Solutions to Homework 5

Solutions to Multiple Choice Questions

1. **E** – Statistical inference is about inferring from the sample data, parameters that described an assumed model for the data. For example, an ideal normal model will have process parameters μ and σ, and we infer knowledge of these unknowns from corresponding values of $\bar{X}$ and s from statistical data.
2. **C** – Estimation uses statistical data to propose reasonable values for the parameters of a model. Estimation is a form of statistical inference that looks for functions of the data that when evaluated, provide reasonable values for the unknown parameters of a proposed model for the data. For example, the mean μ of male height in the world is technically unknown, since it would involve taking the height of every male on the planet, and then calculating the average. But with a reasonably large sample size, we can estimate with a high degree of confidence what we would expect the true mean to be.
3. **E** – Confidence intervals are computed from the data and have a good chance to include the true value of the parameter of interest. Confidence intervals at a chosen confidence level, say 95%, are evaluated from the data and have a 95% chance to include the true mean, i.e. if their computation is repeated for different datasets the proportion of time that they include the true mean will be approximately 95%. Generally we deal with 90%, 95%, and 99% confidence levels.
4. **D** – When testing statistical hypotheses the focus is on not falsely rejecting the null hypothesis. We want the percentage of times the null hypothesis is falsely rejected (Type I Error) to be typically a small percentage such as 5% or 1%. The percentage is set prior to the application of the testing procedure to the data. In this way, the test protects us from falsely rejecting the null hypothesis.
5. **E** –If the confidence level of a confidence interval is, say 95%, then the chances that the true value does not lie within this interval are 5%. Therefore there is a 5% chance that we will falsely reject the null hypothesis. This means we end up with a significance test on the significance level equal to one minus the confidence level.
6. **B** – $\bar{X}$ is a function evaluated from sample data, and as such is a sample statistic, while μ is an unknown parameter of the model we assume for the entire population.

Solutions to Problems

Answer to Question 1

We use the following equation to determine 95% confidence intervals:

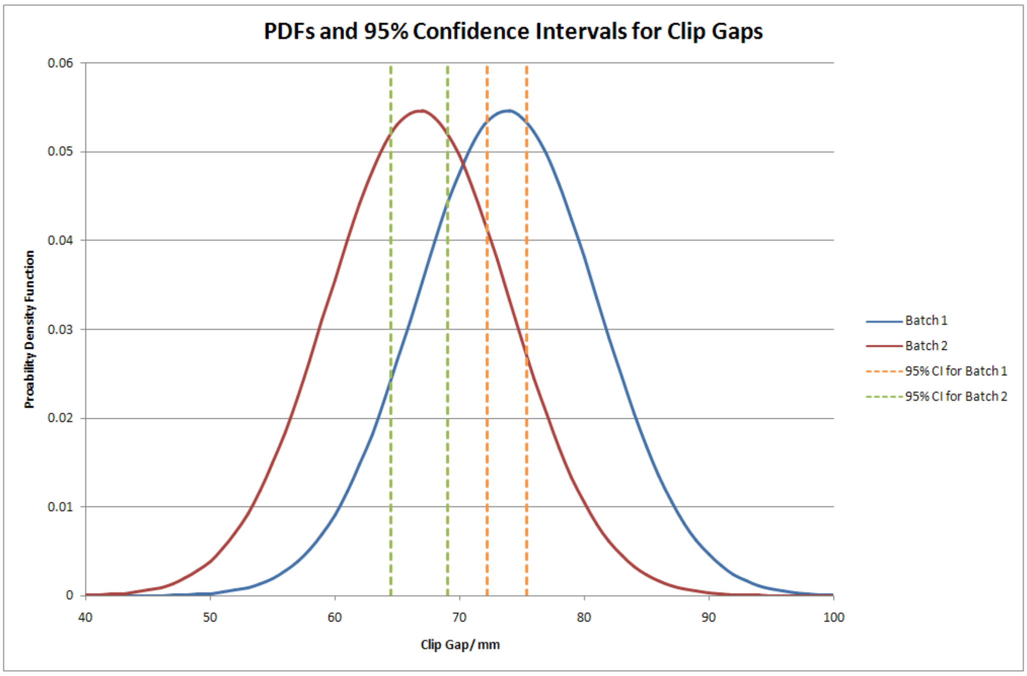
$$\left[\bar{X} - \frac{1.96\sigma}{\sqrt{n}}, \qquad \bar{X} + \frac{1.96\sigma}{\sqrt{n}}\right]$$

