}{(1+\beta\alpha)^2} = \frac{(1+\beta\alpha)^2}{(1+\beta\alpha)^2} = 1$$