Statistical Papers 48, 419-436 (2007)

Statistical Papers
© Springer-Verlag 2007

Influence diagnostics for the Grubbs's model

Víctor H. Lachos[1], Filidor Vilca[2], Manuel Galea[3]

[1] Departamento de Estatística, Universidade de São Paulo, Caixa Postal 66281, CEP 05315-970, São Paulo, Brasil
[2] Departamento de Estatística, Universidade Estadual de Campinas, Caixa Postal 6065, CEP 13083-970, Campinas, São Paulo, Brasil
[3] Departamento de Estadística, Universidad de Valparaíso, Casilla 5030, Valparaiso, Chile
(corresponding author, e-mail: manuel.galea@uv.cl)

Received: November 22, 2004; revised version: May 31, 2005

Abstract In this paper we consider applications of local influence (Cook, 1986) to evaluate small perturbations in the model or in data sets of several measuring devices, assuming Grubbs's model. Different perturbation schemes are investigated and an application is considered to two real data sets.

Key words : Local influence, measuring devices, Grubbs's model.

1 Introduction

Comparing measuring devices which varies in pricing, fastness and other features, such as efficiency, has been of growing interest in many engineering and scientific applications. Grubbs (1948, 1973, 1983) proposed a model for n items, each measured on p instruments. The main object of this paper is the study of local influence and diagnostic in the Grubbs's measurement model used to assess the relative quality of several measuring devices (or instruments) when measuring the same unknown quantity x in a common group of individuals or experimental units. As pointed out by a referee, this model could be seen as a particular case of the usual linear mixed model (LMM, Verbeke and Molenberghs, 2000) but, in the context of this work, is more interesting to consider this model as a particular case of the Barnett model (Barnett, 1969) since we proposed a perturbation scheme that allows to compare those models. Besides, the Barnett model is not a particular case of the usual LMM.

Outliers and detection of influent observations is an important step in the analysis of a data set. There are several ways of evaluating the influence of perturbations in the data set and in the model given the parameter estimates. Important reviews can be found in the books by Cook and Weisberg (1982) and Chatterjee and Hadi (1988) and in the paper by Cook (1986). On the other hand, there are just a few works in the literature for diagnostic and influence of observations in models with measurement errors. Kelly (1984) considered a diagnostic procedure in the structural linear model based on the influence function. Tanaka et al. (1991) also consider the influence function introduced by Hampel for evaluating the influence of observations in the analysis of covariance structures. Zhao and Lee (1998) define leverage of one observation and Cook's distance in a simultaneous equation model. Rather than eliminating cases, the approach proposed by Cook (1986) is a general method for evaluating, under the maximum likelihood estimators, the influence of small perturbations in the model or data set. Additional results on local influence and applications in linear regression and mixed models can be found in Beckman et al. (1987), Lawrance (1988), Thomas and Cook (1990), Tsai and Wu (1992), Paula (1993), Galea et al. (1997) and Lesaffre and Verbeke (1998). Zhao and Lee (1998) and Kwan and Fung (1998) apply the local influence approach for factor analysis and simultaneous equations. Recently, Galea et al. (2002) apply the local influence method in functional and structural comparative calibration models. In this paper, we applied the approach of local influence to the Grubbs's measurement model. Several perturbation schemes are considered, such as, case perturbation and response perturbation.

The paper is organized as follows. In Section 2 the Grubbs's measurement model is considered and in Section 3 the main concepts of local influence are revised. In Section 4 model curvatures are considered for different perturbation schemes and in Section 5 an illustration of the methodology is presented for two real data set.

2 The Grubbs's Model

Suppose that we have at our disposal $p \geq 2$ instruments for measuring a characteristic of interest x in a group of n experimental units. Let x_i the true (unknown) value in unit i and y_{ij} the measured value obtained with instrument j in unit i, $i = 1, \ldots, n$ and $j = 1, \ldots, p$. A model typically considered in the literature see, Grubbs (1973, 1983), for such situation is given, in matrix notation, by

$$\begin{aligned} \mathbf{Y}_i &= \boldsymbol{a} + \mathbf{1}_p x_i + \boldsymbol{\epsilon}_i \qquad (2.1) \\ &= \boldsymbol{a} + \mathbf{K}\mathbf{U}_i, \end{aligned}$$

where $\boldsymbol{a}=(0,\boldsymbol{\alpha}^\top)^\top=(0,\alpha_2,\ldots,\alpha_p)^\top$ is $p\times 1$ vector, $\mathbf{K}=(\mathbf{1}_p,\mathbf{I}_p)$ is a $p\times(p+1)$ matrix, $\mathbf{Y}_i=(y_{i1},\ldots,y_{ip})^\top$ and $\boldsymbol{\epsilon}_i=(\epsilon_{i1},\ldots,\epsilon_{ip})^\top$ are $p\times 1$ random vectors $\mathbf{U}_i=(x_i,\boldsymbol{\epsilon}_i^\top)^\top$ is of dimension $(p+1)\times 1$, $\mathbf{1}_p$ is a $p\times 1$ vector of ones and $\mathbf{I}_p$ denotes the identity matrix of dimension p, $i=1,\ldots n$. As in Bedrick (2001), to eliminate redundancy we set that $\alpha_1=0$. Finally, it is considered that the random vectors $\mathbf{U}_1,\ldots,\mathbf{U}_n$ are independent and identically distributed $N_{p+1}(\boldsymbol{\eta},\boldsymbol{\Psi})$, where