$\bar{X}$ = Sample mean
σ = Standard deviation
n = Sample size

The value of 1.96 is used since the area under the standard normal curve between the limits of -1.96 and 1.96 is exactly 95%. We are told in the problem the sample mean for the second batch is now 66.75, the standard deviation remains the same at 7.3, and the sample size is 40. Therefore our 95% confidence interval will be:

$$\left[66.75 - \frac{1.96(7.3)}{\sqrt{40}}, \qquad 66.75 + \frac{1.96(7.3)}{\sqrt{40}}\right]$$

$$[64.49, 69.01]$$



Looking at the graph above, this means that we are 95% confident that the true mean μ of the second batch lies within the dashed green lines. There is only a 5% chance that the true mean lies outside the interval. We also note from the graph that the confidence intervals for the two batches are disjoint. Therefore we conclude that the clip gaps from the second batch are smaller than those from the first.

Answer to Question 2

For the first part of the question, we want to choose a sample size, such that we are 90% confident that the percentage of customers whose accounts turn over more than €100,000 annually will have a confidence interval of ±1%. For example, if our sample size is 1,000 we are 90% confident that 10% ± 1% of customers have an annual turnover of at least €100,000. To determine the sample size we note that the range will be given by the following equation:

$$\left[\hat{p} - 1.645\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \quad \hat{p} + 1.645\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

$\hat{p}$	=	Mean proportion of customers with an annual turnover of at least €100,000
n	=	Sample size

The value of 1.645 is used since the area under the standard normal curve between the limits of -1.645 and 1.645 is exactly 90%. Since we want the half length to be 1% this means:

$$1.645\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.01$$

But we don't know what $\hat{p}$ is, so we need to determine the worst case scenario i.e. the value of $\hat{p}$ that will require the largest sample size *n*. To do this we first rewrite the above equation with *n* as a function of $\hat{p}$.

$$1.645\sqrt{\frac{\hat{p} - \hat{p}^2}{n}} = 0.01$$

$$1.645\sqrt{\hat{p} - \hat{p}^2} = 0.01\sqrt{n}$$

$$164.5\sqrt{\hat{p} - \hat{p}^2} = \sqrt{n}$$

$$\sqrt{n} = 164.5\sqrt{\hat{p} - \hat{p}^2}$$

$$n = (164.5)^2\left(\sqrt{\hat{p} - \hat{p}^2}\right)^2$$

$$n = 27060.25(\hat{p} - \hat{p}^2)$$

We now have the sample size *n* written as a function of $\hat{p}$. We differentiate this function and let the derivative equal to zero. This will tell us for which value of $\hat{p}$ the sample size *n* will be a maximum.

$$n = 27060.25(\hat{p} - \hat{p}^2)$$

$$\frac{dn}{d\hat{p}} = 27060.25(1 - 2\hat{p})$$

$$\frac{dn}{d\hat{p}} = 27060.25(1 - 2\hat{p}) = 0$$

$$27060.25(1 - 2\hat{p}) = 0$$

$$1 - 2\hat{p} = 0$$

$$\hat{p} = 0.5$$

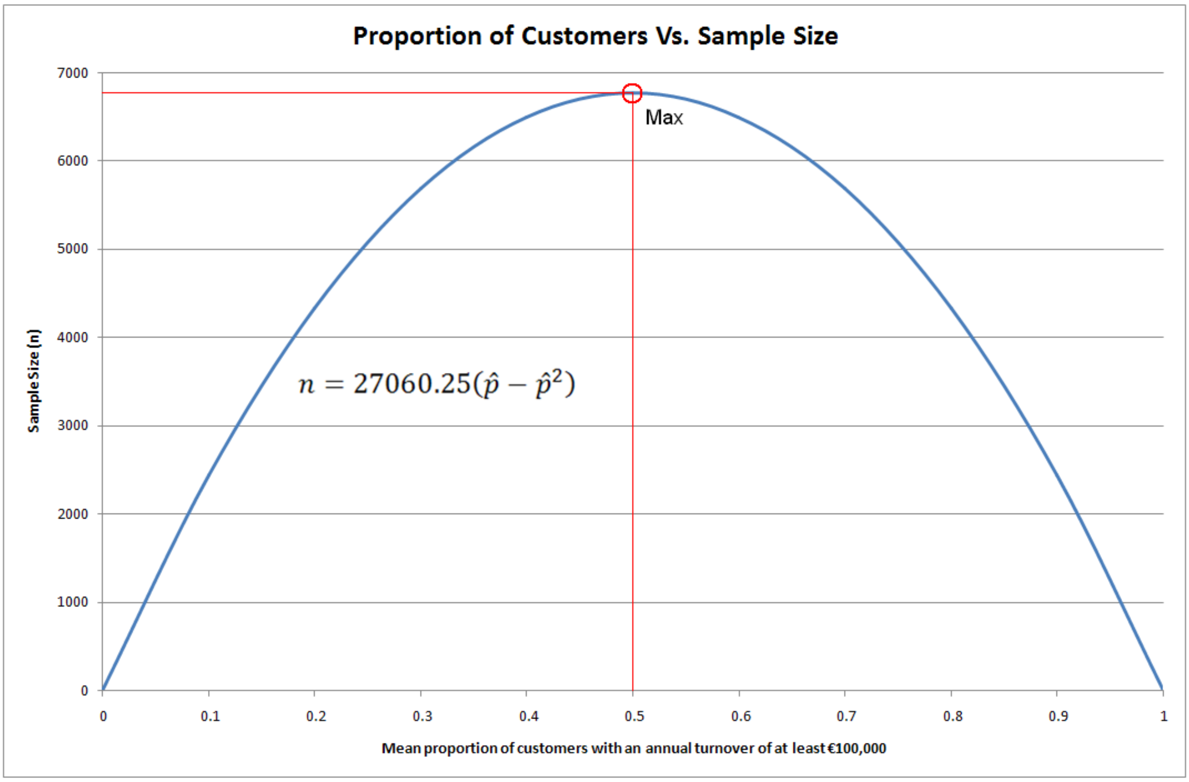
So when 50% of the customers have an annual turnover of at least €100,000 is when we will need the largest sample size. The actual size of the largest sample size will be determined from:

