Discrete Random Variables

1. On a roulette wheel there are 37 numbers 0,1,,36. 18 numbers are black. If I bet 1 on black, I win 1 if a black number comes up, otherwise I lose my stake. Let X denote my winnings on one bet.

   (i) Calculate E(X) and Var(X)

   Suppose I make 6 such bets. Let Y denote my total winnings.

   (ii) Derive the distribution of Y.

   (iii) Calculate E(Y) and Var(Y)

2. The probability distribute of discrete random variable X is tabulated below. There are 5 possible outcome of X, i.e. 1, 2, 3, 4 and 5.

x_i	1	2	3	4	5
$p(x_i)$	0.30	0.20	k	0.10	0.20

   a. (1 Mark) Compute the value of k.

   b. (1 Mark) What is the expected value of X?

   c. (1 Mark) Given that $E(X^2) = 9.5$, compute the variance of X.

1. The probability distribution of discrete random variable X is tabulated below. There are 6 possible outcome of X, i.e. 0, 1, 2, 4 ,8 and 10.

x_i	0	1	2	4	8	10
$P(x_i)$	0.25	0.15	0.25	0.15	k	0.10

   i. (1 marks) Compute the value for k.

   ii. (3 marks) Determine the expected value $E(X)$.

   iii. (2 marks) Evaluate $E(X^2)$.

   iv. (3 marks) Compute the variance of random variable X.

2. Suppose X is a random variable with

   - $E(X^2) = 3.6$
   - $P(X = 2) = 0.6$
   - $P(X = 3) = 0.1$

   (a) The random variable takes just one other value besides 2 and 3. This value is greater than 0. What is this value?

   (b) What is the variance of X?

3. Consider the random variables X and Y. Both X and Y take the values 0, 1 and 2. The joint probabilities for each pair are given by the following table.

	$X=0$	$X=1$	$X=2$
$Y=0$	0.1	0.15	0.1
$Y=1$	0.1	0.1	0.1
$Y=2$	0.2	0.05	0.1

Compute the $E(U)$ expected value of U, where $U = X - Y$.

Given:

Suppose X is a random variable with

- $E(X^2) = 3.6$
- $P(X = 2) = 0.6$
- $P(X = 3) = 0.1$

Questions:

(a) The random variable takes just one other value besides 2 and 3. This value is greater than 0. What is this value?

(b) What is the variance of X?

Part a

- Determine the missing value (let's call it k).

- First we determine the probability of that value.
- We know that $E(X^2) = 3.6$. Let use the approach for computing $E(X^2)$.

x_i	2	3	k
x_i^2	4	9	k^2
$p(x_i)$	0.6	0.1	

Part a

- Determine the missing value (let's call it k).

- First we determine the probability of that value.
- We know that $E(X^2) = 3.6$. Let use the approach for computing $E(X^2)$.

x_i	2	3	k
x_i^2	4	9	k^2
$p(x_i)$	0.6	0.1	**0.3**

x_i	2	3	k
x_i^2	4	9	k^2
$p(x_i)$	0.6	0.1	0.3

$$E(X^2) = \sum x_i^2 \cdot p(x_i) = 3.6$$

x_i	2	3	k
x_i^2	4	9	k^2
$p(x_i)$	0.6	0.1	0.3

$$E(X^2) = \sum x_i^2 \cdot p(x_i) = 3.6$$

$$(4 \times 0.6) + (9 \times 0.1) + (k^2 \times 0.3) = 3.6$$

x_i	2	3	k
x_i^2	4	9	k^2
$p(x_i)$	0.6	0.1	0.3

- $E(X^2) = \sum x_i^2 \cdot p(x_i) = 3.6$
- $(4 \times 0.6) + (9 \times 0.1) + (k^2 \times 0.3) = 3.6$
- $2.4 + 0.9 + (k^2 \times 0.3) = 3.6$
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- $3.3 + 0.3k^2 = 3.6$
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$$(4 \times 0.6) + (9 \times 0.1) + (k^2 \times 0.3) = 3.6$$

$$2.4 + 0.9 + (k^2 \times 0.3) = 3.6$$

$$3.3 + 0.3k^2 = 3.6$$

$$0.3k^2 = 0.3$$

$$k^2 = 1 \qquad \text{Therefore } k = 1$$

Part b
Compute the variance of X

$$\text{Var}(x) = E(X^2) - \{E(X)\}^2$$

- We already know $E(X^2) = 3.6$
- Need to compute $E(X)$.

