Today's outline - January 24, 2023

- The EPR paradox
- Bell's inequality
- Experimental tests of Bell's inequality
- Unitary transformations
- No clone theorem

Reading Assignment: Reiffel: 5.1-5.2 Wong: ??

Homework Assignment #02:
due Thursday, January 26, 2023

Homework Assignment #03:
due Thursday, February 02, 2023

Einstein Podolsky Rosen paradox review

Suppose a pair of photons are generated in the entangled state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

The first photon is sent to Alice and the second to Bob who are far apart

Now Alice measures her photon and sees that it is the the $|0\rangle$ state which forces the original state to collapse: $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \longrightarrow |00\rangle$

When Bob now measures his photon he will get $|0\rangle$ with 100% certainty

This so-called "spooky action at a distance" profoundly bothers Einstein, Podolsky, and Rosen who postulate that there must be a hidden local variable that cannot be measured.

This implies that when the two photons are created, there is some additional hidden state that is created with a random value along with the two photon which determines the outcome of the measurements.

If such a theory is correct, then the result of the measurements is determined before the photons are separated and no possible violations of causality can occur

Bell's inequality

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ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

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I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty. There have been attempts [3] to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible. These attempts have been examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.

II. Formulation

With the example advocated by Bohm and Aharonov [6], the EPR argument is the following. Consider a pair of spin one-half particles formed somehow in the singlet spin state and moving freely in opposite directions. Measurements can be made, say by Stern-Gerlach magnets, on selected components of the spins $\vec{\sigma}_1$ and $\vec{\sigma}_2$. If measurement of the component $\vec{\sigma}_1 \cdot \vec{a}$, where $\vec{a}$ is some unit vector, yields the value $+1$ then, according to quantum mechanics, measurement of $\vec{\sigma}_2 \cdot \vec{a}$ must yield the value -1 and vice versa. Now we make the hypothesis [2], and it seems one at least worth considering, that if the two measurements are made at places remote from one another the orientation of one magnet does not influence the result obtained with the other. Since we can predict in advance the result of measuring any chosen component of $\vec{\sigma}_2$, by previously measuring the same component of $\vec{\sigma}_1$, it follows that the result of any such measurement must actually be predetermined. Since the initial quantum mechanical wave function does *not* determine the result of an individual measurement, this predetermination implies the possibility of a more complete specification of the state.

Let this more complete specification be effected by means of parameters λ. It is a matter of indifference in the following whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous. However, we write as if λ were a single continuous parameter. The result A of measuring $\vec{\sigma}_1 \cdot \vec{a}$ is then determined by $\vec{a}$ and λ, and the result B of measuring $\vec{\sigma}_2 \cdot \vec{b}$ in the same instance is determined by $\vec{b}$ and λ, and

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$$A(\vec{a}, \lambda) = \pm 1, \; B(\vec{b}, \lambda) = \pm 1. \tag{1}$$

The vital assumption [2] is that the result B for particle 2 does not depend on the setting $\vec{a}$, of the magnet for particle 1, nor A on $\vec{b}$.

If $\rho(\lambda)$ is the probability distribution of λ then the expectation value of the product of the two components $\vec{\sigma}_1 \cdot \vec{a}$ and $\vec{\sigma}_2 \cdot \vec{b}$ is

$$P(\vec{a}, \vec{b}) = \int d\lambda\, \rho(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda) \tag{2}$$

This should equal the quantum mechanical expectation value, which for the singlet state is

$$\langle \vec{\sigma}_1 \cdot \vec{a}\; \vec{\sigma}_2 \cdot \vec{b} \rangle = -\vec{a} \cdot \vec{b}. \tag{3}$$

But it will be shown that this is not possible.

Some might prefer a formulation in which the hidden variables fall into two sets, with A dependent on one and B on the other; this possibility is contained in the above, since λ stands for any number of variables and the dependences thereof of A and B are unrestricted. In a complete physical theory of the type envisaged by Einstein, the hidden variables would have dynamical significance and laws of motion; our λ can then be thought of as initial values of these variables at some suitable instant.

III. Illustration

The proof of the main result is quite simple. Before giving it, however, a number of illustrations may serve to put it in perspective.

Firstly, there is no difficulty in giving a hidden variable account of spin measurements on a single particle. Suppose we have a spin half particle in a pure spin state with polarization denoted by a unit vector $\vec{p}$. Let the hidden variable be (for example) a unit vector $\vec{\lambda}$ with uniform probability distribution over the hemisphere $\vec{\lambda} \cdot \vec{p} > 0$. Specify that the result of measurement of a component $\vec{\sigma} \cdot \vec{a}$ is

$$\text{sign}\; \vec{\lambda} \cdot \vec{a}', \tag{4}$$

where $\vec{a}'$ is a unit vector depending on $\vec{a}$ and $\vec{p}$ in a way to be specified, and the sign function is $+1$ or -1 according to the sign of its argument. Actually this leaves the result undetermined when $\lambda \cdot a' = 0$, but as the probability of this is zero we will not make special prescriptions for it. Averaging over $\vec{\lambda}$ the expectation value is