$$n = 27060.25(0.5 - 0.5^2)$$

$$n = 27060.25(0.5 - 0.25)$$

$$n = 27060.25(0.25)$$

$$n \cong 6765$$



So we need a minimum sample size of 6,765 if we want to be at least 90% confident that the value we obtain for the percentage of customers with an annual turnover of at least €100,000 has a confidence interval of ±1%.

For the second part of the question, we want to choose a sample size, such that we are 90% confident that the percentage of customers whose accounts turn over more than €100,000 annually will have a confidence interval of ±2%. For example, if our sample size is 1,000 we are 90% confident that 10% ± 2% of customers have an annual turnover of at least €100,000. To determine the sample size again we note that the range will be given by the following equation:

$$\left[\hat{p} - 1.645\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \quad \hat{p} + 1.645\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

$\hat{p}$ = Mean proportion of customers with an annual turnover of at least €100,000
n = Sample size

The value of 1.645 is used since the area under the standard normal curve between the limits of -1.645 and 1.645 is exactly 90%. Since we want the half length to be 2% this means:

$$1.645\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.02$$

But we don't know what $\hat{p}$ is, so we need to determine the worst case scenario i.e. the value of $\hat{p}$ that will require the largest sample size *n*. To do this we first rewrite the above equation with *n* as a function of $\hat{p}$.

$$1.645\sqrt{\hat{p} - \hat{p}^2} = 0.02\sqrt{n}$$

$$82.25\sqrt{\hat{p} - \hat{p}^2} = \sqrt{n}$$

$$\sqrt{n} = 82.25\sqrt{\hat{p} - \hat{p}^2}$$

$$n = (82.25)^2\left(\sqrt{\hat{p} - \hat{p}^2}\right)^2$$

$$n = 6765.0625(\hat{p} - \hat{p}^2)$$

We now have the sample size *n* written as a function of $\hat{p}$. Again, like in the first part of this question, this function is a maximum when $\hat{p} = 0.5$. The actual size of the largest sample size will be determined from:

$$n = 6765.0625(0.5 - 0.5^2)$$

$$n \cong 1691$$

So we need a minimum sample size of 1,691 if we want to be at least 90% confident that the value we obtain for the percentage of customers with an annual turnover of at least €100,000 has a confidence interval of ±2%.

For the third part of the question, we want to choose a sample size, such that we are 95% confident that the percentage of customers whose accounts turn over more than €100,000 annually will have a confidence interval of ±1%. For example, if our sample size is 1,000 we are 95% confident that 10% ± 1% of customers have an annual turnover of at least €100,000. To determine the sample size again we note that the range will be given by the following equation:

$$\left[\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \quad \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

$\hat{p}$ = Mean proportion of customers with an annual turnover of at least €100,000
n = Sample size

The value of 1.96 is used since the area under the standard normal curve between the limits of -1.96 and 1.96 is exactly 95%. Since we want the half length to be 1% this means:

$$1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.01$$

But we don't know what $\hat{p}$ is, so we need to determine the worst case scenario i.e. the value of $\hat{p}$ that will require the largest sample size *n*. To do this we first rewrite the above equation with *n* as a function of $\hat{p}$.

