

# 1 Application to MCS

Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.

## 2 Grubbs' Data

For the Grubbs data the  $\hat{\beta}$  estimated are  $\hat{\beta}_0$  and  $\hat{\beta}_1$  respectively. Leaving the fourth case out, i.e.  $k = 4$  the corresponding estimates are  $\hat{\beta}_0^{-4}$  and  $\hat{\beta}_1^{-4}$

$$Y^{-Q} = \hat{\beta}^{-Q} X^{-Q} \quad (1)$$

When considering the regression of case-wise differences and averages, we write  $D^{-Q} = \hat{\beta}^{-Q} A^{-Q}$

	F	C	D	A
1	793.80	794.60	-0.80	794.20
2	793.10	793.90	-0.80	793.50
3	792.40	793.20	-0.80	792.80
4	794.00	794.00	0.00	794.00
5	791.40	792.20	-0.80	791.80
6	792.40	793.10	-0.70	792.75
7	791.70	792.40	-0.70	792.05
8	792.30	792.80	-0.50	792.55
9	789.60	790.20	-0.60	789.90
10	794.40	795.00	-0.60	794.70
11	790.90	791.60	-0.70	791.25
12	793.50	793.80	-0.30	793.65

$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (2)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of differences on averages can be fitted with the view to exploring some characteristics of the data.

When considering the regression of case-wise differences and averages, we write

$$D^{-Q} = \hat{\beta}^{-Q} A^{-Q} \quad (3)$$

Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.

For the Grubbs data the  $\hat{\beta}$  estimated are  $\hat{\beta}_0$  and  $\hat{\beta}_1$  respectively. Leaving the fourth case out, i.e.  $k = 4$  the corresponding estimates are  $\hat{\beta}_0^{-4}$  and  $\hat{\beta}_1^{-4}$

$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (4)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of differences on averages can be fitted with the view to exploring some characteristics of the data.

```
Call: lm(formula = D ~ A)
```

```
Coefficients: (Intercept)          A
-37.51896      0.04656
```

## 2.1 Influence measures using R

R provides the following influence measures of each observation.

	dfb.1_	dfb.A	dffit	cov.r	cook.d	hat
1	0.42	-0.42	-0.56	1.13	0.15	0.18
2	0.17	-0.17	-0.34	1.14	0.06	0.11
3	0.01	-0.01	-0.24	1.17	0.03	0.08
4	-1.08	1.08	1.57	0.24	0.56	0.16
5	-0.14	0.14	-0.24	1.30	0.03	0.13
6	-0.00	0.00	-0.11	1.31	0.01	0.08
7	-0.04	0.04	-0.08	1.37	0.00	0.11
8	0.02	-0.02	0.15	1.28	0.01	0.09
9	0.69	-0.68	0.75	2.08	0.29	0.48
10	0.18	-0.18	-0.22	1.63	0.03	0.27
11	-0.03	0.03	-0.04	1.53	0.00	0.19
12	-0.25	0.25	0.44	1.05	0.09	0.12

## 3 Application to MCS

Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.

For the Grubbs data the  $\hat{\beta}$  estimated are  $\hat{\beta}_0$  and  $\hat{\beta}_1$  respectively. Leaving the fourth case out, i.e.  $k = 4$  the corresponding estimates are  $\hat{\beta}_0^{-4}$  and  $\hat{\beta}_1^{-4}$

$$Y^{-Q} = \hat{\beta}^{-Q} X^{-Q} \quad (5)$$

When considering the regression of case-wise differences and averages, we write

$$D^{-Q} = \hat{\beta}^{-Q} A^{-Q} \quad (6)$$

	F	C	D	A
1	793.80	794.60	-0.80	794.20
2	793.10	793.90	-0.80	793.50
3	792.40	793.20	-0.80	792.80
4	794.00	794.00	0.00	794.00
5	791.40	792.20	-0.80	791.80
6	792.40	793.10	-0.70	792.75
7	791.70	792.40	-0.70	792.05
8	792.30	792.80	-0.50	792.55
9	789.60	790.20	-0.60	789.90
10	794.40	795.00	-0.60	794.70
11	790.90	791.60	-0.70	791.25
12	793.50	793.80	-0.30	793.65

$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (7)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of differences on averages can be fitted with the view to exploring some characteristics of the data.

When considering the regression of case-wise differences and averages, we write

$$D^{-Q} = \hat{\beta}^{-Q} A^{-Q} \quad (8)$$

Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.