$$\langle \vec{\sigma} \cdot \vec{a} \rangle = 1 - 2\theta'/\pi, \tag{5}$$

where θ' is the angle between $\vec{a}'$ and $\vec{p}$. Suppose then that $\vec{a}'$ is obtained from $\vec{a}$ by rotation towards $\vec{p}$ until

$$1 - \frac{2\theta'}{\pi} = \cos\theta \tag{6}$$

where θ is the angle between $\vec{a}$ and $\vec{p}$. Then we have the desired result

$$\langle \vec{\sigma} \cdot \vec{a} \rangle = \cos\theta \tag{7}$$

So in this simple case there is no difficulty in the view that the result of every measurement is determined by the value of an extra variable, and that the statistical features of quantum mechanics arise because the value of this variable is unknown in individual instances.

"On the Einstein Podolsky Rosen paradox," J.S. Bell, *Physics* **1**, 195-200 (1964).

Bell's thought experiment

The pair of photons are emitted in an entangled state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Alice and Bob have polarizers which can be set to vertical or $\pm 60^\circ$ from vertical

If O_θ is a 1-qubit observable with two basis vectors with results (eigenvalues) ± 1

$$|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle \longrightarrow +1$$
$$|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle \longrightarrow -1$$

According to quantum mechanics, what is the probability of Alice and Bob obtaining the same value when they make their individual measurements, $O_{\theta_1} \otimes I$ and $I \otimes O_{\theta_2}$?

Start with the projectors for each of the measureable states $|v_i\rangle$ and $|v_i^\perp\rangle$

$$P^{v_i} = |v_i\rangle\langle v_i|, \qquad P^{v_i^\perp} = |v_i^\perp\rangle\langle v_i^\perp|$$

For the two measurements to result in $|v_1\rangle|v_2\rangle$ or $|v_1^\perp\rangle|v_2^\perp\rangle$, the projector must be

$$P = \left(P^{v_1} \otimes I\right)\left(I \otimes P^{v_2}\right) + \left(P^{v_1^\perp} \otimes I\right)\left(I \otimes P^{v_2^\perp}\right) = \left(P^{v_1} \otimes P^{v_2}\right) + \left(P^{v_1^\perp} \otimes P^{v_2^\perp}\right) = P^{v_1 v_2} + P^{v_1^\perp v_2^\perp}$$

Quantum mechanics prediction

Now expand each of the two projection operators $P^{v_1 v_2}$ and $P^{v_1^\perp v_2^\perp}$ recalling that $|v\rangle = +\cos\theta|0\rangle + \sin\theta|1\rangle$ and $|v^\perp\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

$$\begin{aligned}
P^{v_1 v_2} &= P^{v_1} \otimes P^{v_2} = (|v_1\rangle\langle v_1| \otimes |v_2\rangle\langle v_2|) \\
&= |v_1\rangle|v_2\rangle(\cos\theta_1\cos\theta_2\langle 00| + \cos\theta_1\sin\theta_2\langle 01| + \sin\theta_1\cos\theta_2\langle 10| + \sin\theta_1\sin\theta_2\langle 11|) \\
P^{v_1^\perp v_2^\perp} &= P^{v_1^\perp} \otimes P^{v_2^\perp} = (|v_1^\perp\rangle\langle v_1^\perp| \otimes |v_2^\perp\rangle\langle v_2^\perp|) \\
&= |v_1^\perp\rangle|v_2^\perp\rangle(\sin\theta_1\sin\theta_2\langle 00| - \sin\theta_1\cos\theta_2\langle 01| - \cos\theta_1\sin\theta_2\langle 10| + \cos\theta_1\cos\theta_2\langle 11|)
\end{aligned}$$

Using these projection operators, measure the probability of Alice and Bob getting the same answer when applied to $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ by applying $P = P^{v_1 v_2} + P^{v_1^\perp v_2^\perp}$

$$\begin{aligned}
P|\psi\rangle &= \frac{1}{\sqrt{2}}|v_1\rangle|v_2\rangle(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + \frac{1}{\sqrt{2}}|v_1^\perp\rangle|v_2^\perp\rangle(\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2) \\
&= \frac{1}{\sqrt{2}}\cos(\theta_1 - \theta_2)[|v_1\rangle|v_2\rangle + |v_1^\perp\rangle|v_2^\perp\rangle] \quad \longrightarrow \quad \langle\psi|P|\psi\rangle = \cos^2(\theta_1 - \theta_2)
\end{aligned}$$

The probability of $|\psi\rangle$ being found in the $+1$ eigenspace generated by $\{|v_1\rangle|v_2\rangle, |v_1^\perp\rangle|v_2^\perp\rangle\}$

Photon polarization example

The three polarizations for each filter represent three different observables, M_{0°, M_{+60°, and M_{-60°

