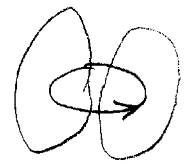
Chapter 11 - lecture 2



Now that we've done the electric dipole, we can do the magnetic dipole following much the same procedure.

1) Find the potential, $\vec{A}$. This time $V=0$ since the current loop which is the magnetic dipole is electrically neutral.

   $\vec{A}$ is a vector, so use the symmetry of the loop to symplify the integral. (to have a scalar integral).

2) Find $\mathscr{r}$ using the law of cosines. It is a little trickier than for the electric dipole case due to the geometry of the problem.

3) Use the dipole approximation ($b \ll r$ in this case) to allow a binomial expansion of expression for $\mathscr{r}$, $1/\mathscr{r}$.

4) To handle the argument of the cosine function invoke a 2nd approximation, $b \ll \lambda$.

5) Finally, make the far field approximation, $r \gg \lambda$. This simplifies the expression for $\vec{A}$, leaving only one term.

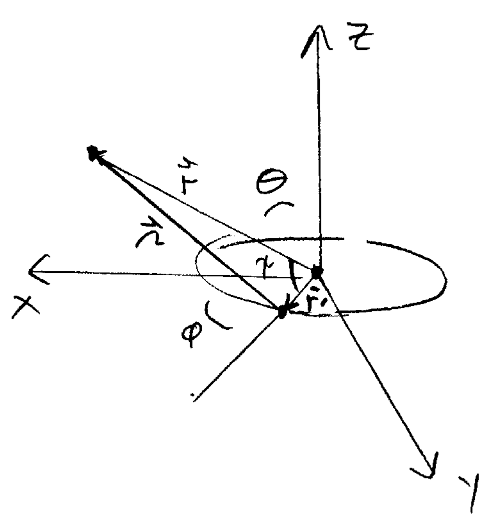
OK - let's roll.

$$I(t) = I_0 \cos \omega t$$

$$\vec{m} = \pi b^2 I(t)\, \hat{z} \qquad \text{magnetic dipole moment}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{I_0 \cos(\omega t_r)}{\eta}\, d\vec{l}\,'$$

Pick an observation point in the xz plane. Or, define the coordinate axes so that the observation point is in the xz plane. The origin is located at the center of the current loop.

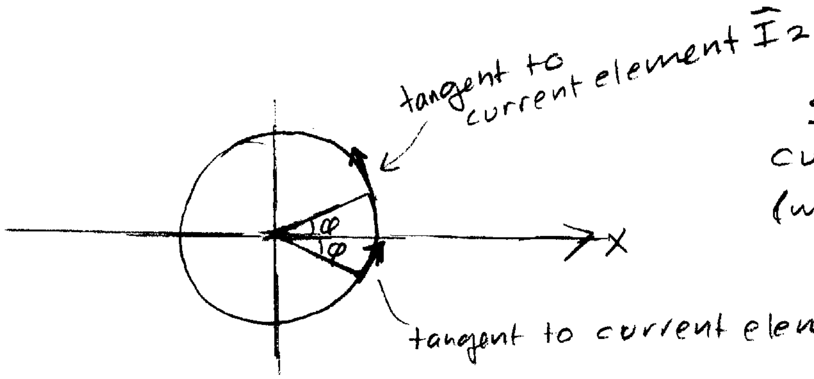


$$\eta = [r^2 + r'^2 - 2\,\vec{r}\cdot\vec{r}\,']^{1/2}$$

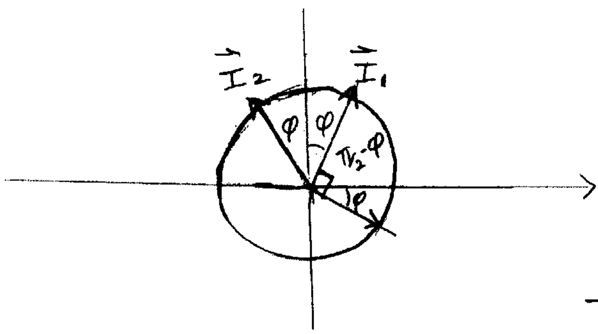
$r' = b$ all current elements are b from the origin

$$\eta = r\left[1 + (b/r)^2 - \frac{2rb\cos\psi}{r^2}\right]^{1/2}$$

Also, let's check the symmetry



Symmetrically located current elements (with respect to x-axis).



