Today's outline - April 11, 2023

- Superoperators
- Examples
- Operator sum decomposition
- Superconducting qubits
- Tunable qubits

Reading assignment: Reiffel: 11.1

Homework Assignment #07:
Due Tuesday, April 25, 2023

Exam #2 Tuesday, April 18, 2023
Covers Chapters 6-10

Please send me your choice of paper to present if you have not yet done so

Superoperators

Consider a unitary operator U acting on a system X such that its action on $|\psi\rangle$ is

$$|\psi\rangle \mapsto U|\psi\rangle$$

Since the density operator for pure state $|\psi\rangle$ is $\rho = |\psi\rangle\langle\psi|$, U has the effect

$$\rho \mapsto U|\psi\rangle\langle\psi|U^\dagger = U\rho U^\dagger$$

Things are more complicated in the general case when $X = A \otimes B$ and $|\psi\rangle \in X$, now

$$\rho_A = \mathrm{Tr}(\rho) \mapsto \rho'_A = \mathrm{Tr}(U|\psi\rangle\langle\psi|U^\dagger)$$

In the case when $U = U_A \otimes U_B$ then the person who controls subsystem A can obtain ρ'_A directly using ρ_A and U as $\rho'_A = U_A \rho_A U_A^\dagger$

However for a general unitary operator, it is not possible to deduce ρ'_A from ρ_A and U alone as ρ'_A depends on the initial state $|\psi\rangle$ of the entire system

Example 10.4.1

Let $X = A \otimes B$, where A and B are single qubit systems

Define $\rho_A = |0\rangle\langle 0|$ and take $U = C_{not}$ with B as the control and A the target

$$U = |00\rangle\langle 00| + |11\rangle\langle 01| + |10\rangle\langle 10| + |01\rangle\langle 11|$$

Consider the following states, each of which is consistent with ρ_A, and compute ρ'_A

$$|\psi_0\rangle = |00\rangle, \quad \rho_A = \mathrm{Tr}_B(|00\rangle\langle 00|) = \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{1}\langle ik|00\rangle\langle 00|jk\rangle|j\rangle\langle i| = |0\rangle\langle 0|$$

$$\rho'_A = \mathrm{Tr}_B(U|00\rangle\langle 00|U) = \mathrm{Tr}_B(|00\rangle\langle 00|) = |0\rangle\langle 0|$$

$$|\psi_1\rangle = |01\rangle, \quad \rho_A = \mathrm{Tr}_B(|01\rangle\langle 01|) = \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{1}\langle ik|01\rangle\langle 01|jk\rangle|j\rangle\langle i| = |0\rangle\langle 0|$$

$$\rho'_A = \mathrm{Tr}_B(U|01\rangle\langle 01|U) = \mathrm{Tr}_B(|11\rangle\langle 11|) = \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{1}\langle ik|11\rangle\langle 11|jk\rangle|j\rangle\langle i| = |1\rangle\langle 1|$$

Example 10.4.1 (cont.)

$$U = |00\rangle\langle 00| + |11\rangle\langle 01| + |10\rangle\langle 10| + |01\rangle\langle 11|$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$$

$$\begin{aligned}
\rho_A &= \mathrm{Tr}_B\left(\frac{1}{2}\left(|00\rangle\langle 00| + |00\rangle\langle 01| + |01\rangle\langle 00| + |01\rangle\langle 01|\right)\right) \\
&= \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{1}\frac{1}{2}\langle ik|\left(|00\rangle\langle 00| + |00\rangle\langle 01| + |01\rangle\langle 00| + |01\rangle\langle 01|\right)|jk\rangle|j\rangle\langle i| \\
&= \left(\frac{1}{2}+\frac{1}{2}\right)|0\rangle\langle 0| = |0\rangle\langle 0| \\
\rho_A' &= \mathrm{Tr}_B\left(\frac{1}{2}U\left(|00\rangle\langle 00| + |00\rangle\langle 01| + |01\rangle\langle 00| + |01\rangle\langle 01|\right)U\right) \\
&= \mathrm{Tr}_B\left(\frac{1}{2}\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right)\right) \\
&= \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{1}\frac{1}{2}\langle ik|\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right)|jk\rangle|j\rangle\langle i| \\
&= \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}I
\end{aligned}$$