Each observable has only two outcomes, the photon passing through (outcome P) or the photon being absorbed (outcome A)

We can now compute the probabilities for all different settings of the two polarizers (remember Alice and Bob choose the settings randomly and measure at any time they like)

$$\langle\psi|O_{\theta_1} \otimes O_{v_2}|\psi\rangle = \cos^2(\theta_1 - \theta_2)$$

$\theta_1 - \theta_2$	$\cos(\theta_1 - \theta_2)$	Probability
0°	1	1
$\pm 60^\circ$	$+\frac{1}{2}$	$\frac{1}{4}$
$\pm 120^\circ$	$-\frac{1}{2}$	$\frac{1}{4}$

If the polarizers are set randomly and independently, they will be the same $\frac{1}{3}$ of the time with 100% probability of the measurements agreeing and be different $\frac{2}{3}$ of the time with 25% probability of the measurements agreeing

The overall probability of measurements agreeing is thus $\frac{1}{3} \cdot 1 + \frac{2}{3} \cdot \frac{1}{4} = \frac{1}{2}$

Consequences of a local hidden variable

Suppose there is a local hidden state associated with each photon which determines the result of the measurement in each of the three polarizer settings

There can only be 2^3 such states for this kind of system

We know that when both filters are in the same position the two measurements of an EPR pair must coincide such that if Alice's measurements are to be *PAP*, then Bob's must also be *PAP* so we enumarate the 9 possible filter settings and see what the local hidden variables predict

	Polarizer		
	$\nearrow$	$\uparrow$	$\nwarrow$
h_0	P	P	P
h_1	P	P	A
h_2	P	A	P
h_3	P	A	A
h_4	A	P	P
h_5	A	P	A
h_6	A	A	P
h_7	A	A	A

$$\{(\nearrow\nearrow),(\nearrow\uparrow),(\nearrow\nwarrow),(\uparrow\nearrow),(\uparrow\uparrow),(\uparrow\nwarrow),(\nwarrow\nearrow),(\nwarrow\uparrow),(\nwarrow\nwarrow)\}$$

If the hidden state is h_0 or h_7 measurements agree for all possible filter settings but for the other 6 hidden states $\frac{5}{9}$ of the measurements will agree giving total probability $1 \cdot \frac{2}{8} + \frac{5}{9} \cdot \frac{6}{8} = \frac{8}{12}$

This does not match the quantum mechanics result of $\frac{1}{2}$

Bell's inequality

The previous is a special case of Bell's inequality, which is a more general derivation

If we have two detectors with three polarizations each, a, b, and c we define the following probabilities

P_{xy}: the observed probabilities of the two EPR photons interacting the same way with the first polarizer set to x and the second set to y or the first set to y and the second set to x

According to a local hidden variable theory, the result of a measurement is determined by the value of the hidden state h

Since the measurements of the two photons are identical if the filter settings are the same ($P_{xx} \equiv 1$), both photons must be described by the same hidden variable

Define P^h_{xy} to be 1 if the results of the two measurements agree on states with hidden variable h and 0 otherwise

Finally, let w_h be the probability with which the EPR source emits photons of kind h

Bell's inequality (cont.)

The sum of the observed probabilities the three combinations $P_{ab} + P_{ac} + P_{bc}$ is given by

$$P_{ab} + P_{ac} + P_{bc} = \sum_h w_h \left(P^h_{ab} + P^h_{ac} + P^h_{bc} \right)$$

However, as we saw from the simple example, for every possible local hidden state h, the result of measuring the two photons will be the same for one or more of the three combinations, P^h_{ab}, P^h_{ac}, or P^h_{bc} and this forces the sum to be greater than 1 for any values of a, b, and c

$$P^h_{ab} + P^h_{ac} + P^h_{bc} \geq 1 \quad \longrightarrow \quad P_{ab} + P_{ac} + P_{bc} > 1$$

This is Bell's inequality and provides an experimentally testable condition

For the angle between a and b being θ and the angle between b and c being ϕ we have that

$$P_{ab} + P_{ac} + P_{bc} = \cos^2\theta + \cos^2(\theta + \phi) + \cos^2\phi > 1$$

Testing Bell's inequality

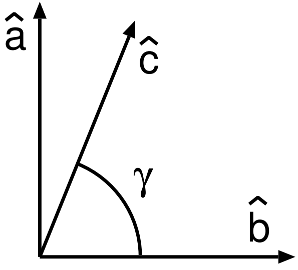
$$P_{ab} + P_{ac} + P_{bc} = \cos^2(\theta_a - \theta_b) + \cos^2(\theta_a - \theta_c) + \cos^2(\theta_b - \theta_c) > 1$$

Take the worst case, that of $\theta_a = \frac{\pi}{2}$, $\theta_b = 0$, and $\theta_c = \gamma$

$$P_{ab} = \cos^2 \frac{\pi}{2} = 0$$

$$P_{bc} = \cos^2(-\gamma) = \cos^2 \gamma$$

$$P_{ac} = \cos^2(\frac{\pi}{2} - \gamma) = \sin^2 \gamma$$



$$P_{ab} + P_{ac} + P_{bc} = 0 + \cos^2 \gamma + \sin^2 \gamma = 1 \not> 1$$

All other cases give answers that are less than 1 and thus an experimental result predicted by quantum mechanics would rule out the presence of any local hidden variables

Since Bell's paper, there have been many efforts to demonstrate the failure of this inequality

Bell's inequality tested

for trapping as b decreases is reflective of the incorporation of periodic components into the sequence of numbers generated.

