Now let's show that the radiation fields transport energy. Recall that the energy density in EM fields is $\frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) \equiv \rho$ (here ρ = energy density, not charge density)

Then $$\partial_t \left(\int_{\text{Volume}} dV \, \rho \right) = - \oint_{\text{Surf}} (\text{power flux } \underbrace{\vec{S}}_{\text{energy/area/time}}) \cdot dA$$

(Just conservation of energy.)

The LHS is $$\int dV \left(\epsilon_0 \vec{E} \cdot \dot{\vec{E}} + \frac{1}{\mu_0} \vec{B} \cdot \dot{\vec{B}} \right)$$

$$= \frac{1}{\mu_0} \int dV \left(\cancel{\epsilon_0} \, \vec{E} \cdot \frac{(\vec{\nabla} \times \vec{B})}{\cancel{\epsilon_0}} - \vec{B} \cdot (\vec{\nabla} \times \vec{E}) \right)$$

$$= -\frac{1}{\mu_0} \int dV \; \vec{\nabla} \cdot (\vec{E} \times \vec{B})$$

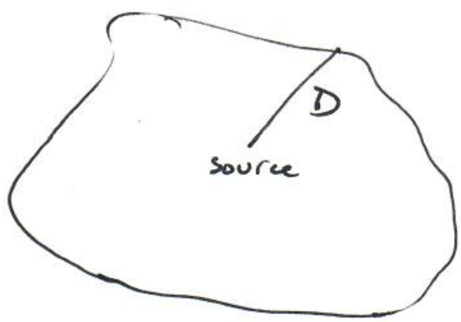
$$= -\frac{1}{\mu_0} \oint_{\text{Surf}} dA \cdot (\vec{E} \times \vec{B})$$

So $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ is the power flux

"Poynting vector"

Since photons have $E = |p|c$, the momentum flux rate is $\vec{S}/c$

Now look at a surface far away from some source



The total power flowing through the surface is

$$\frac{1}{\mu_0} \int_{\substack{surf \\ @ large D}} (\vec{E} \times \vec{B}) \cdot dA \qquad \begin{aligned} E &= E_{rad} + \mathcal{O}(1/D^2) \sim 1/D \\ B &= B_{rad} + \mathcal{O}(1/D^2) \sim 1/D \end{aligned}$$

$$\sim \frac{1}{\mu_0} \int (D^2 \sin\theta \, d\theta \, d\phi) \left(E_{rad} \times B_{rad} \sim 1/D^2 \right)$$

$$\sim \text{finite, indep of } D \text{ as } D \to \infty$$

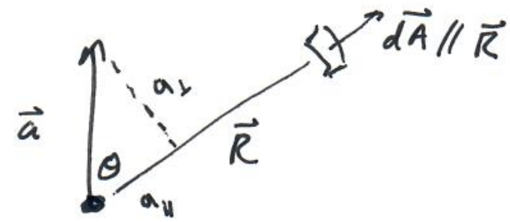
The contribution of any $1/r^2$ fields $\to 0$ in the limit.

So only the radiation fields can transport energy far from the source.

Let's look at some nonrelativistic ($|\vec{\beta}| = \frac{|\vec{v}|}{c} \ll 1$)
large distance radiation effects.

Here $\vec{E} \simeq \left(\frac{-e\vec{a}_\perp}{4\pi\epsilon_0 c^2 R} \right)_{ret}$, $\vec{B} = \frac{(\hat{n}\times\vec{E})_{ret}}{c}$

First let's compute the ~~charge~~ power radiated through a solid angle. Say that at t' the acceleration is along $\hat{z}$:



Here $|\vec{a}_\perp| = |\vec{a}| \sin\theta$

and since $\vec{E} \perp \vec{B}$, $\vec{B} \perp \vec{R}$, $\vec{E} \perp \vec{R}$,

$$(\vec{E}\times\vec{B}) \cdot dA = |E||B|R^2 \, d\Omega$$

So $$\frac{dP}{d\Omega} = \frac{1}{\mu_0}|E||B|R^2 = \frac{e^2 a^2 \sin^2\theta}{4\pi\epsilon_0 \cdot 4\pi c^3}$$

maximized @ $\theta = \pi/2$, $R \perp a$, with $\vec{E} \parallel \vec{a}$.

This is the power/$d\Omega$ <u>emitted</u> at $t' = t - R/c$

Using $\int_0^{\pi} \sin^2\theta(\sin\theta \, d\theta) = \int_{-1}^{1} (1-\cos^2\theta)\, d\cos\theta$

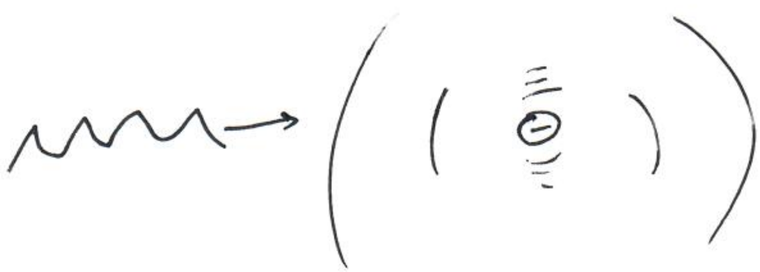
$$= 2 - \frac{2}{3} = \frac{4}{3},$$

We obtain the Larmor formula,

$$\boxed{P = \frac{2e^2a^2}{4\pi\epsilon_0 \cdot 3c^3}} \qquad \left(\frac{e^2a^2}{6\pi\epsilon_0 c^3}\right)$$

This is the power radiated by the charge at some t' when its acceleration was $|\vec{a}|$. It passes through the sphere at $t = t' + R/c$.

Thomson scattering



an electromagnetic wave strikes a free electron, causing it to accelerate and radiate.

the acceleration is $\vec{a} = e\vec{E}/m_e$

(Newton's law + coulomb's law)

So the total power radiated is $\frac{2e^2a^2}{4\pi\epsilon_0 \cdot 3c^3} = \frac{2e^4|\vec{E}|^2}{3m_e^2c^3 \cdot 4\pi\epsilon_0}$