$$\boldsymbol{\eta}=\begin{pmatrix}\mu_x\\0\end{pmatrix}\quad\text{and}\quad\boldsymbol{\Psi}=\begin{pmatrix}\phi_x&0\\0&D(\boldsymbol{\phi})\end{pmatrix},\tag{2.2}$$

with $D(\boldsymbol{\phi})=\operatorname{diag}(\phi_1,\ldots,\phi_p)$ and $\boldsymbol{\phi}=(\phi_1,\ldots,\phi_p)^\top$. Thus, $\mathbf{Y}_1,\ldots,\mathbf{Y}_n$ are independent and identically distributed with according to the $N_p(\boldsymbol{\mu},\boldsymbol{\Sigma})$, where

$$\boldsymbol{\mu}=\boldsymbol{a}+\mathbf{1}\mu_x=\boldsymbol{\mu}(\boldsymbol{\theta})\quad\text{and}\quad\boldsymbol{\Sigma}=\phi_x\mathbf{1}_p\mathbf{1}_p^\top+D(\boldsymbol{\phi})=\boldsymbol{\Sigma}(\boldsymbol{\theta}),\tag{2.3}$$

with $\boldsymbol{\theta}=(\mu_x,\boldsymbol{\alpha}^\top,\phi_x,\boldsymbol{\phi}^\top)^\top$. The log-likelihood function is given by

$$\ell(\boldsymbol{\theta})=\sum_{i=1}^{n}l_i(\boldsymbol{\theta}),\tag{2.4}$$

where $l_i(\boldsymbol{\theta})=(-p/2)\log(2\pi)-\frac{1}{2}\log|\boldsymbol{\Sigma}|-\frac{1}{2}\|\mathbf{T}_i\|^2$, with $\|\mathbf{T}_i\|^2=(\mathbf{Y}_i-\boldsymbol{\mu})^\top\boldsymbol{\Sigma}^{-1}(\mathbf{Y}_i-\boldsymbol{\mu})$, $i=1,\ldots,n$.

Inference for such model is considered in Grubbs (1948, 1973, 1983), Christensen and Blackwood (1993) and recently by Bedrick (2001). Thus, the main object of this paper is to consider the approach of local influence in the Grubbs's measurement model given in (2.1). To obtain the maximum likelihood estimators we used the EM-algorithm.

3 Local Influence

Let $l(\boldsymbol{\theta})$ denote the log-likelihood function from the postulated model (here $\boldsymbol{\theta}=(\mu_x,\boldsymbol{\alpha}^\top,\phi_x,\boldsymbol{\phi}^\top)^\top$) and let $\boldsymbol{\omega}$ be a $q\times 1$ vector of perturbation restricted to some open subset of $\mathbb{R}^q$. The perturbations are made in the likelihood function such that it takes form $l(\boldsymbol{\theta}|\boldsymbol{\omega})$. Denoting the vector of no perturbation by $\boldsymbol{\omega}_0$, we assume $l(\boldsymbol{\theta}|\boldsymbol{\omega}_0)=l(\boldsymbol{\theta})$. To asses the influence of the perturbations on the maximum likelihood estimate of $\boldsymbol{\theta}$, one may consider the likelihood displacement

$$LD(\boldsymbol{\omega})=2[l(\widehat{\boldsymbol{\theta}})-l(\widehat{\boldsymbol{\theta}}_\omega)],$$

where $\widehat{\boldsymbol{\theta}}_\omega(\widehat{\boldsymbol{\theta}})$ denotes the maximum likelihood estimator under the model $l(\boldsymbol{\theta}|\boldsymbol{\omega})(l(\boldsymbol{\theta}))$. The idea of local influence (Cook, 1986) is concerned in characterizing the behavior of $LD(\boldsymbol{\omega})$ at $\boldsymbol{\omega}_0$. The procedure consists in selecting a unit direction $\boldsymbol{d}$, $||\boldsymbol{d}|| = 1$, and then to consider the plot of $LD(\boldsymbol{\omega}_0 + a\boldsymbol{d})$ against a with $a \in R$. This plot is called *lifted line*. Notice that since $LD(\boldsymbol{\omega}_0) = 0$, $LD(\boldsymbol{\omega}_0 + a\boldsymbol{d})$ has a local minimum at $a = 0$. Each lifted line can be characterized by considering the normal curvature $C_d(\boldsymbol{\theta})$ around $a = 0$. The suggestion is to consider the direction $\boldsymbol{d}_{\max}$ corresponding to the largest curvature $C_{d\max}(\boldsymbol{\theta})$. The index plot of $\boldsymbol{d}_{\max}$ may reveal those observations that under small perturbations exert notable influence on $LD(\boldsymbol{\omega})$. Cook (1986) showed that the normal curvature at the direction $\boldsymbol{d}$ takes the form

