Tutorial Sheet 6

Q6.1

A time series of 100 observations produced sample auto-correlation of $r_1 = -0.49$, $r_2 = 0.31$, $r_3 = -0.21$, $r_4 = 0.11$ and $|r_k| < 0.09$ for $k > 4$. On this basis alone, what ARIMA model would we tentatively specify for the series?

Q6.2

A stationary time series of 121 observations produced sample partial auto-correlation of $\hat{\phi}_{11} = 0.8$, $\hat{\phi}_{22} = -0.6$, $\hat{\phi}_{33} = 0.08$, $\hat{\phi}_{44} = 0.00$ On this basis alone, what ARIMA model would we tentatively specify for the series?

Q6.3

A time series of 169 observations produced sample auto-correlation of $r_1 = 0.41$, $r_2 = 0.32$, $r_3 = 0.26$, $r_4 = 0.21$ and $r_5 = 0.16$ On this basis alone, what ARIMA model would we tentatively specify for the series?

Q6.4

The sample acf for a series and its first difference are given in the following table. Here $n = 100$.

lag	1	2	3	4	5	6
acf for Y_t	0.97	0.97	0.93	0.85	0.80	0.71
acf for ∇Y_t	-0.42	0.18	-0.02	0.07	-0.10	-0.09

Based on this information alone, which ARIMA model(s) would we consider for the series?

Q6.5

The sample pacf for a series of length 64 are given in the following table.

lag	1	2	3	4	5
pacf	0.47	-0.34	0.20	0.02	-0.06

Based on this information alone, which ARIMA model(s) would we consider for the series?

Q6.6

Simulate an AR(1) series with $n = 48$ and $\phi = 0.7$.

(a) Calculate the theoretical auto-correlations at lag 1 and lag 5 for this model.

(b) Calculate the sample auto-correlations at lag 1 and lag 5 and compare the values with their theoretical values. Quantify the comparisons using standard error (se) formulae.

(c) Repeat the simulation of the series and calculation of r_1 and r_5 many times and form the sampling distributions of r_1 and r_5.

Describe how the precision of the estimate varies with different samples selected under identical conditions.

How well does the large sample variance approximate the variance in your sampling distribution?

Q6.7

Simulate an MA(1) time series with $n = 60$ and $\theta = 0.5$

(a) Calculate the theoretical auto-correlation at lag1 for this model.

(b) Calculate the auto-correlation at lag 1 and compare the value with its theoretical value.

(c) Quantify the comparisons using the standard error formulae.

(d) Repeat the simulation of the series and calculation of r_1 many times and form the sampling distributions of r_1.

Describe how the precision of the estimate varies with different samples selected under identical conditions.

How well does the large sample variance approximate the variance in your sampling distribution?

Q6.8

Simulate an AR(1) time series with $n = 48$ and calculate theoretical ρ_1 and ρ_5.

(a) with $\phi = 0.9$.

(b) with $\phi = 0.6$.

(c) with $\phi = 0.3$.

(d) Calculate r_1 and r_5 for (a), (b) and (c) above and the precision of the estimates.

Describe, in general, how the precision of the estimates varies with the value of ϕ.

Q6.9

Simulate 3 AR(1) time series with $\phi = 0.6$ and 3 different sample sizes.

With each one, estimate ρ_1 with r_1 and quantify the error.

(a) $n = 24$,

(b) $n = 60$.

(c) $n = 120$.

Q6.10

Simulate 3 MA(1) time series with $\theta = 0.7$.

and 3 different sample sizes.

With each one, estimate ρ_1 with r_1 and quantify the error..

(a) $n = 24$,

(b) $n = 60$.

(c) $n = 120$.

Q6.11

Simulate a stationary time series of length $n = 36$ according to an AR(1) model with $\phi = 0.95$.

(a) Plot the series and calculate the sample acf and pacf and describe them.

(b) Perform the augmented Dicky-Fuller test with k chosen by the software.

Q6.12

The data file named "deere1" contains 82 consecutive values for the amount of deviation (in 0.000025 inch units) from a specified target value that an industrial machining process at Deere & Co. produced under specified operating conditions.

(a) Display the time series plot of this series and comment on any unusual points.

(b) Calculate the sample acf for this series and comment on the results.

(c) Now replace the unusual value by a much more typical value and recalculate the sample acf.

(d) Calculate the sample pacf on the revised series and comment on the results.

Q6.13

The datafile named "robot" contains a time series obtained from an industrial robot. The robot was put through a sequence of maneuvers, and the distance from a desired ending point was recorded in inches. This was repeated 324 times to form the time series.

(a) Display the time series plot of the data. Based on this information, do these data appear to come from a stationary or non-stationary process?

(b) Calculate and plot the sample acf and pacf for these data. Based on this additional information, do these data appear to come from a stationary or non-stationary process?

(c) Calculate and interpret the sample eacf.

(d) Use the best subsets ARMA approach to specify a model for these data. Compare these results with what you discovered in parts (a), (b) and (c).

Q6.14

Calculate and interpret the sample eacf for the logarithms of the LA rainfall series.

Do the results confirm that the logs are white noise?

Q6.15

Calculate and interpret the sample eacf for the colour property time series.

Do the results suggest the same model specified by the sample pacf.