Example 10.4.2

Consider the operator $U_{switch} = |00\rangle\langle 00| + |10\rangle\langle 01| + |01\rangle\langle 10| + |11\rangle\langle 11|$ acting on single qubit systems A and B

Suppose that system A is initially in state $\rho_A = |\psi\rangle\langle\psi|$ and system B is in state $|0\rangle\langle 0|$

Because $\rho_B = |0\rangle\langle 0|$ the state of the system initially can be described as $(a|0\rangle + b|1\rangle)|0\rangle = a|00\rangle + b|10\rangle$ where $|a|^2 + |b|^2 = 1$

$$\begin{aligned}
\rho'_A &= \mathrm{Tr}_B \left(U \left(|a|^2 |00\rangle\langle 00| + a\bar{b}|10\rangle\langle 00| + \bar{a}b|00\rangle\langle 10| + |b|^2 |10\rangle\langle 10| \right) U \right) \\
&= \mathrm{Tr}_B \left(|a|^2 |00\rangle\langle 00| + a\bar{b}|01\rangle\langle 00| + \bar{a}b|00\rangle\langle 01| + |b|^2 |01\rangle\langle 01| \right) \\
&= \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{1} \langle ik| \left(|a|^2 |00\rangle\langle 00| + a\bar{b}|01\rangle\langle 00| + \bar{a}b|00\rangle\langle 01| + |b|^2 |01\rangle\langle 01| \right) |jk\rangle |j\rangle\langle i| \\
&= (|a|^2 + |b|^2)|0\rangle\langle 0| = |0\rangle\langle 0|
\end{aligned}$$

Note that U_{switch} is not reversible

Superoperators

Suppose that $\mathcal{D}_A$ is the set of all density operators for subsystem A

If A and B are not entangled and B is initially in a state $|\phi_B\rangle$ then the action of U on X is

$$U : X \to X$$
$$|\psi\rangle \mapsto U|\psi\rangle$$

which induces the action of the superoperator $S_U^{\phi_B}$ where

$\rho_A = \mathrm{Tr}(|\psi\rangle\langle\psi|)$ and $\rho'_A = \mathrm{Tr}(U|\psi\rangle\langle\psi|U^\dagger)$

$$S_U^{\phi_B} : \mathcal{D}_A \to \mathcal{D}_A$$
$$\rho_A \mapsto \rho'_A$$

Consider a density operator that is a probabilistic mixture of other density operators

$$\rho = \sum_i p_i \rho_i$$

The effect of a superoperator on this mixture is the sum of its effect on the individual density operators

$$S : \rho \mapsto \sum_i p_i S(\rho_i)$$

Operator sum decomposition

In general superoperators are not reversible, of the form $U\rho U^\dagger$ where U is unitary or even of the form $A\rho A^\dagger$ where A is a linear operator

It is, however, possible to write all superoperators as a sum of linear operators $A_1, \ldots, A_K$

$$S(\rho) = \sum_{i=0}^{K-1} A_i \rho A_i^\dagger$$

This sum is called an operator sum decomposition for S and is not generally unique

In order to obtain the operator sum decomposition for S_U^ϕ, let $\{|\beta_i\rangle\}$ be a basis for B and let $A_i = \langle\beta_i|U|\phi\rangle : A \to A$ so we have

$$S_U^\phi(\rho) = \mathrm{Tr}_B\left(U(\rho \otimes |\phi\rangle\langle\phi|)U^\dagger\right) = \sum_{i=0}^{K-1} \langle\beta_i|U(\rho \otimes |\phi\rangle\langle\phi|)U^\dagger|\beta_i\rangle$$

For the pure state $\rho = |\psi\rangle\langle\psi|$ the tensor product is separable and for a mixed state which is a probabilistic sum of pure states can be similarly separated, so

$$S_U^\phi(\rho) = \sum_{i=0}^{K-1} \langle\beta_i|U|\phi\rangle\rho\langle\phi|U^\dagger|\beta_i\rangle = \sum_{i=0}^{K-1} A_i \rho A_i^\dagger$$

Operator sum decomposition

$$S_U^\phi(\rho) = \sum_{i=0}^{K-1} \langle \beta_i | U | \phi \rangle \rho \langle \phi | U^\dagger | \beta_i \rangle = \sum_{i=0}^{K-1} A_i \rho A_i^\dagger$$