Computing $E(X)$

x_i	2	3	1
$p(x_i)$	0.6	0.1	0.3

$$E(X) = \sum x_i \cdot p(x_i)$$

$$E(X) = (2 \times 0.6) + (3 \times 0.1) + (1 \times 0.3) = 1.8$$

Part b
Compute the variance of X

$$\text{Var}(X) = E(X^2) - \{E(X)\}^2$$

$$\text{Var}(X) = 3.6 - \{1.8\}^2$$

$$\text{Var}(X) = 3.6 - 3.24 = \mathbf{0.36}$$

Consider the random variables X and Y. X takes the values 0,1 and 2. Y takes the values 0 and 1. The joint probabilities for each pair are given by the following table.

	$X = 0$	$X = 1$	$X = 2$
$Y = 0$	0.1	0.4	0.1
$Y = 1$	0.1	0.1	0.2

- Compute the expected values of X and Y.
- Compute the $E(X|Y = 1)$

Compute $E(X)$ **and** $E(Y)$**.**
First compute the marginal distributions.

	$X = 0$	$X = 1$	$X = 2$	
$Y = 0$	0.1	0.4	0.1	
$Y = 1$	0.1	0.1	0.2	

Compute $E(X)$ **and** $E(Y)$**.**
First compute the marginal distributions.

	$X=0$	$X=1$	$X=2$	
$Y=0$	0.1	0.4	0.1	0.6
$Y=1$	0.1	0.1	0.2	0.4
	0.2	0.5	0.3	

Compute $E(X)$

x_i	0	1	2
$p(x_i)$	0.20	0.50	0.30

$$E(X) = \sum x_i \cdot p(x_i)$$

Compute $E(X)$

x_i	0	1	2
$p(x_i)$	0.20	0.50	0.30

$$E(X) = \sum x_i \cdot p(x_i)$$

$$E(X) = (0 \times 0.2) + (1 \times 0.5) + (2 \times 0.3) \; = 1.1$$

Compute $E(X)$

y_i	0	1
$p(y_i)$	0.60	0.40

$$E(Y) = \sum y_i \cdot p(y_i)$$

Compute $E(X)$

y_i	0	1
$p(y_i)$	0.60	0.40

$$E(Y) = \sum y_i \cdot p(y_i)$$

$$E(Y) = (0 \times 0.6) + (1 \times 0.4) = 0.4$$

Consider the random variables X and Y. Both X and Y take the values 0, 1 and 2. The joint probabilities for each pair are given by the following table.

	$X=0$	$X=1$	$X=2$
$Y=0$	0.1	0.15	0.1
$Y=1$	0.1	0.1	0.1
$Y=2$	0.2	0.05	0.1

Compute the $E(U)$ expected value of U, where $U = X - Y$.

Compute $X - Y$

	$X=0$	$X=1$	$X=2$
$Y=0$	0.1	0.15	0.1
$Y=1$	0.1	0.1	0.1
$Y=2$	0.2	0.05	0.1

Compute $X-Y$

	U	$X=0$	U	$X=1$	U	$X=2$
$Y=0$	0	0.1	1	0.15	2	0.1
$Y=1$	-1	0.1	0	0.1	1	0.1
$Y=2$	-2	0.2	-1	0.05	0	0.1

Determine the probability of each outcome of U.

u_i	-2	-1	0	1	2
p (u_i)					

$$E(U) = \sum u_i \cdot p(u_i)$$

u_i	-2	-1	0	1	2
p (u_i)	0.20	0.15	0.30	0.25	0.10
$u_i \cdot p(u_i)$					

$$E(U) = \sum u_i \cdot p(u_i)$$

u_i	-2	-1	0	1	2
p (u_i)	0.20	0.15	0.30	0.25	0.10
$u_i \cdot p(u_i)$	-0.40	-0.15	0.00	0.25	0.20

$$E(U) = \sum u_i \cdot p(u_i)$$

u_i	-2	-1	0	1	2
p (u_i)	0.20	0.15	0.30	0.25	0.10
$u_i \cdot p(u_i)$	-0.40	-0.15	0.00	0.25	0.20

$$E(U) = \sum u_i \cdot p(u_i)$$

u_i	-2	-1	0	1	2
p (u_i)	0.20	0.15	0.30	0.25	0.10
$u_i \cdot p(u_i)$	-0.40	-0.15	0.00	0.25	0.20

$$E(U) = -0.10$$

$$E(U) = \sum u_i \cdot p(u_i)$$

u_i	-2	-1	0	1	2
p (u_i)	0.20	0.15	0.30	0.25	0.10
$u_i \cdot p(u_i)$	-0.40	-0.15	0.00	0.25	0.20

$$E(U) = -0.10$$