Solutions to Homework 4

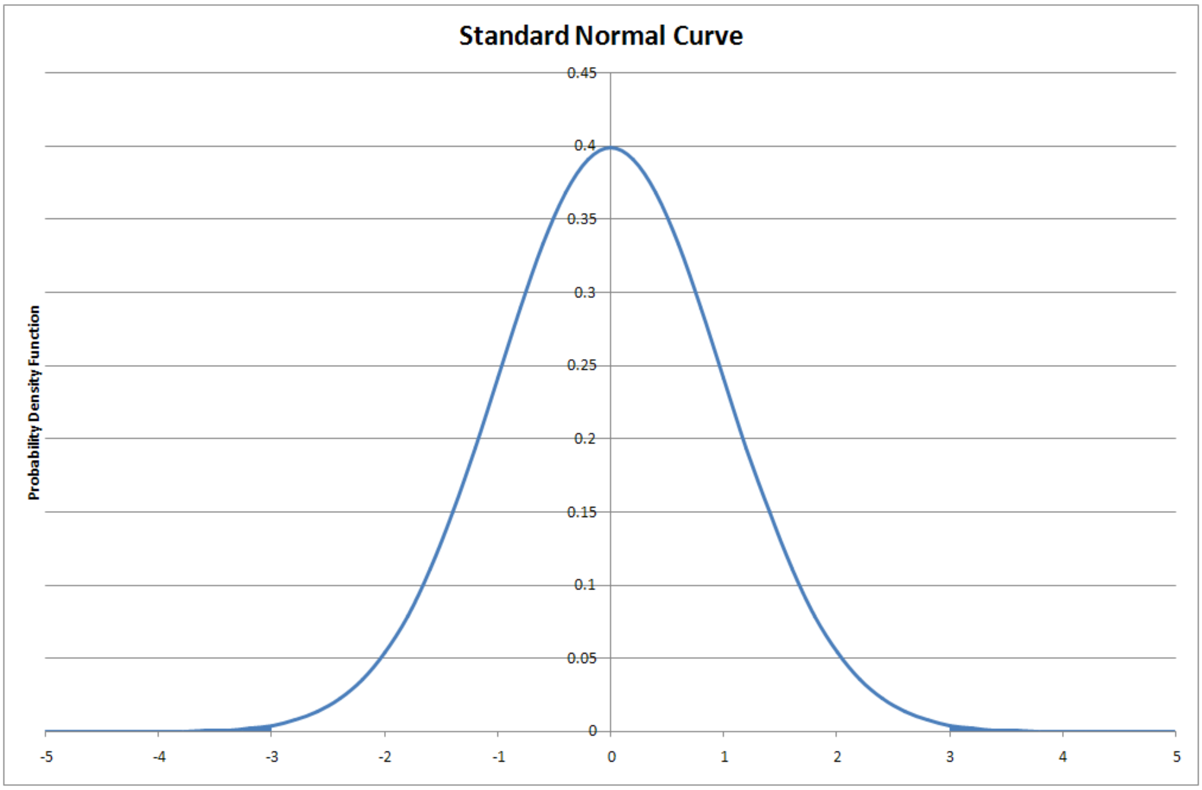
Solutions to Multiple Choice Questions

1. **D** – When determining the central horizontal line we must eliminate unusually large or small values which may be indicative of the process being out of control. The control charts are designed to detect unusual (from the point of view of chance variation) behavior of the process. Thus the iterative procedure of determining the mean, range, and standard deviation of the data is attempting to eliminate the influence of unusually large or small data.
2. **A** – The upper and lower control limits are usually set at 3σ between which 99.7% of the data will occur, provided the process is in control.
3. **B** – An out-of-control point on a control chart is unlikely to happen (3 in a 1000) so it is considered statistically significant for detecting an assignable cause of variation.
4. **B** – If there are *n* observations, and in each the chances of a defect are *p*, then on average we should observe *np* defects, so that *np* stands for the average number of defects, also known as the expected number of defects.

