Poisson Expected Value and Variance

If the random variable X has a Poisson distribution with parameter m, we write

$$X \sim Poisson(m)$$

- Expected Value of X : E(X) = m
- Variance of X : $\text{Var}(X) = m$
- Standard Deviation of X : $SD(X) = \sqrt{m}$

Poisson Distribution : Example

- The number of faults in a fibre optic cable were recorded for each kilometre length of cable.
- The mean number of faults was found to be 4 faults per kilometre.
- The standard deviation of the number of faults was found to be 2 faults per kilometre.
- Is the Poisson Distribution is a useful technique for modelling the number of faults in fibre optic cable?
- (Looking at the last slide, the answer is yes, because the variance and mean are equal).

Poisson Approximation of the Binomial

- The Poisson distribution can sometimes be used to approximate the binomial distribution
- When the number of observations n is large, and the success probability p is small, the $B(n,p)$ distribution approaches the Poisson distribution with the parameter given by $m = np$.
- This is useful since the computations involved in calculating binomial probabilities are greatly reduced.
- As a rule of thumb, n should be greater than 50 with p very small, such that np should be less than 5.
- If the value of p is very high, the definition of what constitutes a "success" or "failure" can be switched.

Poisson Approximation: Example

- Suppose we sample 1000 items from a production line that is producing, on average, 0.1% defective components.
- Using the binomial distribution, the probability of exactly 3 defective items in our sample is

$$P(X=3) = {}^{1000}C_3 \times 0.001^3 \times 0.999^{997}$$

Poisson Approximation: Example

Lets compute each of the component terms individually.

- ${}^{1000}C_3$

$$ {}^{1000}C_3 = \frac{1000 \times 999 \times 998}{3 \times 2 \times 1} = 166,167,000 $$

- 0.001^3

$$ 0.001^3 = 0.000000001 $$

- 0.999^{997}

$$ 0.999^{997} = 0.36880 $$

Multiply these three values to compute the binomial probability $P(X = 3) = 0.06128$

Poisson Approximation: Example

- Lets use the Poisson distribution to approximate a solution.
- First check that $n \geq 50$ and $np < 5$ (Yes to both).
- We choose as our parameter value $m = np = 1000 \times 0.001 = 1$

$$P(X = 3) = \frac{e^{-1} \times 1^3}{3!} = \frac{e^{-1}}{6} = \frac{0.36787}{6} = 0.06131$$

Compare this answer with the Binomial probability P(X = 3) = 0.06128. Very good approximation, with much less computation effort.

Poisson Approximation of the Binomial

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