

MS4218 Mid-term examination

Answer all 3 questions.

15% of total marks.

Q 1

Suppose that

$$E(X) = 4, \text{ } Var(X) = 16,$$

$$E(Y) = 2, \text{ } Var(Y) = 9, \text{ and}$$

$$Corr(X, Y) = 0.3.$$

Find:

(a) $Var(X + Y)$.

(b) $Corr(X + Y, X - Y)$.

3 marks.

Q 2

A particular AR(2) process is given by:

$$Y_t = 0.3Y_{t-1} - 0.9Y_{t-2} + e_t.$$

Given that the first two theoretical auto-correlation values are

$$\rho_1 = \frac{\phi_1}{1 - \phi_2}$$

and

$$\rho_2 = \frac{\phi_1^2 + \phi_2(1 - \phi_2)}{1 - \phi_2},$$

- (i) Calculate ρ_1 and ρ_2 for this particular process.
- (ii) Use the Yule-Walker recursive equations to find ρ_3 and ρ_4 .
- (iii) Find the roots of the characteristic equation.
- (iv) Show why the process is stationary.

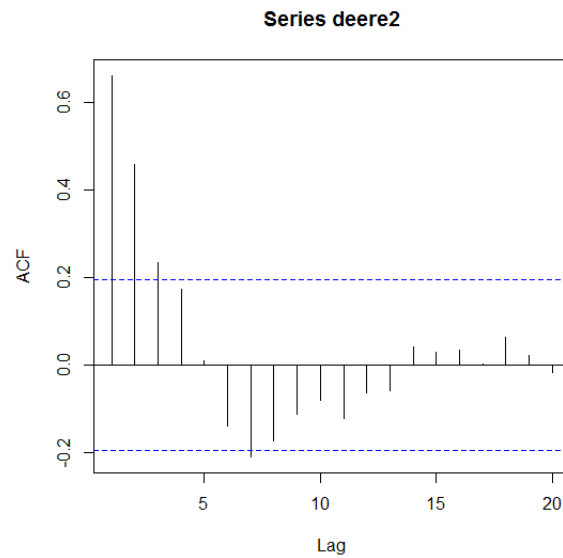
7 marks.

Q 3

The diagram below shows the plot of the sample auto-correlation function for a particular dataset.

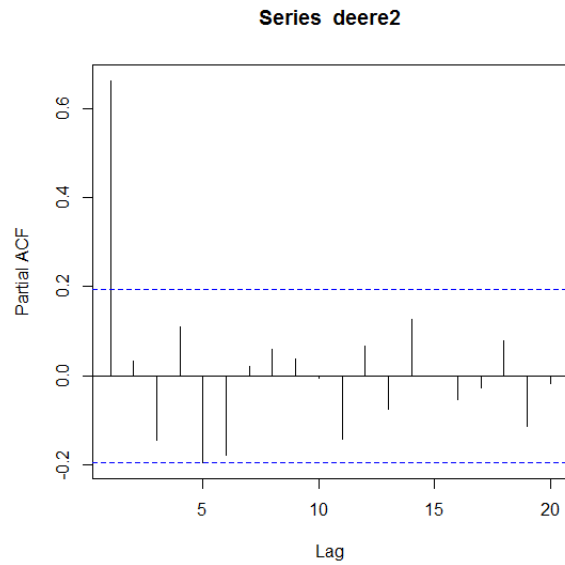
(a) Describe the plot below and the type of model(s) that is(are) suggested by the plot to model the auto-correlation (acf).

What change could be made to this particular plot which might give a different result?



Parts (b) and (c) on next page.

(b) The partial auto-correlation function for the same data is shown below. Describe the plot and the type of model that is suggested by the plot to model the partial auto-correlation (PACF).



(c) Name and briefly explain one technique that would help you to decide further which model might be preferable to use.

5 marks.