$$1.96\sqrt{\hat{p} - \hat{p}^2} = 0.01\sqrt{n}$$

$$196\sqrt{\hat{p} - \hat{p}^2} = \sqrt{n}$$

$$\sqrt{n} = 196\sqrt{\hat{p} - \hat{p}^2}$$

$$n = (196)^2\left(\sqrt{\hat{p} - \hat{p}^2}\right)^2$$

$$n = 38416(\hat{p} - \hat{p}^2)$$

We now have the sample size *n* written as a function of $\hat{p}$. Again, like in the first part of this question, this function is a maximum when $\hat{p} = 0.5$. The actual size of the largest sample size will be determined from:

$$n = 38416(0.5 - 0.5^2)$$

$$n = 9604$$

So we need a minimum sample size of 9,604 if we want to be at least 95% confident that the value we obtain for the percentage of customers with an annual turnover of at least €100,000 has a confidence interval of ±1%.

For the final part of the question, we want to choose a sample size, such that we are 95% confident that the percentage of customers whose accounts turn over more than €100,000 annually will have a confidence interval of ±2%. For example, if our sample size is 1,000 we are 95% confident that 10% ± 2% of customers have an annual turnover of at least €100,000. To determine the sample size again we note that the range will be given by the following equation:

$$\left[\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \quad \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

$\hat{p}$ = Mean proportion of customers with an annual turnover of at least €100,000
n = Sample size

The value of 1.96 is used since the area under the standard normal curve between the limits of -1.96 and 1.96 is exactly 95%. Since we want the half length to be 1% this means:

$$1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.02$$

But we don't know what $\hat{p}$ is, so we need to determine the worst case scenario i.e. the value of $\hat{p}$ that will require the largest sample size *n*. To do this we first rewrite the above equation with *n* as a function of $\hat{p}$.

$$1.96\sqrt{\hat{p} - \hat{p}^2} = 0.02\sqrt{n}$$

$$98\sqrt{\hat{p} - \hat{p}^2} = \sqrt{n}$$

$$\sqrt{n} = 98\sqrt{\hat{p} - \hat{p}^2}$$

$$n = (98)^2\left(\sqrt{\hat{p} - \hat{p}^2}\right)^2$$

$$n = 9604(\hat{p} - \hat{p}^2)$$

We now have the sample size *n* written as a function of $\hat{p}$. Again, like in the first part of this question, this function is a maximum when $\hat{p} = 0.5$. The actual size of the largest sample size will be determined from:

$$n = 9604(0.5 - 0.5^2)$$

$$n = 2401$$

So we need a minimum sample size of 2,401 if we want to be at least 95% confident that the value we obtain for the percentage of customers with an annual turnover of at least €100,000 has a confidence interval of ±2%.

Answer to Question 3

The null hypothesis H_O is that the theoretical mean, $\mu = 0$. Therefore, the alternative hypothesis H_a is that the theoretical mean, $\mu \neq 0$. We decide to pick a significance level of 5%. Also, because the null hypothesis we are checking only uses the equality symbol (=) the test is two-sided. Since the sample size is greater than 30, the standardised statistics for this test is:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$\bar{X}$	=	Sample mean
μ	=	Theoretical mean
σ	=	Standard deviation (assume same as sample standard deviation s)
n	=	Sample size
Z	=	Number of standard deviations from the mean

We are told in the problem the sample mean is 0.6, the standard deviation is 4.15, and the sample size is reduced from 52 to 49 (3 values were exceptional and thus rejected). The null hypothesis also claims that the theoretical mean is 0.