Moving tangents so their tails are located at the origin.

The $I\hat{x}$ components cancel.
The $I\hat{y}$ components add.

So, for the integral

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{I_0 \cos(\omega t r)}{r} \, d\vec{l}$$

Only the y-component of the current elements contribute to $\vec{A}$.

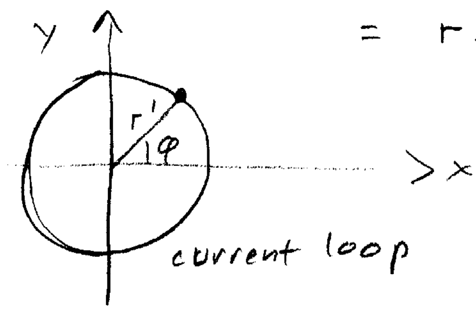
$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{I_0 \cos(\omega t r)}{r} \cos\varphi \, dl$$

Next let's find $\cos\psi$ in $\imath = \sqrt{r^2 + r'^2 - 2\vec{r}\,\vec{r}'\cos\psi}$

$$\vec{r}\cdot\vec{r}\,' = (x_r\hat{x} + z_r\hat{z})\cdot(x_{r'}\hat{x} + y_{r'}\hat{y})$$

$$= (r\sin\theta\,\hat{x} + r\cos\theta\,\hat{z})\cdot(b\cos\varphi\,\hat{x} + b\sin\varphi\,\hat{y})$$

$$= rb\sin\theta\cos\varphi$$



$$\left.\begin{aligned} \imath &\approx r\left(1 - \tfrac{b}{r}\sin\theta\cos\varphi\right) \\ \frac{1}{\imath} &\approx \frac{1}{r}\left(1 + \tfrac{b}{r}\sin\theta\cos\varphi\right) \end{aligned}\right\}\ \text{Binomial Expansion}$$

$$\cos[\underbrace{\omega t - \omega/c\,\imath}_{\omega t - \omega/c\, r + \omega/c\, b\sin\theta\cos\varphi}]$$

$$= \omega(t - r/c) + \frac{2\pi b}{\lambda}\sin\theta\cos\varphi$$

Let $\alpha \equiv \omega(t - r/c)$, $a \equiv \sin\theta\cos\varphi$, $\beta \equiv \frac{\omega b}{c}a$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

By approximation 2, $b \ll c/\omega$, β is small.

$$\cos(\alpha+\beta) = \cos\alpha - \beta\sin\alpha$$

$$\vec{A} = \frac{\mu_0 I_0}{4\pi}\int_0^{2\pi}\left(\cos\alpha - \beta\sin\alpha\right)\frac{1}{r}\left(1+\frac{b}{r}a\right)\cos\varphi\,(b\,d\varphi)\,\hat{y}$$

$$= \frac{\mu_0 I_0 b\hat{y}}{4\pi r}\int_0^{2\pi}\Bigg(\cancelto{0,\ \varphi\text{ integration}}{\cos\alpha} - \frac{\omega b}{c}\sin\theta\cos\varphi\sin\alpha$$

$$+ \frac{b}{r}\sin\theta\cos\varphi\cos\alpha - \overset{\text{negligable 2nd order}}{\underbrace{\cancel{\frac{b}{r}\frac{\omega b}{c}}}_{2\pi(b/r)(b/\lambda)}}\sin\theta\cos\varphi\sin\alpha\Bigg)\cos\varphi\,d\varphi$$

$$= \frac{\mu_0 I_0 b}{4\pi r}\left(-\frac{\omega b}{c}\sin\theta\sin\alpha + \frac{b}{r}\sin\theta\cos\alpha\right)\int_0^{2\pi}\cos^2\varphi\,d\varphi\ \hat{y}$$

$$= \frac{\mu_0(I_0 b^2\pi)}{4\pi}\left(\frac{\sin\theta}{r}\right)\left(-\frac{\omega}{c}\sin[\omega(t-r/c)] + \frac{1}{r}\cos[\omega(t-r/c)]\right)\hat{y}$$

For observation points in xz plane, $\vec{A}$ is in the $\hat{y}$ direction. In general, $\vec{A}$ is in the $\hat{\phi}$ direction.

