0.1 Demidenko

Standardised residuals are typicallu used to detect outliers. However, the presence of outliers does not necessarily affect the model fit or any related statistical inference.

Leverage, defined as the identification of data points that influence the fitted values, are detected by exploring large values of the diagonal elements of the projection matrix (also known as the hat matrix.

Cook and Weisber suggested analysin the standardised squared distance between the OLS estimate and the estimate after case deletion. This has become known as Cook's Distance.

The goal of Demidenko's paper is generalize several common measures of influence for the fixed effects parameters of an LME model.

The LME model is typciall estimated using restricted maximum likelihood (REML) which simultaneously produces an estimate of $\boldsymbol{D}$ and $\boldsymbol{\beta}$.

The hat matrix is

$$\boldsymbol{H} = \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'$$

Leverage - page 895

Leverage is the partial derivate of the predicted value with respect to the corresponding dependent variable.

Hence the $i-$th leverage indicates how the predicted value of the ith case is influenced by the ith observation.

Leverage Matrix for the LME Model $n_i \times n_i$

$$\boldsymbol{H}_i = \frac{\partial \hat{\boldsymbol{y}}_i}{\partial \boldsymbol{y}_i}$$

0.2 Lesaffre's paper.

Lesaffre considers the case-weight perturbation approach.

Cook's 86 describes a local approach wherein each case is given a weight w_i and the effect on the parameter estimation is measured by perturbing these weights. Choosing weights close to zero or one corresponds to the global case-deletion approach.

Lesaffre describes the displacement in log-likelihood as a useful metric to evaluate local influence

Lesaffre describes a framework to detect outlying observations that matter in an LME model. Detection should be carried out by evaluating diagnostics C_i , $C_i(\alpha)$ and $C_i(D, \sigma^2)$.

Lesaffre defines the total local influence of individual i as

$$C_i = 2|\triangle\prime_i L^{-1} \triangle_i|. \tag{1}$$

The influence function of the MLEs evaluated at the ith point IF_i, given by

$$IF_i = -L^{-1} \triangle_i \tag{2}$$

can indicate how $th\hat{e}ta$ changes as the weight of the ith subject changes.

The manner by which influential observations distort the estimation process can be determined by inspecting the interpretable components in the decomposition of the above measures of local influence.

Lesaffre comments that there is no clear way of interpreting the information contained in the angles, but that this doesn't mean the information should be ignored.

0.3 The extended likelihood

The desire to have an entirely likelihood-based justification for estimates of random effects, in contrast to Henderson's equation, has motivated Pawitan (2001, page 429) to define the *extended likelihood*. He remarks "In mixed effects modelling the extended likelihood has been called *h-likelihood* (for hierarchical likelihood) by Lee and Nelder (1996), while in smoothing literature it is known as the *penalized likelihood* (e.g. Green and Silverman 1994)." The extended likelihood can be written $L(\beta, \theta, b|y) = p(y|b; \beta, \theta)p(b; \theta)$ and adopting the same distributional assumptions used by Henderson (1950) yields the log-likelihood function

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\begin{eqnarray*}
\ell_h(\beta,\theta,b|y)
& = \displaystyle -\frac{1}{2} \left\{ \log|\Sigma| + (y - X \beta -Zb)'\Sigma^{-1}
&  \hspace{0.5in} \left. + \log|D| + b^\prime D^{-1}b \right\}.
\end{eqnarray*}
Given $\theta$, differentiating with respect to $\beta$ and $b$ returns Henderson's
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The LME model as a general linear model

Henderson's equations in (**??**) can be rewritten $(T'W^{-1}T)\delta = T'W^{-1}y_a$ using

```
\[
\delta = \begin{pmatrix}{\beta \cr b},
\ y_{a} = \begin{pmatrix}{
y \cr \psi
},
\ T = \begin{pmatrix}{
X & Z  \cr
0 & I
},
\ \textrm{and} \ W = \begin{pmatrix}{
```

```
\Sigma & 0  \cr

0 &  D },

\]

where \cite{Lee:Neld:Pawi:2006} describe $\psi = 0$ as quasi-data with mean $\mathr
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