$$Z = \frac{0.6 - 0}{\frac{4.15}{\sqrt{49}}}$$

$$Z = \frac{\sqrt{49}(0.6)}{4.15}$$

$$Z = \frac{7(0.6)}{4.15}$$

$$Z = 1.01$$

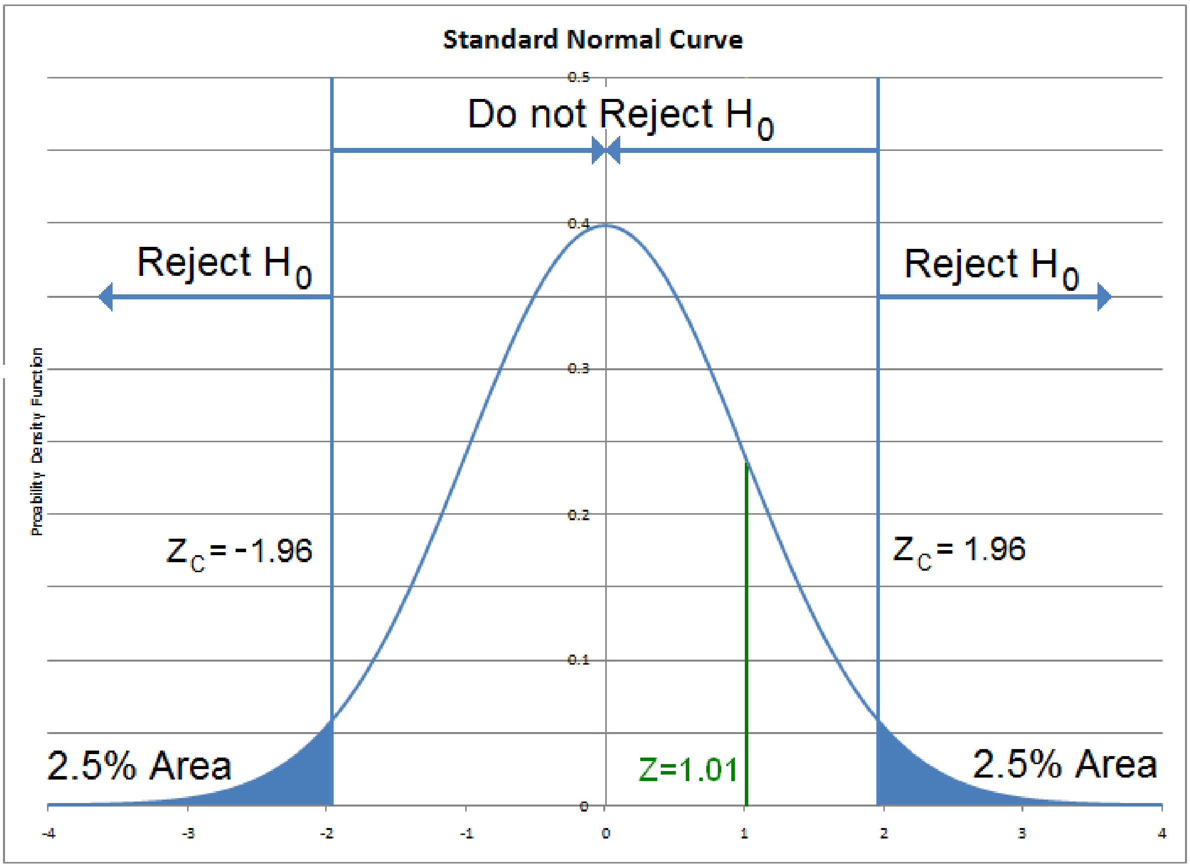
Since the test is two-sided and we have a significance level of 5% we will have two cutoff Z values; -1.96 and 1.96. We use these values since the area under the standard normal curve outside these two limits is 5%. Since the test statistics lies between these two limits we do not reject the null hypothesis. The *p*-value or observed significance level can be determined from the normal distribution table (given on the second last page) for the Z value of 1.01. We calculate the area outside the limits of -1.01 and 1.01:

$$Area = 2[1 - Z(1.01)]$$

$$Area = 2[1 - 0.8438)]$$

$$Area = 0.3124$$

Therefore our *p*-value is 31.24%, which is fairly large. This means that if the null hypothesis is true, and the theoretical mean is 0 then we have a 31.24% chance of observing values which are at least as extreme as 0.6 the sample mean. If the *p*-value is less than 5% this means, that if the null hypothesis was true, and the theoretical mean was 0, then we would have less than a 5% chance of observing values at least as extreme as 0.6, the sample mean. Comparing the *p*-value with the significance level is an alternative way of testing the null hypothesis; since our *p*-value is significantly larger than 5% we do not reject the null hypothesis. In the below graph, we see that the Z value 1.01, corresponds to the green line. Since this is between the cutoff values of -1.96 and 1.96 (the area under the curve between the limits of -1.96 and 1.96 is 95%) we do not reject the null hypothesis.



Answer to Question 4

The null hypothesis H_O is that the theoretical mean, $\mu \geq 100$. Therefore the alternative hypothesis H_a is that the theoretical mean, $\mu < 100$. We decide to pick a significance level of 5%. Also, because the null hypothesis we are checking includes the greater-than symbol (>) the test is one-sided. Since the sample size is greater than 30, the standardised statistics for this test is:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$\bar{X}$	=	Sample mean
μ	=	Theoretical mean
σ	=	Standard deviation (assume same as sample standard deviation *s*)
n	=	Sample size
Z	=	Number of standard deviations from the mean

We are told in the problem the sample mean is 95.53, the standard deviation is 24, and the sample size is 250. The null hypothesis also claims that the theoretical mean is 100.