To summarize the motivation and principal conclusion of this Letter, we restate[1] that for values of b where numerically generated sequences *appear* to be chaotic, it has not been settled whether those sequences "are truly chaotic, or whether, in fact, they are really periodic, but with exceedingly large periods and very long transients required to settle down." On the one hand, Grossman and Thomae[5,6] have suggested that (only) the parameter value $b=1$ generates pure chaos [see the discussion following Eq. (31) of Ref. 5 and the correlations plotted in their Fig. 9]. On the other hand, for certain *other* values of b, numberical results of Lorenz (reported in Ref. 1) "strongly suggest that the sequences are truly chaotic." The purpose of this communication was to use an independent and exact result from the statistical-mechanical theory of $d=1$ random walks to test the randomness of the parabolic map for parameter values where the existence of "true chaos" is still an open question.

Our results strongly support the conclusions of Grossmann and Thomae.

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Experimental Test of Bell's Inequalities Using Time-Varying Analyzers

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Correlations of linear polarizations of pairs of photons have been measured with time-varying analyzers. The analyzer in each leg of the apparatus is an acousto-optical switch followed by two linear polarizers. The switches operate at incommensurate frequencies near 50 MHz. Each analyzer amounts to a polarizer which jumps between two orientations in a time short compared with the photon transit time. The results are in good agreement with quantum mechanical predictions but violate Bell's inequalities by 5 standard deviations.

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Bell's inequalities apply to any correlated measurement on two correlated systems. For instance, in the optical version of the Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*,[1] a source emits pairs of photons (Fig. 1). Measurements of the correlations of linear polarizations are performed on two photons belonging to the same pair. For pairs emitted in suitable states, the correlations are strong. To account for these correlations, Bell[2] considered theories which invoke common properties of both members of the

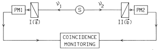


FIG. 1. Optical version of the Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*. The pair of photons ν_1 and ν_2 is analyzed by linear polarizers I and II (in orientations $\vec{a}$ and $\vec{b}$) and photomultipliers. The coincidence rate is monitored.

pair. Such properties are referred to as supplementary parameters. This is very different from the quantum mechanical formalism, which does not involve such properties. With the addition of a reasonable locality assumption, Bell showed that such classical-looking theories are constrained by certain inequalities that are not always obeyed by quantum mechanical predictions.

Several experiments of increasing accuracy[3-5] have been performed and clearly favor quantum mechanics. Experiments using pairs of visible photons emitted in atomic radiative cascades seem to achieve a good realization of the ideal *Gedankenexperiment*.[5] However, all these experiments have been performed with static setups, in which polarizers are held fixed for the whole duration of a run. Then, one might question Bell's locality assumption, that states that the results of the measurement by polarizer II does not depend on the orientation $\vec{a}$ of polarizer I (and vice versa), nor does the way in which pairs are emitted depend on $\vec{a}$ or $\vec{b}$. Although highly reasonable, such a locality condition is not prescribed by any fundamental physical law. As pointed out by Bell,[2] it is possible, in such experiments, to reconcile supplementary-parameter theories and the experimentally verified predictions of quantum mechanics: "The settings of the instruments are made sufficiently in advance to allow them to reach some mutual rapport by exchange of signals with velocity less than or equal to that of light." If such interactions existed, Bell's locality condition would no longer hold for static experiments, nor would Bell's inequalities.

Bell thus insisted upon the importance of "experiments of the type proposed by Bohm and Aharonov,[6] in which the settings are changed during the flight of the particles." In such a "timing experiment," the locality condition would then become a consequence of Einstein's causality, preventing any faster-than-light influence.

In this Letter, we report the results of the first experiment using variable polarizers. Following our proposal,[7] we have used a modified scheme (Fig. 2). Each polarizer is replaced by a setup involving a switching device followed by two polarizers in two different orientations: $\vec{a}$ and $\vec{a}'$ on side I, and $\vec{b}$ and $\vec{b}'$ on side II. Such an optical switch is able to rapidly redirect the incident light from one polarizer to the other one. If the two switches work at random and are uncorrelated, it is possible to write generalized Bell's inequalities in a form similar to Clauser-Horne-

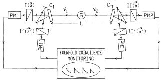


FIG. 2. Timing experiment with optical switches. Each switching device (C_I, C_{II}) is followed by two polarizers in two different orientations. Each combination is equivalent to a polarizer switched fast between two orientations.