Finally, with the far-field approximation,

$$\frac{1}{r} << \frac{1}{\lambda} \Rightarrow$$

$$\vec{A} = \frac{-\mu_o m_o \omega}{4\pi c}\left(\frac{\sin\theta}{r}\right)\sin[\omega(t-r/c)]\,\hat{\varphi}$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} = \frac{\mu_o m_o \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t-r/c)]\,\hat{\varphi}$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\varphi)\right]\hat{r} - \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\varphi)\right]\hat{\theta}$$

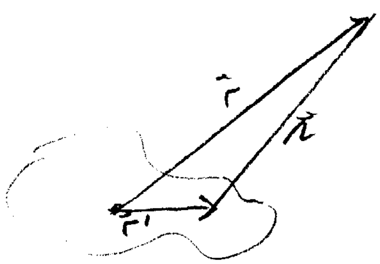
$$= \left[\frac{1}{r\cancel{\sin\theta}}\frac{\sin[\omega(t-r/c)]}{r}\,2\cancel{\sin\theta}\cos\theta\;\hat{r}\right.$$

$$\left. - \frac{1}{r}\sin\theta\cos[\omega(t-r/c)](-\omega/c)\,\hat{\theta}\right]\left(\frac{-\mu_o m_o \omega}{4\pi c}\right)$$

The second term dominates, 1^{st} term $1/r^2$; 2^{nd} $\frac{2\pi}{\lambda r}$

$$\vec{B} = \frac{-\mu_o m_o \omega^2}{4\pi c^2}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t-r/c)]\,\hat{\theta}$$

Finally, we'll find the radiation from an arbitrary source. Once again, the procedure is similar to what was done for the oscillating dipoles. The main difference is that instead of a known source with a definite functional form, such as a loop with $I(t) = I_0 \cos\omega t$, there is an arbitrary (unknown) charge distribution, $\rho(\vec{r}', t_r) = \rho(\vec{r}', t - \mathscr{r}/c)$. One argument of ρ is the retarded time. The $\mathscr{r}$ may be written in terms of $\vec{r}, \vec{r}'$ using a binomial expansion.



$$\mathscr{r} = r\left[1 + (r'/r)^2 - \frac{2\vec{r}\cdot\vec{r}'}{r^2}\right]^{1/2}$$

$$\mathscr{r} \approx r\left(1 - \frac{\vec{r}\cdot\vec{r}'}{r^2}\right)$$

$$\frac{1}{\mathscr{r}} \approx \frac{1}{r}\left(1 + \frac{\vec{r}\cdot\vec{r}'}{r^2}\right)$$

$$\rho(\vec{r}\,', t_r) \approx \rho\left(\vec{r}\,', t - r/c + \frac{\hat{r}\cdot\vec{r}\,'}{c}\right)$$

We may deal with this further with a Taylor expansion.

Remember how the expansion goes:

$$f(x+h) = \sum_n \frac{h^n}{n!} \frac{d^n f(x)}{dx^n}$$

expansion point

small interval from expansion point

h must be small or this approximation is not good. If h is small, successive powers of h decrease

$h^n < h^{n+1}$, etc.

Going back to the expansion of problem 10.12 when the current density $J(t_r)$ was expanded,

$$\vec{J}(t_r) = \vec{J}(t) + (t_r - t)\dot{\vec{J}}(t) + \frac{(t_r - t)^2}{2}\ddot{\vec{J}}(t)$$

Expansion done around time t

At an interval $\Delta t = t_r - t$ away from t.