Solutions to Problems

Answer to Question 1

- If we assume the process follows the standard normal model and has a mean 0 and standard deviation of 1, then we need to find the total area outside the limits of -3 and +3 as shown below in the graph. We consult the Z-table (given on the last page) and find that the total area to the left of 3 is 0.9987. Hence the total area under the curve greater than 3 is 1 - 0.9987 = 0.0013. By symmetry the total area outside the limits of -3 and +3 is simply 0.0013 + 0.0013 = 0.0026. Therefore, the probability of a point occurring outside the 3σ limits is 0.0026.
- Since the probability of a day occurring outside the 3σ limits is 0.0026, in 1,000 days we expect 2.6 instances of the point occurring outside the 3σ limits. Rounding this up, the frequency is expected to be 3 out of 1,000 days.
- We need to find the total area outside the limits of -2 and +2. We consult the Z-Table and find that the total area to the left of 2 is 0.9772. Hence the total area under the curve greater than 2 is 1 - 0.9772 = 0.0228. By symmetry the total area outside the limits of -2 and +2 is simply 0.0228 + 0.0228 = 0.0456. Therefore, the probability of a point occurring outside the 2σ limits is 0.0456.
- Since the probability of a day occurring outside the 2σ limits is 0.0456, in 1,000 days we expect 45.6 instances of the point occurring outside the 2σ limits. Rounding this up, the frequency is expected to be 46 out of 1,000 days.



Answer to Question 2

- From the AIAG chart on the following page we obtain the following for a subsample size of 7:

    - $A_2 = 0.42$
    - $D_3 = 0.08$
    - $D_4 = 1.92$

- The control limits for the $\bar{X}$ control chart are given by the following equations:

$LCL_{\bar{X}} = \bar{X} - A_2\bar{R}$ $\qquad$ $UCL_{\bar{X}} = \bar{X} + A_2\bar{R}$

$\bar{X}$ = mean
$\bar{R}$ = average range

Since we are told the mean is 78 and the average range is 6.3, and since we know the value of A_2 is 0.42 for a subsample size of 7, the controls limits for the $\bar{X}$ control chart will be:

$LCL_{\bar{X}} = 78 - (0.42)(6.3)$ $\qquad$ $UCL_{\bar{X}} = 78 + (0.42)(6.3)$

$LCL_{\bar{X}} = 78 - (0.42)(6.3)$ $\qquad$ $UCL_{\bar{X}} = 78 + (0.42)(6.2)$

$LCL_{\bar{X}} = 78 - 2.646$ $\qquad$ $UCL_{\bar{X}} = 78 + 2.646$

$LCL_{\bar{X}} = 75.354$ $\qquad$ $UCL_{\bar{X}} = 80.646$

The control limits for the R control chart are given by the following equations:

$LCL_R = D_3\bar{R}$ $\qquad$ $UCL_R = D_4\bar{R}$

Since we are told the range is 6.3, and since we know the values of D_3 and D_4 are 0.08 and 1.92 respectively, for a sample size of 7, the controls limits for the R control chart will be:

$LCL_R = (0.08)(6.3)$ $\qquad$ $UCL_R = (1.92)(6.3)$

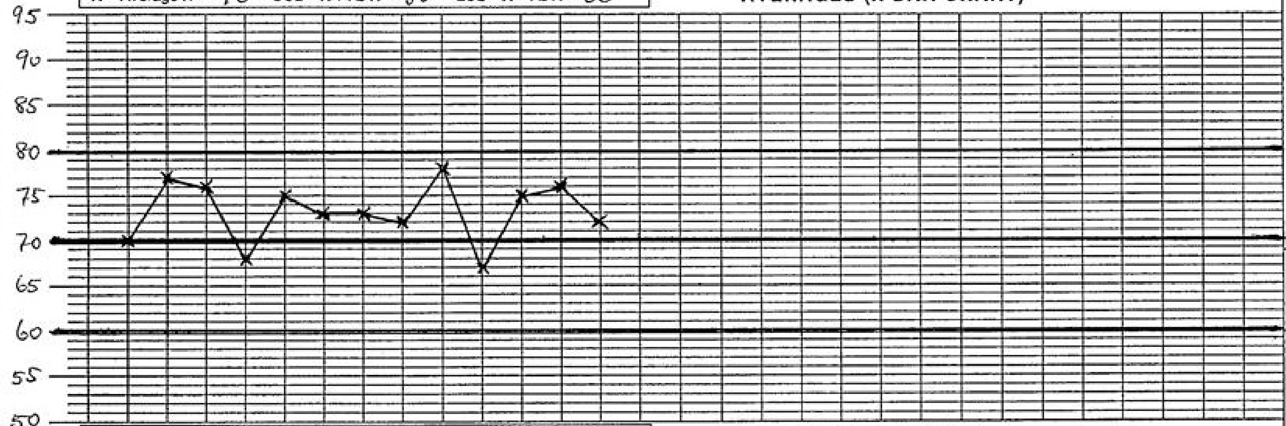
$LCL_R = 0.504$ $\qquad$ $UCL_R = 12.096$

