Tutorial Sheet 9

Q9.1

An AR(1) model has $Y_t = 12.2$, $\phi = -0.5$ and $\mu = 10.8$,

(a) Find $\hat{Y}_t(1)$.

(b) Calculate $\hat{Y}_t(2)$.

(c) Calculate $\hat{Y}_t(10)$.

Q9.2

Suppose that annual sales in millions of dollars of the Acme Corporation follow the AR(2) model

$$Y_t = 5 + 1.1Y_{t-1} - 0.5Y_{t-2} + e_t,$$

with $\sigma_e^2 = 2$.

(a) If sales for 2005, 2006 and 2007 were \$9 million, \$11 million and \$10 million, respectively forecast sales for 2008 and 2009.

(b) Calculate 95% prediction limits for your forecast in part (a) for 2008.

Q9.3

For the Dubuque temperature data the estimated cosine trend is

$$\hat{\mu}_t = 46.266 + (-26.7079)Cos(2\pi t) + (-2.1697)Sin(2\pi t)$$

with $t =$ January 1964 as the starting value.

(a) Forecast the average monthly temperature in Dubuque, Iowa for April 1976.

(b) Calculate 95% prediction limits, given that the estimate of the $\sqrt{\gamma_0}$ for this model is 3.719°F, and compare the result with the forecast in part (a).

(c) What are the forecasts for April 1977 and for April 2007?

Q9.4

The seasonal means model *without* an intercept is found on page 22 of TSLecture3.

(a) Forecast the average monthly temperature in Dubuque, Iowa, for April 1976.

(b) Find a 95% prediction interval for that April forecast, given that the estimate for $\sqrt{\gamma_0}$ for this model is 3.419°F, and compare it with the forecast in Q9.3(b).

(c) What are the forecasts for April 1977 and for April 2007?

Q9.5

Simulate an AR(1) process with $\phi = 0.8$ and $\mu = 100$.

Simulate 48 values but set aside the last 8 values to compare forecasts to actual values.

(a) Using the first 40 values of the series, find the values for the maximum likelihood estimates of ϕ and μ.

(b) Using the estimated model, forecast the next 8 values of the series and plot the series together with the eight forecasts. Place a horizontal line at the estimate of the process mean.

(c) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?

Q9.6

Simulate an AR(2) process with $\phi_1 = 1.5$, $\phi_2 = -0.75$ and $\mu = 100$.

Simulate 52 values but set aside the last 12 values to compare forecasts to actual values.

(a) Using the first 40 values of the series, find the values for the maximum likelihood estimates of the ϕ's and μ.

(b) Using the estimated model, forecast the next 12 values of the series and plot the series together with the 12 forecasts. Place a horizontal line at the estimate of the process mean.

(c) Compare the 12 forecasts with the actual values set aside.

(d) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?

Q9.7

Simulate an MA(1) process with $\theta = 0.6$ and $\mu = 100$.

Simulate 36 values but set aside the last 4 values to compare forecasts to actual values.

(a) Using the first 32 values of the series, find the values for the maximum likelihood estimates of θ and μ.

(b) Using the estimated model, forecast the next 4 values of the series and plot the series together with the four forecasts. Place a horizontal line at the estimate of the process mean.

(c) Compare the four forecasts with the actual values set aside.

(d) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?

Q9.8

Simulate an MA(2) process with $\theta_1 = 1$, $\theta_2 = -0.6$ and $\mu = 100$.

Simulate 36 values but set aside the last 4 values to compare forecasts to actual values.

(a) Using the first 32 values of the series, find the values for the maximum likelihood estimates of the θ's and μ.

(b) Using the estimated model, forecast the next 4 values of the series and plot the series together with the four forecasts. Place a horizontal line at the estimate of the process mean.

(c) Compare the four forecasts with the actual values set aside.

(d) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?

Q9.9

Simulate an ARMA(1,1) process with $\phi = 0.7$, $\theta = -0.5$ and $\mu = 100$.