Each term in the operator sum decomposition is Hermitian and positive but does not necessarily have trace one

$$\mathrm{Tr}(A_i \rho A_i^\dagger) \geq 0$$

However, a density operator, ρ_{decomp}, can be constructed by normalizing

$$\rho_{decomp} = \frac{A_i \rho A_i^\dagger}{\mathrm{Tr}(A_i \rho A_i^\dagger)}$$

The trace of the associated superoperator is one so

$$\mathrm{Tr}\left(S_U^\phi(\rho)\right) = \sum_{i=0}^{K-1} \mathrm{Tr}(A_i \rho A_i^\dagger) \equiv 1$$

Thus the superoperator is a probabilistic mixture of the normalized operators with $p_i = \mathrm{Tr}(A_i \rho A_i^\dagger)$

$$S_U^\phi(\rho) = \sum_{i=0}^{K-1} p_i \frac{A_i \rho A_i^\dagger}{\mathrm{Tr}(A_i \rho A_i^\dagger)}$$

This is reminiscent of the possible measurement outcomes of ρ by operator O with projectors P_j

$$\rho' = \sum_{j=0}^{K-1} p_j \frac{P_j \rho P_j^\dagger}{\mathrm{Tr}(P_j \rho P_j^\dagger)}$$

Operator sum decomposition

If A_i is the operator obtained in the operator sum decomposition for S_U^ϕ when using basis $\{|\beta_i\rangle\}$ for the B subsystem

Suppose that after $U : A \otimes B \rightarrow A \otimes B$ is applied to ρ, subsystem B were measured with respect to the projectors $P_i = |\beta_i\rangle\langle\beta_i|$ for the $K = 2^k$ basis elements of B, the best description of subsystem A after this measurement is a probabilistic mixture of mixed states

$$\rho' = \sum_{i=0}^{K-1} p_i \rho_i, \qquad \rho_i = \mathrm{Tr}_B \left(\frac{(I \otimes P_i)U(\rho \otimes |\phi\rangle\langle\phi|)U^\dagger(I \otimes P_i^\dagger)}{\mathrm{Tr}\left((I \otimes P_i)U(\rho \otimes |\phi\rangle\langle\phi|)U^\dagger(I \otimes P_i^\dagger)\right)} \right)$$

$$\text{and} \qquad p_i = \mathrm{Tr}\left((I \otimes P_i)U(\rho \otimes |\phi\rangle\langle\phi|)U^\dagger(I \otimes P_i^\dagger)\right)$$

$$\text{but} \qquad \mathrm{Tr}_B\left((I \otimes |\beta_i\rangle\langle\beta_i|)U\rho \otimes |\phi\rangle\langle\phi|U^\dagger(I \otimes |\beta_i\rangle\langle\beta_i|)\right) = \langle\beta_i|U\rho \otimes |\phi\rangle\langle\phi|U^\dagger|\beta_i\rangle$$

So the density operator $\rho' = \sum_i p_i \rho_i \equiv S_U^\phi(\rho)$

Example 10.4.3

Find the operator sum decomposition for C_{not} and $|\phi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

The C_{not} operator U can be written as

$$U = X \otimes |1\rangle\langle 1| + I \otimes |0\rangle\langle 0|$$

If the two systems are initially unentangled and A and B are in states

$$\rho = |\psi\rangle\langle\psi|, \quad \rho' = |\phi\rangle\langle\phi|$$

$$S_U^{\phi} = \mathrm{Tr}\left(U(\rho \otimes |\phi\rangle\langle\phi|)U^{\dagger}\right) = A_0 \rho A_0^{\dagger} + A_1 \rho A_1^{\dagger}$$

