0.0.1 Leverage

Leverage can be defined through the projection matrix that results from a transformation of the model with the inverse of the Cholesky decomposition of $\boldsymbol{V}$, or an oblique projector: $\boldsymbol{Y} = \boldsymbol{H}\hat{\boldsymbol{Y}}$.

While H is idempotent, it is generally not symmetric and thus not a projection matrix in the narrow sense.

$$h_{ii} = x_i'(X'X)^{-1}x_i$$

The trace of $\boldsymbol{H}$ equals the rank of $\boldsymbol{X}$. If V_{ij} denotes the element in row i, column j of $\boldsymbol{V}^{-1}$, then for a model containing only an intercept the diagonal elements of $\boldsymbol{H}$.

$$h_{ii} = \frac{\sum v_{ij}}{\sum \sum v_{ij}}$$

Nobre Singer : Mixed Model Residuals

Usually one assumes

- $b_i \sim N_q(0, G) i = 1, ..., m$
- $e_i \sim N_{n_i}(0, \sigma_i)$
- b_i and e_i independent
- G and σ_i are $(q \times q)$ and $(n_i \times n_i)$ positive denite matrices with elements expressed as functions of a vector of covariance parameters θ not functionally related to β
- If $\sigma_i = I_{n_i}\sigma^2$: homoskedastic conditional independence model

$$\begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{e} \end{bmatrix} \sim \mathcal{N}_{qm+n}$$

$$\boldsymbol{Q} = \boldsymbol{V}^{-1} - \boldsymbol{V}^{-1}\boldsymbol{X}(\boldsymbol{X}^T\boldsymbol{V}^{-1}\boldsymbol{X})^{-1}$$

Sensitivity and residual analysis of the underlying assumptions constitute important tools for evaluating the fit of any model to given data.

Generalized Leverage