$$C_d(\boldsymbol{\theta}) = 2|\boldsymbol{d}^\top \boldsymbol{\Delta}^\top \mathbf{L}^{-1} \boldsymbol{\Delta} \boldsymbol{d}|, \tag{3.1}$$

where $-\mathbf{L}$ is the observed Fisher information matrix for the postulated model ($\boldsymbol{\omega} = \boldsymbol{\omega}_0$) and $\boldsymbol{\Delta}$ is the $p \times q$ matrix with elements

$$\Delta_{ij} = \frac{\partial^2 l(\boldsymbol{\theta}|\boldsymbol{\omega})}{\partial\theta_i \partial\omega_j},$$

evaluated at $\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}$ and $\boldsymbol{\omega} = \boldsymbol{\omega}_0, i = 1, ..., p$ and $j = 1, ..., q$. Therefore, the maximization of (3.1) is equivalent to finding the largest absolute eigenvalue of the matrix $\mathbf{B} = \boldsymbol{\Delta}^\top \mathbf{L}^{-1} \boldsymbol{\Delta}$ and, $\boldsymbol{d}_{\max}$ is the corresponding eigenvector. In some situations, it may be of interest to assess the influence on a subset $\boldsymbol{\theta}_1$ of $\boldsymbol{\theta} = (\boldsymbol{\theta}_1^\top, \boldsymbol{\theta}_2^\top)^\top$. For example, one may have interest on $\boldsymbol{\theta}_1 = \boldsymbol{\alpha}$ or $\boldsymbol{\theta}_1 = \boldsymbol{\phi}$. In such situations, the curvature at the direction $\boldsymbol{d}$ is given by

$$C_d(\boldsymbol{\theta}_1) = 2|\boldsymbol{d}^\top \boldsymbol{\Delta}^\top (\mathbf{L}^{-1} - \mathbf{B}_{22}) \boldsymbol{\Delta} \boldsymbol{d}|, \tag{3.2}$$

where,

$$\mathbf{B}_{22} = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_{22}^{-1} \end{pmatrix},$$

and $\mathbf{L}_{22}$ is obtained from the partition of $\mathbf{L}$ according to the partition of $\boldsymbol{\theta}$. The eigenvector $\boldsymbol{d}_{\max}$ corresponds to the largest absolute eigenvalue of the matrix $\mathbf{B} = \boldsymbol{\Delta}^\top (\mathbf{L}^{-1} - \mathbf{B}_{22}) \boldsymbol{\Delta}$.

Other important direction, according to Escobar and Meeker (1992) (see also Verbeke and Molenberghs, 2000) is $\boldsymbol{d} = \boldsymbol{e}_{in}$, which corresponds to the $i-th$ position, where there is a one. In that case, the normal curvature, called the total local influence of individual i, is given by $C_i = 2|\boldsymbol{e}_{in}^\top \mathbf{B} \boldsymbol{e}_{in}| = 2|b_{ii}|$, where b_{ii} is the $i - th$ element diagonal of $\mathbf{B}$, $i = 1, ..., n$. Verbeke and Molenberghs (2000) propose consider the $i - th$ observation influential if C_i is larger than the cutoff value $2\sum_{i=1}^{q} C_i/q$. We use $\boldsymbol{d}_{\max}$ and C_i as diagnostics for local influence.

4 Curvature Derivation

In this section we derive the observed information matrix and the $\boldsymbol{\Delta}$ matrix for different schemes of perturbations.

4.1 The observed information matrix

From (2.4) following that the matrix of second derivatives with respect to $\boldsymbol{\theta}$ is given by:

$$\mathbf{L} = \frac{\partial^2 l(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^\top}\Big|_{\boldsymbol{\theta}=\widehat{\boldsymbol{\theta}}} = \begin{pmatrix} \mathbf{L}_{11} & \mathbf{0} \\ \mathbf{0}^\top & \mathbf{L}_{22} \end{pmatrix}, \tag{4.1}$$

where $\widehat{\boldsymbol{\theta}}$ is the estimator of maximum likelihood of $\boldsymbol{\theta}=(\boldsymbol{\theta}_1^\top, \boldsymbol{\theta}_2^\top)^\top$, where $\boldsymbol{\theta}_1 = (\mu_x, \boldsymbol{\alpha}^\top)^\top$ and $\boldsymbol{\theta}_2 = (\phi_x, \boldsymbol{\phi}^\top)^\top$. The elements of this matrix are given in the appendix A.

4.2 Perturbation of cases

We consider the model (2.1) and weights vector $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_n)^\top$. The log-likelihood function for the perturbed model is given by

$$\ell(\boldsymbol{\theta}/\boldsymbol{\omega}) = \sum_{i=1}^{n} \omega_i \ell_i(\boldsymbol{\theta}), \tag{4.2}$$

where $\ell_i(\boldsymbol{\theta})$, $i = 1, \ldots, n$, as defined in (2.5), with $\boldsymbol{\theta} = (\mu_x, \boldsymbol{\alpha}^\top, \phi_x, \boldsymbol{\phi}^\top)^\top$. Note that here $\boldsymbol{\omega}_0 = \mathbf{1}_n$.