Where $A_0 = \langle 0|U|\phi\rangle$ and $A_1 = \langle 1|U|\phi\rangle$ so applying these operators to the state of A

$$\begin{aligned}
A_0|\psi\rangle &= \sum_{i=0}^{1} \langle\alpha_i|\langle 0|U|\psi\rangle|\phi\rangle|\alpha_i\rangle \\
&= \langle 0|\langle 0|(X \otimes |1\rangle\langle 1| + I \otimes |0\rangle\langle 0|)|\psi\rangle|\phi\rangle|0\rangle + \langle 1|\langle 0|(X \otimes |1\rangle\langle 1| + I \otimes |0\rangle\langle 0|)|\psi\rangle|\phi\rangle|1\rangle \\
&= \big(\langle 0|\langle 0|(X \otimes |1\rangle\langle 1|)|\psi\rangle|\phi\rangle + \langle 0|\langle 0|(I \otimes |0\rangle\langle 0|)|\psi\rangle|\phi\rangle\big)|0\rangle \\
&\quad + \big(\langle 1|\langle 0|(X \otimes |1\rangle\langle 1|)|\psi\rangle|\phi\rangle + \langle 1|\langle 0|(I \otimes |0\rangle\langle 0|)|\psi\rangle|\phi\rangle\big)|1\rangle
\end{aligned}$$

Example 10.4.3 (cont.)

$$\begin{aligned}
A_0|\psi\rangle &= \Big(\langle 0|\cancel{\langle 0|}(X\otimes \cancel{|1\rangle}\langle 1|)|\psi\rangle|\phi\rangle + \langle 0|\langle 0|(I\otimes |0\rangle\langle 0|)|\psi\rangle|\phi\rangle\Big)|0\rangle \\
&\quad + \Big(\langle 1|\cancel{\langle 0|}(X\otimes \cancel{|1\rangle}\langle 1|)|\psi\rangle|\phi\rangle + \langle 1|\langle 0|(I\otimes |0\rangle\langle 0|)|\psi\rangle|\phi\rangle\Big)|1\rangle \\
&= \langle 0|\langle 0|(I\otimes |0\rangle\langle 0|)|\psi\rangle|\phi\rangle\Big)|0\rangle + \langle 1|\langle 0|(I\otimes |0\rangle\langle 0|)|\psi\rangle|\phi\rangle\Big)|1\rangle \\
&= \langle 0|\psi\rangle\langle 0|\phi\rangle|0\rangle + \langle 1|\psi\rangle\langle 0|\phi\rangle|1\rangle = a_0\langle 0|\phi\rangle|0\rangle + a_1\langle 0|\phi\rangle|1\rangle = \langle 0|\phi\rangle|\psi\rangle
\end{aligned}$$

Recall that $|\phi\rangle\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ so $\qquad A_0|\psi\rangle = \langle 0|\phi\rangle|\psi\rangle = \frac{1}{\sqrt{2}}|\psi\rangle \quad\longrightarrow\quad A_0 = \frac{1}{\sqrt{2}}I$

Similarly for A_1 we have

$$\begin{aligned}
A_1|\psi\rangle &= \langle 0|\langle 1|(X\otimes |1\rangle\langle 1|)|\psi\rangle|\phi\rangle\Big)|0\rangle + \langle 1|\langle 1|(X\otimes |1\rangle\langle 1|)|\psi\rangle|\phi\rangle\Big)|1\rangle \\
&= \langle 0|X|\psi\rangle\langle 1|\phi\rangle|0\rangle + \langle 1|X|\psi\rangle\langle 1|\phi\rangle|1\rangle = a_1\langle 1|\phi\rangle|0\rangle + a_0\langle 1|\phi\rangle|1\rangle = \langle 1|\phi\rangle X|\psi\rangle \\
&= \frac{1}{\sqrt{2}}X|\psi\rangle \qquad\longrightarrow\qquad A_1 = \frac{1}{\sqrt{2}}X
\end{aligned}$$

Example 10.4.4

Find the operator sum decomposition for U_{switch} and $|\phi\rangle = |0\rangle$

$$U_{switch} = |00\rangle\langle 00| + |10\rangle\langle 01| + |01\rangle\langle 10| + |11\rangle\langle 11|$$

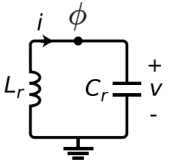
$$S_U^\phi = \mathrm{Tr}_B\left(U(\rho \otimes |\phi\rangle\langle\phi|)U^\dagger\right) = A_0\rho A_0^\dagger + A_1\rho A_1^\dagger, \qquad A_i = \langle i|U|\phi\rangle$$

