## 0.1 The CPJ Paper

### 0.1.1 Case-Deletion results for Variance components

**CPJ** examines case deletion results for estimates of the variance components, proposing the use of one-step estimates of variance components for examining case influence. The method describes focuses on REML estimation, but can easily be adapted to ML or other methods.

This paper develops their global influences for the deletion of single observations in two steps: a one-step estimate for the REML (or ML) estimate of the variance components, and an ordinary case-deletion diagnostic for a weighted regression problem ( conditional on the estimated covariance matrix) for fixed effects.

### 0.1.2 CPJ Notation

$$\boldsymbol{C} = \boldsymbol{H}^{-1} = \begin{bmatrix} c_{ii} & \boldsymbol{c}_i' \\ \boldsymbol{c}_i & \boldsymbol{C}_{[i]} \end{bmatrix}$$

**CPJ** noted the following identity:

$$\boldsymbol{H}_{[i]}^{-1} = \boldsymbol{C}_{[i]} - \frac{1}{c_{ii}} \boldsymbol{c}_{[i]} \boldsymbol{c}_{[i]}'$$

**CPJ** use the following as building blocks for case deletion statistics.

- $\breve{x}_i$

- $\breve{z}_i$

- $\breve{z}_i j$

- $\breve{y}_i$

- $p_i i$

- $m_i$

All of these terms are a function of a row (or column) of $\boldsymbol{H}$ and $\boldsymbol{H}_{[i]}^{-1}$

## 0.2 Matrix Notation for Case Delection

### 0.2.1 Case deletion notation

For notational simplicity, $\boldsymbol{A}(i)$ denotes an $n \times m$ matrix $\boldsymbol{A}$ with the $i$-th row removed, $a_i$ denotes the $i$-th row of $\boldsymbol{A}$, and $a_{ij}$ denotes the $(i,j)-$th element of $\boldsymbol{A}$.

### 0.2.2 Partitioning Matrices

Without loss of generality, matrices can be partitioned as if the $i-$th omitted observation is the first row; i.e. $i = 1$.

## 0.3 CPJ's Three Propositions

**Proposition 1**

$$\boldsymbol{V}^{-1} = \begin{bmatrix} \nu^{ii} & \lambda_i' \\ \lambda_i & \Lambda_{[i]} \end{bmatrix}$$

$$\boldsymbol{V}_{[i]}^{-1} = \boldsymbol{\Lambda}_{[i]} - \frac{\lambda_i \lambda_i'}{\lambda_i}$$

### 0.3.1 Proposition 2

(i) $\boldsymbol{X}_{[i]}^T \boldsymbol{V}_{[i]}^{-1} \boldsymbol{X}_{[i]} = \boldsymbol{X}' \boldsymbol{V}^{-1} \boldsymbol{X}$

(ii) $= (\boldsymbol{X}' \boldsymbol{V}^{-1} \boldsymbol{Y})^{-1}$

(iii) $\boldsymbol{X}_{[i]}^T \boldsymbol{V}_{[i]}^{-1} \boldsymbol{Y}_{[i]} = \boldsymbol{X}' \boldsymbol{V}^{-1} \boldsymbol{Y}$

## 0.3.2 Proposition 3

This proposition is similar to the formula for the one-step Newtown Raphson estimate of the logistic regression coefficients given by Pregibon (1981) and discussed in Cook Weisberg.

## 0.4 CPJ's Three Propositions

**Proposition 1**

$$V^{-1} = \begin{bmatrix} \nu^{ii} & \lambda_i' \\ \lambda_i & \Lambda_{[i]} \end{bmatrix}$$

$$V_{[i]}^{-1} = \Lambda_{[i]} - \frac{\lambda_i \lambda_i'}{\lambda_i}$$

### 0.4.1 Proposition 2

(i) $X_{[i]}^T V_{[i]}^{-1} X_{[i]} = X' V^{-1} X$

(ii) $= (X' V^{-1} Y)^{-1}$

(iii) $X_{[i]}^T V_{[i]}^{-1} Y_{[i]} = X' V^{-1} Y$

### 0.4.2 Proposition 3

This proposition is similar to the formula for the one-step Newtown Raphson estimate of the logistic regression coefficients given by pregibon (1981) and discussed in Cook Weisberg.

## 0.5 The CPJ Paper

### 0.5.1 Case-Deletion results for Variance components

?examines case deletion results for estimates of the variance components, proposing the use of one-step estimates of variance components for examining case influence. The method describes focuses on REML estimation, but can easily be adapted to ML or other methods.

Christensen developed their global influences for the deletion of single observations in two steps: a one-step estimate for the REML (or ML) estimate of the variance components, and an ordinary case-deletion diagnostic for a weighted resgression problem ( conditional on the estimated covariance matrix) for fixed effects. Lesaffre's approach accords with that proposed by Christensen et al when applied in a repeated measurement context, with a large sample size.

## 0.5.2   CPJ Notation

$$C = H^{-1} = \begin{bmatrix} c_{ii} & c_i' \\ c_i & C_{[i]} \end{bmatrix}$$

? noted the following identity:

$$H_{[i]}^{-1} = C_{[i]} - \frac{1}{c_{ii}} c_{[i]} c_{[i]}'$$

? use the following as building blocks for case deletion statistics.