$$Z = \frac{95.53 - 100}{\frac{24}{\sqrt{250}}}$$

$$Z = \frac{\sqrt{250}(95.53 - 100)}{24}$$

$$Z = \frac{\sqrt{250}(-4.47)}{24}$$

$$Z = -2.94$$

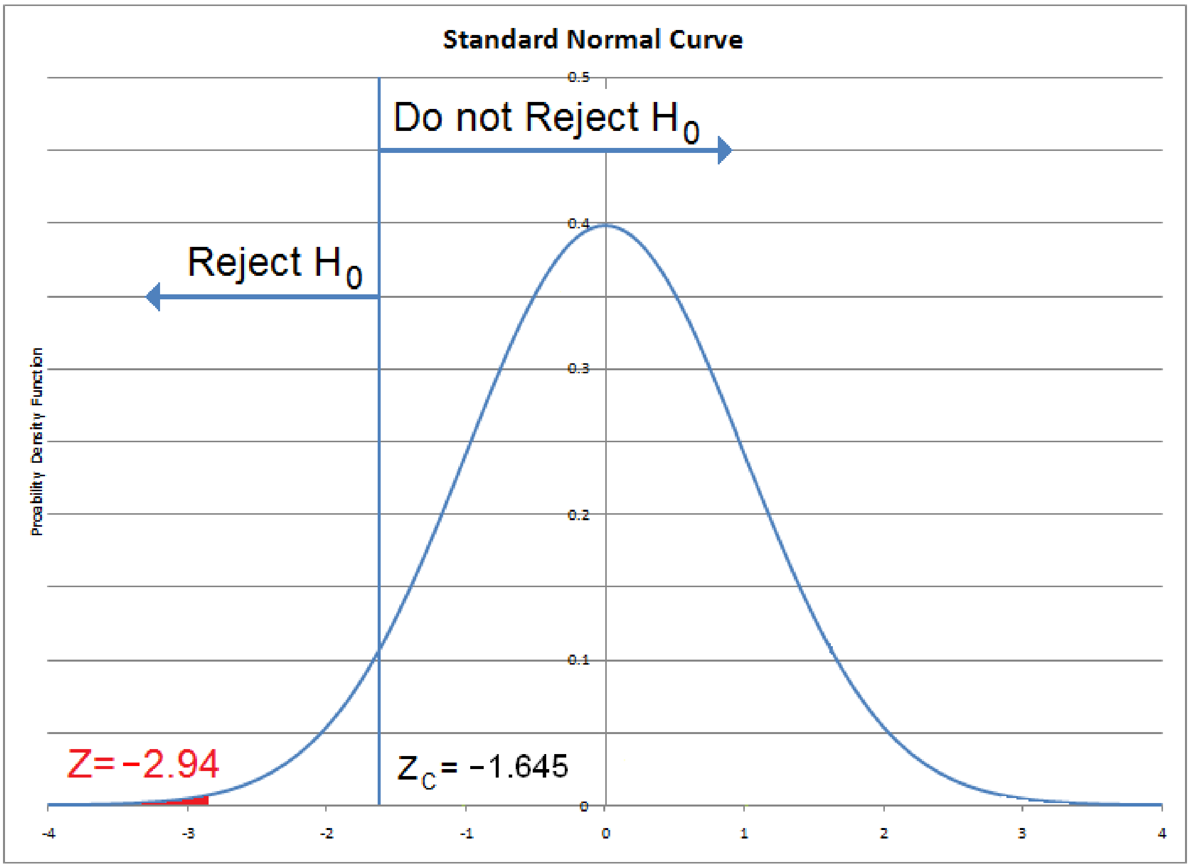
Since the test is one-sided, and we are using the greater-than symbol, and we have a significance level of 5% we will have a single cutoff Z value of -1.645. We use this value since the area under the standard normal curve from minus infinity to -1.645 is 5%. Since the test statistics lies to the left of -1.645 we reject the null hypothesis. The *p*-value or observed significance level can be determined from the normal distribution table for the Z value of 2.94. Since our null hypothesis is one-sided we calculate the area under the curve from minus infinity to -2.94. Due to symmetry this is the same as the area under the curve from 2.94 to plus infinity.

$$Area = 1 - Z(2.94)$$

$$Area = 1 - 0.9984$$

$$Area = 0.0016$$

Therefore our *p*-value is 0.16%, which is very small. This means that if the null hypothesis is true, and the theoretical mean is greater than or equal to €100, then we would only have a 0.16% chance of observing values at least as low as €95.53, the sample mean. Comparing the *p*-value with the significance level is an alternative way of testing the null hypothesis; since our *p*-value is significantly smaller than 5% we reject the null hypothesis. In the below graph, we see that the test statistic is -2.94, and is lower than the cutoff Z value of -1.645 (the area under the curve to the left of -1.645 is 5%). Therefore we reject the null hypothesis. Also, the area under the curve shown in red is equivalent to the *p*-value of 0.16%.