CONTROL CHART

PLANT	DEPT.	OPERATION	DATE CONTROL LIMITS CALCULATED	ENGINEERING SPECIFICATION	PART NO.
CHICAGO	105	BEND CLIP	5-4	50 to 90 mm	E2BB 123
MACH. NO.	**DATES**	**CHARACTERISTIC**		**SAMPLE SIZE/FREQUENCY**	**PART NAME**
030	6-8 to 6-16	CLIP GAP DIM "A"		5 / 2HRS	RETAINER

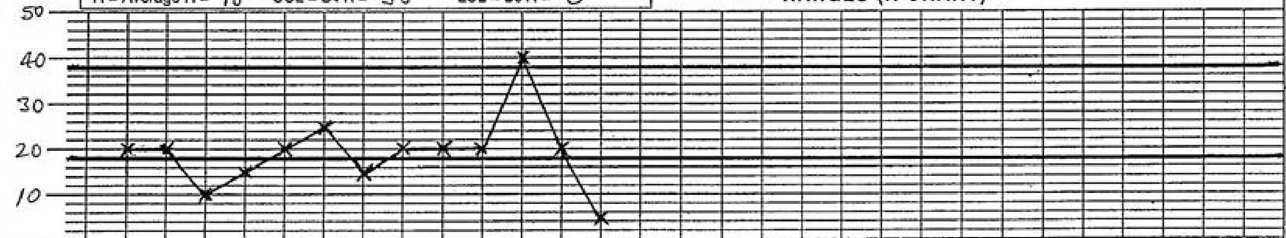
$\bar{\bar{X}}$ = Average $\bar{X}$ = 70 UCL = $\bar{\bar{X}} + A_2\bar{R}$ = 80 LCL = $\bar{\bar{X}} - A_2\bar{R}$ = 60

AVERAGES (X BAR CHART)



$\bar{R}$ = Average R = 18 UCL = $D_4\bar{R}$ = 38 LCL = $D_3\bar{R}$ = 0 *

RANGES (R CHART)



DATE / TIME		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
READINGS 1		65	75	75	60	70	60	75	60	65	60	80	85	70	65	90
2		70	85	80	70	75	75	80	70	80	70	75	75	70	70	80
3		65	75	80	70	65	75	65	80	85	60	90	85	75	85	80
4		65	85	70	75	85	85	75	75	85	80	50	65	75	75	75
5		85	65	75	65	80	70	70	75	75	65	80	70	70	60	85
SUM		350	385	380	340	375	365	365	360	390	335	375	380	360		
$\bar{X}$ = SUM / NO. OF READINGS		70	77	76	68	75	73	73	72	78	67	75	76	72		
R = HIGHEST - LOWEST		20	20	10	15	20	25	15	20	20	20	40	20	5		

* For sample sizes of less than seven, there is no lower control limit for ranges.

