#### **CS 481**

# Artificial Intelligence Language Understanding

**February 7, 2023** 

### **Announcements / Reminders**

- Please follow the Week 04 To Do List instructions
- Quiz #04 due on Sunday (02/12/23) at 11:59 PM CST
- PA #01 due on Monday (02/20/23) at 11:59 PM CST

#### Exam dates:

• Midterm: 03/02/2023 during Thursday lecture time

■ Final: 04/27/2023 during Thursday lecture time

### @them.iit.edu | Hackathon 2023





Linktree with interest form and application for executive board





https://linktr.ee/them.at.iit

### **Plan for Today**

- Spelling: Minimum Edit Distance
- Parts of Speech tagging introduction (if time permits)

### Spelling: Real-world Problems

- Non-word error detection
  - graffe instead of giraffe
- Isolated-word error correction
- Context-dependent error detection and correction
  - typos
    - three instead of there
  - homophone or near-homophones
    - dessert instead of desert or piece for peace

### **How Similar are Two Strings?**

- The user typed "graffe". Which string is closest?
  - graf
  - graft
  - grail
  - giraffe

Why? Spell checking

### **How Similar are Two Strings?**

- Why? Computational Biology:
  - Align two sequences of nucleotides:

AGGCTATCACCTGACCTCCAGGCCGATGCCC
TAGCTATCACGACCGCGGTCGATTTGCCCGAC

Resulting alignment:

```
-AGGCTATCACCTGACCTCCAGGCCGA--TGCCC---
TAG-CTATCAC--GACCGC--GGTCGATTTGCCCGAC
```

### **How Similar are Two Strings?**

■ The user typed "graffe". Which string is closest?

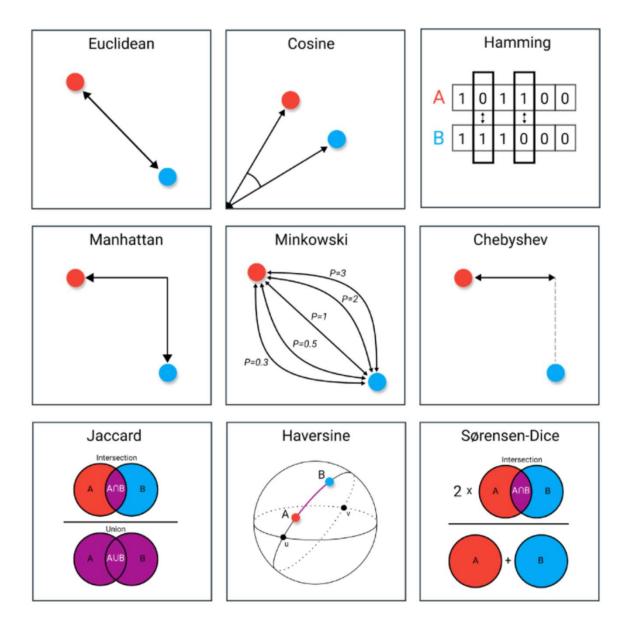
- graf deleted "i" deleted "fe"
- graft deleted "i" "e" and substituted "f"
- grail deletion and substitution
- giraffe correct form (we need to insert
  "i")

### Alignment

Given two sequences, an alignment is a correspondence between substrings of the two sequences.

Alignment is made up of edits.

# Distance Measures | String Distance?



Source: https://towardsdatascience.com/9-distance-measures-in-data-science-918109d069fa

#### **Edits**

One string can be transformed to another by a sequence of edits (delete, insert,

**s**ubstitute).

### **Edits with Costs: Edit Distance**

#### Each edit operation can have its cost:

cost(d) = cost(i) = cost(s) = 1

**Edit distance = 5** 

#### **Edits with Costs: Levenshtein Distance**

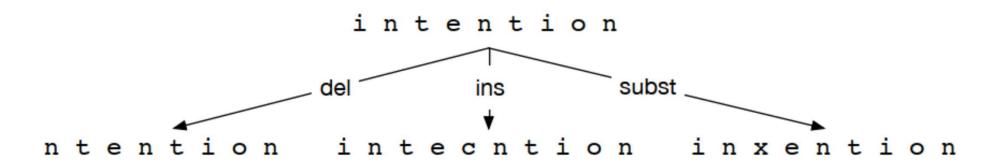
#### Each edit operation can have its cost:

cost(d) = cost(i) = 1 | cost(s) = cost(d) + cost(i) = 2

Levenshtein edit distance = 8

### Searching for Minimum Edit Path

String transformation (a sequence of edits) can be represented with a tree:



#### Solution: Minimum Edit Path found via tree search:

- Initial state (root): the word we're transforming
- Operators / actions: insert, delete, substitute
- Goal state: the word we're trying to get to
- Path cost: what we want to minimize the number of edits

#### **Edit Path**

One of the edit paths (we want minimum # of edits):

```
intention
               ← delete i
ntention

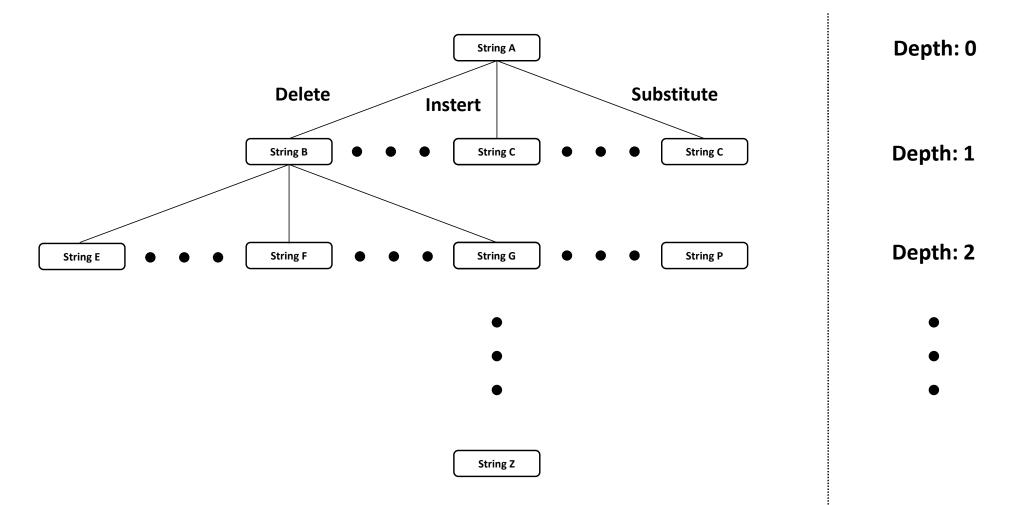
→ substitute n by e

etention

→ substitute t by x

exention
               ← insert u
exenution
               substitute n by c
execution
```

# Finding Minimum Edit Path /w Search



Quickly becomes unmanageable and impossible to search with brute force!

### **Minimum Edit Distance: Definition**

- For two strings:
  - X of length n
  - Y of length m
- We define D(i, j)
  - the edit distance between X[1..i] and Y[1..j]
  - i.e., the first i characters of X and the first j characters of Y
  - The edit distance between X and Y is thus D(n, m)

### **MED: Dynamic Programming**

- Dynamic programming: A tabular computation of D(n, m)
  - Solving problems by combining solutions to subproblems.
- Bottom-up approach
  - we compute D(i,j) for small i, j
  - and then compute larger D(i, j) based on previously computed smaller values
    - i.e., compute D(i, j) for all i (0 < i < n) and j (0 < j < m)</p>

### Minimum Edit Distance: Pseudocode

function MIN-EDIT-DISTANCE(source, target) returns min-distance

```
n \leftarrow \text{LENGTH}(source)
m \leftarrow \text{LENGTH}(target)
Create a distance matrix D[n+1,m+1]
# Initialization: the zeroth row and column is the distance from the empty string
D[0,0] = 0
for each row i from 1 to n do
   D[i,0] \leftarrow D[i-1,0] + del-cost(source[i])
for each column j from 1 to m do
   D[0,j] \leftarrow D[0,j-1] + ins-cost(target[j])
# Recurrence relation:
for each row i from 1 to n do
     for each column j from 1 to m do
        D[i,j] \leftarrow MIN(D[i-1,j] + del\text{-}cost(source[i]),
                         D[i-1,j-1] + sub-cost(source[i], target[j]),
                         D[i, j-1] + ins-cost(target[j])
# Termination
return D[n,m]
```

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# Recurrence relation:
for each row i from 1 to n do
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        D[i,j] \leftarrow MIN(D[i-1,j] + del\text{-}cost(source[i]),
                         D[i-1,j-1] + sub-cost(source[i], target[j]),
                         D[i, j-1] + ins-cost(target[j])
# Termination
return D[n,m]
```

### Distance Matrix (m+1 x n+1): Setup

	m	???	???	???	???	???	???	???	???	???
ers)	m-1	???	???	???	???	???	???	???	???	???
characters)	<b>m</b> -2	???	???	???	???	???	???	???	???	???
	m-3	???	???	???	???	???	???	???	???	???
B (m		???	???	???	???	???	???	???	???	???
string		???	???	???	???	???	???	???	???	???
rce s	3	???	???	???	???	???	???	???	???	???
source	2	???	???	???	???	???	???	???	???	???
	1	???	???	???	???	???	???	???	???	???
#	0	1	2	3			n-3	n-2	n-1	n
	#				target str	ing ( <mark>n</mark> ch	aracters			

# - empty string

### Distance Matrix: Levenshtein Distance

source	2	???	???	???	???	???	???	???	???	???
se string	3	???	???	???	???	???	???	???	???	???
ing (m		???	???	???	???	???	???	???	???	???
	m-3	???	???	???	???	???	???	???	???	???
characters)	m-2	???	???	???	???	???	???	???	???	???
ers)	m-1	???	???	???	???	???	???	???	???	???
	m	???	???	???	???	???	???	???	???	???

$$\begin{aligned} \textit{distance}[i,j] &= min \begin{cases} \textit{distance}[i-1,j] + insertionCost(target_{i-1}) \\ \textit{distance}[i-1,j-1] + substitutionCost(source_{j-1}, target_{i-1}) \\ \textit{distance}[i,j-1] + deletionCost(source_{j-1}) \\ \end{aligned}$$

### Distance Matrix: Levenshtein Distance

	m	???	???	???	???	???	???	???	???	???	
ers)	m-1	???	???	???	???	???	???	???	???	???	
characters)	m-2	???	???	???	???	???	???	???	???	???	
	m-3	???	???	???	???	???	???	???	???	???	
B (m		???	???	???	???	???	???	???	???	???	
string		???	???	???	???	???	???	???	???	???	
rce s	3	???	???	???	???	???	???	???	???	???	
source	2	???	???	???	???	???	???	???	???	???	
	1	???	???	???	???	???	???	???	???	???	
#	0	1	2	3			n-3	n-2	n-1	n	
	#	target string (n characters)									

