Homework 6

Here we present solutions to the problems posted in the sixth homework assignment. The solutions and related commentary are put in italics. Remember that problems can have several different but correct ways of solving them.

Multiple choice questions

1. Student-t distribution is used

   A when data do not follow the normal model;

   B when there is too many data to account for the normal model;

   C when the standard deviation of the model is known;

   D for testing percentages and proportions;

   ***E** for the data that follow the normal model with unknown standard deviation specially when sample sizes are small.

   Solution: *Student-t distribution is the distribution of the standardized statistic*

   $$\frac{\bar{X} - \mu}{s/\sqrt{n}}$$

   that is used instead of the normal distribution because of presence of the estimated standard deviation s instead of the unknown model standard deviation σ. For large samples, the difference between these two disapear.

2. The paired sample test applies to

   A testing the difference of two means for two samples taken independently from two different populations;

   ***B** testing the mean difference between elements of sampled pairs of values;

   C testing equality of standard deviations for two samples;

   D testing proportions between two populations;

   E pooled sample made of two samples of the same size.

   Solution: *The paired sample test is used on pairs of data that have something in common and when we test for the difference between the means of elements of these pairs.*

3. If two samples are taken independently and are of different and small sizes, then

   A it is impossible to test for the equality of their means;

   ***B** one can use Student-t distribution to test the difference of their means if their variances are equal;

   C one can use approximate normal distribution to test the difference of their means;

D one can use Student-t distribution to test the difference of their means if their variances are not equal;

E one can used the paired sample test to test the difference of their means.

Solution: *Two sample test is using Student-t distribution to test for the difference in the means of two indpenedent samples taken from different populations. Samples can be of different sizes. The test statistics is based on the so called pooled standard deviation and has the form*

$$T = \frac{\bar{X}_1 - \bar{X}_2}{s_p\sqrt{1/n_1 + 1/n_2}}.$$

4. The observed significance level (also called p-value)

   A is always smaller than the significance level α;

   B represents our believe that the null hypothesis is correct;

   ***C** can be used for testing by rejecting the null hypothesis if it is smaller than the significance level α;

   D is equal to the power of a test;

   E is equal to the chances of Type I Error.

   Solution: *Instead of test statistic and the critical value for it, one can equivalently use the p-value and reject the null hypothesis if it is smaller than the decided significance level.*

5. The significance level α is

   ***A** equal to chances of rejecting the null hypothesis given that it is true;

   B is equal to chances of Type II Error;

   C is equal to one minus the chances of Type II Error;

   D is always set to 5%;

   E is smaller more data are available.

   Solution: *The significance level α represents chances of making Type I Error which is rejecting the null hypothesis while it is true.*

Problems

1. A sample of scores one an examination given in Business Statistics are:

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Males:   72 69 98 66 85 76 79 80 77
Females: 81 67 90 78 81 80 76
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   At the 5% significance level test if there is a difference in the mean scores between female and male students. In the process of doing this

- write down the test statistic that is used for this purpose of testing;
- identify the distribution of this statistic;
- find from the table the critical value for this test;
- evaluate test statistic;
- compare the evaluated value with the critical value;
- write the conclusion to the test.

The following values can be found usefull when solving the problem: the sample mean for males is 78 and sample standard deviation is 9.49, the sample mean for females is 79 and sample standard deviation is 6.88.

Solution: *This is a typical problem of testing for the difference of means of two independent samples. We are testing*

$$\mathbf{H}_0 : \mu_1 = \mu_2$$

vs.

$$\mathbf{H}_a : \mu_1 \neq \mu_2,$$

i.e. the two-sided version of the test.

- *The test statistics that is used for this purpose is*

$$T = \frac{\bar{X}_1 - \bar{X}_2}{s_p\sqrt{1/n_1 + 1/n_2}},$$

where s_p is the so-called pooled standard deviations and is obtained from the standard deviations for each sample according to the formula

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}.$$

- *This statistic follows the Student-t distribution with $n_1 + n_2 - 2$ degrees of freedom. In our case 9+7-2=14.*
- *Thus the critical value for this test can be found in the table of Student-t distribution (see the handout link on our homepage) and it is for the two-sided equal to 2.145.*
- *We evaluate our test statistic starting with the evaluation of the pooled standard deviation:*

$$s_p = \sqrt{(8 * 9.49^2 + 6 * 6.88^2)/14} = \sqrt{71.75} = 8.47.$$

Then

$$T = \frac{78 - 79}{8.47\sqrt{1/9 + 1/7}} = -0.2342755.$$

For the two sided test in order to reject the null hypothesis we should be either below negative critical value -2.145 or above positive critical value which is 2.145.

- *We conclude that there is no significant difference in scoring on the business statistics exam between female and male students.*

2.
- Cigarette Tar Content Problem have been used in the lecture to illustrate importance of accounting for variation in the process of decision making. Here we will discuss the problem for more comprehensive data set of 100 measurements of the tar content of a certain type of cigarette. The average value has been reported 14.1 and standard deviation was reported equal to 0.3 (all in milligrams). Test for the difference between the mean of this sample (14.1 mg/cg) and the average tar content claimed by the cigarette manufacturer, $\mu = 14.0$ at the 5% significance level ($\alpha = 0.05$).
- Assume that another brand of cigaretes has been also studied and a sample of 50 cigarettes has been taken and the tar content has been measured yielding the mean 14.2 and sample standard deviation 1.3 has been reported. Test the same hypothesis as before for this new data set.
- In view of the obtained results, discuss if looking only at the mean tar content is a sufficient criterium of the quality of cigarettes.

Solution:

- *Here we deal with relatively large sample sizes thus we can consider large sample test as discussed in the lecture. We are testing*

$$\mathbf{H}_0 : \mu \leq 14.0$$

vs.

$$\mathbf{H}_a : \mu > 14.0$$

at 5% significance level so that the critical value for this test is 1.645. The test statistic is given by

$$Z = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{14.1 - 14.0}{0.3/10} = 3.33$$

and the critical value is exceeded and we have the right to reject the null hypothesis.

- *For the same test but for other brand of cigarettes we have*

$$Z = \frac{14.2 - 14.0}{1.3/\sqrt{50}} = 1.09$$

and this value does not exceed the critical value of 1.645. For the other brand we can not reject the null hypothesis.

- *We note that the lack of the conclusion for the second brand is due to the larger variability of their product. In order to test this brand more cigarettes has to be tested to account for large variability. This emphasize the fact that for testing the quality of the product it is very important to analyze variability as measured by standard deviation together with the mean.*