Shimony-Holt inequalities[8]:

$$-1 \leqslant S \leqslant 0,$$

with

$$S = \frac{N(\vec{a},\vec{b})}{N(\infty,\infty)} - \frac{N(\vec{a},\vec{b}')}{N(\infty,\infty')} + \frac{N(\vec{a}',\vec{b})}{N(\infty',\infty)} + \frac{N(\vec{a}',\vec{b}')}{N(\infty',\infty')} - \frac{N(\vec{a}',\infty)}{N(\infty',\infty)} - \frac{N(\infty,\vec{b})}{N(\infty,\infty)}.$$

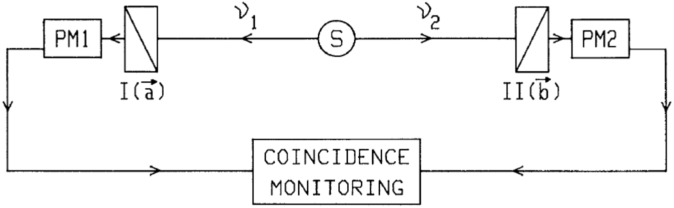
The quantity S involves (i) the four coincidence counting rates [$N(\vec{a},\vec{b})$, $N(\vec{a}',\vec{b})$, etc.] measured in a *single run;* (ii) the four corresponding coincidence rates [$N(\infty,\infty)$, $N(\infty',\infty)$, etc.] with all polarizers removed; and (iii) two coincidence rates [$N(\vec{a}',\infty)$, $N(\infty,\vec{b})$] with a polarizer removed on each side. The measurements (ii) and (iii) are performed in auxiliary runs.

In this experiment, switching between the two channels occurs about each 10 ns. Since this delay, as well as the lifetime of the intermediate level of the cascade (5 ns), is small compared to L/c (40 ns), a detection event on one side and the corresponding change of orientation on the other side are separated by a spacelike interval.

The switching of the light is effected by acousto-optical interaction with an ultrasonic standing wave in water.[9] As sketched in Fig. 3 the incidence angle is equal to the Bragg angle, $\theta_B = 5 \times 10^{-3}$ rad. It follows that light is either transmitted straight ahead or deflected at an angle $2\theta_B$. The light is completely transmitted when the amplitude of the standing wave is null, which occurs twice during an acoustical period. A quarter of a period later, the amplitude of the standing wave is maximum and, for a suitable value of the acoustical power, light is then fully

"Experimental test of Bell's inequalities using time-varying analyzers,", A. Aspect, J. Dalibard, and G. Roger, *Phys. Rev. Lett.* **49**, 1804-1807 (1982).

The EPR experiment

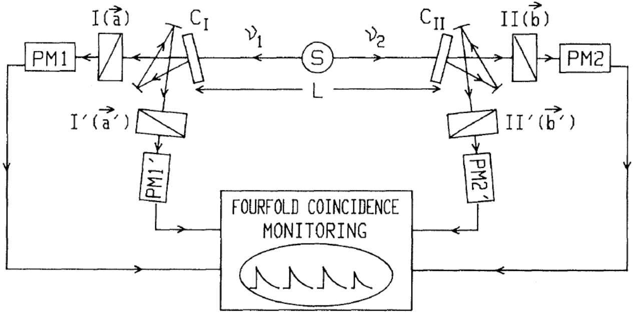


The nominal EPR experiment with photons has two photons emitted by a single source and their polarizations measured by two polarizers $\hat{a}$ and $\hat{b}$ to measure their correlation

while a number of experiments of this kind yielded the expected result, the fact that the polarizers are static is problematic and could be argued to violate Bell's conditions

what is needed is a system where the relative orientation of $\hat{a}$ and $\hat{b}$ is randomized and unknown at the time of photon emission

"Randomized" EPR experiment

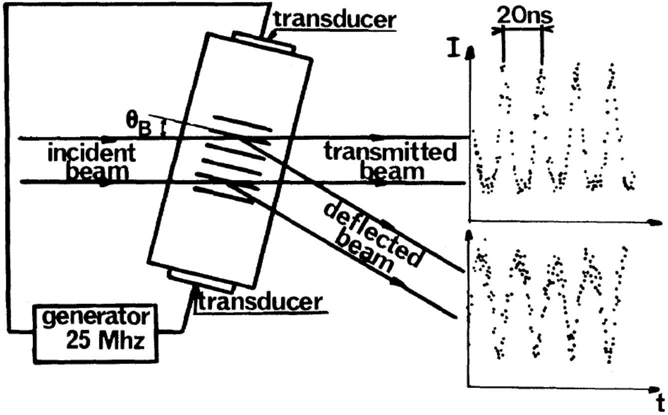


this experiment used two different orientations of the polarizers on each side and used fast switches to randomly and in an uncorrelated manner switch between the two possibilities

the switches work on a time scale of $\sim$10 ns while the transit time of the photons, $L/c \approx 40$ ns

measurements are taken with all 4 polarizers in place, only two in place and none in place

