

# 1 The extended likelihood

The desire to have an entirely likelihood-based justification for estimates of random

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\begin{eqnarray*}
\ell_h(\beta, \theta, b|y)
&= \displaystyle -\frac{1}{2} \left\{ \log|\Sigma| + (y - X\beta - Zb)' \Sigma^{-1} \right. \\
&\quad \left. + \log|D| + b' D^{-1} b \right\}.
\end{eqnarray*}
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Given  $\theta$ , differentiating with respect to  $\beta$  and  $b$  returns Henderson's

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\subsubsection{The LME model as a general linear model}
Henderson's equations in (\ref{Henderson:Equations}) can be rewritten  $(T' W$ 
\[
\Delta = \begin{pmatrix} \beta \\ b \end{pmatrix},
y_a = \begin{pmatrix} y \\ \psi \end{pmatrix},
T = \begin{pmatrix} X & Z \\ 0 & I \end{pmatrix},
\text{and } W = \begin{pmatrix} \Sigma & 0 \\ 0 & D \end{pmatrix},
\]
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where \cite{Lee:Neld:Pawi:2006} describe  $\psi = 0$  as quasi-data with mean  $\mathbf{0}$

## 2 Lesaffre's paper.

Lesaffre considers the case-weight perturbation approach.

Cook's 86 describes a local approach wherein each case is given a weight  $w_i$  and the effect on the parameter estimation is measured by perturbing these weights. Choosing weights close to zero or one corresponds to the global case-deletion approach.

Lesaffre describes the displacement in log-likelihood as a useful metric to evaluate local influence

Lesaffre describes a framework to detect outlying observations that matter in an LME model. Detection should be carried out by evaluating diagnostics  $C_i$ ,  $C_i(\alpha)$  and  $C_i(D, \sigma^2)$ .

Lesaffre defines the total local influence of individual  $i$  as

$$C_i = 2|\Delta_i L^{-1} \Delta_i|. \quad (1)$$

The influence function of the MLEs evaluated at the  $i$ th point  $IF_i$ , given by

$$IF_i = -L^{-1} \Delta_i \quad (2)$$

can indicate how  $\hat{\theta}$  changes as the weight of the  $i$ th subject changes.

The manner by which influential observations distort the estimation process can be determined by inspecting the interpretable components in the decomposition of the above measures of local influence.

Lesaffre comments that there is no clear way of interpreting the information contained in the angles, but that this doesn't mean the information should be ignored.

## References