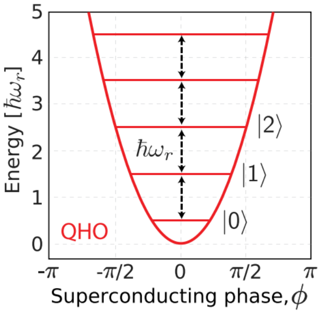
$$\begin{aligned} A_0|\psi\rangle = \sum_{i=0}^{1} \langle\alpha_i|\langle 0|U|\psi\rangle|\phi\rangle|\alpha_i\rangle &= \langle 00|(|00\rangle\langle 00| + \cancel{|10\rangle\langle 01|} + \cancel{|01\rangle\langle 10|} + \cancel{|11\rangle\langle 11|})|\psi\phi\rangle|0\rangle \\ &+ \langle 10|(\cancel{|00\rangle\langle 00|} + |10\rangle\langle 01| + \cancel{|01\rangle\langle 10|} + \cancel{|11\rangle\langle 11|})|\psi\phi\rangle|1\rangle \\ = \langle 00|\psi\phi\rangle|0\rangle + \langle 01|\psi\phi\rangle|1\rangle &= \langle 0|\psi\rangle\langle 0|\phi\rangle|0\rangle + \langle 0|\psi\rangle\cancel{\langle 1|\phi\rangle}|1\rangle = |0\rangle\langle 0|\psi\rangle \longrightarrow A_0 = |0\rangle\langle 0| \end{aligned}$$

$$\begin{aligned} A_1|\psi\rangle = \sum_{i=0}^{1} \langle\alpha_i|\langle 1|U|\psi\rangle|\phi\rangle|\alpha_i\rangle &= \langle 01|(\cancel{|00\rangle\langle 00|} + \cancel{|10\rangle\langle 01|} + |01\rangle\langle 10| + \cancel{|11\rangle\langle 11|})|\psi\phi\rangle|0\rangle \\ &+ \langle 11|(\cancel{|00\rangle\langle 00|} + \cancel{|10\rangle\langle 01|} + \cancel{|01\rangle\langle 10|} + |11\rangle\langle 11|)|\psi\phi\rangle|1\rangle \\ = \langle 10|\psi\phi\rangle|0\rangle + \langle 11|\psi\phi\rangle|1\rangle &= \langle 1|\psi\rangle\langle 0|\phi\rangle|0\rangle + \langle 1|\psi\rangle\cancel{\langle 1|\phi\rangle}|1\rangle = |0\rangle\langle 1|\psi\rangle \longrightarrow A_1 = |0\rangle\langle 1| \end{aligned}$$

The superconducting qubit

To understand the superconducting qubit, it is useful to start with a simple harmonic oscillator





This can be modeled by a superconducting LC circuit where the charge carriers are Cooper pairs with a harmonic oscillator Hamiltonian whose variables are the reduced flux or phase at the island ϕ and the reduced charge n

$$\mathcal{H} = 4E_C n^2 + \frac{1}{2}E_L\phi^2, \quad E_C = \frac{e^2}{2C}, \quad E_L = \frac{\Phi_0^2}{4\pi^2 L}$$

where E_C is the charging energy to add each electron of the Cooper pair to the island and E_L is the inductive energy

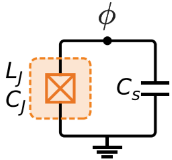
The eigenvalues for the states $|k\rangle$ are $E_k = \hbar\omega_r(k + \frac{1}{2})$

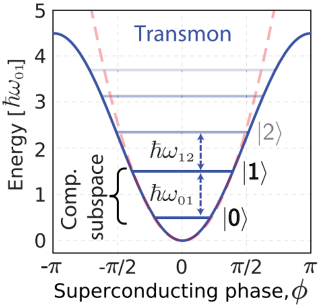
This is not suitable for a qubit since the excitation frequency ω_r can excite states above $|1\rangle$ and only two states are needed

"A quantum engineer's guide to superconducting qubits," P. Krantz, M. Kjaergaard, F. Yan, T.P. Orlando, S. Gustavsson, and W.D. Oliver, *Appl. Phys. Rev.* **6**, 021318 (2019).

