

Tutorial Sheet 7

Q7.1

From a series of length 100, we have computed $r_1 = 0.8$, $r_2 = 0.5$, $r_3 = 0.4$, $\bar{Y} = 2$, and a sample variance of 5.

If we assume that an AR(2) model with a constant term is appropriate, how can we get simple estimates of ϕ_1 , ϕ_2 , θ_0 and σ_e^2 ?

Q7.2

Assuming that the following data arise from a stationary process, calculate method-of-moments estimates of μ , γ_0 and ρ_1 : 6, 5, 4, 6, 4.

Q7.3

Simulate an MA(1) series with $\theta = 0.8$ and $n = 48$.

- (a) Find the method-of-moments estimate of θ .
- (b) Find the conditional least squares estimate of θ and compare it with part (a).
- (c) Find the maximum likelihood estimate of θ and compare it with parts (a) and (b).

Q7.4

Simulate an MA(1) series with $\theta = -0.6$ and $n = 120$.

- (a) Find the maximum likelihood estimate of θ .
- (b) Repeat many times with a new simulated series using the same parameter and sample size.
- (c) Form the sampling distribution of the maximum likelihood estimates of θ .
- (d) Are the estimates approximately unbiased?
- (e) Calculate the variance of your sampling distribution and compare with the large sample result given in class.

Q7.5

Simulate an AR(1) series with $\phi = 0.7$ and $n = 100$.

- (a) Find the maximum likelihood estimate of ϕ .
- (b) Repeat many times with a new simulated series using the same parameter and sample size.
- (c) Form the sampling distribution of the maximum likelihood estimates of ϕ .
- (e) Calculate the variance of your sampling distribution and compare with the large sample result given in class.

Q7.6

Simulate an AR(2) series with $\phi_1 = 0.6$, $\phi_2 = 0.3$ and $n = 60$.

- (a) Find the method-of-moments estimates of ϕ_1 and ϕ_2 .
- (b) Find the conditional least squares estimates of ϕ_1 and ϕ_2 and compare them with part (a).
- (c) Find the maximum likelihood estimates of ϕ_1 and ϕ_2 and compare them with parts (a) and (b).

Q7.7

Simulate an ARMA(1,1) series with $\phi = 0.7$, $\theta = 0.4$ and $n = 72$.

- (a) Find the method-of-moments estimates of ϕ and θ .
- (b) Find the conditional least squares estimates of ϕ and θ and compare them with part (a).
- (c) Find the maximum likelihood estimates of ϕ and θ and compare them with parts (a) and (b).

Q7.8

Consider the AR(1) model for the colour property time series.

See TSlecture6 pages 41 and 42, as well as Question 6.15.

- (a) Find the method-of-moments estimate of ϕ .
- (b) Find the maximum likelihood estimate of ϕ and compare it with part (a).

Q7.9

Consider using an AR(1) model or an AR(4) model for the difference of the logarithms of the oil price series.

See TSlecture5, pages 4, 5 and 6, and TSlecture6, pages 27, 28, 29 and 30.

- (a) Estimate both of these models using maximum likelihood and compare it with the results using the AIC criteria.
- (b) Consider also an MA(1) model for the difference of the logs. Estimate this model by maximum likelihood and compare to your results in part (a). This was the model suggested in TSlecture6 Page 30.

Q7.10

The datafile `robot` contains a time series obtained from an industrial robot. The robot was put through a sequence of maneuvers, and the distance from a desired ending point was recorded in inches. This was repeated 324 times to form the time series. See Question 6.13 in last week's lab.

- (a) Estimate the parameters of an $AR(1)$ model for these data.
- (b) Estimate the parameters of an $IMA(1,1)$ model for these data.
- (c) Compare results from (a) and (b) in terms of AIC.