```
\begin{aligned} \textit{distance}[\textit{col}, \textit{row}] &= min \begin{cases} \textit{distance}[\textit{col} - 1, \textit{row}] + insertion\textit{Cost}(target_{col-1}) \\ \textit{distance}[\textit{col}, \textit{row} - 1] + substitution\textit{Cost}(source_{row-1}, target_{col-1}) \\ \textit{distance}[\textit{col}, \textit{row} - 1] + \textit{deletion}\textit{Cost}(source_{row-1}) \end{aligned}
```

#### Distance Matrix: Levenshtein Distance

	m	???	???	???	???	???	???	???	???	???	
ers)	m-1	???	???	???	???	???	???	???	???	???	
characters)	<b>m</b> -2	???	???	???	???	???	???	???	???	???	
	m-3	???	???	???	???	???	???	???	???	???	
<b>E</b> ( <b>m</b>		???	???	???	???	???	???	???	???	???	
string		???	???	???	???	???	???	???	???	???	
rce s	3	???	???	???	???	???	???	???	???	???	
source	2	???	???	???	???	???	???	???	???	???	
	1	???	???	???	???	???	???	???	???	???	
#	0	1	2	3			n-3	n-2	n-1	n	
	#	target string (n characters)									

$$\begin{aligned} \textit{distance}[i,j] &= min \begin{cases} \textit{distance}[i-1,j] + 1 \\ \textit{distance}[i-1,j-1] + 2 \end{cases} \\ \textit{distance}[i,j-1] + 1 \end{aligned} \text{ 2 if different characters } \\ \textit{0 if same characters} \end{aligned}$$

### **Edit Distance Matrix: Calculations**

	m	???	???	???	???	???	???	???	???	???		
ers)	m-1	???	???	???	???	???	???	???	???	???		
characters)	m-2	???	???	???	???	???	???	???	???	???		
	m-3	???	???	???	???	???	???	???	???	???		
<b>B</b> ( <b>B</b>		???	???	???	???	???	???	???	???	???		
string		???	???	???	???	???	???	???	???	???		
rce s	3	???	???	???	???	???	???	???	???	???		
source	2	???	???	???	???	???	???	???	???	???		
	1	???	???	???	???	???	???	???	???	???		
#	0	1	2	3			n-3	n-2	n-1	n		
	#	target string (n characters)										

 $\square \uparrow \square$  - insertion

□ 7 □ - substitution

☐ ↑ ☐ - deletion

### Minimum Edit Distance: Pseudocode

function MIN-EDIT-DISTANCE(source, target) returns min-distance

```
n \leftarrow \text{LENGTH}(source)
m \leftarrow \text{LENGTH}(target)
Create a distance matrix D[n+1,m+1]
# Initialization: the zeroth row and column is the distance from the empty string
D[0,0] = 0
for each row i from 1 to n do
   D[i,0] \leftarrow D[i-1,0] + del-cost(source[i])
for each column j from 1 to m do
   D[0,j] \leftarrow D[0,j-1] + ins-cost(target[j])
# Recurrence relation:
for each row i from 1 to n do
     for each column j from 1 to m do
        D[i,j] \leftarrow MIN(D[i-1,j] + del\text{-}cost(source[i]),
                         D[i-1,j-1] + sub-cost(source[i], target[j]),
                         D[i, j-1] + ins-cost(target[j])
# Termination
return D[n,m]
```

### **Edit Distance Matrix: Initialization 1**

n	???	???	???	???	???	???	???	???	???	???
O	???	???	???	???	???	???	???	???	???	???
i	???	???	???	???	???	???	???	???	???	???
t	???	???	???	???	???	???	???	???	???	???
n	???	???	???	???	???	???	???	???	???	???
е	???	???	???	???	???	???	???	???	???	???
t	???	???	???	???	???	???	???	???	???	???
n	???	???	???	???	???	???	???	???	???	???
i	???	???	???	???	???	???	???	???	???	???
#	0	???	???	???	???	???	???	???	???	???
	#	е	X	е	С	u	t	i	0	n

### Minimum Edit Distance: Pseudocode

function MIN-EDIT-DISTANCE(source, target) returns min-distance

```
n \leftarrow \text{LENGTH}(source)
m \leftarrow \text{LENGTH}(target)
Create a distance matrix D[n+1,m+1]
# Initialization: the zeroth row and column is the distance from the empty string
D[0,0] = 0
for each row i from 1 to n do
   D[i,0] \leftarrow D[i-1,0] + del\text{-}cost(source[i])
for each column j from 1 to m do
   D[0,j] \leftarrow D[0,j-1] + ins-cost(target[j])
# Recurrence relation:
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        D[i,j] \leftarrow MIN(D[i-1,j] + del\text{-}cost(source[i]),
                         D[i-1,j-1] + sub-cost(source[i], target[j]),
                         D[i,j-1] + ins-cost(target[j])
# Termination
return D[n,m]
```

### **Edit Distance Matrix: Initialization 2**

n	9	???	???	???	???	???	???	???	???	???
0	8	???	???	???	???	???	???	???	???	???
i	7	???	???	???	???	???	???	???	???	???
t	6	???	???	???	???	???	???	???	???	???
n	5	???	???	???	???	???	???	???	???	???
е	4	???	???	???	???	???	???	???	???	???
t	3	???	???	???	???	???	???	???	???	???
n	2	???	???	???	???	???	???	???	???	???
i	1	???	???	???	???	???	???	???	???	???
#	0	???	???	???	???	???	???	???	???	???
	#	е	X	е	С	u	t	i	0	n

### Minimum Edit Distance: Pseudocode

function MIN-EDIT-DISTANCE(source, target) returns min-distance

```
n \leftarrow \text{LENGTH}(source)
m \leftarrow \text{LENGTH}(target)
Create a distance matrix D[n+1,m+1]
# Initialization: the zeroth row and column is the distance from the empty string
D[0,0] = 0
for each row i from 1 to n do
   D[i,0] \leftarrow D[i-1,0] + del-cost(source[i])
for each column j from 1 to m do
   D[0,j] \leftarrow D[0,j-1] + ins-cost(target[j])
# Recurrence relation:
for each row i from 1 to n do
     for each column j from 1 to m do
        D[i,j] \leftarrow MIN(D[i-1,j] + del\text{-}cost(source[i]),
                         D[i-1,j-1] + sub-cost(source[i], target[j]),
                         D[i,j-1] + ins-cost(target[j])
# Termination
return D[n,m]
```

### **Edit Distance Matrix: Initialization 3**

n	9	???	???	???	???	???	???	???	???	???
0	8	???	???	???	???	???	???	???	???	???
i	7	???	???	???	???	???	???	???	???	???
t	6	???	???	???	???	???	???	???	???	???
n	5	???	???	???	???	???	???	???	???	???
е	4	???	???	???	???	???	???	???	???	???
t	3	???	???	???	???	???	???	???	???	???
n	2	???	???	???	???	???	???	???	???	???
i	1	???	???	???	???	???	???	???	???	???
#	0	1	2	3	4	5	6	7	8	9
	#	е	X	e	С	u	t	i	0	n

### Minimum Edit Distance: Pseudocode

function MIN-EDIT-DISTANCE(source, target) returns min-distance