So, for that expansion of the current density, the valid region is for when $t_r - t$ is small, or, the retarded time is close to the actual time (not far from source).

$x + h \rightarrow t + (t_r - t) = t_r$ (in this case)

$\vec{J}(t_r)$ is being approximated using $J(t)$.

Now, back to the case at hand:

$$\rho\left(\vec{r}', t - r/c + \frac{\hat{r}\cdot\vec{r}'}{c}\right)$$

We want an approximation good in the far-field limit, this time. In the expression $t - r/c + \frac{\hat{r}\cdot\vec{r}'}{c}$, it could be considered the far-field if $r \gg r'$ and $\lambda \gg r'$, or in other words, if $\frac{\hat{r}\cdot\vec{r}'}{c}$ is a small correction to $t - r/c$.

Let $t_0 \equiv t - r/c$, then,

$$\rho(\vec{r}', t_0 + \Delta t) \approx \rho(\vec{r}', t_0) + \dot{\rho}(\vec{r}', t_0)\Delta t + \ddot{\rho}\,\frac{\Delta t^2}{2} + \ldots$$

This is

$$\rho(\vec{r}\,', t - \imath/c) = \rho(\vec{r}\,', t_0) + \dot{\rho}(\vec{r}\,'; t_0)\left(\frac{\hat{r}\cdot\vec{r}\,'}{c}\right) + \ldots$$

Now we can write an expression for the potential, good in the far-field limit:

Always true:

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,', t - \imath/c)}{\imath}\, d\tau'$$

In the far field:

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{1}{r}\left(1 + \frac{\vec{r}\cdot\vec{r}\,'}{r^2}\right)\left(\rho(\vec{r}\,'; t_0) + \dot{\rho}(r'; t_0)\left(\frac{\hat{r}\cdot\vec{r}\,'}{c}\right) + \ldots\right) d\tau'$$

$$= \frac{1}{4\pi\epsilon_0 r}\int \left[\left(1 + \frac{\hat{r}\cdot\vec{r}\,'}{r}\right)\left(\rho + \dot{\rho}\left(\frac{\hat{r}\cdot\vec{r}\,'}{c}\right) + \ldots\right)\right] d\tau'$$

The portion in square brackets is,

$$\left(1 + \epsilon_1\right)\left(\rho + \dot{\rho}\left(\frac{2\pi}{\omega}\right)\epsilon_2 + \ddot{\rho}\left(\frac{2\pi}{\omega}\right)^2 \frac{\epsilon_2^2}{2} + \ldots\right)$$

$$\epsilon_1 \equiv \frac{\hat{r}\cdot\vec{r}\,'}{r} \quad , \quad \epsilon_2 \equiv \frac{\hat{r}\cdot\vec{r}\,'}{\left(\frac{2\pi c}{\omega}\right)} = \frac{\hat{r}\cdot\vec{r}\,'}{\lambda}$$

ϵ_1 and ϵ_2 are small, dimensionless parameters.

The portion in square brackets is:

$$\rho + \dot{\rho}\left(\frac{2\pi}{\omega}\right)\epsilon_2 + \ddot{\rho}\left(\frac{2\pi}{\omega}\right)^2\frac{\epsilon_2^2}{2} + \dddot{\rho}\left(\frac{2\pi}{\omega}\right)^3\frac{\epsilon_2^3}{6} + \dots$$

$$+\ \epsilon_1\rho + \epsilon_1\epsilon_2\,\dot{\rho}\left(\frac{2\pi}{\omega}\right) + \frac{\epsilon_1\epsilon_2^2}{2}\,\ddot{\rho}\left(\frac{2\pi}{\omega}\right)^2 + \dots$$