ACTION
ON SPECIAL CAUSES

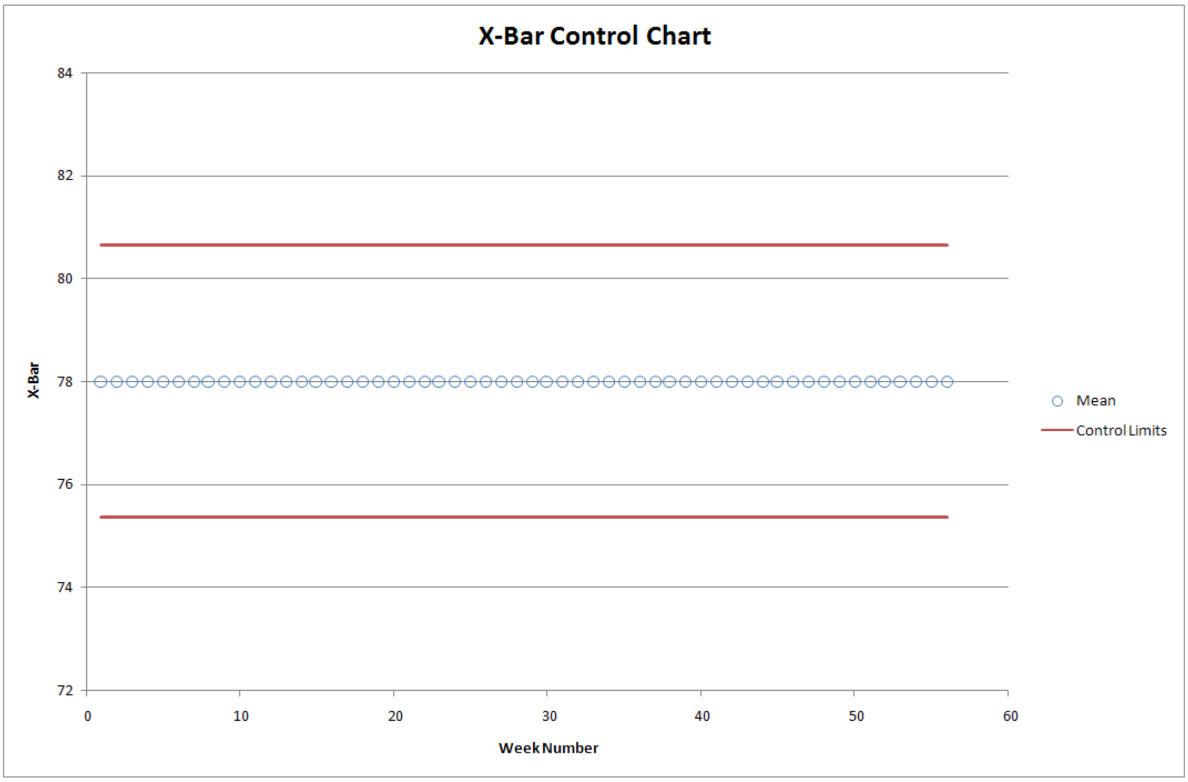
- ANY POINT OUTSIDE OF THE CONTROL LIMITS
- A RUN OF 7 POINTS ALL ABOVE OR ALL BELOW THE CENTRAL LINE
- A RUN OF 7 POINTS UP OR DOWN
- ANY OTHER OBVIOUSLY NON-RANDOM PATTERN

