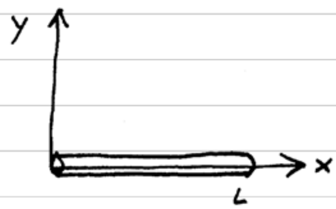
Integrating over areas, volumes, lines

lines

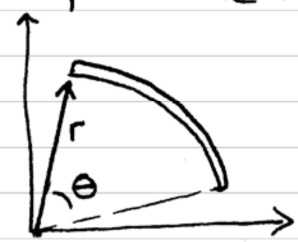
cartesian line integral



$$dl = dx$$

$$\int_0^L dx = x\Big|_0^L = L$$

polar line integral



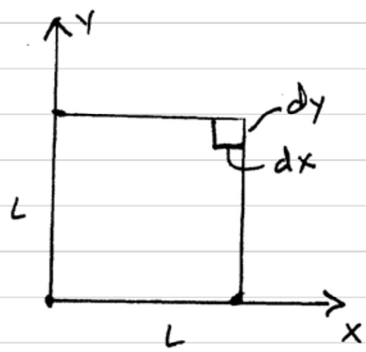
$$dl = ds = r\,d\theta$$

$$\int_{\theta_i}^{\theta_f} r\,d\theta = r\int_{\theta_i}^{\theta_f} d\theta = r\Big|_{\theta_i}^{\theta_f}$$

$$= r(\theta_f - \theta_i)$$

For a circle: $r(2\pi - 0) = 2\pi r$

areas



$$dA = dx\,dy$$

Area of Square $= \int_0^L dx \int_0^L dy$

$$= \left(x\Big|_0^L\right)\left(y\Big|_0^L\right)$$

$$A = L^2$$

$$S = r\theta$$
$$ds = r\,d\theta$$

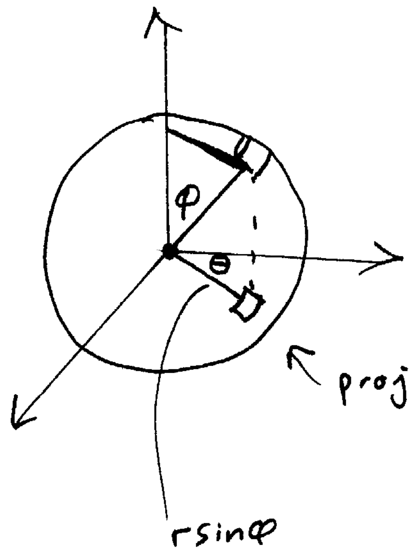
$$dA = dr\,ds = dr\,(r\,d\theta)$$

$$= r\,dr\,d\theta$$

Area of Circle $= \int_0^R r\,dr \int_0^{2\pi} d\theta$

$$A = \left(\frac{r^2}{2}\Big|_0^R\right)\left(\theta\Big|_0^{2\pi}\right) = \pi R^2$$

VOLUMES



sideview

$ds = r\,d\varphi$

dr

projection:

$ds' = r'd\theta = r\sin\varphi\, d\theta$

dr

$$dv = dr\,ds\,ds' = dr\,(r\,d\varphi)(r\sin\varphi\, d\varphi)$$

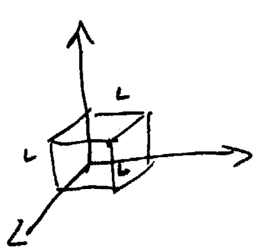
$$dV = r^2 \sin\varphi\, d\varphi\, d\theta\, dr$$

Volume of sphere $= \displaystyle\int_0^R r^2 dr \int_0^{2\pi} d\theta \int_0^{\pi} \sin\varphi\, d\varphi$

$$= \left(\frac{r^3}{3}\Big|_0^R\right)\left(\theta\Big|_0^{2\pi}\right)\left(-\cos\varphi\Big|_0^{\pi}\right)$$

$$V = (R^3/3)(2\pi)(2) = 4/3\,\pi R^3$$

Cartesian



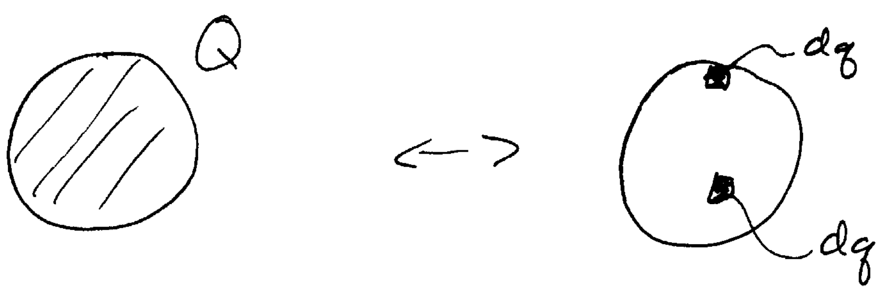
$$dv = dx\,dy\,dz$$

$$V = \int_0^L dx \int_0^L dy \int_0^L dz = L^3$$

Putting this in table form

	cartesian	polar
line, dl	$dl = dx$	$dl = r\,d\theta$
area, dA	$dA = dx\,dy$	$dA = r^2 d\theta\,dr$
volume, dV	$dV = dx\,dy\,dz$	$dV = r^2 \sin\varphi\, d\varphi\, d\theta\, dr$

To find an electric field, at say point P, from a charge <u>distribution</u> which is continuous superposition may be used, and contributions from small elements of charge, dq, may be summed.



$\Sigma dq = Q$

If a total charge, Q, is uniformly distributed along a line of charge of length L, the charge can be written:

$$Q = \underbrace{\left(\frac{\text{charge}}{\text{length}}\right)}_{\text{we'll call this } \lambda}(\text{total length}) = \lambda L$$

The charge of a small section of the line is $dq = \lambda\, dx$ (differential charge element).

Similarly, if Q is uniformly distributed on an area, A:

$$Q = \underbrace{\left(\frac{\text{charge}}{\text{area}}\right)}_{\sigma}(\text{total area}) = \sigma A$$

$$dq = \sigma\, dA$$

For a charge Q uniformly distributed over a volume:

$$Q = \underbrace{\left(\frac{\text{charge}}{\text{volume}}\right)}_{\rho}(\text{total volume}) = \rho V$$

$$dq = \rho\, dV$$

or, in table form:

	total charge	differential charge
length	$Q = \lambda L$	$dq = \lambda dx$
area	$Q = \sigma A$	$dq = \sigma dA$
volume	$Q = \rho V$	$dq = \rho dV$