The superconducting qubit

The first two states can be made distinct by introducing a nonlinear circuit element





Replacing the inductor with a Josephson junction, effectively a nonlinear capacitor, adds anharmonicity to the Hamiltonian

$$\mathcal{H} = 4E_C n^2 - E_J \cos\phi, \quad E_C = \frac{e^2}{2(C_s + C_J)}, \quad E_J = \frac{I_c \Phi_0}{2\pi}$$

where $E_J \gg E_C$ is the Josephson energy and I_c is the critical current of the junction

The Hamiltonian now has anharmonic terms that can be seen by expanding the second term

$$E_J \cos\phi = \frac{1}{2}E_J\phi^2 - \frac{1}{4}E_J\phi^4 + \mathcal{O}(\phi^6)$$

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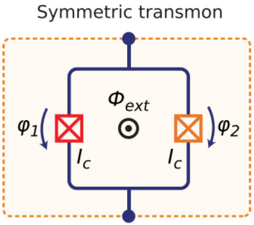
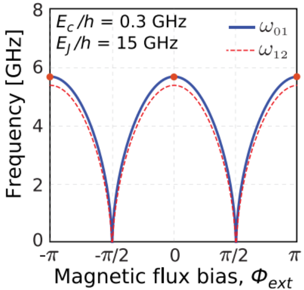
Tunable qubits





The goal of fast gate operations and high-fidelity has led to qubit circuits which have tunable frequencies

One of the most common implementations is to replace the single Josephson junction with a loop containing two identical junctions making a DC-SQUID

There is interference between the two arms and the the critical current of the junctions can be decreased by applying an external magnetic field through the loop

E_J, and thus the frequencies of the interlevel transitions change with the magnetic flux Φ_{ext}

However, this introduces noise when the slope of the curve is steep and when the frequencies become degenerate

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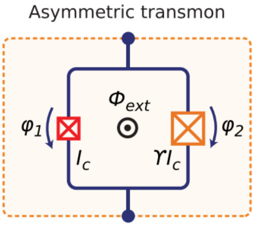
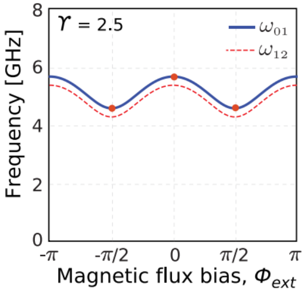
Tunable qubits





The problem of increased noise can be avoided by making the two junctions asymmetric giving a Hamiltonian

$$\mathcal{H} = 4E_C n^2 - (E_{J1} + E_{J2})\sqrt{\cos^2\varphi_e + d^2\sin^2\varphi_e}\,\cos\phi$$

where $\varphi_e = \pi\Phi_{ext}/\Phi_0$, $\gamma = E_{J2}/E_{J1}$, and $d = (\gamma - 1)/(\gamma + 1)$ is the junction asymmetry parameter

When $|d| \to 1$ the single junction transmon is recovered and when $d = 0$ we have the symmetric transmon

The noise is supressed by limiting the slope of the tuning curve and eliminating transition frequency degeneracies

The tunability is sufficient for many purposes, such as matching qubit frequencies, which is important for certain specialized gates

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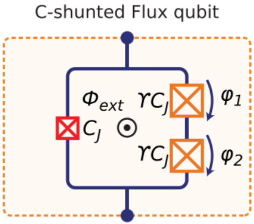
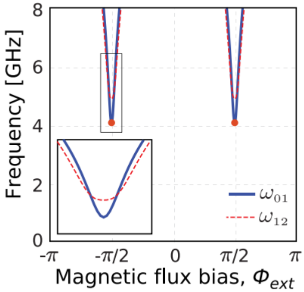
Tunable qubits





The transmon has limited anharmonicity so the sinusoidal dependence on Φ_{ext} cannot be eliminated

The invention of the flux qubit with three or more junctions has the benefit of strong anharmonicity

$$\mathcal{H} \approx 4E_C n^2 - E_J \cos(2\phi + \varphi_e) - 2\gamma E_J \cos\phi$$

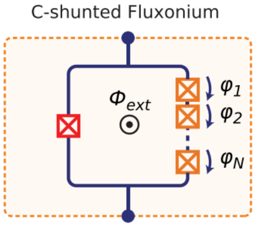
The operating points are at $\varphi_e = \pi + 2\pi k$ where k is an integer