Keeping only 1^{st} order terms in ϵ :

$$\rho + \epsilon_1\rho + \epsilon_2\left(\frac{2\pi}{\omega}\right)\dot{\rho}$$

$$V = \frac{1}{4\pi\epsilon_0 r}\int \rho(\vec{r}\,',t_0)\,d\tau' + \frac{\hat{r}}{4\pi\epsilon_0 r^2}\cdot\int \vec{r}\,'\rho(r',t_0)\,d\tau'$$

$$+\ \frac{\hat{r}}{4\pi\epsilon_0 r c}\cdot\int \vec{r}\,'\dot{\rho}(\vec{r}\,',t_0)\,d\tau$$

$$V = \frac{1}{4\pi\epsilon_0 r}\left[\int \rho(\vec{r}\,', t_0)\, d\tau' + \frac{\hat{r}}{r}\cdot\int \vec{r}\,'\rho(\vec{r}\,', t_0)\, d\tau'\right.$$

$$\left. + \frac{\hat{r}}{c}\cdot\frac{d}{dt}\int \vec{r}\,'\rho(\vec{r}\,', t_0)\, d\tau'\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{r} + \frac{\hat{r}\cdot\vec{p}(t_0)}{r^2} + \frac{\hat{r}\cdot\dot{\vec{p}}(t_0)}{rc}\right]$$

since $\vec{p}(t_0) \equiv \int \vec{r}\,'\rho(\vec{r}\,', t_0)\, d\tau'$ is the dipole moment

(see chapter 3 Eq. 3.98)

Now, what about $\vec{A}$?

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi}\int \frac{J(\vec{r}\,', t - \eta/c)}{\eta}\, d\tau'$$

$$\frac{\vec{J}(\vec{r}\,', t - \eta/c)}{\eta} \approx \frac{1}{r}\left(1 + \epsilon_1\right)\left(\vec{J} + \dot{\vec{J}}\left(\frac{2\pi}{\omega}\right)\epsilon_2 + \ddot{\vec{J}}\left(\frac{2\pi}{\omega}\right)^2\frac{\epsilon_2^2}{2} + \ldots\right)$$

We know that $\int \vec{J}(\vec{r}\,', t_r)\, d\tau' = \frac{d}{dt}\int \vec{r}\,'\rho(\vec{r}\,', t_r)\, d\tau'$

(see problem 5.7)

Then,

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\frac{d}{dt}\int (1+\epsilon_1)\left(\frac{\vec{r}\,'}{r}\right)\rho(\vec{r}\,',t_r)\,d\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{d}{dt}\int (1+\epsilon_1)\,\epsilon_1\,(\rho + \dot{\rho}(\tfrac{2\pi}{\omega})\epsilon_2 + \dots)\,d\tau'$$

There is only one 1st order term:

$$\vec{A} \approx \frac{\mu_0}{4\pi}\frac{d}{dt}\int \epsilon_1\,\rho(\vec{r}\,',t_0)\,d\tau' = \frac{\mu_0}{4\pi r}\frac{d}{dt}\int \vec{r}\,'\rho(\vec{r}\,',t_0)\,d\tau'$$

$$= \frac{\mu_0}{4\pi}\frac{\dot{\vec{p}}}{r}$$

Now the fields can be found.

$$\vec{E} = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}$$

Terms in $\vec{E}$ that drop off faster than $1/r$ cannot contribute radiated power, as discussed at the beginning of the chapter.

First lets look at $\vec{\nabla} V$:

$$\vec{\nabla} V = \frac{1}{4\pi\epsilon_0} \vec{\nabla} \left[\frac{Q}{r} + \frac{\hat{r} \cdot \mathbf{p}(t_0)}{r^2} + \frac{\hat{r} \cdot \dot{\vec{p}}(t_0)}{rc} \right]$$

$-\frac{Q}{r^2}$

All $\vec{\nabla}\left(\frac{\hat{r}\cdot\mathbf{p}}{r^2}\right)$ terms drop off at <u>least</u> as fast as $\frac{1}{r^2}$