```
n \leftarrow \text{LENGTH}(source)
m \leftarrow \text{LENGTH}(target)
Create a distance matrix D[n+1,m+1]
# Initialization: the zeroth row and column is the distance from the empty string
D[0,0] = 0
for each row i from 1 to n do
   D[i,0] \leftarrow D[i-1,0] + del-cost(source[i])
for each column j from 1 to m do
   D[0,j] \leftarrow D[0,j-1] + ins-cost(target[j])
# Recurrence relation:
for each row i from 1 to n do
     for each column j from 1 to m do
        D[i,j] \leftarrow MIN(D[i-1,j] + del\text{-}cost(source[i]),
                         D[i-1,j-1] + sub\text{-}cost(source[i], target[j]),
                         D[i,j-1] + ins-cost(target[j])
# Termination
return D[n,m]
```

### **Edit Distance Matrix: Populate**

n	9	???	???	???	???	???	???	???	???	???
0	8	???	???	???	???	???	???	???	???	???
i	7	???	???	???	???	???	???	???	???	???
t	6	???	???	???	???	???	???	???	???	???
n	5	???	???	???	???	???	???	???	???	???
е	4	???	???	???	???	???	???	???	???	???
t	3	???	???	???	???	???	???	???	???	???
n	2	???	???	???	???	???	???	???	???	???
i	1	0 + <mark>2</mark> = 2	???	???	???	???	???	???	???	???
#	0	1	2	3	4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

$$\begin{aligned} \textit{distance}[i,j] &= min \begin{cases} \textit{distance}[i-1,j] + insertionCost(target_{i-1}) \\ \textit{distance}[i-1,j-1] + substitutionCost(source_{j-1}, target_{i-1}) \\ \textit{distance}[i,j-1] + deletionCost(source_{j-1}) \\ \end{aligned}$$

### **Edit Distance Matrix: Populate**

n	9	8	9	10	11	12	11	10	9	8
0	8	7	8	9	10	11	10	9	8	9
i	7	6	7	8	9	10	9	8	9	10
t	6	5	6	7	8	9	8	9	10	11
n	5	4	5	6	7	8	9	10	11	10
е	4	3	4	5	6	7	8	9	10	9
t	3	4	5	6	7	8	7	8	9	8
n	2	3	4	5	6	7	8	7	8	7
i	1	2	3	4	5	6	7	6	7	8
#	0	1	2	3	4	5	6	7	8	9
	#	е	X	е	C	u	t	i	0	n

$$\begin{aligned} \textit{distance}[i,j] &= min \begin{cases} \textit{distance}[i-1,j] + insertionCost(target_{i-1}) \\ \textit{distance}[i-1,j-1] + substitutionCost(source_{j-1}, target_{i-1}) \\ \textit{distance}[i,j-1] + deletionCost(source_{j-1}) \\ \end{aligned}$$

### Minimum Edit Path with Backtrace

n	9	<b>√</b> 8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	∠ <b>←</b> ↓11	<b>∠</b> ←↓12	<b>↓</b> 11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8
0	8	<b>↓</b> 7	<b>∠</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9
i	7	<b>√</b> 6	<b>∠</b> ←↓7	<b>∠</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9	←10
t	6	<b>√</b> 5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>⊬</b> ←↓9	<b>∠</b> 8	<b>←</b> 9	←10	<b>←</b> ↓11
n	5	<b>↓</b> 4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ↓10
е	4	<b>∠</b> 3	<b>←</b> 4	<b>∠</b> ←5	<b>←</b> 6	<b>←</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9
t	3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>Ľ</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>√</b> 8
n	2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>↓</b> 7	<b>⊬</b> ←√8	<b>Ľ</b> 7
i	1	<b>∠</b> ←↓2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> 6	<b>←</b> 7	<b>←</b> 8
#	0	1	2	3	4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

Idea: while populating, add "pointers" ( $\psi \leftarrow \lor$ ) to indicate which cell did we come from. Use pointers to "backtrace" by following the minimum edit path.

### Minimum Edit Path with Backtrace

n	9	<b>√</b> 8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ←↓12	<b>↓</b> 11	<b>↓</b> 10	<b>↓</b> 9	<b>∠</b> 8
O	8	<b>↓</b> 7	<b>∠</b> ←↓8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9
i	7	<b>√</b> 6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9	←10
t	6	<b>√</b> 5	<b>∠</b> ←↓6	<b>⊬</b> ←↓7	<b>∠</b> ←↓8	<b>⊬</b> ←↓9	<b>∠</b> 8	<b>←</b> 9	←10	<b>←</b> ↓11
n	5	<b>↓</b> 4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ↓10
е	4	<b>∠</b> 3	<b>←</b> 4	<b>∠</b> ←5	<b>←</b> 6	<b>←</b> 7	<b>←</b> ↓8	<b>⊬</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9
t	3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>L</b> 7	<b>←</b> ↓8	<b>⊬</b> ←↓9	<b>√</b> 8
n	2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>⊬</b> ←√7	<b>⊬</b> ←↓8	<b>√</b> 7	<b>⊬</b> ←√8	<b>Ľ</b> 7
i	1	<b>∠</b> ←↓2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←√6	<b>∠</b> ←↓7	<b>∠</b> 6	<b>←</b> 7	<b>←</b> 8
#	0	1	2	3	4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

 $\psi\leftarrow \ \lor$  - which cell did we come from?

red - minimum edit cost

n	9	<b>√</b> 8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ←↓12	<b>↓</b> 11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8
O	8	<b>↓</b> 7	<b>∠</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9
i	7	<b>√</b> 6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9	←10
t	6	<b>√</b> 5	<b>∠</b> ←↓6	<b>⊬</b> ←↓7	<b>∠</b> ←↓8	<b>⊬</b> ←↓9	<b>∠</b> 8	<b>←</b> 9	←10	<b>←</b> ↓11
n	5	<b>↓</b> 4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ↓10
е	4	<b>∠</b> 3	<b>←</b> 4	<b>∠</b> ←5	<b>←</b> 6	<b>←</b> 7	<b>←</b> ↓8	<b>⊬</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9
t	3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>L</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>√</b> 8
n	2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>⊬</b> ←√7	<b>⊬</b> ←↓8	<b>√</b> 7	<b>∠</b> ←√8	<b>Ľ</b> 7
i	1	<b>∠</b> ←↓2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←√6	<b>∠</b> ←↓7	<b>∠</b> 6	<b>←</b> 7	<b>←</b> 8
#	0	1	2	3	4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

 $\psi\leftarrow \ \lor$  - which cell did we come from?

n	9	<b>√</b> 8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ←↓12	<b>↓</b> 11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8
0	8	<b>↓</b> 7	<b>∠</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	∠ <b>←</b> ↓11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9
i	7	<b>√</b> 6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9	←10
t	6	<b>√</b> 5	<b>∠</b> ←↓6	<b>⊬</b> ←↓7	<b>⊬</b> ←√8	<b>⊬</b> ←↓9	<b>∠</b> 8	<b>←</b> 9	←10	<b>←</b> ↓11
n	5	<b>↓</b> 4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ↓10
е	4	<b>∠</b> 3	<b>←</b> 4	<b>∠</b> ←5	<b>←</b> 6	<b>←</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9
t	3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>L</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>√</b> 8
n	2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>⊬</b> ←√7	<b>⊬</b> ←↓8	<b>√</b> 7	<b>∠</b> ←√8	<b>Ľ</b> 7
i	1	<b>∠</b> ←↓2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←√6	<b>∠</b> ←↓7	<b>∠</b> 6	<b>←</b> 7	<b>←</b> 8
#	0	1	2	3	4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

 $\psi\leftarrow \ \lor$  - which cell did we come from?

n	9	<b>√</b> 8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ←↓12	<b>↓</b> 11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8
O	8	<b>↓</b> 7	<b>∠</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9
i	7	<b>√</b> 6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9	←10
t	6	<b>√</b> 5	<b>∠</b> ←↓6	<b>⊬</b> ←↓7	<b>∠</b> ←↓8	<b>⊬</b> ←↓9	<b>∠</b> 8	<b>←</b> 9	←10	<b>←</b> ↓11
n	5	<b>↓</b> 4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>⊬</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ↓10
е	4	<b>∠</b> 3	<b>←</b> 4	<b>∠</b> ←5	<b>←</b> 6	<b>←</b> 7	<b>←</b> ↓8	<b>⊬</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9
t	3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>L</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>√</b> 8
n	2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>⊬</b> ←√7	<b>⊬</b> ←↓8	<b>√</b> 7	<b>∠</b> ←√8	<b>Ľ</b> 7
i	1	<b>∠</b> ←↓2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←√6	<b>∠</b> ←↓7	<b>∠</b> 6	<b>←</b> 7	<b>←</b> 8
#	0	1	2	3	4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

 $\psi\leftarrow \ensuremath{\,\subset}$  - which cell did we come from?

n	9	<b>√</b> 8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ←↓12	<b>↓</b> 11	<b>↓</b> 10	<b>↓</b> 9	<b>∠</b> 8
O	8	<b>√</b> 7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>V</b> 10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9
i	7	<b>√</b> 6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9	←10
t	6	<b>√</b> 5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>∠</b> ←↓9	<b>∠</b> 8	<b>←</b> 9	←10	<b>←</b> ↓11
n	5	<b>↓</b> 4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ↓10
е	4	<b>∠</b> 3	<b>←</b> 4	<b>∠</b> ←5	<b>←</b> 6	<b>←</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9
t	3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>Ľ</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>√</b> 8
n	2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>√</b> 7	<b>∠</b> ←↓8	<b>Ľ</b> 7
i	1	<b>∠</b> ←↓2	<b>∠</b> ←√3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> 6	<b>←</b> 7	<b>←</b> 8
#	0	1	1 2		4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

 $\psi\leftarrow \ \lor$  - which cell did we come from?

n	9	<b>√</b> 8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ←↓12	<b>↓</b> 11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8
0	8	<b>↓</b> 7	<b>∠</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	∠ <b>←</b> ↓11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9
i	7	<b>√</b> 6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9	←10
t	6	<b>√</b> 5	<b>∠</b> ←↓6	<b>⊬</b> ←↓7	<b>∠</b> ←↓8	<b>⊬</b> ←↓9	<b>∠</b> 8	<b>←</b> 9	←10	<b>←</b> ↓11
n	5	<b>↓</b> 4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>⊬</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ↓10
е	4	<b>∠</b> 3	<b>←</b> 4	<b>∠</b> ←5	<b>←</b> 6	<b>←</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9
t	3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>Ľ</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>√</b> 8