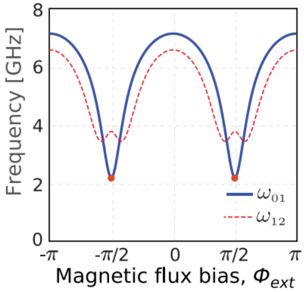
At these points, half a superconducting flux quantum threads the qubit loop and the coherence time is enhanced

The anharmonicity of the flux qubit is much higher than that of the transmon for equivalent values of E_J/E_C

"A quantum engineer's guide to superconducting qubits," P. Krantz, M. Kjaergaard, F. Yan, T.P. Orlando, S. Gustavsson, and W.D. Oliver, *Appl. Phys. Rev.* **6**, 021318 (2019).

Tunable qubits





The effects seen in the flux qubit can be further enhanced by appropriate choice of circuit design parameters

The fluxonium qubit is an example of a recent development where many junctions (up to 100) are added to one side of the loop

$$\mathcal{H} \approx 4E_C n^2 - E_J \cos(\phi + \varphi_e) + \frac{1}{2} E_L \phi^2$$

where $E_L = (\gamma N) E_J$ is the superinductance contributed by the array of N junctions

Long coherence and high anharmonicity are possible at the operating point

An additional benefit is the possibility of incorporating plasmon and fluxon states which have the potential to be used for quantum information processing

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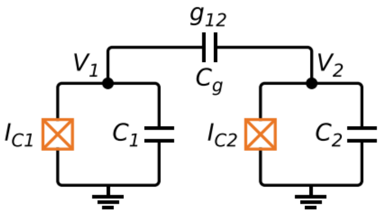
Coupling qubits



Generating entanglement between qubits requires an interaction Hamiltonian that couples their degrees of freedom

For superconducting qubits the coupling Hamiltonian takes the general form

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_{int}$$

where $\mathcal{H}_1$ and $\mathcal{H}_2$ are the Hamiltonians of the individual qubits and $\mathcal{H}_{int}$ is the physical coupling by electric or magnetic field

Capacitive coupling is achieved by placing a capacitor between the voltage nodes of two qubits to give $\mathcal{H}_{int} = C_g V_1 V_2$

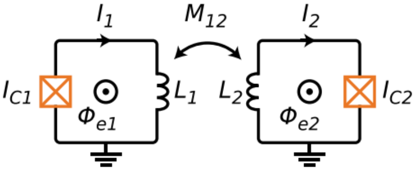
When the coupling capacitance $C_g \ll C_1, C_2$ the effective Hamiltonian becomes

$$\mathcal{H} = \sum_{i=1,2} \left[4E_{Ci} n_i^2 - E_{Ji} \cos\phi\right] + 4e^2 \frac{C_g}{C_1 C_2} n_1 n_2$$

"A quantum engineer's guide to superconducting qubits," P. Krantz, M. Kjaergaard, F. Yan, T.P. Orlando, S. Gustavsson, and W.D. Oliver, *Appl. Phys. Rev.* **6**, 021318 (2019).

Coupling qubits

For inductive coupling, a mutual inductance between the two qubits is required with the interaction Hamiltonian $\mathcal{H}_{int} = M_{12} I_1 I_2$



where M_{12} is the mutual inductance and I_1 and I_2 are the current operators of the qubit loops

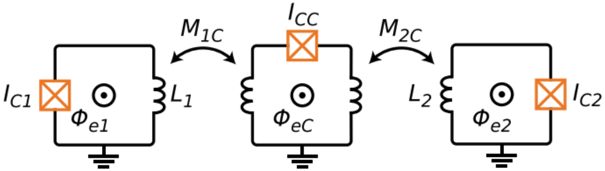
The system Hamiltonian thus becomes

$$\mathcal{H} = \sum_{i=1,2} \left[4E_{Ci} n_i^2 + \tfrac{1}{2} E_{Li} \phi_i^2 - E_{Ji} \cos\phi\right] + M_{12}(I_{c1}\sin\phi_1)(I_{c2}\sin\phi_2)$$

The strength of the coupling depends on the mutual inductance as well as the matrix element of the current operators which are governed by the Josephson relations

$$I_i = I ci \sin\phi_i, \quad V = \frac{\hbar}{2e}\frac{d\phi}{dt}$$

Tunable inductive coupling can be achieved with a third qubit



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