Simulate 50 values but set aside the last 10 values to compare forecasts to actual values.

(a) Using the first 40 values of the series, find the values for the maximum likelihood estimates of ϕ, θ and μ.

(b) Using the estimated model, forecast the next 10 values of the series and plot the series together with the 10 forecasts. Place a horizontal line at the estimate of the process mean.

(c) Compare the 10 forecasts with the actual values set aside.

(d) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?

Q9.10

Simulate an IMA(1,1) process with $\theta = 0.8$ and $\theta_0 = 0$.

Simulate 35 values but set aside the last five values to compare forecasts to actual values.

(a) Using the first 30 values of the series, find the values for the maximum likelihood estimates of θ.

(b) Using the estimated model, forecast the next 5 values of the series and plot the series together with the five forecasts.

(c) Compare the five forecasts with the actual values set aside.

(d) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?

Q9.11

Simulate an IMA(1,1) process with $\theta = 0.8$ and $\theta_0 = 10$.

Simulate 35 values but set aside the last five values to compare forecasts to actual values.

(a) Using the first 30 values of the series, find the values for the maximum likelihood estimates of θ.

(b) Using the estimated model, forecast the next 5 values of the series and plot the series together with the five forecasts.

(c) Compare the five forecasts with the actual values set aside.

(d) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?

Q9.12

Simulate an IMA(2,2) process with $\theta_1 = 1$, $\theta_2 = -0.75$ and $\theta_0 = 0$.

Simulate 45 values but set aside the last five values to compare forecasts to actual values.

(a) Using the first 40 values of the series, find the values for the maximum likelihood estimates of θ_1 and θ_2.

(b) Using the estimated model, forecast the next 5 values of the series and plot the series together with the five forecasts.

(c) Compare the five forecasts with the actual values set aside.

(d) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?

Q9.13

Simulate an IMA(2,2) process with $\theta_1 = 1$, $\theta_2 = -0.75$ and $\theta_0 = 10$.

Simulate 45 values but set aside the last five values to compare forecasts to actual values.

(a) Using the first 40 values of the series, find the values for the maximum likelihood estimates of θ_1 and θ_2.

(b) Using the estimated model, forecast the next 5 values of the series and plot the series together with the five forecasts.

(c) Compare the five forecasts with the actual values set aside.

(d) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?

Q9.14 The data file names "deere3" contains 57 consecutive values form a complex machine tool process at Deere & Co. The values given are deviations from a target value in units of ten millionths of an inch. The process employs a control mechanism that resets some of the parameters of the machine tool depending on the magnitude of the deviation from target of the last item produced.

(a) Using an AR(1) model for this series, forecast the next 10 values.

(b) Plot the series with 95% forecast limits and interpret the results.

Q9.15

The time series in the data file "days" contains accounting data from Winegard Co. of Burlington, Iowa. The data are the number of days until Winegard receives payment for 130 consecutive orders from particular distributor of Winegard products. Replace the obvious outliers at times 63, 106 and 129 with the much more typical value of 35 days.

(a) Use an MA(2) model to forecast the next 10 values of this modified series.

(b) Plot the series, the forecasts and 95% forecast limits, and interpret the results.

Q9.16

The time series in the data file "robot" gives the final position in the "x-direction" after an industrial robot has finished a planned set of exercises. The measurements are expressed as deviations from a target position. The robot is put through this planned set of exercises in the hope that its behaviour is repeatable and thus predictable.

(a) Using an IMA(1,1) model to forecast the next 5 values and obtain 95% forecast limits also.

(b) Plot the forecasts, 95% forecast limits and actual values and interpret the results.

(c) Using an ARMA(1,1) model to forecast the next 5 values and obtain 95% forecast limits also and compare with results in part (a).

Q9.17

The Canadian hare abundance file is labelled "hare".

Plot the original abundance values together with the squares of the forecasts and squares of the forecast limits.