n	2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>⊬</b> ←√7	<b>⊬</b> ←√8	<b>√</b> 7	<b>∠</b> ←√8	<b>Ľ</b> 7
i	1	<b>∠</b> ←↓2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←√6	<b>∠</b> ←↓7	<b>∠</b> 6	<b>←</b> 7	<b>←</b> 8
#	0	1	2	3	4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

 $\psi\leftarrow \ \lor$  - which cell did we come from?

n	9	<b>√</b> 8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ←↓12	<b>↓</b> 11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8
0	8	<b>↓</b> 7	<b>∠</b> ←↓8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9
i	7	<b>√</b> 6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9	←10
t	6	<b>√</b> 5	<b>∠</b> ←↓6	<b>⊬</b> ←↓7	<b>∠</b> ←↓8	<b>⊬</b> ←↓9	<b>∠</b> 8	<b>←</b> 9	←10	<b>←</b> ↓11
n	5	<b>↓</b> 4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>⊬</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ↓10
е	4	<b>∠</b> 3	<b>←</b> 4	<b>∠</b> ←5	<b>←</b> 6	<b>←</b> 7	<b>←</b> ↓8	<b>⊬</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9
t	3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>L</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>√</b> 8
n	2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>⊬</b> ←√7	<b>⊬</b> ←↓8	<b>√</b> 7	<b>∠</b> ←√8	<b>Ľ</b> 7
i	1	<b>∠</b> ←↓2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←√6	<b>∠</b> ←↓7	<b>∠</b> 6	<b>←</b> 7	<b>←</b> 8
#	0	1	1 2		4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

 $\psi\leftarrow \ensuremath{\,\subset}$  - which cell did we come from?

n	9	<b>√</b> 8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ←↓12	<b>↓</b> 11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8
0	8	<b>√</b> 7	<b>∠</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9
i	7	<b>√</b> 6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9	←10
t	6	<b>√</b> 5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>⊬</b> ←↓9	<b>∠</b> 8	<b>←</b> 9	←10	<b>←</b> ↓11
n	5	<b>↓</b> 4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ↓10
е	4	<b>∠</b> 3	<b>←</b> 4	<b>∠</b> ←5	<b>←</b> 6	<b>←</b> 7	<b>←</b> ↓8	<b>⊬</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9
t	3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>L</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>√</b> 8
n	2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>↓</b> 7	<b>⊬</b> ←↓8	<b>Ľ</b> 7
i	1	<b>∠</b> ←↓2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> 6	<b>←</b> 7	<b>←</b> 8
#	0	1	2	3	4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

 $\psi\leftarrow \ensuremath{\,\subset}$  - which cell did we come from?

n	9	<b>√</b> 8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	∠ <b>←</b> ↓11	<b>∠</b> ←↓12	<b>↓</b> 11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8
0	8	<b>↓</b> 7	<b>∠</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9
i	7	<b>√</b> 6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9	←10
t	6	<b>√</b> 5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>⊬</b> ←↓9	<b>∠</b> 8	<b>←</b> 9	←10	<b>←</b> ↓11
n	5	<b>↓</b> 4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>⊬</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ↓10
е	4	<b>∠</b> 3	<b>←</b> 4	<b>∠</b> ←5	<b>←</b> 6	<b>←</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9
t	3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>Ľ</b> 7	<b>←</b> ↓8	<b>⊬</b> ←↓9	<b>√</b> 8
n	2	<b>∠</b> ←√3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> +√8	<b>√</b> 7	<b>⊬</b> ←√8	<b>Ľ</b> 7
i	1	<b>∠</b> ←↓2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> 6	<b>←</b> 7	<b>←</b> 8
#	0	1	1 2		4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

 $\psi\leftarrow \ensuremath{\,\subset}$  - which cell did we come from?

n	9	<b>√</b> 8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ←↓12	<b>↓</b> 11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8
O	8	<b>√</b> 7	<b>∠</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9
i	7	<b>√</b> 6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9	←10
t	6	<b>√</b> 5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>∠</b> ←↓9	<b>∠</b> 8	<b>←</b> 9	←10	<b>←</b> ↓11
n	5	<b>↓</b> 4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ↓10
е	4	<b>∠</b> 3	<b>←</b> 4	<b>∠</b> ←5	<b>←</b> 6	<b>←</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9
t	3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>Ľ</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>√</b> 8
n	2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>√</b> 7	<b>∠</b> ←↓8	<b>Ľ</b> 7
i	1	<b>∠</b> ←↓2	<b>∠</b> ←√3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> 6	<b>←</b> 7	<b>←</b> 8
#	0	1	1 2		4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

 $\psi\leftarrow \ \lor$  - which cell did we come from?

n	9	<b>√</b> 8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ←↓12	<b>↓</b> 11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8
0	8	<b>↓</b> 7	<b>∠</b> ←↓8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	∠ <b>←</b> ↓11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9
i	7	<b>√</b> 6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9	←10
t	6	<b>√</b> 5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>⊬</b> ←↓9	<b>∠</b> 8	<b>←</b> 9	←10	<b>←</b> ↓11
n	5	<b>√</b> 4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>⊬</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ↓10
е	4	<b>∠</b> 3	<b>←</b> 4	<b>∠</b> ←5	<b>←</b> 6	<b>←</b> 7	<b>←</b> ↓8	<b>⊬</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9
t	3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>L</b> 7	<b>←</b> ↓8	<b>⊬</b> ←↓9	<b>√</b> 8
n	2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>↓</b> 7	<b>⊬</b> ←√8	<b>Ľ</b> 7
i	1	<b>∠</b> ←↓2	<b>⊬</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> 6	<b>←</b> 7	<b>←</b> 8
#	0	1	2	3	4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

 $\psi\leftarrow \ensuremath{\,\subset}$  - which cell did we come from?

n	9	<b>√</b> 8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ←↓12	<b>↓</b> 11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8
0	8	<b>↓</b> 7	<b>∠</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	∠ <b>←</b> ↓11	<b>↓</b> 10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9
i	7	<b>√</b> 6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9	<b>∠</b> 8	<b>←</b> 9	←10
t	6	<b>√</b> 5	<b>∠</b> ←↓6	<b>⊬</b> ←↓7	<b>∠</b> ←↓8	<b>⊬</b> ←↓9	<b>∠</b> 8	<b>←</b> 9	←10	<b>←</b> ↓11
n	5	<b>↓</b> 4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>∠</b> ←↓8	<b>⊬</b> ←↓9	<b>∠</b> ←↓10	<b>∠</b> ←↓11	<b>∠</b> ↓10
е	4	<b>∠</b> 3	<b>←</b> 4	<b>∠</b> ←5	<del>←</del> 6	<b>←</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>∠</b> ←↓10	<b>√</b> 9
t	3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←↓6	<b>∠</b> ←↓7	<b>⊬</b> ←√8	<b>L</b> 7	<b>←</b> ↓8	<b>∠</b> ←↓9	<b>√</b> 8
n	2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>∠</b> ←↓6	<b>⊬</b> ←√7	<b>⊬</b> ←↓8	<b>√</b> 7	<b>∠</b> ←√8	<b>Ľ</b> 7
i	1	<b>∠</b> ←↓2	<b>∠</b> ←↓3	<b>∠</b> ←↓4	<b>∠</b> ←↓5	<b>⊬</b> ←√6	<b>∠</b> ←↓7	<b>∠</b> 6	<b>←</b> 7	<b>←</b> 8
#	0	1	2	3	4	5	6	7	8	9
	#	е	X	е	С	u	t	i	0	n

Final minimum edit path.

## **Time and Space Complexity**

Time:

$$O(n * m)$$

Space:

Backtrace time complexity:

$$O(n + m)$$

### Weighted Edit Distance

- Why would we add weights to the computation?
- Spell Correction:
  - some letters are more likely to be mistyped than others
- Biology:
  - certain kinds of deletions or insertions are more likely than others

# Weighted Edit Distance

### **Confusion matrix for spelling errors:**

	sub[X, Y] = Substitution of X (incorrect) for Y (correct)																									
X								_				Y	(co	rrect)	)			•								
	a	b	С	d	e	f	g	h	i	j	k	1	m	n	0	p	q	r	S	t	u	v	w	х	У	Z
a	0	0	7	1	342	0	0	2	118	0	1	0	0	3	76	0	0	1	35	9	9	0	1	0	5	0
b	0	0	9	9	2	2	3	1	0	0	0	5	11	5	0	10	0	0	2	1	0	0	8	0	0	0
С	6	5	0	16	0	9	5	0	0	0	1	0	7	9	1	10	2	5	39	40	1	3	7	1	1	0
d	1	10	13	0	12	0	5	5	0	0	2	3	7	3	0	1	0	43	30	22	0	0	4	0	2	0
c	388	0	3	11	0	2	2	0	89	0	0	3	0	5	93	0	0	14	12	6	15	0	1	0	18	0
f	0	15	0	3	1	0	5	2	0	0	0	3	4	1	0	0	0	6	4	12	0	0	2	0	0	0
g	4	1	11	11	9	2	0	0	0	1	1	3	0	0	2	1	3	5	13	21	0	0	1	0	3	0
h	1	8	0	3	0	0	0	0	0	0	2	0	12	14	2	3	0	3	1	11	0	0	2	0	0	0
i	103	0	0	0	146	0	1	0	0	0	0	6	0	0	49	0	0	0	2	1	47	0	2	1	15	0
j	0	1	1	9	0	0	1	0	0	0	0	2	1	0	0	0	0	0	5	0	0	0	0	0	0	0
k	1	2	8	4	1	1	2	5	0	0	0	0	5	0	2	0	0	0	6	0	0	0	. 4	0	0	3
1	2	10	1	4	0	4	5	6	13	0	1	0	0	14	2	5	0	11	10	2	0	0	0	0	0	0
m	1	3	7	8	0	2	0	6	0	0	4	4	0	180	0	6	0	0	9	15	13	3	2	2	3	0
n	2	7	6	5	3	0	1	19	1	0	4	35	78	0	0	7	0	28	5	7	0	0	1	2	0	2
0	91	1	1	3	116	0	0	0	25	0	2	0	0	0	0	14	0	2	4	14	39	0	0	0	18	0
р	0	11	1	2	0	6	5	0	2	9	0	2	7	6	15	0	0	1	3	6	0	4	1	0	0	0
q	0	0	1	0	0	0	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
r	0	14	0	30	12	2	2	8	2	0	5	8	4	20	1	14	0	0	12	22	4	0	0	1	0	0
s	11	8	27	33	35	4	0	1	0	1	0	27	0	6	1	7	0	14	0	15	0	0	5	3	20	1
t	3	4	9	42	7	5	19	5	0	1	0	14	9	5	5	6	0	11	37	0	0	2	19	0	7	6
u	20	0	0	0	44	0	0	0	64	0	0	0	0	2	43	0	0	4	0	0	0	0	2	0	8	0
v	0	0	7	0	0	3	0	0	0	0	0	1	0	0	1	0	0	0	8	3	0	0	0	0	0	0
w	2	2	1	0	1	0	0	2	0	0	1	0	0	0	0	7	0	6	3	3	1	0	0	0	0	0
х	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0
У	0	0	2	0	15	0	1	7	15	0	0	0	2	0	6	1	0	7	36	8	5	0	0	1	0	0
7.	0	0	0	7	0	0	0	0	0	0	0	7	5	0	0	0	0	2	21	3	0	0	0	0	3	0

### **Parts of Speech**

- Idea:
  - classify words according to their grammatical categories
- Categories = part of speech, word classes, POS,POS tags
- Basic categories / tags:
  - noun, verb, pronoun, preposition, adverb, conjunction, participle, article

### Parts of Speech: Closed vs. Open

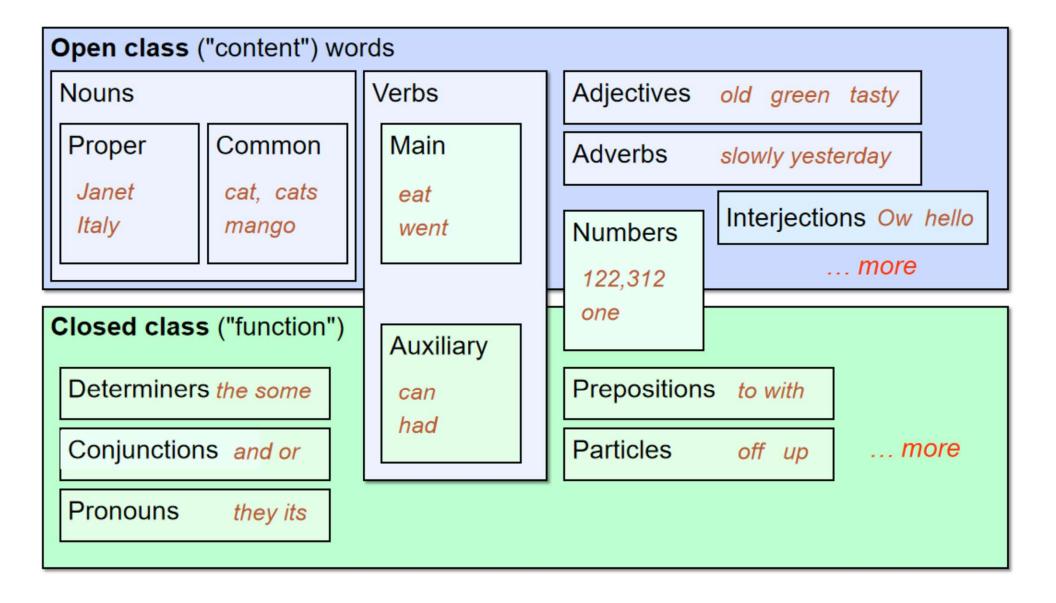
#### Closed class:

- relatively fixed set new members rarely added
- usually function words: short, frequent words with grammatical function:
  - **determiners:** *a, an, the*
  - pronouns: she, he, l
  - prepositions: on, under, over, near, by

#### Open class:

- word sets where new members are constantly created
- usually content words: nouns, verbs, adjectives, adverbs
- new words | examples: nouns (iPhone), verbs (to google)

## Parts of Speech: Closed vs. Open



## Parts of Speech Tagging

- Assigning a part-of-speech (POS) to each word in a text.
- Words often have more than one POS.
  - example: book
    - VERB: Book that flight
    - NOUN: Hand me that book

## Sample Tagged Sentence

There/PRO were/VERB 70/NUM children/NOUN there/ADV ./PUNC

Preliminary/ADJ findings/NOUN were/AUX reported/VERB in/ADP today/NOUN 's/PART New/PROPN England/PROPN Journal/PROPN of/ADP Medicine/PROPN

## Parts of Speech: Tagset Example

### Parts of Speech in the Universal Dependencies tagset

	Tag	Description	Example		
	ADJ	Adjective: noun modifiers describing properties	red, young, awesome		
Class	ADV	Adverb: verb modifiers of time, place, manner	very, slowly, home, yesterday		
	NOUN	words for persons, places, things, etc.	algorithm, cat, mango, beauty		
Open	VERB	words for actions and processes	draw, provide, go		
O	PROPN	Proper noun: name of a person, organization, place, etc	Regina, IBM, Colorado		
	INTJ	Interjection: exclamation, greeting, yes/no response, etc.	oh, um, yes, hello		
	ADP	Adposition (Preposition/Postposition): marks a noun's	in, on, by, under		
S		spacial, temporal, or other relation			
Closed Class Words	AUX	Auxiliary: helping verb marking tense, aspect, mood, etc.,	can, may, should, are		
≥	CCONJ	Coordinating Conjunction: joins two phrases/clauses	and, or, but		
ass	DET	Determiner: marks noun phrase properties	a, an, the, this		
$\Box$	NUM	Numeral	one, two, first, second		
seq	PART	Particle: a preposition-like form used together with a verb	up, down, on, off, in, out, at, by		
CP CP	PRON	Pronoun: a shorthand for referring to an entity or event	she, who, I, others		
	<b>SCONJ</b>	Subordinating Conjunction: joins a main clause with a	that, which		
		subordinate clause such as a sentential complement			
et	PUNCT	Punctuation	; , ()		
Other	SYM	Symbols like \$ or emoji	\$, %		
	X	Other	asdf, qwfg		

## Parts of Speech: Tagset Example

### Penn Treebank Parts-of-speech tags:

Tag Description	Example	Tag	Description	Example	Tag	Description	Example
CC coord. conj.	and, but, or	NNP	proper noun, sing.	IBM	TO	"to"	to
CD cardinal number	one, two	NNPS	proper noun, plu.	Carolinas	UH	interjection	ah, oops
DT determiner	a, the	NNS	noun, plural	llamas	VB	verb base	eat
EX existential 'there'	there	PDT	predeterminer	all, both	VBD	verb past tense	ate
FW foreign word	mea culpa	POS	possessive ending	's	VBG	verb gerund	eating
IN preposition/	of, in, by	PRP	personal pronoun	I, you, he	VBN	verb past partici-	eaten
subordin-conj						ple	
JJ adjective	yellow	PRP\$	possess. pronoun	your, one's	VBP	verb non-3sg-pr	eat
JJR comparative adj	bigger	RB	adverb	quickly	VBZ	verb 3sg pres	eats
JJS superlative adj	wildest	RBR	comparative adv	faster	WDT	wh-determ.	which, that
LS list item marker	1, 2, One	RBS	superlatv. adv	fastest	WP	wh-pronoun	what, who
MD modal	can, should	RP	particle	up, off	WP\$	wh-possess.	whose
NN sing or mass noun	llama	SYM	symbol	+,%, &	WRB	wh-adverb	how, where

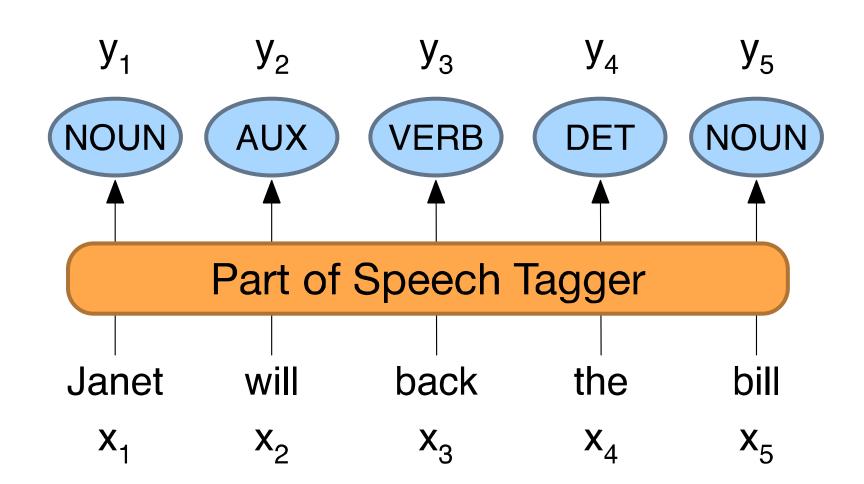
## Parts of Speech Tagging: Motivation

- Can be useful for other NLP tasks
  - Parsing: POS tagging can improve syntactic parsing
  - MT: reordering of adjectives and nouns (say from Spanish to English)
  - Sentiment or affective tasks: may want to distinguish adjectives or other POS
  - Text-to-speech (how do we pronounce "lead" or "object"?)
- Or linguistic or language-analytic computational tasks
  - Need to control for POS when studying linguistic change like creation of new words, or meaning shift
  - Or control for POS in measuring meaning similarity or difference

## Part of Speech Tagging

#### Task:

■ Map sequence  $x_1,...,x_n$  of words to  $y_1,...,y_n$  of POS tags



## Parts of Speech: Tagset Example

### Parts of Speech in the Universal Dependencies tagset

	Tag	Description	Example		
	ADJ	Adjective: noun modifiers describing properties	red, young, awesome		
Open Class	ADV	Adverb: verb modifiers of time, place, manner	very, slowly, home, yesterday		
	NOUN	words for persons, places, things, etc.	algorithm, cat, mango, beauty		
ben	VERB	words for actions and processes	draw, provide, go		
O	<b>PROPN</b>	Proper noun: name of a person, organization, place, etc	Regina, IBM, Colorado		
	INTJ	Interjection: exclamation, greeting, yes/no response, etc.	oh, um, yes, hello		
	ADP	Adposition (Preposition/Postposition): marks a noun's	in, on, by, under		
S		spacial, temporal, or other relation			
ord	AUX	Auxiliary: helping verb marking tense, aspect, mood, etc.,	can, may, should, are		
≥	CCONJ	Coordinating Conjunction: joins two phrases/clauses	and, or, but		
Closed Class Words	DET	Determiner: marks noun phrase properties	a, an, the, this		
D C	NUM	Numeral	one, two, first, second up, down, on, off, in, out, at, by		
seq	PART	Particle: a preposition-like form used together with a verb			
Clo	PRON	Pronoun: a shorthand for referring to an entity or event	she, who, I, others		
	<b>SCONJ</b>	Subordinating Conjunction: joins a main clause with a	that, which		
		subordinate clause such as a sentential complement			
ı	PUNCT	Punctuation	;,()		
Other	SYM	Symbols like \$ or emoji	\$, %		
	X	Other	asdf, qwfg		

## Parts of Speech: Tagset Example

### Penn Treebank Parts-of-speech tags:

Tag Description	Example	Tag	Description	Example	Tag	Description	Example
CC coord. conj.	and, but, or	NNP	proper noun, sing.	IBM	TO	"to"	to
CD cardinal number	one, two	NNPS	proper noun, plu.	Carolinas	UH	interjection	ah, oops
DT determiner	a, the	NNS	noun, plural	llamas	VB	verb base	eat
EX existential 'there'	there	PDT	predeterminer	all, both	VBD	verb past tense	ate
FW foreign word	mea culpa	POS	possessive ending	's	VBG	verb gerund	eating
IN preposition/	of, in, by	PRP	personal pronoun	I, you, he	VBN	verb past partici-	eaten
subordin-conj						ple	
JJ adjective	yellow	PRP\$	possess. pronoun	your, one's	VBP	verb non-3sg-pr	eat
JJR comparative adj	bigger	RB	adverb	quickly	VBZ	verb 3sg pres	eats
JJS superlative adj	wildest	RBR	comparative adv	faster	WDT	wh-determ.	which, that
LS list item marker	1, 2, One	RBS	superlatv. adv	fastest	WP	wh-pronoun	what, who
MD modal	can, should	RP	particle	up, off	WP\$	wh-possess.	whose
NN sing or mass noun	llama	SYM	symbol	+,%, &	WRB	wh-adverb	how, where

## Sample Tagged Sentence

There/PRO were/VERB 70/NUM children/NOUN there/ADV ./PUNC

Preliminary/ADJ findings/NOUN were/AUX reported/VERB in/ADP today/NOUN 's/PART New/PROPN England/PROPN Journal/PROPN of/ADP Medicine/PROPN

## **POS Tagging: Challenges**

- Roughly 15% of word types are ambiguous
  - Hence 85% of word types are unambiguous
  - Janet is always PROPN, hesitantly is always ADV
- But those 15% tend to be very common.
- Effectively around ~60% of word tokens are ambiguous
  - For example: back
    - earnings growth took a back/ADJ seat
    - a small building in the back/NOUN
    - a clear majority of senators back/VERB the bill
    - enable the country to buy back/PART debt
    - I was twenty-one back/ADV then

## **POS Tagging: Challenges**

- How many tags are correct? (Tag accuracy)
  - about 97%
  - different taggers perform similarly, human accuracy about the same
- But baseline is 92%!
- Baseline is performance of stupidest possible method
  - "Most frequent class baseline" is an important baseline for many tasks
    - tag every word with its most frequent tag
    - (and tag unknown words as nouns)
  - Partly easy because many words are unambiguous

# Sources of Tagging Information

### Janet will back the bill

**AUX/NOUN/VERB?** 

**NOUN/VERB?** 

- Prior probabilities of word/tag
  - "will" is usually an AUX
- Identity of neighboring words
  - "the" means the next word is probably not a verb
- Morphology and wordshape:

■ Prefixes unable: un- ② ADJ

■ Suffixes importantly: -ly ② ADJ

Capitalization
Janet:
CAP PROPN

## **Standard POS Tagging Models**

- Supervised Machine Learning Algorithms:
  - Hidden Markov Models
  - Conditional Random Fields (CRF) / Maximum Entropy Markov Models (MEMM)
  - Neural sequence models (RNNs or Transformers)
  - Large Language Models (like BERT), finetuned
- All required a hand-labeled training set, all about equal performance (97% on English)
  - All make use of information sources we discussed
  - Via human created features: HMMs and CRFs
  - Via representation learning: Neural LMs

## **Conditional Probability**

$$P(A \mid B) = \frac{P(A \land B)}{P(B)}$$

where P(B) > 0

# Part of Speech: Conditional Probability

$$P(Category = NOUN \mid word = flies) = \frac{P(word = flies, Category = NOUN)}{P(Word = flies)}$$

where 
$$P(Word = flies) > 0$$

# Part of Speech: Conditional Probability

$$P(C = NOUN \mid w = flies) = \frac{P(w = flies, C = NOUN)}{P(w = flies)}$$

where 
$$P(w = flies) > 0$$

## **Most Frequent Class Tagging**

- Basic idea: calculate conditional probabilities for all possible categories and compare.
- For simplicity, let's assume only two lexical categories:
   NOUN and VERB.

$$P(C = NOUN \mid w = flies) = \frac{P(w = flies, C = NOUN)}{P(w = flies)}$$

VS.

$$P(C = VERB \mid w = flies) = \frac{P(w = flies, C = VERB)}{P(w = flies)}$$

## **Most Frequent Class Tagging**

Say we have some corpus with 1 273 000 words.

The word *flies* appears 1000 times in the corpus:

- 400 times as a NOUN
- 600 times as a VERB

#### So:

```
P(w = flies) = 1000/1 \ 273 \ 000 = 0.0008

P(w = flies, C = NOUN) = 400/1 \ 273 \ 000 = 0.0003

P(w = flies, C = VERB) = 600/1 \ 273 \ 000 = 0.0005
```

## **Most Frequent Class Tagging**

#### With:

$$P(w = flies) = 1000/1 273 000 = 0.0008$$
  
 $P(w = flies, C = NOUN) = 400/1 273 000 = 0.0003$   
 $P(w = flies, C = VERB) = 600/1 273 000 = 0.0005$ 

$$P(C = NOUN \mid w = flies) = rac{P(w = flies, C = NOUN)}{P(w = flies)} = rac{0.0003}{0.0008}$$
 $VS.$ 
 $P(C = VERB \mid w = flies) = rac{P(w = flies, C = VERB)}{P(w = flies)} = rac{0.0005}{0.0008}$ 

With this approach flies will ALWAYS be tagged as a VERB.

## Bayes' Rule

$$P(A \mid B) = \frac{P(B \mid A) * P(A)}{P(B)}$$

Given a sequence of words (a "sentence"):

$$W_1, W_2, W_3, ..., W_T$$

there is going to be a corresponding sequence of lexical categories:

$$C_1, C_2, C_3, ..., C_T$$

What is most likely sequence of categories?

To answer this we would want to find a conditional probability:

$$P(C_1, C_2, C_3, ..., C_T | w_1, w_2, w_3, ..., w_T)$$

In other words: what is the probability of having a sequence of lexical categories

$$C_1, C_2, C_3, ..., C_T$$

**GIVEN** that the sequence of words is

$$w_1, w_2, w_3, ..., w_T$$
?

The probability we are looking for

$$P(C_1, C_2, C_3, ..., C_T | w_1, w_2, w_3, ..., w_T)$$

will require a lot of data, which we most likely won't have. We can use Bayes' Theorem:

$$P(C_1, C_2, C_3, ..., C_T \mid w_1, w_2, w_3, ..., w_T) =$$

$$= \frac{P(w_1, w_2, w_3, \dots, w_T \mid C_1, C_2, C_3, \dots, C_T) * P(C_1, C_2, C_3, \dots, C_T)}{P(w_1, w_2, w_3, \dots, w_T)}$$

In order to find the most likely sequence:

$$C_1, C_2, C_3, ..., C_T$$

we need to maximize (most likely sequence!):

$$P(C_1, C_2, C_3, ..., C_T \mid w_1, w_2, w_3, ..., w_T) =$$

$$= \frac{P(w_1, w_2, w_3, \dots, w_T \mid C_1, C_2, C_3, \dots, C_T) * (C_1, C_2, C_3, \dots, C_T)}{P(w_1, w_2, w_3, \dots, w_T)}$$

#### **Maximizing:**

$$P(C_1, C_2, C_3, ..., C_T \mid w_1, w_2, w_3, ..., w_T)$$

in practice means maximizing the numerator:

$$\frac{P(w_1, w_2, w_3, \dots, w_T \mid C_1, C_2, C_3, \dots, C_T) * P(C_1, C_2, C_3, \dots, C_T)}{P(w_1, w_2, w_3, \dots, w_T)}$$

as denominator  $P(w_1, w_2, w_3, \dots, w_T)$  will not change:

#### **Estimating:**

$$P(w_1, w_2, w_3, ..., w_T \mid C_1, C_2, C_3, ..., C_T) * P(C_1, C_2, C_3, ..., C_T)$$

using counts once again requires a lot of data that we will likely not have.

Alternative: approximate it with N-grams (here bigrams):

$$P(C_1, C_2, C_3, ..., C_T) = \prod_{i=1}^T P(C_i \mid all \ categories \ preceding \ C_i)$$

$$P(C_1, C_2, C_3, ..., C_T) \cong \prod_{i=1}^T P(C_i \mid C_{i-})$$

#### **Estimating:**

$$P(w_1, w_2, w_3, ..., w_T \mid C_1, C_2, C_3, ..., C_T) * P(C_1, C_2, C_3, ..., C_T)$$

#### Approximate it with N-grams (here bigrams):

$$P(C_1, C_2, C_3, ..., C_T) = \prod_{i=1}^T P(C_i \mid all \ categories \ preceding \ C_i)$$

$$P(C_1, C_2, C_3, ..., C_T) \cong \prod_{i=1}^T P(C_i \mid C_{i-1})$$

$$P(w_1, w_2, w_3, ..., w_T \mid C_1, C_2, C_3, ..., C_T) \cong \prod_{i=1}^T P(w_i \mid C_i)$$

#### With approximations:

$$P(w_{1}, w_{2}, w_{3}, ..., w_{T} \mid C_{1}, C_{2}, C_{3}, ..., C_{T}) * P(C_{1}, C_{2}, C_{3}, ..., C_{T}) \cong$$

$$\cong \prod_{i=1}^{T} P(w_{i} \mid C_{i}) * P(C_{i} \mid C_{i-1})$$

and we want to maximize:

$$\prod_{i=1}^{T} P(\mathbf{w_i} \mid \mathbf{C_i}) * P(\mathbf{C_i} \mid \mathbf{C_{i-1}})$$

Individual probabilities can now be estimated using corpus counts!