The delta matrix is given by

$$\boldsymbol{\Delta} = (\boldsymbol{\Delta}_1, \ldots, \boldsymbol{\Delta}_n), \tag{4.3}$$

where $\boldsymbol{\Delta}_i = \dfrac{\partial \ell_i(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$, $i = 1, \ldots, n$, with elements

$$\frac{\partial l_i(\boldsymbol{\theta})}{\partial \gamma} = -\frac{1}{2}\Big[\frac{\partial \log|\boldsymbol{\Sigma}|}{\partial \gamma} + \frac{\partial \|\mathbf{T}_i\|^2}{\partial \gamma}\Big], \quad \gamma = \mu_x, \boldsymbol{\alpha}, \phi_x, \boldsymbol{\phi}. \tag{4.4}$$

The components $\dfrac{\partial \log|\boldsymbol{\Sigma}|}{\partial \gamma}$ and $\dfrac{\partial \|\mathbf{T}_i\|^2}{\partial \gamma}$ are presented in Appendix A.

Note that, by (4.1), the normal curvature at the direction $\boldsymbol{d}$ takes the form

$$C_d(\boldsymbol{\theta}) = C_d(\mu_x, \boldsymbol{\alpha}) + C_d(\phi_x, \boldsymbol{\phi}). \tag{4.5}$$

See Verbeke and Molenberghs (2000).

4.3 Response perturbation

We considering here, of following perturbation schemes in the response variable

$$\mathbf{Y}_{\omega_i} = \mathbf{Y}_i + \mathbf{S}_y\,\omega_i\,, \;\; i = 1, \ldots, n, \tag{4.6}$$

where $\mathbf{S}_y = (s_1, \ldots, s_p)^\top$ is a vector $p \times 1$, with s_j, scale factor corresponding to the $j-th$ instrument, $j = 1, \ldots, p$. In this case $\boldsymbol{\omega}_o = (0, \ldots, 0)^\top_{n\times 1}$. The log-likelihood function for the perturbed model $\ell(\boldsymbol{\theta}/\boldsymbol{\omega})$ is given by

$$\ell(\boldsymbol{\theta}/\boldsymbol{\omega}) = \sum_{i=1}^{n} \ell_i(\boldsymbol{\theta}/\omega_i), \tag{4.7}$$

where

$$\ell_i(\boldsymbol{\theta}/\omega_i) = (-p/2)\log(2\pi) - \frac{1}{2}\log|\boldsymbol{\Sigma}| - \frac{1}{2}\|\mathbf{T}_{\omega_i}\|^2\,,$$

and $\|\mathbf{T}_{\omega_i}\|^2 = (\mathbf{Y}_{\omega_i} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{Y}_{\omega_i} - \boldsymbol{\mu})$ with $\boldsymbol{\Sigma}$, $\boldsymbol{\mu}$ as in (2.3) and $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_n)^\top$. Then the $i-th$ column of the $\boldsymbol{\Delta}$ matrix, $\boldsymbol{\Delta}_i$, have elements given by,

$\boldsymbol{\Delta}_{\mu_x i} = \mathbf{1}_p^\top \boldsymbol{\Sigma}^{-1}\mathbf{S}_y$, $\boldsymbol{\Delta}_{\alpha i} = \mathbb{I}_{(p)}\boldsymbol{\Sigma}^{-1}\mathbf{S}_y$, $\boldsymbol{\Delta}_{\phi_x i} = \frac{c^{-2}}{\phi_x}(\mathbf{Y}_{\omega_i} - \boldsymbol{\mu})^\top \mathbf{M}\mathbf{S}_y$ and

$$\begin{aligned}\boldsymbol{\Delta}_{\phi i} = {} & \mathrm{D}(\mathbf{Y}_{\omega_i} - \boldsymbol{\mu})\mathrm{D}^{-2}(\boldsymbol{\phi})\mathbf{S}_y + c^{-2}\phi_x\mathrm{D}^{-2}(\boldsymbol{\phi})\mathbf{1}_p(\mathbf{Y}_{\omega_i} - \boldsymbol{\mu})^\top \mathbf{M}\mathbf{S}_y \\ & - c^{-1}\phi_x\mathrm{D}^{-2}(\boldsymbol{\phi})(\mathbf{Y}_{\omega_i} - \boldsymbol{\mu})\mathbf{1}_p^\top \mathrm{D}^{-1}(\boldsymbol{\phi})\mathbf{S}_y \\ & - c^{-1}\phi_x(\mathbf{Y}_{\omega_i} - \boldsymbol{\mu})^\top \mathrm{D}^{-1}(\boldsymbol{\phi})\mathbf{1}_p\mathrm{D}^{-2}(\boldsymbol{\phi})\mathbf{S}_y, \;\; i = 1, \ldots, n,\end{aligned}$$