Acoustical Bragg switching



an acoustic generator is used to actuate the switches which are standing waves in water

Bragg diffraction occurs when the amplitude is maximum, twice each period

Bell's inequality confirmed

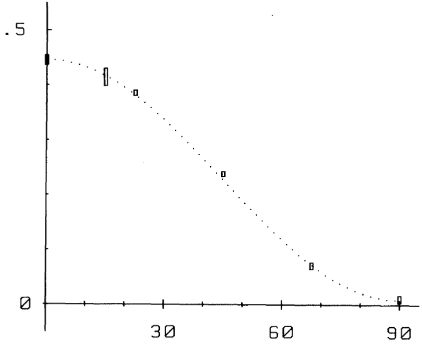
The results of the three different correlation experiments are used to compute the quantity S, which is corresponds to Bell's inequality

$$S = \frac{N(\hat{a},\hat{b})}{N(\infty,\infty)} - \frac{N(\hat{a},\hat{b}')}{N(\infty,\infty')} + \frac{N(\hat{a}',\hat{b})}{N(\infty',\infty)} + \frac{N(\hat{a}',\hat{b}')}{N(\infty',\infty')} - \frac{N(\hat{a}',\infty)}{N(\infty',\infty)} - \frac{N(\infty,\hat{b})}{N(\infty,\infty)}$$

for $(\hat{a},\hat{b}') = 67.5^\circ$ and all others 22.5° we have

$$S_{expt} = 0.101 \pm 0.020$$
$$S_{QM} = 0.112$$

The coincidence rates as a function of angle between polarizers also follows the quantum prediction



Latest Bell tests

Since 1982 many groups have improved on these experiments and removed any loopholes that were present in the original experiments

The first is the "locality" loophole that the result of a measurement at one polarizer does not depend on the orientation of the other This can be solved by ensuring that the choice of polarizer orientation is done while the two photons are in flight to Alice and Bob

While this was done in the 1982 experiment, there were only a limited number of orientations available

This was solved in 1998 with genuine random numbers used to select orientations

The second is the "detection" loophole that due to the low fraction of detected pairs in all the experiments, one could not be sure that the photons being detected were representative of all photons

This was solved in 2013 with high quantum efficiency detectors

Finally in 2015, three papers came out which closed both loopholes simultaneously

Quantum operators

All quantum operators, U, are unitary transformations which must respect the following

$$U|\psi\rangle = U(a_1|\psi_1\rangle + \cdots + a_k|\psi_k\rangle) = a_1 U|\psi_1\rangle + \cdots + a_k U|\psi_k\rangle$$

$$\langle U\phi|U\psi\rangle = \langle\phi|U^\dagger U|\psi\rangle = \langle\phi|I|\psi\rangle = \langle\phi|\psi\rangle$$

A unitary transformation is its own inverse ($U^\dagger \equiv U^{-1}$), maps one orthonormal basis to another orthonormal basis, is reversible, and does not change the outcome of a measurement

The product of two unitary transformations is also unitary so $U_1 \otimes U_2$ is a unitary transformation in the combined space $X_1 \otimes X_2$

The general unitary operator must be able to take the $|0\rangle$ state to any general state on the Bloch sphere

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \longrightarrow |\psi\rangle = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ e^{i\phi}\sin\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ e^{i\phi}\sin\left(\frac{\theta}{2}\right) \end{pmatrix}$$

General unitary operator

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ e^{i\phi}\sin\left(\frac{\theta}{2}\right) \end{pmatrix} \longrightarrow \quad a = \cos\left(\frac{\theta}{2}\right), \quad c = e^{i\phi}\sin\left(\frac{\theta}{2}\right)$$

The other two constants are determined by the property of the unitary matrix that $U^\dagger U \equiv I$

$$U^\dagger U = \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^*a + b^*b & a^*c + b^*d \\ c^*a + d^*b & c^*c + d^*d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Take the top two equations to solve for b and d

$$1 = a^*a + b^*b = \cos^2\left(\frac{\theta}{2}\right) + |b|^2$$

$$|b|^2 = 1 - cos^2\left(\frac{\theta}{2}\right) = sin^2\left(\frac{\theta}{2}\right)$$

$$b = -e^{i\lambda}\sin\left(\frac{\theta}{2}\right)$$

$$0 = a^*c + b^*d = \cos\left(\frac{\theta}{2}\right) e^{i\phi}\sin\left(\frac{\theta}{2}\right) - e^{-i\lambda}\sin\left(\frac{\theta}{2}\right) d$$

$$d = \frac{\cos\left(\frac{\theta}{2}\right) e^{i\phi}\sin\left(\frac{\theta}{2}\right)}{e^{-i\lambda}\sin\left(\frac{\theta}{2}\right)} = e^{i\phi+i\lambda}\cos\left(\frac{\theta}{2}\right)$$

where arbitrary choices for the sign and phase factor of b have been made

Quantum gates

The general unitary operator is thus with the three real parameters

$$U = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -e^{i\lambda}\sin\left(\frac{\theta}{2}\right) \\ e^{i\phi}\sin\left(\frac{\theta}{2}\right) & e^{i\phi+i\lambda}\cos\left(\frac{\theta}{2}\right) \end{pmatrix}, \quad 0 \le \phi \le 2\pi, \quad 0 \le \theta \le \pi, \quad 0 \le \lambda \le 2\pi$$

