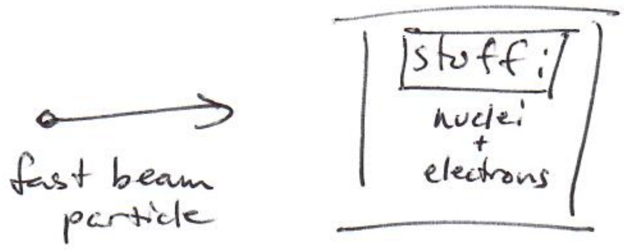
Collisions, Energy loss, & Scattering

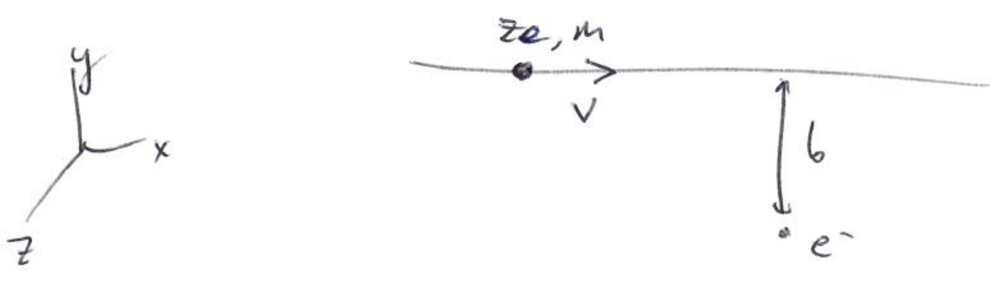
Last unit on the properties & dynamics of single charges.



Nuclei are heavy & cause large-angle quasi-elastic scattering. Electrons are light and can absorb energy without significant deflection. Both are important effects.

energy transfer in a coulomb collision

start w/ heavy beam particle & atomic e^-.



Atomic e^- have typical speeds $\sim 1\%$ c.
Assume beam is much faster.

Then the transverse momentum kick is (for small deflection)

$$\Delta p \approx \int_{-\infty}^{\infty} dt \, (E_y)(Ze)$$

see last lecture

$$= \int_{-\infty}^{\infty} \frac{Ze^2}{4\pi\epsilon_0} \frac{\gamma b \, dt}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

can absorb γ into t - result indep of γ!

$$= \frac{2Ze^2}{4\pi\epsilon_0 b v}$$

If the e^- is nonrelativistic, it will gain energy

$$\Delta E_{e^-} \cong \frac{(\Delta p)^2}{2m_e} = \frac{2Z^2e^4}{(4\pi\epsilon_0)^2 m_e v^2 b^2}$$

The beam deflection angle is

$$\frac{\Delta p}{p} \cong \theta = \frac{2Ze^2}{4\pi\epsilon_0 p b v}$$

We need $\Delta p / p \ll 1$ for the approximation used to compute Δp to hold (small deflection) and $p = \gamma m v$, so

$$\frac{z Z e^2}{4\pi\epsilon_0 \gamma m v^2 b} \ll 1$$

$$\text{or} \quad b_{min} = \frac{Z e^2}{4\pi\epsilon_0 \gamma m v^2} \quad \text{(ignore } z\text{'s)}$$

When $b \gtrsim b_{min}$, our calculations are breaking.

Another way to see it: we want the electron to move a small distance cf. b.

The duration of the collision is $\Delta t \simeq \dfrac{b}{\gamma v}$

(stare at E_y - the γ is due to the "packing up" of the transverse fields w/ γ.)

During the collision, the e^- moves a distance

$$d \sim \left(\frac{\Delta p}{2m}\right) \times \Delta t = \frac{Z e^2}{\gamma m v^2} = b_{min}.$$

↑ avg velocity

There's also a b_{max}. If the collision lasts too long, the fact that the electron is bound to a nucleus will matter, and it becomes a 3-body problem. The nucleus moves too and its mass enters the ΔE formula.

$$\text{require } \Delta t \lesssim \frac{1}{\omega} \qquad \omega = \text{orbital frequency of the } e^-.$$

$$\Delta t \sim \frac{b}{\gamma v} \qquad \left(\text{cf. } E_y \sim \frac{\gamma b}{(b^2+\gamma^2 v^2 t^2)^{3/2}} - \text{field is important over times} \lesssim \frac{b}{\gamma v} \right)$$

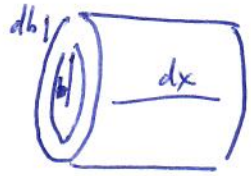
$$\Rightarrow \quad b_{max} = \frac{\gamma v}{\omega}$$

(numbers: $\omega \sim \dfrac{1\% c}{1\,\text{Å}}$ ← atomic e^- speed, ← atomic radius $\sim 3\times 10^{16}$ Hz

for $\gamma v \sim c$, would get $b_{max} = 10^{-8}$ m $= 100$ Å)

If a block of matter has N atoms / unit vol and Z_1 e^-/atom,

$$dn = 2\pi b \, db \, dx \, N Z_1 \quad \text{electrons}$$

in 

← this material.

So the energy loss is

$$\frac{dE}{dx} = 2\pi N Z_1 \int_{b_{min}}^{b_{max}} \Delta E(b) \, b \, db$$

$$\simeq 2\pi N Z_1 \int_{b_{min}}^{b_{max}} b \, db \, \frac{2 Z^2 e^4}{m v^2} \frac{1}{b^2}$$

$$= \frac{4\pi N Z_1 Z^2 e^4}{m v^2} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

with $b_{max}/b_{min} \approx \dfrac{\gamma^2 m v^3}{Z e^2 \omega}$

The log forgives all our $\approx$ in the precise form of $b_{min, max}$.