This may have a term that goes as $\sim \frac{1}{r}$

$$\vec{\nabla}\left(\frac{\hat{r}\cdot\dot{\vec{p}}(t_0)}{rc}\right) = \frac{1}{r^2 c}\vec{\nabla}(\vec{r}\cdot\dot{\mathbf{p}}(t_0)) + \cancel{(\vec{r}\cdot\dot{\mathbf{p}}(t_0))\vec{\nabla}\left(\frac{1}{r^2 c}\right)}$$

too fast

$$\frac{1}{r^2 c}\vec{\nabla}(\vec{r}\cdot\dot{\mathbf{p}}(t_0)) = \frac{1}{r^2 c}\int \vec{\nabla}\left(\vec{r}\cdot\vec{r}'\dot{\rho}(\vec{r}',t_0)\right) d\tau'$$

$$\vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f$$

$$\Rightarrow \frac{1}{r^2 c}\int (\vec{r}\cdot\vec{r}')\vec{\nabla}\dot{\rho}(\vec{r}',t_0)\, d\tau'$$

$$+ \frac{1}{r^2 c}\int \dot{\rho}\,\vec{\nabla}(\vec{r}\cdot\vec{r}')\, d\tau'$$

$$\vec\nabla(\vec r\cdot\vec r') = \hat x \frac{\partial xx'}{\partial x} + \hat y \frac{\partial yy'}{\partial y} + \hat z \frac{\partial zz'}{\partial z}$$

$$\vec\nabla(\vec r\cdot\vec r') = \vec r'$$

In Chapter 10 (see lecture 2) we discovered:

$$\vec\nabla\rho(\vec r', t_r) = -\dot\rho/c\ \hat{\imath} \quad \Rightarrow \quad \vec\nabla\rho(\vec r', t_0) = -\dot\rho/c\ \hat r$$

$$\vec\nabla\dot\rho(\vec r', t_0) = -\frac{\ddot\rho}{c}\hat r \quad , \quad \text{etc.}$$

Then

$$-\vec\nabla V \approx \frac{1}{4\pi\epsilon_0}\left[\frac{1}{r^2 c}\vec r\cdot\int \vec r'(-\ddot\rho/c)\,\hat r\, d\tau' + \frac{1}{r^2 c}\cancel{\int \vec r'\rho\, d\tau'}\right]$$

falls fast

$$-\vec\nabla V = \frac{1}{4\pi\epsilon_0 c^2}\frac{\hat r}{r}\cdot\left(\frac{d^2}{dt^2}\int \vec r'\cdot\rho\, d\tau'\right)\hat r$$

$$= \frac{1}{4\pi\epsilon_0 c^2}\frac{\hat r\cdot\ddot{\vec p}}{r}\hat r$$

$$-\frac{\partial \vec{A}}{\partial t} = \frac{-\mu_0}{4\pi r}\frac{\partial}{\partial t}\dot{\vec{p}}(t_0) = -\frac{\mu_0}{4\pi r}\ddot{\vec{p}}(t_0)$$

Finally, we have $\vec{E}$,

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

$$= \frac{1}{4\pi\epsilon_0 c^2}\left(\frac{\hat{r}\cdot\ddot{\vec{p}}}{r}\right)\hat{r} - \frac{\mu_0}{4\pi r}\ddot{\vec{p}}$$

$$= \frac{\mu_0}{4\pi r}\left[\left(\hat{r}\cdot\ddot{\vec{p}}\right)\hat{r} - \ddot{\vec{p}}\,\right]$$

using BAC-CAB $\rightarrow$ $\overset{B}{\hat{r}}(\overset{A}{\hat{r}}\cdot\overset{C}{\ddot{\vec{p}}}) - \overset{C}{\ddot{\vec{p}}}(\overset{A}{\hat{r}}\cdot\overset{B}{\hat{r}})$

$$\vec{E} = \frac{\mu_0}{4\pi r}\left[\hat{r} \times (\hat{r} \times \ddot{\vec{p}})\right]$$

$\vec{E}$ is perpendicular to $\hat{r}$ direction of propagation, and we'll soon see, also perpendicular to $\vec{B}$.