## **POS Tagging: Simple Tagset**

Let's assume we have a simple tagset:

- N NOUN
- V VERB
- ART ARTICLE
- P PREPOSITION

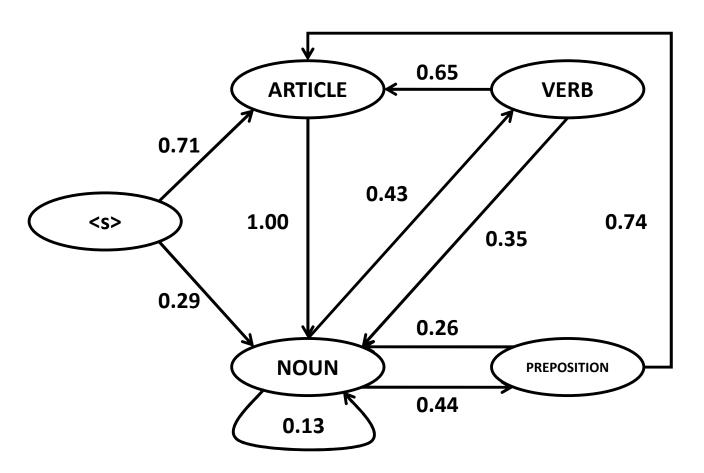
and a some synthetic corpus.

#### **Estimations with corpus counts:**

$$P(C_i = VERB \mid C_{i-1} = NOUN) = \frac{Count (NOUN \text{ at position } i - \text{ and } VERB \text{ at } i)}{Count(NOUN \text{ at position } i-)}$$

#### Sample bigram probabilities from our synthetic corpus:

Category	Count at i	Pair	Count at i,i+1	P(Bigram)	Estimate
<s></s>	300	<s>, ARTICLE</s>	213	P(ARTICLE  <s>)</s>	0.71
<s></s>	300	<s>, NOUN</s>	87	P(NOUN  <s>)</s>	0.29
ARTICLE	558	ARTICLE, NOUN	558	P(NOUN ARTICLE)	1.00
NOUN	833	NOUN, VERB	358	P (VERB   NOUN)	0.43
NOUN	833	NOUN, NOUN	108	P (NOUN   NOUN)	0.13
NOUN	833	NOUN, PREPOSITION	366	P(PREPOSITION NOUN)	0.44
VERB	300	VERB, NOUN	75	P (NOUN   VERB)	0.35
VERB	300	VERB, ARTICLE	194	P(ARTICLE VERB)	0.65
PREPOSITION	307	PREPOSITION, ARTICLE	226	P (ARTICLE   PREPOSITION)	0.74
PREPOSITION	307	PREPOSITION, NOUN	81	P(NOUN PREPOSITION)	0.26



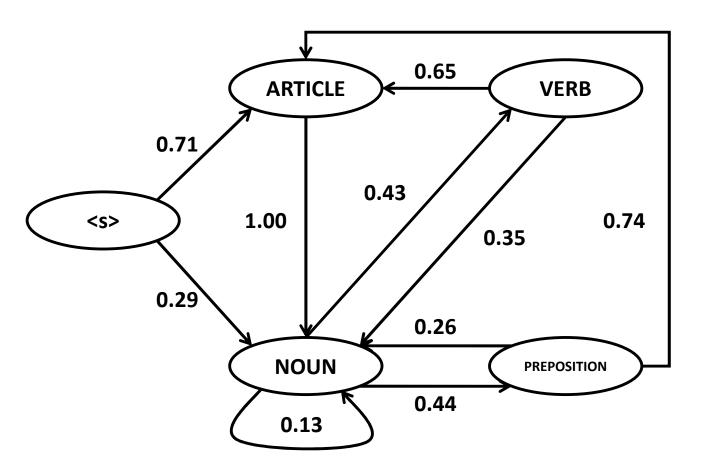
P(Bigram)	Estimate
P(ARTICLE  <s>)</s>	0.71
P(NOUN  <s>)</s>	0.29
P(NOUN ARTICLE)	1.00
P(VERB NOUN)	0.43
P (NOUN   NOUN)	0.13
P (PREPOSITION   NOUN)	0.44
P(NOUN VERB)	0.35
P (ARTICLE   VERB)	0.65
P(ARTICLE PREPOSITION)	0.74
P(NOUN PREPOSITION)	0.26

Consider a following sequence of categories (tags):

<s>, ARTICLE, NOUN, VERB, NOUN

What's the probability of its occurence in our synthetic corpus?

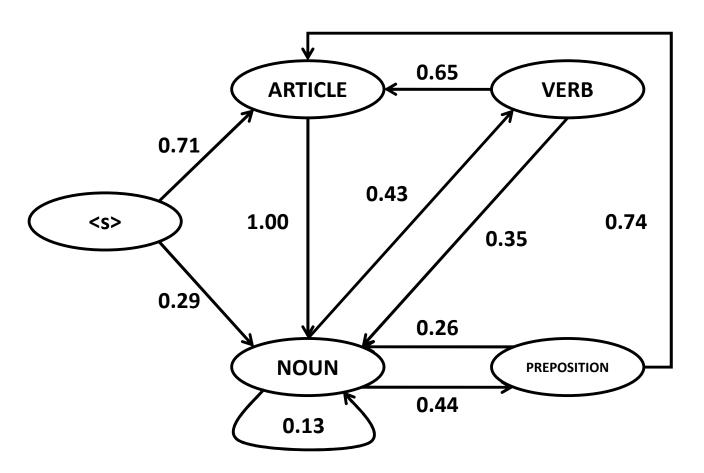
## Hidden Markov Model (HMM)



P(Bigram)	Estimate
P(ARTICLE  <s>)</s>	0.71
P(NOUN  <s>)</s>	0.29
P(NOUN ARTICLE)	1.00
P(VERB NOUN)	0.43
P (NOUN   NOUN)	0.13
P (PREPOSITION   NOUN)	0.44
P(NOUN VERB)	0.35
P (ARTICLE   VERB)	0.65
P(ARTICLE PREPOSITION)	0.74
P(NOUN PREPOSITION)	0.26

The word "Hidden" in Hidden Markov Model means that for a specific sequence (of words) it is unclear what state the model is in.

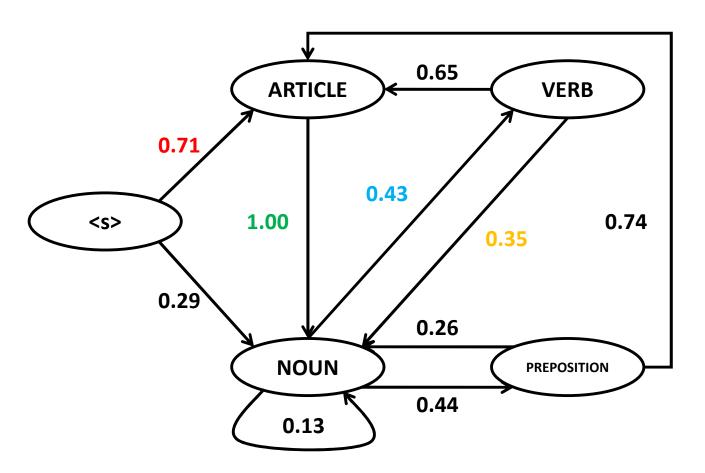
The word *flies* could be generated from state NOUN and state VERB.



P(Bigram)	Estimate
P(ARTICLE  <s>)</s>	0.71
P(NOUN  <s>)</s>	0.29
P(NOUN ARTICLE)	1.00
P(VERB NOUN)	0.43
P (NOUN   NOUN)	0.13
P (PREPOSITION   NOUN)	0.44
P(NOUN VERB)	0.35
P (ARTICLE   VERB)	0.65
P(ARTICLE PREPOSITION)	0.74
P(NOUN PREPOSITION)	0.26

Probability of occurrence of a sequence of categories (tags):

$$P(C_1, C_2, C_3, ..., C_T) \cong \prod_{i=1}^T P(C_i \mid C_{i-1})$$



P(Bigram)	Estimate
P(ARTICLE  <s>)</s>	0.71
P(NOUN  <s>)</s>	0.29
P(NOUN ARTICLE)	1.00
P(VERB NOUN)	0.43
P (NOUN   NOUN)	0.13
P (PREPOSITION   NOUN)	0.44
P(NOUN VERB)	0.35
P (ARTICLE   VERB)	0.65
P(ARTICLE PREPOSITION)	0.74
P(NOUN PREPOSITION)	0.26

#### Probability of occurrence of a sequence of categories (tags):

P(<s>, ARTICLE, NOUN, VERB, NOUN) =

 $\cong P(ART|<s>) * P(N|ART) * P(V|N) * P(N|V) = 0.71 * 1.00 * 0.43 * 0.35 = 0.107$ 

# Synthetic Corpus: Word/Tag Counts

#### Summary of selected word counts in the synthetic corpus:

Word/Tag	N	V	ART	Р	TOTAL
flies	21	23	0	0	44
fruit	49	5	1	0	55
like	10	30	0	21	61
а	1	0	201	0	202
the	1	0	300	2	303
flower	53	15	0	0	68
flowers	42	16	0	0	58
birds	64	1	0	0	65
others	592	210	56	284	1142
TOTAL	833	300	558	307	1998

#### From the table we can calculate lexical generation probabilities P(w|C) estimates:

$$P(the|ART) = 300/558 = 0.54$$

$$P(a|ART) = 201/558 = 0.36$$

$$P(flies|N) = 21/833 = 0.025$$

$$P(a|N) = 1/833 = 0.001$$

$$P(flies|V) = 23/300 = 0.076$$

$$P(flower|N) = 53/833 = 0.063$$