- $\breve{x}_i$

- $\breve{z}_i$

- $\breve{z}_i j$

- $\breve{y}_i$

- $p_i i$

- $m_i$

All of these terms are a function of a row (or column) of $H$ and $H_{[i]}^{-1}$

## 0.6 The CPJ Paper

### 0.6.1 Case-Deletion results for Variance components

**?** examines case deletion results for estimates of the variance components, proposing the use of one-step estimates of variance components for examining case influence. The method describes focuses on REML estimation, but can easily be adapted to ML or other methods.

This paper develops their global influences for the deletion of single observations in two steps: a one-step estimate for the REML (or ML) estimate of the variance components, and an ordinary case-deletion diagnostic for a weighted regression problem ( conditional on the estimated covariance matrix) for fixed effects.

### 0.6.2 CPJ Notation

$$\boldsymbol{C} = \boldsymbol{H}^{-1} = \begin{bmatrix} c_{ii} & \boldsymbol{c}_i' \\ \boldsymbol{c}_i & \boldsymbol{C}_{[i]} \end{bmatrix}$$

**?** noted the following identity:

$$\boldsymbol{H}_{[i]}^{-1} = \boldsymbol{C}_{[i]} - \frac{1}{c_{ii}} \boldsymbol{c}_{[i]} \boldsymbol{c}_{[i]}'$$

**?** use the following as building blocks for case deletion statistics.

- $\breve{x}_i$

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- $\breve{y}_i$

- $p_i i$

- $m_i$

All of these terms are a function of a row (or column) of $\boldsymbol{H}$ and $\boldsymbol{H}_{[i]}^{-1}$

# 0.7 Matrix Notation for Case Deletion

## 0.7.1 Case deletion notation

For notational simplicity, $\boldsymbol{A}(i)$ denotes an $n \times m$ matrix $\boldsymbol{A}$ with the $i$-th row removed, $a_i$ denotes the $i$-th row of $\boldsymbol{A}$, and $a_{ij}$ denotes the $(i,j)-$th element of $\boldsymbol{A}$.

## 0.7.2 Partitioning Matrices

Without loss of generality, matrices can be partitioned as if the $i-$th omitted observation is the first row; i.e. $i = 1$.

## 0.8   Matrix Notation for Case Deletion

### 0.8.1   Case deletion notation

For notational simplicity, $\boldsymbol{A}(i)$ denotes an $n \times m$ matrix $\boldsymbol{A}$ with the $i$-th row removed, $a_i$ denotes the $i$-th row of $\boldsymbol{A}$, and $a_{ij}$ denotes the $(i,j)-$th element of $\boldsymbol{A}$.

### 0.8.2   Partitioning Matrices

Without loss of generality, matrices can be partitioned as if the $i-$th omitted observation is the first row; i.e. $i = 1$.

## 0.9   CPJ's Three Propositions

**Proposition 1**

$$
\boldsymbol{V}^{-1} = \begin{bmatrix} \nu^{ii} & \lambda_i' \\ \lambda_i & \Lambda_{[i]} \end{bmatrix}
$$

$$
\boldsymbol{V}_{[i]}^{-1} = \boldsymbol{\Lambda}_{[i]} - \frac{\lambda_i \lambda_i'}{\lambda_i}
$$

### 0.9.1   Proposition 2

(i)  $\boldsymbol{X}_{[i]}^{T} \boldsymbol{V}_{[i]}^{-1} \boldsymbol{X}_{[i]} = \boldsymbol{X}' \boldsymbol{V}^{-1} \boldsymbol{X}$

(ii)  $= (\boldsymbol{X}' \boldsymbol{V}^{-1} \boldsymbol{Y})^{-1}$

(iii)  $\boldsymbol{X}_{[i]}^{T} \boldsymbol{V}_{[i]}^{-1} \boldsymbol{Y}_{[i]} = \boldsymbol{X}' \boldsymbol{V}^{-1} \boldsymbol{Y}$

## 0.9.2 Proposition 3

This proposition is similar to the formula for the one-step Newtown Raphson estimate of the logistic regression coefficients given by Pregibon (1981) and discussed in Cook Weisberg.

**?** describes three propositions that are required for efficient case-deletion in LME models. The first proposition decribes how to efficiently update $V$ when the $i$th element is deleted.

$$V_{[i]}^{-1} = \Lambda_{[i]} - \frac{\lambda\lambda\prime}{\nu ii} \tag{1}$$

The second of christensen's propostions is the following set of equations, which are variants of the Sherman Wood bury updating formula.

$$X_{[i]}'V_{[i]}^{-1}X_{[i]} = X'V^{-1}X - \frac{\hat{x}_i\hat{x}_i'}{s_i} \tag{2}$$

$$(X_{[i]}'V_{[i]}^{-1}X_{[i]})^{-1} = (X'V^{-1}X)^{-1} + \frac{(X'V^{-1}X)^{-1}\hat{x}_i\hat{x}_i'(X'V^{-1}X)^{-1}}{s_i - \bar{h}_i} \tag{3}$$

$$X_{[i]}'V_{[i]}^{-1}Y_{[i]} = X\prime V^{-1}Y - \frac{\hat{x}_i\hat{y}_i'}{s_i} \tag{4}$$