Answer to Question 5

We let μ represent the theoretical weight of loaves, and we set the null hypothesis as μ = 800, and the alternative hypothesis as μ < 800. We decide to pick a significance level of 5%. Also, because the null hypothesis we are checking only uses the equality symbol (=) the test is two-sided. Since the sample size is less than 30 the standardised statistics for this test is:

$$T = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$$

$\bar{X}$	=	Sample mean
μ	=	Theoretical mean
s	=	Sample Standard deviation (assume same as sample standard deviation *s*)
n	=	Sample size
T	=	Student's t-distribution with *n-1* degrees of freedom

We are told in the problem the sample mean is 792, the sample standard deviation is 25, and the sample size is 10. Because the sample size is less than 30 we should use the Student's t-distribution with *n-1* degrees of freedom (in our case 9 degrees of freedom) instead of the normal distribution. The null hypothesis also claims that the theoretical mean is 800.

$$T = \frac{792 - 800}{\frac{25}{\sqrt{10}}}$$

$$T = -1.01$$

The critical value from the Student's t-distribution (given on the last page) with 9 degrees of freedom for a 95% confidence interval is 2.262. Since the test statistic lies within the limits of -2.262 and 2.262 we must conclude that there is not enough evidence to claim that the loaves are lighter than 800 grams. The 95% confidence interval computed based on the Student's t-distribution is:

$$\left[\bar{X} - \frac{2.262\sigma}{\sqrt{n}}, \qquad \bar{X} + \frac{2.262\sigma}{\sqrt{n}}\right]$$

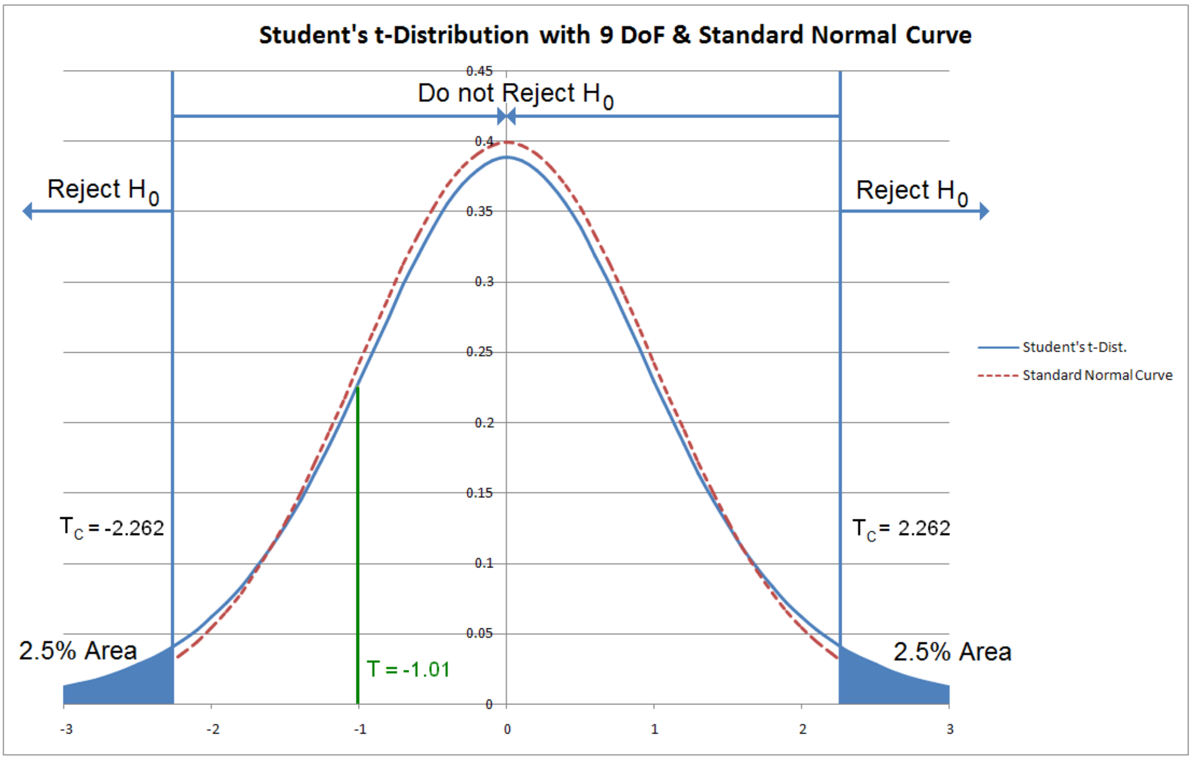
$$\left[792 - \frac{2.262(25)}{\sqrt{10}}, \qquad 792 + \frac{2.262(25)}{\sqrt{10}}\right]$$

