Tutorial Sheet 4a

This will be completed over the next two weeks.

Part 4a

4.1

Use first principles to find the auto-correlation function for the stationary process defined by

$$Y_t = 5 + e_t - \frac{1}{2}e_{t-1} + \frac{1}{4}e_{t-2}.$$

4.2

Verify for an MA(1) process,
that, for $\infty < \theta < \infty$, $\max \rho_1 = 0.5$ and $\min \rho_1 =$ -0.5.
Use R to draw the graph shown in the lecture.

4.3

Describe the important characteristics of the auto-correlation function for MA(1) and MA(2) models.

4.4

Sketch the auto-correlation functions for the following MA(2) models with parameters as specified:

$\theta_1 = 0.5$ and $\theta_2 = 0.4$

$\theta_1 = 1.2$ and $\theta_2 = 0.4$

$\theta_1 = -1$ and $\theta_2 = 0.6$.

Use the function

```
ARMAacf()
```

Type ? before the name to find the arguments it takes.

Use the formulae given in the lecture for auto-correlation to write the code in R to get the same result.

Part 4b

4.5
Describe the important characteristics of the auto-correlation function for AR(1) and AR(2) models.

4.6
Calculate and sketch the auto-correlation function for each of the following AR(1) models. Plot for sufficient lags that the auto-correlation function has nearly died out.

(a)$\phi = 0.6$.

(b)$\phi = 0.6$.

(c)$\phi = 0.95$. Do out to 20 lags.

(d)$\phi = 0.3$.

4.7
Let $\{Y_t\}$ be an AR(1) process with $-1 < \phi < +1$.

(a) Find the auto-covariance function for $W_t = \nabla Y_t = Y_t - Y_{t-1}$ in terms of ϕ and σ_e^2.

(b) In particular, show that $Var(W_t) = \frac{2\sigma_e^2}{1+\phi}$.

4.8
Let $\{Y_t\}$ be an AR(2) process of the special form $Y_t = \phi_2 Y_{t-2} + e_t$.

Use first principles to find the range of values for which the process is stationary.

4.9

Use the recursive Yule-Walker equations: $\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2}$ for $k = 1, 2, 3, \ldots$ to calculate and sketch the auto-correlation functions for the following AR(2) models with parameters as specified.

In each case, specify whether the roots are real or complex. These roots can be confirmed in R with the function

```
polyroot()
```

(a) $\phi_1 = 0.6$ and $\phi_2 = 0.3$.

(b) $\phi_1 = -0.4$ and $\phi_2 = 0.5$.

(c) $\phi_1 = 1.2$ and $\phi_2 = -0.7$.

(d) $\phi_1 = -1$ and $\phi_2 = -0.6$.

(e) $\phi_1 = 0.5$ and $\phi_2 = -0.9$.

(f) $\phi_1 = -0.5$ and $\phi_2 = -0.6$.

Part 4c

4.10
Sketch the auto-correlation function for each of the following ARMA models.

(a)$\phi = 0.7$ and $\theta = 0.4$.

(b)$\phi = 0.6$ and $\theta = -0.4$.

4.11
For the ARMA(1,2) model, $Y_t = 0.8Y_{t-1} + e_t + 0.7e_{t-1} + 0.6e_{t-2}$, show that

(a) $\rho_k = 0.8\rho_{k-1}$ for $k > 2$. Assume, with no loss of generality, that the mean is zero.

(b) $\rho_2 = 0.8\rho_1 + 0.6\sigma_e^2/\gamma_0$.

4.12
Consider two MA(2) processes, one with $\theta_1 = \theta_2 = \frac{1}{6}$, and another with $\theta_1 = -1$ and $\theta_2 = 6$.

(a) Show that these processes have the same auto-correlation function.

(b) How do the roots of the corresponding characteristic polynomials compare?

4.13
Consider the AR(1) model $Y_t = \phi Y_{t-1} + e_t$. Show by taking variances of both sides that, if $|\phi| = 1$, the process cannot be stationary.

4.14
Consider an MA(6) model with $\theta_1 = 0.5, \theta_2 = -0.25, \theta_3 = 0.125, \theta_4 = -0.0625, \theta_5 = 0.03125$ and $\theta_6 = -0.015625$.

Find a much simpler model that has nearly the same ψ weights.