Statistics and Probability

Discrete Random Variables

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Discrete Random Variables

Given:

Suppose X is a random variable with

- $E(X^2) = 3.6$
- $P(X = 2) = 0.6$
- $P(X = 3) = 0.1$

Questions:

(a) The random variable takes just one other value besides 2 and 3. This value is greater than 0. What is this value?

(b) What is the variance of X?

Discrete Random Variables

Part a

- Determine the missing value (let's call it k).
- First we determine the probability of that value.
- We know that $E(X^2) = 3.6$. Let use the approach for computing $E(X^2)$.

x_i	2	3	k
x_i^2	4	9	k^2
$p(x_i)$	0.6	0.1	

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Discrete Random Variables

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x_i^2	4	9	k^2
$p(x_i)$	0.6	0.1	0.3

$$E(X^2) = \sum x_i^2 \cdot p(x_i) = 3.6$$

Discrete Random Variables

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x_i^2	4	9	k^2
$p(x_i)$	0.6	0.1	0.3

$$E(X^2) = \sum x_i^2 \cdot p(x_i) = 3.6$$

$$(4 \times 0.6) + (9 \times 0.1) + (k^2 \times 0.3) = 3.6$$

Discrete Random Variables

x_i	2	3	k
x_i^2	4	9	k^2
$p(x_i)$	0.6	0.1	0.3

$$E(X^2) = \sum x_i^2 \cdot p(x_i) = 3.6$$

$$(4 \times 0.6) + (9 \times 0.1) + (k^2 \times 0.3) = 3.6$$

$$2.4 + 0.9 + (k^2 \times 0.3) = 3.6$$

Discrete Random Variables

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$$3.3 + 0.3k^2 = 3.6$$

Discrete Random Variables

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Discrete Random Variables

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$$0.3k^2 = 0.3$$

$k^2 = 1$ Therefore $k = 1$

Part b

Compute the variance of X

$$\text{Var}(x) = E(X^2) - \{E(X)\}^2$$

- We already know $E(X^2) = 3.6$
- Need to compute $E(X)$.

Discrete Random Variables

Computing $E(X)$

x_i	2	3	1
$p(x_i)$	0.6	0.1	0.3

$$E(X) = \sum x_i \cdot p(x_i)$$

Discrete Random Variables

Computing $E(X)$

x_i	2	3	1
$p(x_i)$	0.6	0.1	0.3

$$E(X) = \sum x_i \cdot p(x_i)$$

$$E(X) = (2 \times 0.6) + (3 \times 0.1) + (1 \times 0.3) = 1.8$$

Discrete Random Variables

Part b

Compute the variance of X

$$\text{Var}(x) = E(X^2) - \{E(X)\}^2$$

$$\text{Var}(x) = 3.6 - \{1.8\}^2$$

Discrete Random Variables

Part b

Compute the variance of X

$$\text{Var}(x) = E(X^2) - \{E(X)\}^2$$

$$\text{Var}(x) = 3.6 - \{1.8\}^2$$

$$\text{Var}(x) = 3.6 - 3.24 = \mathbf{0.36}$$