$$\vec{\nabla} \times \vec{A} = \frac{\mu_0}{4\pi} \vec{\nabla} \times \left(\frac{\dot{\vec{p}}(t_0)}{r} \right)$$

too fast

$$= \frac{\mu_0}{4\pi} \left[\frac{1}{r} \left(\vec{\nabla} \times \dot{\vec{p}} \right) - \cancel{\dot{\vec{p}} \times \vec{\nabla} \left(\frac{1}{r} \right)} \right] \quad \text{product rule 7}$$

$$= \frac{\mu_0}{4\pi r} \left\{ \left(\frac{\partial \dot{p}_z}{\partial t} \frac{\partial t_0}{\partial y} - \frac{\partial \dot{p}_y}{\partial t} \frac{\partial t_0}{\partial z} \right) \hat{x} + \ldots \right\}$$

$$= \frac{\mu_0}{4\pi r} \left\{ \left[\ddot{\vec{p}}_z \left(-\frac{1}{c} \frac{y}{r} \right) - \ddot{p}_y \left(-\frac{z}{cr} \right) \right] \hat{x} + \ldots \right\}$$

but y/r is the $\hat{y}$ component of $\hat{r}$, since,

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{x} + y\hat{y} + z\hat{z}}{r}$$

$$\vec{\nabla} \times \vec{A} = -\frac{\mu_0}{4\pi c r} \left\{ \left(\ddot{\vec{p}}_z \hat{r}_y - \ddot{\vec{p}}_y \hat{r}_z \right) \hat{x} + \ldots \right.$$

$$= -\frac{\mu_0}{4\pi c r} \left(\hat{r} \times \ddot{\vec{p}} \right)$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = -\frac{\mu_0}{4\pi c r} \left(\hat{r} \times \ddot{\vec{p}} \right)$$

$$\vec{E} = \frac{\mu_0}{4\pi r}\left[\hat{r} \times (\hat{r} \times \ddot{\vec{p}})\right] = \frac{\mu_0}{4\pi r}\left[(\hat{r}\cdot\ddot{\vec{p}})\hat{r} - \ddot{\vec{p}}\right]$$

$$\vec{B} = \frac{-\mu_0}{4\pi r c}\left[\hat{r} \times \ddot{\vec{p}}\right]$$

$\vec{E}$ is proportional to the acceleration of the charge

$\vec{E}$ is directed along the projection of $\ddot{\vec{p}}$ in a plane perpendicular to $\vec{r}$

$\vec{B}$ is perpendicular to $\vec{r}$ and to $\vec{E}$

$E/B = c$

If there is no acceleration of charge, there is no radiation.

Let's wrap this up with Griffith's power calculation. Let $\ddot{\vec{p}}(t_0) = \ddot{p}(t_0)\hat{z}$.

$$\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

$$\hat{r} \cdot \ddot{\vec{p}} = \ddot{p}\cos\theta$$

$$(\hat{r} \cdot \ddot{\vec{p}})\,\hat{r} - \ddot{\vec{p}} = \cancel{\ddot{p}\cos\theta\,\hat{r}} - \cancel{\ddot{p}\cos\theta\,\hat{r}} + \ddot{p}\sin\theta\,\hat{\theta}$$

$$\hat{r} \times \ddot{\vec{p}}\,\hat{z} = -\ddot{p}\sin\theta\,(\hat{r} \times \hat{\theta}) = -\ddot{p}\sin\theta\,\hat{\varphi}$$

$$\vec{S} = \frac{1}{\mu_0}\,\vec{E} \times \vec{B} = \frac{1}{\mu_0} E B\,(\hat{\theta} \times \hat{\varphi}) = \frac{1}{\mu_0} E B\,\hat{r}$$

$$= \frac{\mu_0}{(4\pi)^2 c}\left(\frac{\sin\theta}{r}\right)^2 \ddot{\vec{p}}^{\,2}\;\hat{r}$$

$$P \simeq \int \vec{s} \cdot d\vec{a} = \frac{\mu_0\,\ddot{\vec{p}}^{\,2}}{(4\pi)^2 c} \int_0^{2\pi} d\varphi \int_0^{\pi} \sin^2\theta\, d\theta$$

$$= \frac{\mu_0\,\ddot{\vec{p}}^{\,2}}{8\pi c}\left(\frac{4}{3}\right) = \frac{\mu_0\,\ddot{\vec{p}}}{6\pi c}$$