U, θ, ϕ, and λ describe all single qubit gates, with some examples being

Hadamard	$\theta = \frac{\pi}{2}$	$\phi = 0$	$\lambda = \pi$	maps $\vert 0\rangle$ to an equal superposition of $\vert 0\rangle$ and $\vert 1\rangle$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
Pauli-X	$\theta = \pi$	$\phi = 0$	$\lambda = \pi$	a NOT, maps $\vert 0\rangle \rightarrow \vert 1\rangle$ and $\vert 1\rangle \rightarrow \vert 0\rangle$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Phase Shift	$\theta = 0$	ϕ	$\lambda = 0$	leaves $\vert 0\rangle$ unchanged and rotates $\vert 1\rangle$ on Bloch sphere by ϕ	$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$

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LETTERS TO NATURE

A single quantum cannot be cloned

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If a photon of definite polarization encounters an excited atom, there is typically some nonvanishing probability that the atom will emit a second photon by stimulated emission. Such a photon is guaranteed to have the same polarization as the original photon. But is it possible by this or any other process to amplify a quantum state, that is, to produce several copies of a quantum system (the polarized photon in the present case) each having the same state as the original? If it were, the amplifying process could be used to ascertain the exact state of a quantum system: in the case of a photon, one could determine its polarization by first producing a beam of identically polarized copies and then measuring the Stokes parameters[1]. We show here that the linearity of quantum mechanics forbids such replication and that this conclusion holds for all quantum systems.

Note that if photons could be cloned, a plausible argument could be made for the possibility of faster-than-light communication[2]. It is well known that for certain non-separably correlated Einstein–Podolsky–Rosen pairs of photons, once an observer has made a polarization measurement (say, vertical versus horizontal) on one member of the pair, the other one, which may be far away, can be for all purposes of prediction regarded as having the same polarization[3]. If this second photon could be replicated and its precise polarization measured as above, it would be possible to ascertain whether, for example, the first photon had been subjected to a measurement of linear or circular polarization. In this way the first observer would be able to transmit information faster than light by encoding his message into his choice of measurement. The actual impossibility of cloning photons, shown below, thus prohibits superluminal communication by this scheme. That such a scheme must fail for some reason despite the well-established existence of long-range quantum correlations[6–8], is a general consequence of quantum mechanics[9].

A perfect amplifying device would have the following effect on an incoming photon with polarization state $|s\rangle$:

$$|A_0\rangle|s\rangle \rightarrow |A_s\rangle|ss\rangle \qquad (1)$$

Here $|A_0\rangle$ is the 'ready' state of the apparatus, and $|A_s\rangle$ is its final state, which may or may not depend on the polarization of the original photon. The symbol $|ss\rangle$ refers to the state of the radiation field in which there are two photons each having the polarization $|s\rangle$. Let us suppose that such an amplification can in fact be accomplished for the vertical polarization $|\updownarrow\rangle$ and for the horizontal polarization $|\leftrightarrow\rangle$. That is,

$$|A_0\rangle|\updownarrow\rangle \rightarrow |A_{\text{vert}}\rangle|\updownarrow\updownarrow\rangle \qquad (2)$$

and

$$|A_0\rangle|\leftrightarrow\rangle \rightarrow |A_{\text{hor}}\rangle|\leftrightarrow\leftrightarrow\rangle \qquad (3)$$

According to quantum mechanics this transformation should be representable by a linear (in fact unitary) operator. It therefore follows that if the incoming photon has the polarization given by the linear combination $\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle$—for example, it could be linearly polarized in a direction 45° from the vertical, so that $\alpha=\beta=2^{-1/2}$—the result of its interaction with the apparatus will be the superposition of equations (2) and (3):

$$|A_0\rangle(\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle) \rightarrow \alpha|A_{\text{vert}}\rangle|\updownarrow\updownarrow\rangle+\beta|A_{\text{hor}}\rangle|\leftrightarrow\leftrightarrow\rangle \qquad (4)$$

If the apparatus states $|A_{\text{vert}}\rangle$ and $|A_{\text{hor}}\rangle$ are not identical, then the two photons emerging from the apparatus are in a mixed state of polarization. If these apparatus states are identical, then the two photons are in the pure state

$$\alpha|\updownarrow\updownarrow\rangle+\beta|\leftrightarrow\leftrightarrow\rangle \qquad (5)$$

In neither of these cases is the final state the same as the state with two photons both having the polarization $\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle$. That state, the one which would be required if the apparatus were to be a perfect amplifier, can be written as

$$2^{-1/2}(\alpha a^+_{\text{vert}}+\beta a^+_{\text{hor}})^2|0\rangle = \alpha^2|\updownarrow\updownarrow\rangle+2^{1/2}\alpha\beta|\updownarrow\leftrightarrow\rangle+\beta^2|\leftrightarrow\leftrightarrow\rangle$$

which is a pure state different from the one obtained above by superposition [equation (5)].