For the Grubbs data the  $\hat{\beta}$  estimated are  $\hat{\beta}_0$  and  $\hat{\beta}_1$  respectively. Leaving the fourth case out, i.e.  $k = 4$  the corresponding estimates are  $\hat{\beta}_0^{-4}$  and  $\hat{\beta}_1^{-4}$

$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (9)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of

differences on averages can be fitted with the view to exploring some characteristics of the data.

```
Call: lm(formula = D ~ A)
```

```
Coefficients: (Intercept)          A
-37.51896      0.04656
```

When considering the regression of case-wise differences and averages, we write

$$D^{-Q} = \hat{\beta}^{-Q} A^{-Q} \quad (10)$$

### 3.1 Influence measures using R

R provides the following influence measures of each observation.

	dfb.1_	dfb.A	dffit	cov.r	cook.d	hat
1	0.42	-0.42	-0.56	1.13	0.15	0.18
2	0.17	-0.17	-0.34	1.14	0.06	0.11
3	0.01	-0.01	-0.24	1.17	0.03	0.08
4	-1.08	1.08	1.57	0.24	0.56	0.16
5	-0.14	0.14	-0.24	1.30	0.03	0.13
6	-0.00	0.00	-0.11	1.31	0.01	0.08
7	-0.04	0.04	-0.08	1.37	0.00	0.11
8	0.02	-0.02	0.15	1.28	0.01	0.09
9	0.69	-0.68	0.75	2.08	0.29	0.48
10	0.18	-0.18	-0.22	1.63	0.03	0.27
11	-0.03	0.03	-0.04	1.53	0.00	0.19
12	-0.25	0.25	0.44	1.05	0.09	0.12

### 3.2 Influence measures using R

R provides the following influence measures of each observation.

## 4 Grubbs' Data

For the Grubbs data the  $\hat{\beta}$  estimated are  $\hat{\beta}_0$  and  $\hat{\beta}_1$  respectively. Leaving the fourth case out, i.e.  $k = 4$  the corresponding estimates are  $\hat{\beta}_0^{-4}$  and  $\hat{\beta}_1^{-4}$

$$Y^{-Q} = \hat{\beta}^{-Q} X^{-Q} \quad (11)$$

When considering the regression of case-wise differences and averages, we write  $D^{-Q} = \hat{\beta}^{-Q} A^{-Q}$

	F	C	D	A
1	793.80	794.60	-0.80	794.20
2	793.10	793.90	-0.80	793.50
3	792.40	793.20	-0.80	792.80
4	794.00	794.00	0.00	794.00
5	791.40	792.20	-0.80	791.80
6	792.40	793.10	-0.70	792.75
7	791.70	792.40	-0.70	792.05
8	792.30	792.80	-0.50	792.55
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11	790.90	791.60	-0.70	791.25
12	793.50	793.80	-0.30	793.65

$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (12)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of differences on averages can be fitted with the view to exploring some characteristics of the data.

When considering the regression of case-wise differences and averages, we write

$$D^{-Q} = \hat{\beta}^{-Q} A^{-Q} \quad (13)$$

Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.

For the Grubbs data the  $\hat{\beta}$  estimated are  $\hat{\beta}_0$  and  $\hat{\beta}_1$  respectively. Leaving the fourth case out, i.e.  $k = 4$  the corresponding estimates are  $\hat{\beta}_0^{-4}$  and  $\hat{\beta}_1^{-4}$

$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (14)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of differences on averages can be fitted with the view to exploring some characteristics of the data.

Call: `lm(formula = D ~ A)`

Coefficients: (Intercept)                      A

-37.51896      0.04656

When considering the regression of case-wise differences and averages, we write

$$D^{-Q} = \hat{\beta}^{-Q} A^{-Q} \tag{15}$$

## 5 Grubbs' Data

For the Grubbs data the  $\hat{\beta}$  estimated are  $\hat{\beta}_0$  and  $\hat{\beta}_1$  respectively. Leaving the fourth case out, i.e.  $k = 4$  the corresponding estimates are  $\hat{\beta}_0^{-4}$  and  $\hat{\beta}_1^{-4}$

$$Y^{-Q} = \hat{\beta}^{-Q} X^{-Q} \quad (16)$$

When considering the regression of case-wise differences and averages, we write  $D^{-Q} = \hat{\beta}^{-Q} A^{-Q}$

	F	C	D	A
1	793.80	794.60	-0.80	794.20
2	793.10	793.90	-0.80	793.50
3	792.40	793.20	-0.80	792.80
4	794.00	794.00	0.00	794.00
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12	793.50	793.80	-0.30	793.65

$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (17)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of differences on averages can be fitted with the view to exploring some characteristics of the data.

When considering the regression of case-wise differences and averages, we write

$$D^{-Q} = \hat{\beta}^{-Q} A^{-Q} \quad (18)$$

Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.

For the Grubbs data the  $\hat{\beta}$  estimated are  $\hat{\beta}_0$  and  $\hat{\beta}_1$  respectively. Leaving the fourth case out, i.e.  $k = 4$  the corresponding estimates are  $\hat{\beta}_0^{-4}$  and  $\hat{\beta}_1^{-4}$

$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (19)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of

differences on averages can be fitted with the view to exploring some characteristics of the data.

```
Call: lm(formula = D ~ A)
```

```
Coefficients: (Intercept)          A
-37.51896      0.04656
```

When considering the regression of case-wise differences and averages, we write

$$D^{-Q} = \hat{\beta}^{-Q} A^{-Q} \quad (20)$$

## 6 Application to MCS

Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.