Time Series – Tutorial 6

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Question 1

Consider the `hare` series which contains annual abundance of Canadian hares.

a) Based on the time plot and ACF, does this series look stationary?

b) Show using the Box-Cox transformation that a square-root transformation is supported.

c) Plot the transformed (square root) series.

d) Show that the transformed series is more normally distributed than the original.

e) Based on the Dicky-Fuller test, is differencing required?

f) By looking at the ACF and PACF, what possible models may be appropriate?

Question 2

Consider the `color` series which contains the colour property from successive batches of an industrial process.

a) Based on the time plot, does this series look stationary?

b) Apply the Dicky-Fuller test and comment on the result.

c) Plot the differenced series and comment.

d) Based on the ACF and PACF of the differenced series, what model is appropriate?

e) Is your answer to part (d) supported by the EACF? Note use `ar.max = 7` and `ma.max = 7` in the `eacf` function.

f) What models might we consider for the original series (i.e., no differencing) based on the ACF, PACF and EACF?

Question 3

Consider the `tempdub` which contains monthly temperature in Dubuque.

a) Based on the time plot and ACF, does this series look stationary?

b) Apply the Dicky-Fuller test and comment on the result.

c) Transform the series by applying seasonal differencing, i.e.,`diff(tempdub, lag=12)`, plot this transformed series and comment.

d) Based on the ACF and PACF of the differenced series, what model would you suggest?

e) What models are suggested by the `armasubsets` function. Set nar=12 and nma=12.

Question 4

Consider the `robot` which contains measurements obtained from an industrial robot. The robot was put through a sequence of maneuvers, and the distance from a desired ending point was recorded in inches. This was repeated 324 times to form the time series.

a) Does this series appear to be stationary?

b) Based on the ACF and PACF, what model would you suggest?

c) Calculate and interpret the sample EACF.

d) Use the best subsets ARMA approach to suggest a model.

e) Repeat (b), (c) and (d) but for the differenced series.

Question 5

What models would you suggest based on the following information:

a) $n = 100$ with

Lag	1	2	3	4	$k > 4$
ACF	-0.49	0.31	-0.21	0.11	$\lvert r_k\rvert < 0.09$

b) $n = 121$ with

Lag	1	2	3	4	$k > 4$
PACF	0.8	-0.6	0.08	0.00	$\lvert r_k\rvert \approx 0.00$

c) $n = 169$ with

Lag	1	2	3	4	5
ACF	0.41	0.32	0.26	0.21	0.16

d) $n = 100$ with

Lag	1	2	3	4	5	6
ACF for Y_t	0.97	0.97	0.93	0.85	0.80	0.71
ACF for ∇Y_t	-0.42	0.18	-0.02	0.07	-0.10	-0.09

Question 6

Consider the basic Dickey-Fuller test which is based on testing $\alpha = 1$ in the model

$$Y_t = \alpha Y_{t-1} + e_t.$$

a) By subtracting Y_{t-1} from both sides, show that this model can be written as:

$$\nabla Y_t = \beta Y_{t-1} + e_t$$

where $\beta = \alpha - 1$.

Question 7

Consider an augmented Dickey-Fuller test where we assume

$$Y_t = \alpha Y_{t-1} + X_t$$

and X_t is an AR(1) model.

a) Show that $X_t = (1 - \alpha)Y_t + \alpha \nabla Y_t$.

b) By subtracting Y_{t-1} from both sides of $Y_t = \alpha Y_{t-1} + X_t$, substituting $X_t = \phi X_{t-1} + e_t$ and using your answer to part (a), show that this model is can be written as

$$\nabla Y_t = \beta Y_{t-1} + \delta \nabla Y_{t-1} + e_t$$

where $\beta = (\alpha - 1)(1 - \phi)$.

c) Explain why $\beta = 0$ is equivalent to $\alpha = 1$.

Question 8

a) Show that for $k > p$, the partial autocorrelation for an AR(p) model is zero.

Question 9

Consider the situation whereby we wish to estimate Y_t using a linear function of the previous value, Y_{t-1}, i.e., $\hat{Y}_t = \beta Y_{t-1}$, where there is no intercept since we are assuming, without the loss of generality, that $E(Y_t) = 0$.

a) Derive an expression for $\text{Var}(Y_t - \hat{Y}_t)$ which is called the mean squared error. Assume that Y_t is stationary with autocovariance function γ_k.

b) We wish to choose β such that we minimise the above variance, e.g., if $\text{Var}(Y_t - \hat{Y}_t) = 0$ then $\hat{Y}_t = Y_t$ and we have a perfect estimate. By differentiating w.r.t. β, show that this variance is minimised for $\beta = \rho_1$.

c) We may also estimate Y_{t-2} using Y_{t-1} via $\hat{Y}_{t-2} = \beta Y_{t-1}$. Show that $\text{Var}(Y_{t-2} - \hat{Y}_{t-2})$ is also minimised for $\beta = \rho_1$.

d) Using the above results, show that the lag-2 *partial* autocorrelation is given by

$$\begin{aligned} \tau_2 &= \text{Corr}(Y_t - \hat{Y}_t, Y_{t-2} - \hat{Y}_{t-2}) \\ &= \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2}. \end{aligned}$$

Hint: you need to use $\text{Corr}(X, Y) = \frac{\text{Cov}(X,Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$

e) Hence, show that $\tau_2 = 0$ for an AR(1) process and $\tau_2 = \phi_2$ for an AR(2) process.

Note: you may make use of the expressions for ρ_1 and ρ_2 for AR(1) and AR(2) processes in Lecture 4.