where $c = 1 + \phi_x \mathbf{1_p}^\top \boldsymbol{D}^{-1}(\boldsymbol{\phi})\mathbf{1_p}$, $\mathbf{M} = \phi_x \boldsymbol{D}^{-1}(\boldsymbol{\phi})\mathbf{1_p}\mathbf{1_p}^\top \boldsymbol{D}^{-1}(\boldsymbol{\phi})$ and the $\boldsymbol{\Delta}$ matrix is evaluated in $\boldsymbol{\omega} = \boldsymbol{\omega}_0 = (0, \ldots, 0)^\top$ and $\widehat{\boldsymbol{\theta}} = (\widehat{\mu}_x, \widehat{\boldsymbol{\alpha}}^\top, \widehat{\phi}_x, \widehat{\boldsymbol{\phi}}^\top)^\top$.

Another form of the response perturbation is to consider

$$\mathbf{Y}_{\omega_i} = \mathbf{Y}_i + \mathrm{D}(\mathbf{S}_y)\,\boldsymbol{\omega}_i\,, \;\; i = 1, \ldots, n, \tag{4.8}$$

where $D(\mathbf{S}_y) = \mathrm{Diag}(s_1, \ldots, s_p)$ is a diagonal matrix of order p and $\boldsymbol{\omega}_i$ a $p \times 1$ perturbations vector. In this case $\boldsymbol{\omega}_o = (0, \ldots, 0)^\top_{np\times 1}$.

4.4 Perturbation of the multiplicative bias

In this section we consider the follow model perturbed:

$$\mathbf{Y}_i = \boldsymbol{a} + \boldsymbol{b}x_i + \boldsymbol{\epsilon}_i \tag{4.9}$$

where $\boldsymbol{a} = (0, \boldsymbol{\alpha}^\top)^\top$ and $\boldsymbol{b} = (1, \boldsymbol{\omega}^\top)^\top$, $i = 1, \ldots, n$. Under normality $\mathbf{Y}_i \sim N_p(\boldsymbol{\mu}_\omega; \boldsymbol{\Sigma}_\omega)$, where $\boldsymbol{\mu}_\omega = \boldsymbol{a} + \boldsymbol{b}\mu_x$ and $\boldsymbol{\Sigma}_\omega = \phi_x \boldsymbol{b}\boldsymbol{b}^\top + D(\boldsymbol{\phi})$. Note that this model was originally proposed in Barnett (1969), were there is an application to a data set related to vital capacity (see also Bolfarine and

Galea, 1995). The log-likelihood function for the perturbed model is given by

$$\ell(\boldsymbol{\theta}/\boldsymbol{\omega}) = \sum_{i=1}^{n} \ell_i(\boldsymbol{\theta}/\boldsymbol{\omega}), \tag{4.10}$$

where $\ell_i(\boldsymbol{\theta}/\boldsymbol{\omega}) = (-p/2)\log(2\pi) - \frac{1}{2}\log|\boldsymbol{\Sigma}_\omega| - \frac{1}{2}\|\mathbf{T}_{\omega_i}\|^2$, with $\|\mathbf{T}_{\omega_i}\|^2 = (\mathbf{Y}_i - \boldsymbol{\mu}_\omega)^\top \boldsymbol{\Sigma}_\omega^{-1}(\mathbf{Y}_i - \boldsymbol{\mu}_\omega)$. The $\boldsymbol{\Delta}$ matrix is given by

$$\boldsymbol{\Delta} = \sum_{i=1}^{n} \frac{\partial^2 l_i(\boldsymbol{\theta}/\boldsymbol{\omega})}{\partial\boldsymbol{\theta}\partial\boldsymbol{\omega}^\top},$$

where the elements of $\dfrac{\partial^2 l_i(\boldsymbol{\theta}/\boldsymbol{\omega})}{\partial\boldsymbol{\theta}\partial\boldsymbol{\omega}^\top}$ are given in the appendix B.

5 Applications

In this section we presents two applications of the methodology discussed below.

5.1 Grubbs data set

As an first illustration we considering a data set studied by Grubbs (1948). The data are the time of burning for fuses on projectiles as recorded by three different observers, A, B and C, see Figure 7. This data set also where analyzed by Jaech (1985). For this data set we analysis the local influence based in the perturbation of cases with interest in $\boldsymbol{\theta}$. The results are show in the Figure 1. The observations point as more influential are 4 and 17, this observations are the measurement with the largest standard deviation and mean, respectively. The Figure 1(c) show the scatter plot of $C_i(\mu_x, \boldsymbol{\alpha})$ versus $C_i(\phi_x, \boldsymbol{\phi})$. Their respective cutoff values, indicated in the figure by the dashed line, are $2\sum_{i=1}^{n} C_i(\mu_x, \boldsymbol{\alpha})/n = 0.421$ and $2\sum_{i=1}^{n} C_i(\phi_x, \boldsymbol{\phi})/n = 0.614$. Note in this figure that the observations 2, 4, 9, 15 and 17 are highly influential for both part of the parameter and the observation 12 is influential only for the estimation of $(\phi_x, \boldsymbol{\phi})$.