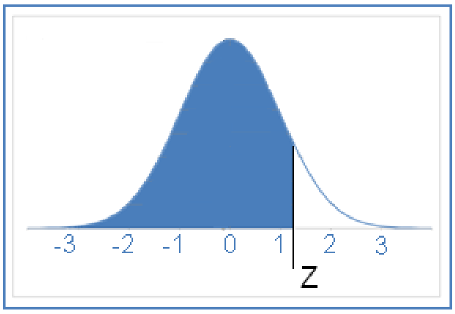
$$[792 - 17.89, \qquad 792 + 17.89]$$

$$[774.11, 809.89]$$

We see (as expected) that 800 is contained within the 95% confidence interval for the sample. This means we cannot exclude 800 as a possible value for the theoretical mean. We do not reject the null hypothesis.

In the below graph, we see that the test statistic is -1.01, and it is between the cutoff values of -2.262 and 2.262. The area under the Student's t-distribution curve with 9 degrees-of-freedom outside the limits of -2.262 and 2.262 is 5%. The *p*-value for this question is equivalent to the area under the blue curve outside the limits of -1.01 and 1.01. However, when we check the Student's t-distrubtion table (given on the last page) for 9 degrees-of-freedom we see that the smallest limit is 1.383, so we are unable to calculate the *p*-value. Also, it should be noted that as we increase the degrees-of-freedom of the Student's t-distribution, the curve starts to matches the standard normal curve. When the degrees-of-freedom is infinity, the Student's t-distribution corresponds to the standard normal distribution. In the below graph, we can see that the blue curve (9 degrees-of-freedom) closely matches the standard normal curve shown in red.

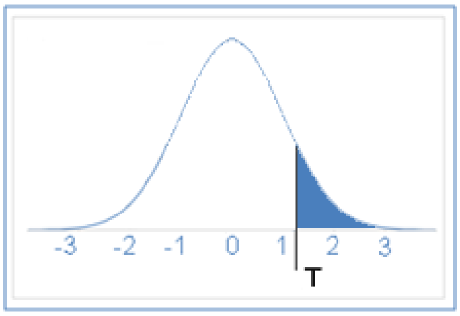




Z-Table (Area Under the Curve[1])

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	$\boxed{0.8438}$	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	$\boxed{0.9984}$	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997

[1] Values used in the problems are boxed in red.



Student's t-Distribution Table[2]

Area	10%	5%	4%	2.5%	2.0%	1%	0.5%	0.25%	0.1%	0.05%
DoF										
1	3.078	6.314	7.916	12.706	15.894	31.821	63.656	127.321	318.289	636.578
2	1.886	2.920	3.320	4.303	4.849	6.965	9.925	14.089	22.328	31.600
3	1.638	2.353	2.605	3.182	3.482	4.541	5.841	7.453	10.214	12.924
4	1.533	2.132	2.333	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	1.476	2.015	2.191	2.571	2.757	3.365	4.032	4.773	5.894	6.869
6	1.440	1.943	2.104	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	1.415	1.895	2.046	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	1.397	1.860	2.004	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	$\boxed{1.383}$	1.833	1.973	$\boxed{2.262}$	2.398	2.821	3.250	3.690	4.297	4.781
10	1.372	1.812	1.948	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	1.363	1.796	1.928	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	1.356	1.782	1.912	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	1.350	1.771	1.899	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	1.345	1.761	1.887	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	1.341	1.753	1.878	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	1.337	1.746	1.869	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	1.333	1.740	1.862	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	1.330	1.734	1.855	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	1.328	1.729	1.850	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	1.325	1.725	1.844	2.086	2.197	2.528	2.845	3.153	3.552	3.85
21	1.323	1.721	1.840	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	1.321	1.717	1.835	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	1.319	1.714	1.832	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	1.318	1.711	1.828	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	1.316	1.708	1.825	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	1.315	1.706	1.822	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	1.314	1.703	1.819	2.052	2.158	2.473	2.771	3.057	3.421	3.689
28	1.313	1.701	1.817	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	1.311	1.699	1.814	2.045	2.150	2.462	2.756	3.038	3.396	3.660
30	1.310	1.697	1.812	2.042	2.147	2.457	2.75	3.030	3.385	3.646
inf	1.282	1.654	1.751	1.960	2.054	2.326	2.576	2.807	3.090	3.290

[2] Values used in the problems are boxed in red.