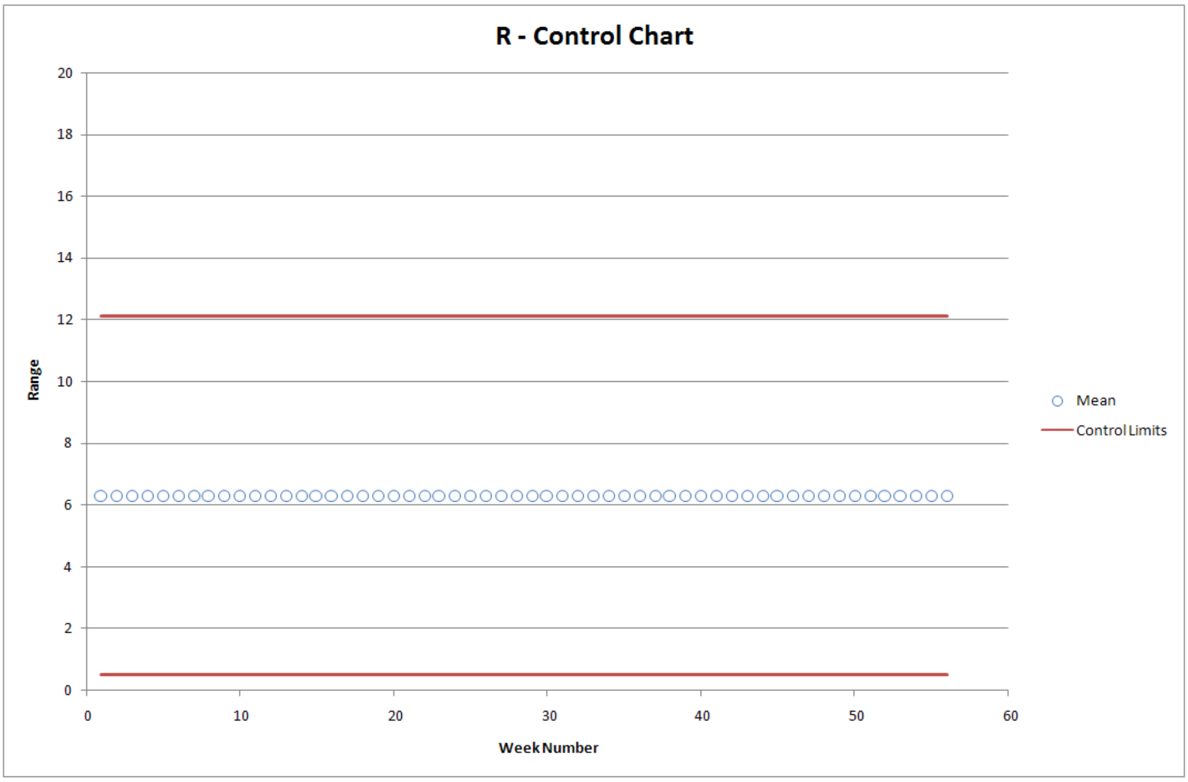
ACTION INSTRUCTIONS

1.
2.
3.
4.
5.

SUBGROUP SIZE	A_2	D_3	D_4
2	1.88	*	3.27
3	1.02	*	2.57
4	.73	*	2.28
5	.58	*	2.11
6	.48	*	2.00
7	.42	.08	1.92
8	.37	.14	1.86
9	.34	.18	1.82
10	.31	.22	1.78

THE PROCESS MUST BE IN CONTROL BEFORE CAPABILITY CAN BE DETERMINED.


X-Bar Control Chart
84
82
80
78
76
74
72
X-Bar
0
10
20
30
40
50
60
Week Number
Mean
Control Limits


R - Control Chart
20
18
16
14
12
10
8
6
4
2
0
Range
0
10
20
30
40
50
60
Week Number
Mean
Control Limits

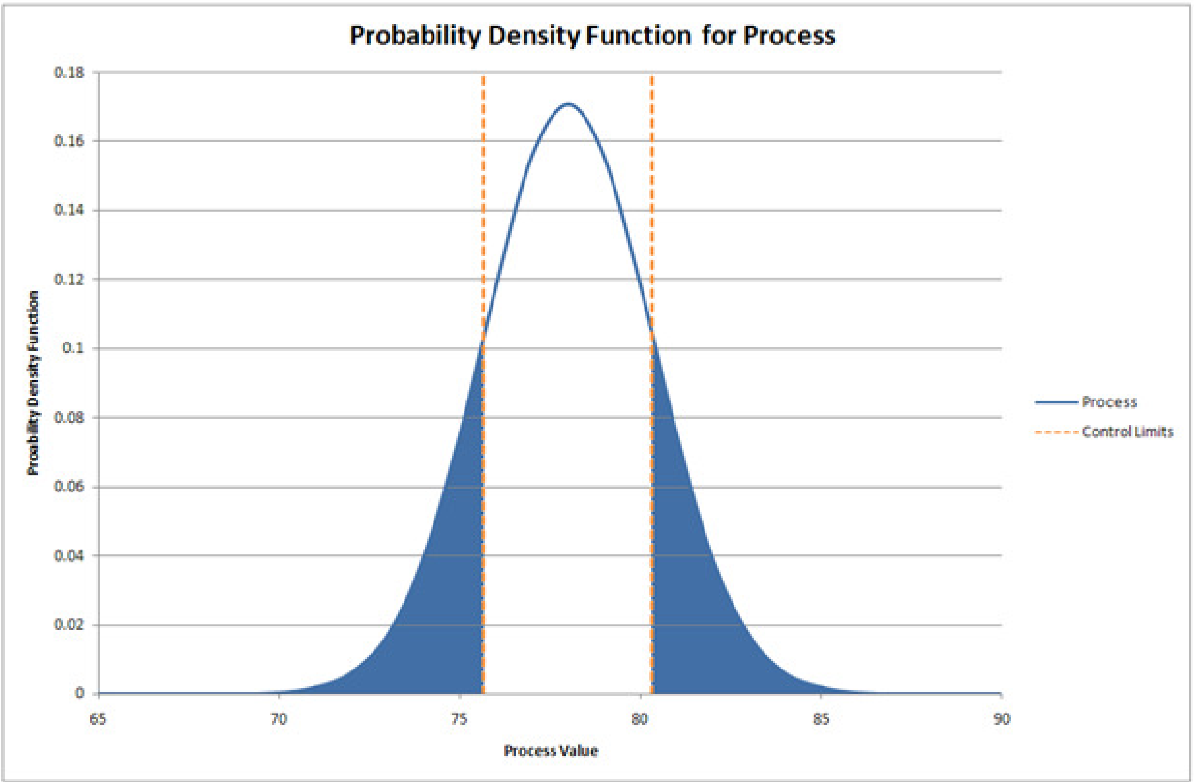
- To determine the standard deviation of the process we use the following equation where *n* is the subsample size, in this case 7:

$$\hat{\sigma} = \frac{A_2\sqrt{n}}{3}\bar{R}$$

$$\hat{\sigma} = \frac{0.42\sqrt{7}}{3}6.3$$

$$\hat{\sigma} = 2.334$$

- We know now the mean of the process which is 78 and the standard deviation of the process which 2.334. The control belt is from 75.354 to 80.646. The non-conformance rate will be equivalent to the area under the curve outside the control limits for the below graph.



The standardised control limits for the process will therefore be given by[1]:

$$SLCL = \frac{75.354 - 78}{2.334} \qquad SUCL = \frac{80.646 - 78}{2.334}$$

$$SLCL = -1.13 \qquad SUCL = 1.13$$

[1] These equations are first introduced in the solutions to Homework 3.

Because the area of the “tails” in the graph on the previous page are symmetric, the percentage of items that fall outside the $\bar{X}$ control belt will be given by the below equation where Z(1.13) is the value obtained from the Z-Table for 1.13.

$$Area = 2[1 - Z(1.13)]$$

$$Area = 2[1 - 0.8708]$$

$$Area = 0.2584$$

Hence, the percentage of items that will fall outside the $\bar{X}$ control belt is 25.8%. This translates to around 2,580 items per 10,000.

Answer to Question 3

- The mean is 74 and the standard deviation 7.3. We want to work out the non-conformance rate if the control limits are set at 50 and 90. Firstly we determine the standardised control limits for the process will therefore be given by:

  $SLL = \frac{50 - 74}{7.3}$ $\qquad$ $SUL = \frac{90 - 74}{7.3}$

  $SLL = -3.29$ $\qquad$ $SUL = 2.19$

  The area of the blue "tails" in the graph on the following page that occur outside the specification limits are given by the following equation:

  $$Area = [1 - Z(2.19)] + [1 - Z(3.29)]$$

  $$Area = [1 - 0.9857] + [1 - 0.9995]$$

  $$Area = [0.0142] + [0.0005]$$

  $$Area = 0.0147$$

  So the non-conformance rate is 1.47%. This translates to around 147 items per 10,000.

- The mean is 67 and the standard deviation 7.3. We want to work out the non-conformance rate if the control limits are set at 50 and 90. Firstly we determine the standardised control limits for the process will therefore be given by:

  $SLL = \frac{50 - 67}{7.3}$ $\qquad$ $SUL = \frac{90 - 67}{7.3}$

  $SLL = -2.33$ $\qquad$ $SUL = 3.15$

  The area of the red "tails" in the graph on the following page that occur outside the specification limits are given by the following equation:

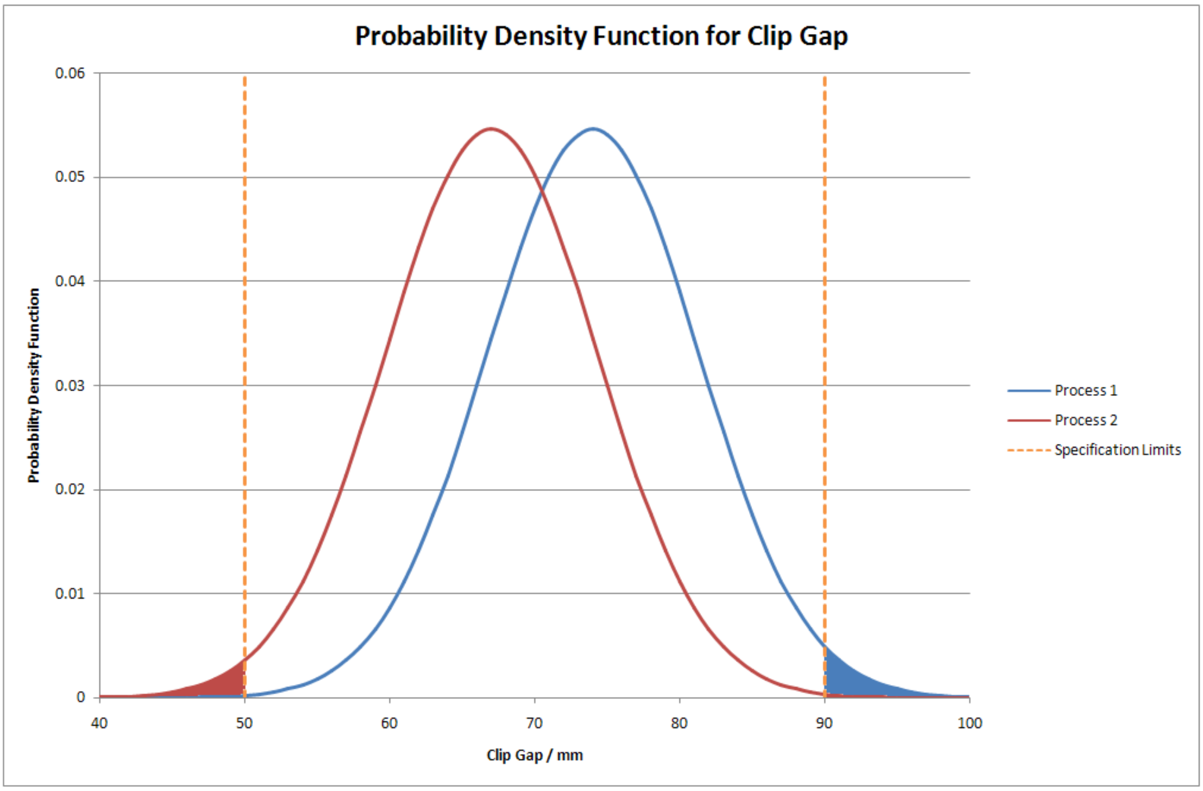
  $$Area = [1 - Z(2.33)] + [1 - Z(3.15)]$$

  $$Area = [1 - 0.9901] + [1 - 0.9992]$$

  $$Area = [0.0099] + [0.0008]$$

  $$Area = 0.0107$$

  So the non-conformance rate is 1.07%. This translates to around 107 items per 10,000.


Probability Density Function for Clip Gap
0.06
0.05
0.04
0.03
0.02
0.01
0
Probability Density Function
40
50
60
70
80
90
100
Clip Gap / mm
Process 1
Process 2
Specification Limits

Answer to Question 4

- When we check the data we notice that point 11 (500.2) is the furthest from the mean. Therefore, we should recalculate the mean and standard deviation excluding this point. Since the summation of all the values is 25,254.7 we can calculate the new mean as follows:

$$\mu = \frac{25254.7 - 500.2}{49}$$

$$\mu = 505.1939$$

We use the following equation for determining standard deviation.:

$$\sigma = \sqrt{\left(\frac{1}{n}\sum_{i=1}^{n} {X_n}^2\right) - \bar{X}^2}$$

We know the summation of the squares of the data values to be 127,561,74, so we must subtract the $(500.2)^2$ component from the summation. Also *n* in this case will be 49 and we must use the new value for the mean:

$$\sigma = \sqrt{\left(\frac{12756174 - 500.2^2}{49}\right) - (505.1939)^2}$$

$$\sigma = 1.76$$

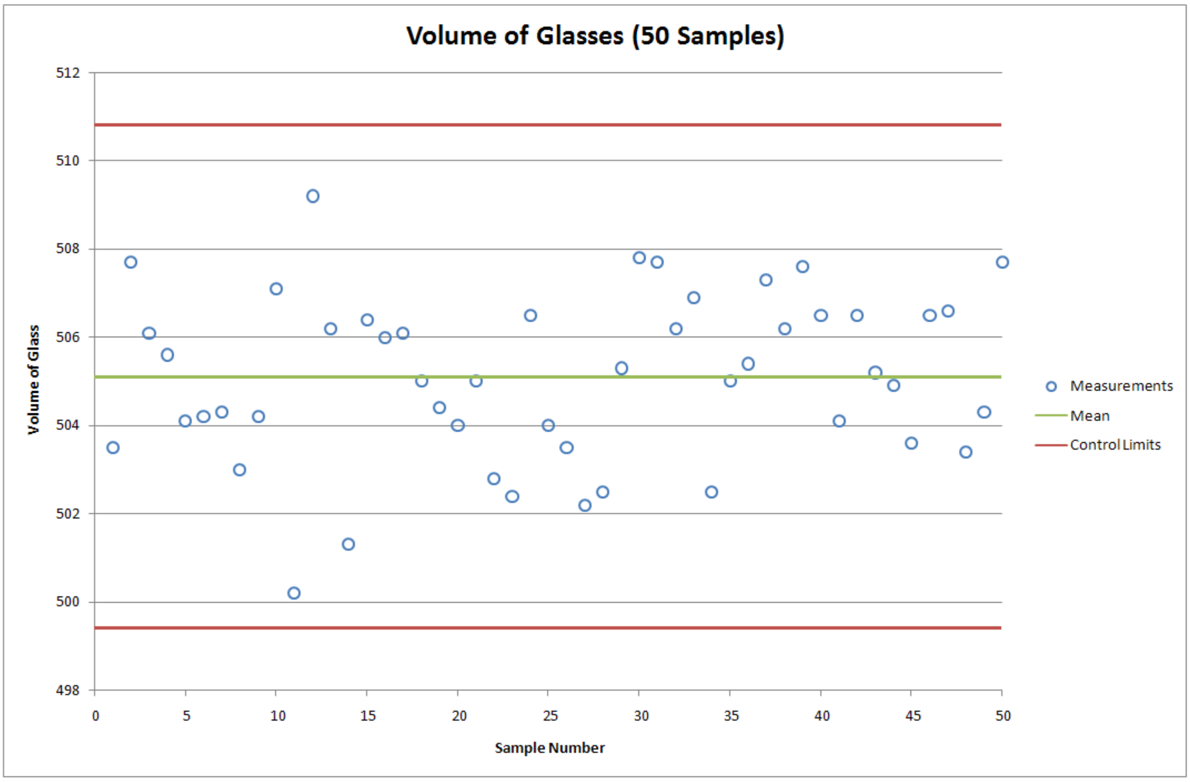
With our new mean and standard deviation, our control limits would be given by:

$LCL = \mu - 3\sigma$	$UCL = \mu + 3\sigma$
$LCL = 505.1939 - 3(1.76)$	$UCL = 505.1939 + 3(1.76)$
$LCL = 499.9139$	$UCL = 510.4739$

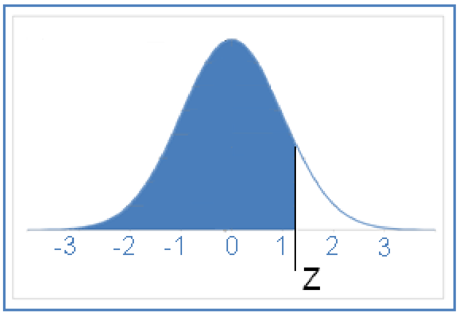
But we notice that point 11 lies within these control limits, implying that we should not have excluded this point. Therefore we calculate the limits using the mean and standard deviation of the entire dataset that were given in the problem.

$LCL = \mu - 3\sigma$	$UCL = \mu + 3\sigma$
$LCL = 505.1 - 3(1.9)$	$UCL = 505.1 + 3(1.9)$
$LCL = 499.4$	$UCL = 510.8$

- The control chart for the individual measurements is shown below:



- By inspecting the chart for any unusual behavior and not finding anyway, we conclude that the process is in control.



Z-Table (Area Under the Curve[2])

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997

[2] Values used in the problems are highlighted in yellow.