Thus no apparatus exists which will amplify an arbitrary polarization. The above argument does not rule out the possibility of a device which can amplify two special polarizations, such as vertical and horizontal. Indeed, any measuring device which distinguishes between these two polarizations, a Nicol prism for example, could be used to trigger such an amplification.

The same argument can be applied to any other kind of quantum system. As in the case of photons, linearity does not forbid the amplification of any given state by a device designed especially for that state, but it does rule out the existence of a device capable of amplifying an arbitrary state.

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COMMUNICATION BY EPR DEVICES

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A recent proposal to achieve faster-than-light communication by means of an EPR-type experimental set-up is examined. We demonstrate that such superluminal communication is not possible. The crucial role of the linearity of the quantum mechanical evolution laws in preventing causal anomalies is stressed.

The existence, according to quantum theory, of correlations between spatially separated systems in EPR-like experiments has suggested to several authors the possibility of message transmission at speeds greater than that of light. The idea is that the correlations subsist between measurement results which do not – as in classical physics – correspond to properties possessed by the systems before the measurement. An experimenter A can therefore choose what kind of experiment to perform at system I and is thus able to influence the probability distribution of outcomes obtained by experimenter B who is measuring on system II. If B were able to recognize this change in the probability distribution he would know what kind of experiment A had decided to perform; and this transmission of information could be used to develop a code for sending messages from A to B (and vice versa). However, it can easily be proved [1] that, due to the fact that the operators representing two measurements at space-like separation commute, all expectation values of physical quantities measured by B remain the same irrespective of A's decisions. Repetition of the experiment therefore will not provide B with any means to discover what A has done. The idea of communication by superluminal velocity thereby seems to be refuted.

There is nevertheless a remaining possibility, recently pointed out by Herbert [2]. The central idea here is to use one single experiment (and not a series of repeated experiments) to transmit one unit of information. In order to ascertain whether or not a change in the probability distribution has taken place a "multiplying device" is included in the experimental set-up. We shall discuss this idea in the context of Bohm's familiar version of the EPR-experiment (see ref. [2] for an exposition in terms of photon polarizations). It will be shown that the laws of quantum theory by virtue of their linearity, prevent such a "quantum communicator" from working.

Suppose that a compound $S = 0$ state decays into two spin 1/2 particles (electrons, say). Experimenter A has the choice to measure either the x-component or the z-component of the spin of electron I. In the path of electron II a "multiplying device" is positioned, in such a way as to ensure that II enters the device *after* A has performed a measurement upon I. The function of the "multiplying device" is to produce a burst of electrons all in exactly the same spin state as the single input electron. The large number N of electrons coming from this device are then examined by B, by means of a Stern–Gerlach apparatus adjusted to measure the x-component of the spin. There are now two possibilities:

(i) A has decided to measure the x-component of the spin of I. Immediately after this measurement II can be described (as far as spin is concerned) with an eigenstate of s_x, and therefore all the electrons emerging from the multiplier will be in this state. The subsequent measurement by B will then have as a result *all* N electrons in either the $s_x = \frac{1}{2}$ or $s_x = -\frac{1}{2}$ channel.

(ii) A has chosen to measure the z-component of the spin of I. Then the electrons emerging from the multiplier will be in an eigenstate of s_z. For each of

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No clone theorem

If it possible to make a quantum "copier" then it is possible to devise a scheme for superluminal information transmission.

Suppose that copier operator, U, acts as on quantum state $|a\rangle$ as

$$U(|a\rangle|0\rangle) \longrightarrow |a\rangle|a\rangle$$

Similarly, it clones orthogonal states $|b\rangle$ as

$$U(|b\rangle|0\rangle) \longrightarrow |b\rangle|b\rangle$$

Can this operator copy a superposition, $|c\rangle$?

$$|c\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle)$$

$$U(|c\rangle|0\rangle) = \frac{1}{\sqrt{2}}\left[U(|a\rangle|0\rangle) + U(|b\rangle|0\rangle)\right] \quad \longrightarrow \quad \frac{1}{\sqrt{2}}(|a\rangle|a\rangle + |b\rangle|b\rangle)$$

but what we really want from the copier is

$$U(|c\rangle|0\rangle) \longrightarrow |c\rangle|c\rangle = \frac{1}{2}(|a\rangle|a\rangle + |a\rangle|b\rangle + |b\rangle|a\rangle + |b\rangle|b\rangle)$$

Even if we account for the different prefactor, the output of the copier differs from the desired result by a factor involving the mixed states of $|a\rangle$ and $|b\rangle$

Thus it is impossible to "clone" a general quantum state