*********Figures 1-2 about here*********

The Figures 2(a) and 2(b) show the index plot of $|d_{max}|$ and C_i for the response perturbation, respectively. Note that this perturbation scheme can be used for identify influential observations, y_{ij}, between experimental unit. Under this perturbation scheme the observations point as more influential are 12 and 17. Moreover, from Likelihood Displacement, figure 2(c), the observations 4 and 17 are globally influential. Note from figure 7(c) that the measurements of the observation 4 had the higher variability, while the measurements of the observations 12 and 17 have higher means. Hence, in this data set, the local influence methods detect like potentially influential, cases with higher means and standard deviations.

The Table 1 present the maximum likelihood estimate for $\boldsymbol{\theta}$ and its standard errors when we dropped some of the most influential observations, according the global/local influence method. It can be observed that there are great differences in some standard errors.

*********Table 1 about here*********

5.2 Barnett data set

In this section we analyze one real data set given in Barnett (1969). Two instruments (standard and new) used for measuring the vital capacity of the human lung and operated by skilled and unskilled operators were compared on a common group of 72 patients. Thus, the following four instruments were compared:

Instrument I : Standard instrument and skilled operator;
Instrument II : Standard instrument and unskilled operator;
Instrument III: New instrument and skilled operator;
Instrument IV: New instrument and unskilled operator.

The Figure 8 present some scatter diagrams of the taken measurement with each instrument.

Figures 3(a) and 3(b) present graphics of local influence for the perturbation of case weights. The cutoff used for C_i equals $2\sum_{i=1}^{n} C_i/n = 0.55$ and has been indicated in the figure by the dashed line. Patients 4, 25 and 67 are found to have relatively a C_i value larger than 0.55 and are therefore considered to be influential for the estimation of the complete parameter vector $\boldsymbol{\theta}$. Similar results are observed in Figure 4 where we present the index plots of $|d_{max}|$ and C_i with interest in $\boldsymbol{\alpha}$ and $\boldsymbol{\phi}$, separately. Next we compare local influence and case deletion diagnostics. Following Zhao and Lee (1998), Cook's distance can be defined by

$$D_i = (\widehat{\boldsymbol{\theta}}_{(i)} - \widehat{\boldsymbol{\theta}})^\top(-\boldsymbol{L})(\widehat{\boldsymbol{\theta}}_{(i)} - \widehat{\boldsymbol{\theta}})/(2p+1), \tag{5.1}$$

$i = 1, ..., n$, where $\widehat{\boldsymbol{\theta}}_{(i)}$ denotes the parameter estimates without case i. Figures 3(c) and 3(d) gives the index plot of D_i and Likelihood Displacement, $LD_i = 2(l(\widehat{\boldsymbol{\theta}}) - l(\widehat{\boldsymbol{\theta}}_{(i)}))$, $i = 1, ..., n$, for the Grubbs's model. Once again cases 4, 25 and 67 are prominent. Note, in figure 8(d), that this observations have one of the major mean and standard deviations, but also it is not a sufficient condition. Hence, diagnostics methods detect, like potentially influential cases, patients with higher measurements, and/or, with greater differences between instruments.

Figures 5 and 6 present graphics of local influence for the perturbation of multiplicative bias. Since the values of d_{max} and C_i are very different for each one of the instruments, we can to conclude that the assumption of equals bias is not plausible in this case and is more appropriate to modify the model incorporating the possibility of a multiplicative bias different of one as is the case in Barnett Models. In effect the maximum log-likelihood for the Grubbs's model is -2074.1 and for the Barnett's model the maximum log-likelihood is -2064.5, corresponding to likelihood ratio statistic of 19.2. This indicates that the Barnett's model fits the data significantly better than the Grubbs's model. Thus, this perturbation scheme serves as a guide in building a revised model.

*********Figures 3-6 about here*********

The Table 2 present the maximum likelihood estimate for $\boldsymbol{\theta}$ and its standard errors when we dropped some of the most influential observations, according the global/local influence method. It can be observed, in all the cases, that estimatives of $\phi_1, ...\phi_4$ are the most affected.

*********Table 2 about here*********

*********Figures 7-8 about here*********

Appendix A: The Observed Information Matrix

In this appendix we present the elements of the observed information matrix. From (2.4), it follows that

$$\frac{\partial l_i(\boldsymbol{\theta})}{\partial \gamma} = -\frac{1}{2}[\frac{\partial \log|\boldsymbol{\Sigma}|}{\partial \gamma} + \frac{\partial \|\mathbf{T}_i\|^2}{\partial \gamma}], \tag{A.1}$$

with $\boldsymbol{\gamma} = \mu_x, \boldsymbol{\alpha}, \phi_x, \boldsymbol{\phi}$, $i = 1, ..., n$. After some algebraic manipulations it follows that

