

0.1 Intervals

Bland and Altman (1999) note the similarity of limits of agreement to confidence intervals, but are clear that they are not the same thing. Interestingly, they describe the limits as “being like a reference interval.”

Limits of agreement have very similar construction to Shewhart control limits. The Shewhart chart is a well known graphical methodology used in statistical process control. Consequently there is potential for misinterpreting the limits of agreement as if equivalent to Shewhart control limits. Importantly the parameters used to determine the limits, the mean and standard deviation, are not based on any sample used for an analysis, but on the process’s historical values, a key difference with Bland-Altman limits of agreement.

Carstensen et al. (2008) regards the limits of agreement as a prediction interval for the difference between future measurements with the two methods on a new individual, but states that it does not fit the formal definition of a prediction interval, since the definition does not consider the errors in estimation of the parameters. Prediction intervals, which are often used in regression analysis, are estimates of an interval in which future observations will fall, with a certain probability, given what has already been observed. Carstensen et al. (2008) offers an alternative formulation, a 95% prediction interval for the difference

$$\bar{d} \pm t_{(0.975, n-1)} S_d \sqrt{1 + \frac{1}{n}} \quad (1)$$

where n is the number of subjects. Only for 61 or more subjects is there a quantile less than 2.

Luiz et al. (2003) describes limits of agreement as tolerance limits. A tolerance interval for a measured quantity is the interval in which a specified fraction of the population’s values lie, with a specified level of confidence.

0.1.1 Purpose of Limits of Agreement

It must be established clearly the specific purpose of the limits of agreement. Bland and Altman (1995) comment that the limits of agreement *how far apart measurements by the two methods were likely to be for most individuals.*, a definition echoed in their 1999 paper:

We can then say that nearly all pairs of measurements by the two methods will be closer together than these extreme values, which we call 95% limits of agreement. These values define the range within which most differences between measurements by the two methods will lie (Bland and Altman, 1999).

? offers an alternative, more specific, definition of the limits of agreement *"a prediction interval for the difference between future measurements with the two methods on a new individual."* Luiz et al. (2003) describes them as tolerance limits.

Importantly they have the same construction as Shewhart Control limits.

0.1.2 Formal definition of limits of agreement

Bland and Altman (1999) note the similarity of limits of agreement to confidence intervals, but are clear that they are not the same thing. Interestingly, they describe the limits as ‘being like a reference interval’.

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$$\bar{d} \pm t_{(0.025, n-1)} s_d \sqrt{1 + \frac{1}{n}}$$

where n is the number of subjects. Carstensen is careful to consider the effect of the sample size on the interval width, adding that only for 61 or more subjects is the quantile less than 2.

Luiz et al. (2003) offers an alternative description of limits of agreement, this time as tolerance limits. A tolerance interval for a measured quantity is the interval in which a specified fraction of the population's values lie, with a specified level of confidence. Barnhart et al. (2007) describes them as a probability interval, and offers a clear description of how they should be used; 'if the absolute limit is less than an acceptable difference d_0 , then the agreement between the two methods is deemed satisfactory'.

The prevalence of contradictory definitions of what limits of agreement strictly are will inevitably attenuate the poor standard of reporting using limits of agreement, as mentioned by Mantha et al. (2000).

Bibliography

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What are Tolerance Intervals?

A tolerance interval is a statistical interval within which, with some confidence level, a specified proportion of a population falls. The Engineering Statistics Handbook describes the difference: Confidence limits are limits within which we expect a given population parameter, such as the mean, to lie. Statistical tolerance limits are limits within which we expect a stated proportion of the population to lie.

It is useful to make the distinction between tolerance intervals and confidence intervals clear. The confidence interval describes a single-valued population parameter, commonly the mean, with a specified confidence level. The tolerance interval, on the other hand, describes the range of data values that includes a specific proportion of the population.

As discussed in Vardeman (1992), the tolerance interval is not as widely used as the confidence interval and prediction interval, largely because of the emphasis placed on these in undergraduate teaching. Furthermore, Vardeman(1992) argues this lack of awareness can lead to misuse of confidence intervals where other types of intervals are more appropriate. Curiously Carstensen et al (2008) describe the Limits of agreement as a prediction interval, although stating that it is formulated in correctly for that purpose.

Why Tolerance Intervals are appropriate?

It is clear from the definition of Tolerance intervals that they function precisely as Bland-Altman intend. Total Deviation Index and Coverage Probability

The Coverage Probability describes the proportion captured within a pre-specified boundary of the absolute paired-measurement differences from two methods of measurement, i.e., the value of k such that $P(|D| \leq k) = p$.

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