$$P(like|V) = 30/300 = 0.1$$

$$P(flower|V) = 15/300 = 0.05$$

$$P(like|P) = 21/307 = 0.068$$

$$P(like|N) = 10/833 = 0.012$$

### Example

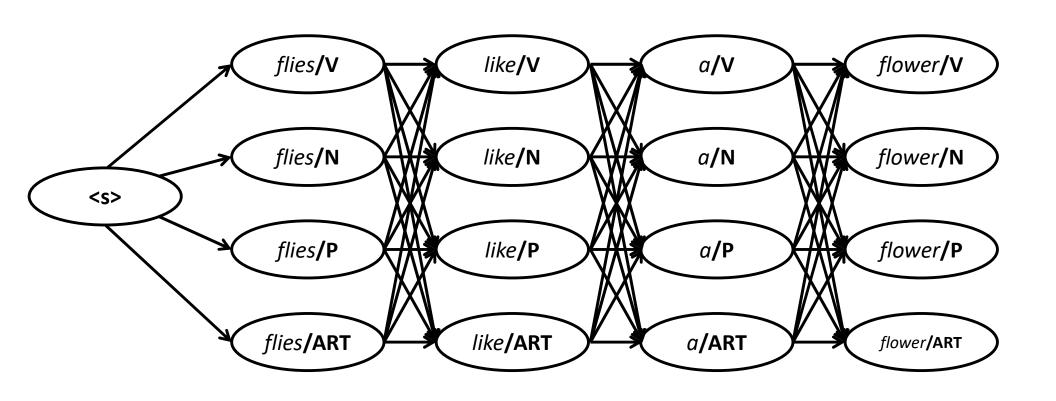
Given our synthetic corpus, what is the most like sequence of categories (tags) corresponding to a sentence:

Flies like a flower

#### We need to maximize:

$$P(w_{1}, w_{2}, w_{3}, ..., w_{T} \mid C_{1}, C_{2}, C_{3}, ..., C_{T}) * P(C_{1}, C_{2}, C_{3}, ..., C_{T}) \cong \prod_{i=1}^{T} P(w_{i} \mid C_{i}) * P(C_{i} \mid C_{i-1})$$

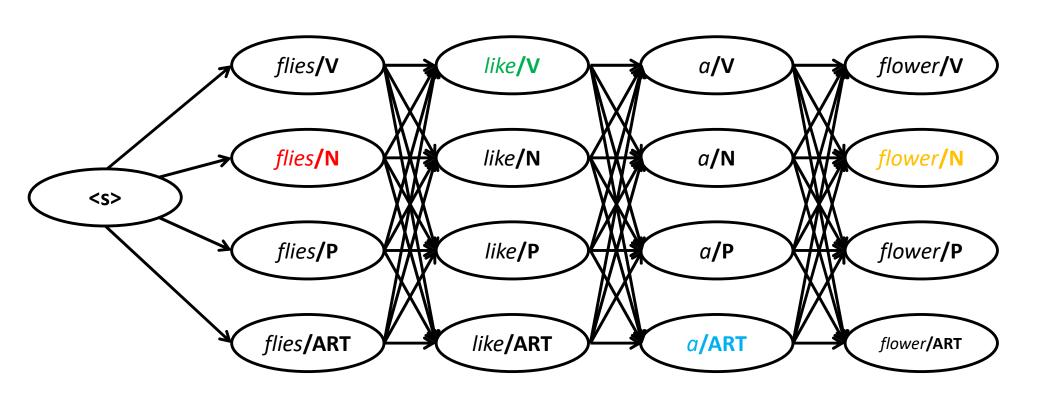
# **Example: All Possible Sequences**



#### Every sequence can be assigned a probability:

$$P(w_1, w_2, w_3, ..., w_T \mid C_1, C_2, C_3, ..., C_T) \cong \prod_{i=1}^T P(w_i \mid C_i)$$

## **Example: All Possible Sequences**



#### Every sequence can be assigned a probability:

$$\prod_{i=1}^{T} P(w_i \mid C_i) = P(flies|N) * P(like|V) * P(a|ART) * P(flower|N)$$

# Synthetic Corpus: Word/Tag Counts

#### Summary of selected word counts in the synthetic corpus:

Word/Tag	N	V	ART	Р	TOTAL
flies	21	23	0	0	44
fruit	49	5	1	0	55
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flowers	42	16	0	0	58
birds	64	1	0	0	65
others	592	210	56	284	1142
TOTAL	833	300	558	307	1998

#### From the table we can calculate lexical generation probabilities P(w|C) estimates:

$$P(the|ART) = 300/558 = 0.54$$

$$(me|AK1) = 300/330 = 0.34$$

$$P(flies|N) = 21/833 = 0.025$$

$$P(flies|V) = 23/300 = 0.076$$

$$P(like|V) = 30/300 = 0.1$$

$$P(like|P) = 21/307 = 0.068$$

$$P(a|ART) = 201/558 = 0.36$$

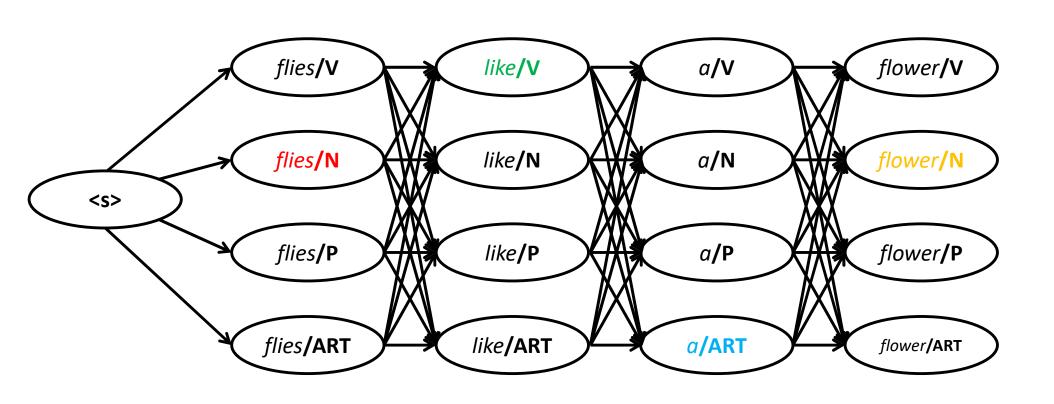
$$P(a|N) = 1/833 = 0.001$$

$$P(flower|N) = 53/833 = 0.063$$

$$P(flower|V) = 15/300 = 0.05$$

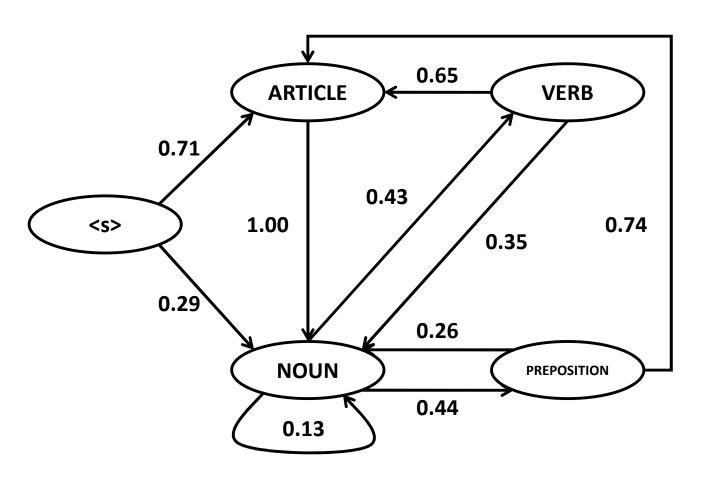
$$P(like|N) = 10/833 = 0.012$$

## **Example: All Possible Sequences**



#### Every sequence can be assigned a probability:

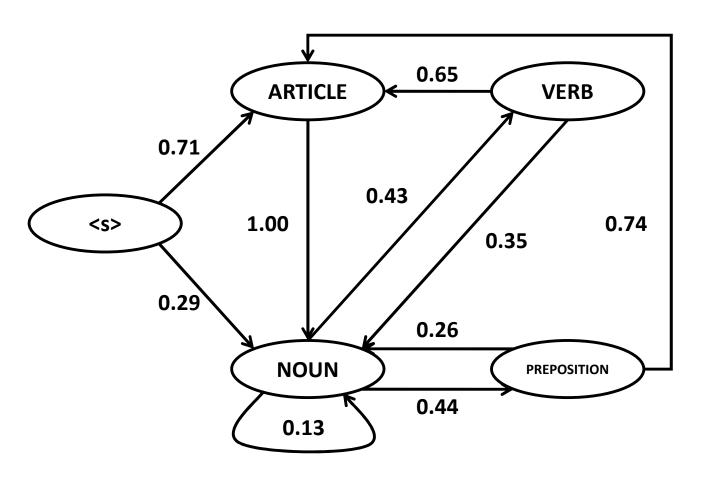
$$\prod_{i=1}^{T} P(w_i \mid C_i) = 0.025 * 0.1 * 0.36 * 0.063 = 5.4 * 0$$



P(Bigram)	Estimate
P(ARTICLE  <s>)</s>	0.71
P(NOUN  <s>)</s>	0.29
P(NOUN ARTICLE)	1.00
P(VERB NOUN)	0.43
P (NOUN   NOUN)	0.13
P (PREPOSITION   NOUN)	0.44
P(NOUN VERB)	0.35
P (ARTICLE   VERB)	0.65
P(ARTICLE PREPOSITION)	0.74
P(NOUN PREPOSITION)	0.26

For any sequence of categories (tags), their probability is:

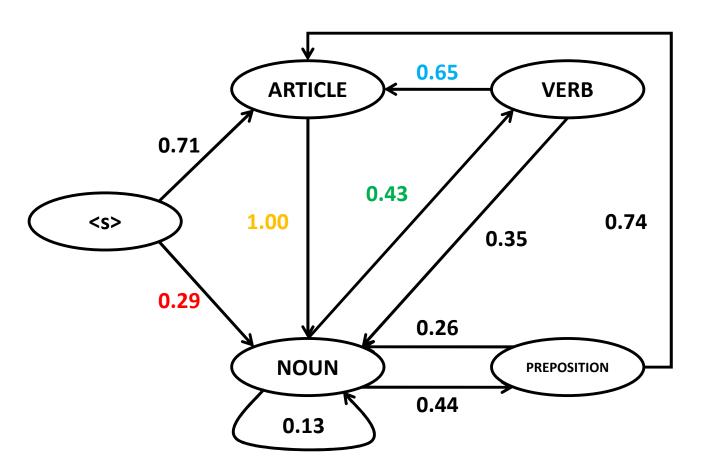
$$P(C_1, C_2, C_3, \dots, C_T) \cong \prod_{i=1}^T P(C_i \mid C_{i-1})$$



P(Bigram)	Estimate
P(ARTICLE  <s>)</s>	0.71
P(NOUN  <s>)</s>	0.29
P(NOUN ARTICLE)	1.00
P(VERB NOUN)	0.43
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P(NOUN VERB)	0.35
P (ARTICLE   VERB)	0.65
P(ARTICLE PREPOSITION)	0.74
P(NOUN PREPOSITION)	0.26

For any sequence of categories (tags), their probability is:

$$\prod_{i=1}^{T} P(C_i \mid C_{i-1}) = P(N \mid < s >) * (V \mid N) * (ART \mid V) * (N \mid ART)$$



P(Bigram)	Estimate
P(ARTICLE  <s>)</s>	0.71
P(NOUN  <s>)</s>	0.29
P(NOUN ARTICLE)	1.00
P(VERB NOUN)	0.43
P (NOUN   NOUN)	0.13
P (PREPOSITION   NOUN)	0.44
P(NOUN VERB)	0.35
P (ARTICLE   VERB)	0.65
P(ARTICLE PREPOSITION)	0.74
P(NOUN PREPOSITION)	0.26

For any sequence of categories (tags), their probability is:

$$\prod_{i=1}^{T} P(C_i \mid C_{i-1}) = 0.29 * 0.43 * 0.65 * 1.00 = 0.081$$

### Example

Given our synthetic corpus, what is the most like sequence of categories (tags) corresponding to a sentence:

Flies like a flower

#### For example:

P(Flies, like, a, flower | N, V, ART, N) \* P(N, V, ART, N)

$$\cong 5.4 * 10^{-5} * 0.081$$