

0.0.1 Example 1: PEFR

- The mathematical indeterminacy in the one-measurement case is well known. Bland and Altman often avoid an explicit model in their Work so it is not clear if they are aware of this problem. In fact, they state (1983) that the considerable extra complexity of such analysis will not be justified if a simple comparison is all that is required and that this is especially true when the results must be conveyed to and used by non-experts, e.g. clinicians.
- We disagree with this comment, and show in our Example 4 that summaries of results need not be complex. We are also preparing a paper for publication that we hope will make such modeling easier for practitioners to understand, perform, and summarize.
- Using the structural equation model, What effect does this problem of indeterminacy have on making practical decisions when comparing two devices? It can have a very important effect, as we illustrate with an example. Keep in mind that we are only considering the one-measurement case if multiple measurements are made on each subject this indeterminacy disappears, although the BA plot alone (now of averages for each subject) still does not capture all the information needed.
- Our example focuses on parameter estimation, not parameter uncertainty. The results we show hold true whether the data sets are based on $N = 10$ or $N = 10,000$ parameter uncertainty is not the issue in parameter indeterminacy.

- Here is our approach. To get estimates of the parameters, we will perform a sensitivity analysis. We do this by pretending that one of the six quantities is known, and then varying this quantity over a range of values. Each fixed value of this quantity allows us to find estimates of the remaining quantities.
- An examination of (4)-(5) reveals that the values of B_0 and a_x only appear in the σ_z parameter is not useful when comparing two methods. The σ_{50} parameter, while important (measuring one aspect of bias), is fairly straightforward to correct. For these reasons, we will simplify our analysis by ignoring this information. This leaves us with four unknown parameters for which we have three estimates, as shown in (5) or We will use the latter form of the variance-covariance matrix, so $(a_m, \sigma_{51}, R_{55}, \sigma_{55})$ are the unknown parameters. One of the strong disagreements We have with the BA method is that the BA method cannot indicate which device is more precise. But we believe that this precision question, and not a measure of agreement, is often a crucial measure by which a device should be judged. For this reason we select a variety of R_{55} values for our sensitivity analysis.
- The R_{55} parameter also has a range of possible values that is independent of the data, making it a natural choice for sensitivity work. The relationship of $(\sigma_{51}, \sigma_{55}, R_{55})$ to (E, R_{55}) is presented in Appendix A. For a set of data, We can obtain the sample variance-covariance matrix S from the differences and averages. We replace E with S in (6) and equate the terms to find estimates of $(\sigma_{51}, B_1, R_{55})$ as a function of R_{55} ; for this S .
- The example We use is from Altman (1995), p. 270, and details are provided in Bland and Altman (1986). We selected this example because Bland and Altman have used it repeatedly to explain their methods. The data set is based on $N = 17$ subjects. PEFR (peak expiratory flow rate) measurements were obtained for each subject, both on a Wright meter (X) and a Mini meter (In fact, several measurements were made on each device for each subject, but Altman used only the first measurement from each device as an example of the BA plot.)
- We will usually suppress the units, liters/min, for brevity. The sample variance-covariance matrix of this set of data is (variance of $Y - X$) $S_{11} = 15027353$, (variance of $(Y + X) / 2$) $S_{22} = 127861324$, and (covariance) $S_{12} = 366.8015$.
- Based on this matrix, We generated the results in Figure 1. We have chosen to display this graph over a wide range of R_{55} values to emphasize that the data themselves provide no evidence of this value. Without additional information, such as that provided by making repeat measurements on each subject, any of the values shown on this graph are equally likely (equal likelihoods) to have given rise to the data observed.