$$\frac{\partial \log|\boldsymbol{\Sigma}|}{\partial \gamma} = 0, \quad \gamma = \mu, \alpha, \quad \frac{\partial \log|\boldsymbol{\Sigma}|}{\partial \phi_x} = c^{-1}\frac{c-1}{\phi_x},$$
$$\frac{\partial \log|\boldsymbol{\Sigma}|}{\partial \boldsymbol{\phi}} = (\mathbf{I}_p - c^{-1}\phi_x \boldsymbol{D}^{-1}(\boldsymbol{\phi}))\boldsymbol{D}^{-1}(\boldsymbol{\phi})\mathbf{1}_p,$$

$$\frac{\partial \|\mathbf{T}_i\|^2}{\partial \mu_x} = -2\mathbf{1}_p^\top \boldsymbol{\Sigma}^{-1}\mathbf{W}_i,$$
$$\frac{\partial \|\mathbf{T}_i\|^2}{\partial \boldsymbol{\alpha}} = -2\mathbb{I}_{(p)}\boldsymbol{\Sigma}^{-1}\mathbf{W}_i,$$
$$\frac{\partial \|\mathbf{T}_i\|^2}{\partial \phi_x} = -\frac{c^{-2}}{\phi_x}\mathbf{W}_i^\top \mathbf{M}\mathbf{W}_i$$
$$\frac{\partial \|\mathbf{T}_i\|^2}{\partial \boldsymbol{\phi}} = -\mathrm{D}(\mathbf{W}_i)\mathrm{D}^{-2}(\boldsymbol{\phi})\mathbf{W}_i - c^{-2}\phi_x \mathbf{W}_i^\top \mathbf{M}\mathbf{W}_i \mathrm{D}^{-2}(\boldsymbol{\phi})\mathbf{1}_p$$
$$+2c^{-1}\phi_x A_i \mathrm{D}^{-2}(\boldsymbol{\phi})\mathbf{W}_i,$$

where $c = 1 + \phi_x \mathbf{1}^\top \boldsymbol{D}^{-1}(\boldsymbol{\phi})\mathbf{1}$, $a_i = \mathbf{W}_i^\top \mathrm{D}^{-1}(\boldsymbol{\phi})\mathbf{1}_p$ and $\boldsymbol{W}_i = \boldsymbol{Y}_i - \boldsymbol{a} - \boldsymbol{b}\mu_x$.

From (A.1) it follows that the per element observed information matrix is given by

$$I_i = I_i(\boldsymbol{\theta}/\boldsymbol{Y}_i) = -\left(\frac{\partial^2 l_i(\boldsymbol{\theta})}{\partial \gamma \partial \boldsymbol{\tau}^\top}\right),$$

where

$$\frac{\partial^2 \ell_i}{\partial \gamma \partial \boldsymbol{\tau}^\top} = -\frac{1}{2}\left[\frac{\partial^2 \log|\boldsymbol{\Sigma}|}{\partial \gamma \partial \boldsymbol{\tau}^\top} + \frac{\partial^2 \|\mathbf{T}_i\|^2}{\partial \gamma \partial \boldsymbol{\tau}^\top}\right]. \tag{A.2}$$

with $\boldsymbol{\tau} = \mu_x, \boldsymbol{\alpha}, \phi_x, \boldsymbol{\phi}$. After some algebraic manipulations we have that

$$\frac{\partial^2 \log|\boldsymbol{\Sigma}|}{\partial \mu_x \partial \gamma^\top} = 0, \quad \gamma = \mu_x, \boldsymbol{\alpha}, \phi_x, \boldsymbol{\phi}, \quad \frac{\partial^2 \log|\boldsymbol{\Sigma}|}{\partial \boldsymbol{\alpha} \partial \gamma^\top} = 0, \quad \gamma = \boldsymbol{\alpha}, \phi_x, \boldsymbol{\phi}$$

$$\frac{\partial^2 \log|\boldsymbol{\Sigma}|}{\partial \phi_x \partial \phi_x} = -\frac{1}{\phi_x^2}\left(\frac{c-1}{c}\right)^2, \quad \frac{\partial^2 \log|\boldsymbol{\Sigma}|}{\partial \phi_x \partial \boldsymbol{\phi}^\top} = -c^{-2}\mathbf{1}_p^\top \mathrm{D}^{-2}(\boldsymbol{\phi}),$$

$$\frac{\partial^2 \log|\boldsymbol{\Sigma}|}{\partial \boldsymbol{\phi} \partial \boldsymbol{\phi}^\top} = -\mathrm{D}^{-2}(\boldsymbol{\phi}) + 2\phi_x c^{-1}\mathrm{D}^{-3}(\boldsymbol{\phi}) - \phi_x c^{-2}\mathrm{D}^{-1}(\boldsymbol{\phi})\mathrm{M}\mathrm{D}^{-1}(\boldsymbol{\phi}),$$

$$\frac{\partial^2 \|\mathbf{T}_i\|^2}{\partial \mu_x \partial \mu_x} = 2\mathbf{1}_p^\top \boldsymbol{\Sigma}^{-1}\mathbf{1}_p,$$

$$\frac{\partial^2 \|\mathbf{T}_i\|^2}{\partial \mu_x \partial \boldsymbol{\alpha}^\top} = 2\mathbf{1}_p' \boldsymbol{\Sigma}^{-1}\mathbb{I}_{(p)}',$$

$$\frac{\partial^2 \|\mathbf{T}_i\|^2}{\partial \mu_x \partial \phi_x} = 2\frac{c^{-2}}{\phi_x}(c-1)\mathbf{1}_p^\top \mathrm{D}^{-1}(\boldsymbol{\phi})\mathbf{W}_i \ ,$$

$$\frac{\partial^2 \|\mathbf{T}_i\|^2}{\partial \mu_x \partial \boldsymbol{\phi}^\top} = 2c^{-1}\mathbf{W}_i^\top \boldsymbol{\Sigma}^{-1}\mathrm{D}^{-1}(\boldsymbol{\phi}) \ ,$$

$$\frac{\partial^2 \|\mathbf{T}_i\|^2}{\partial \boldsymbol{\alpha} \partial \boldsymbol{\alpha}^\top} = 2\mathbb{I}_{(p)}\boldsymbol{\Sigma}^{-1}\mathbb{I}_{(p)}^\top \ ,$$

$$\frac{\partial^2 \|\mathbf{T}_i\|^2}{\partial \boldsymbol{\alpha} \partial \phi_x} = 2\frac{c^{-2}}{\phi_x}\mathbb{I}_{(p)}\mathbf{M}\mathbf{W}_i \ ,$$

$$\begin{aligned}\frac{\partial^2 \|\mathbf{T}_i\|^2}{\partial \boldsymbol{\alpha} \partial \boldsymbol{\phi}^\top} &= 2\mathbb{I}_{(p)}[\boldsymbol{\Sigma}^{-1}\mathrm{D}(\mathbf{W}_i)\mathrm{D}^{-1}(\boldsymbol{\phi}) + c^{-2}\phi_x\mathbf{M}\mathbf{W}_i\mathbf{1}_p^\top \mathrm{D}^{-2}(\boldsymbol{\phi}) \\ &\quad - c^{-1}\phi_x\mathbf{1}_p^\top \mathrm{D}^{-1}(\boldsymbol{\phi})\mathbf{W}_i\mathrm{D}^{-2}(\boldsymbol{\phi})],\end{aligned}$$

$$\frac{\partial^2 \|\mathbf{T}_i\|^2}{\partial \phi_x \partial \phi_x} = 2\frac{c^{-3}}{\phi_x}\mathbf{W}_i^\top \mathbf{M}\mathbf{W}_i\mathbf{1}_p^\top \mathrm{D}^{-1}(\boldsymbol{\phi})\mathbf{1}_p \ ,$$

$$\frac{\partial^2 \|\mathbf{T}_i\|^2}{\partial \phi_x \partial \boldsymbol{\phi}^\top} = -2c^{-3}\mathbf{W}_i^\top \mathbf{M}\mathbf{W}_i\mathbf{1}_p^\top \mathrm{D}^{-2}(\boldsymbol{\phi}) + 2c^{-2}\mathbf{W}_i^\top \mathrm{D}^{-1}(\boldsymbol{\phi})\mathbf{1}_p\mathbf{W}_i^\top \mathrm{D}^{-2}(\boldsymbol{\phi}) \ ,$$

$$\begin{aligned}\frac{\partial^2 \|\mathbf{T}_i\|^2}{\partial \boldsymbol{\phi} \partial \boldsymbol{\phi}^\top} &= 2\mathrm{D}^2(\mathbf{W}_i)D^{-3}(\boldsymbol{\phi}) - 2c^{-3}\phi_x\mathbf{W}_i^\top \mathbf{M}\mathbf{W}_i\mathrm{D}^{-1}(\boldsymbol{\phi})\mathrm{M}\mathrm{D}^{-1}(\boldsymbol{\phi}) \\ &\quad + 2\phi_x c^{-2}\mathrm{D}^{-2}(\boldsymbol{\phi})\mathbf{W}_i\mathbf{W}_i^\top \mathrm{M}\mathrm{D}^{-1}(\boldsymbol{\phi}) + 2c^{-2}\phi_x\mathbf{W}_i^\top \mathbf{M}\mathbf{W}_i\mathrm{D}^{-3}(\boldsymbol{\phi}) \\ &\quad + 2c^{-2}\phi_x\mathrm{D}^{-1}(\boldsymbol{\phi})\mathbf{M}\mathbf{W}_i\mathbf{W}_i^\top \mathrm{D}^{-2}(\boldsymbol{\phi}) - 2c^{-1}\phi_x\mathrm{D}^{-2}(\boldsymbol{\phi})\mathbf{W}_i\mathbf{W}_i^\top \mathrm{D}^{-2}(\boldsymbol{\phi}) \\ &\quad - 4c^{-1}\phi_x\mathbf{1}_p^\top \mathrm{D}^{-1}(\boldsymbol{\phi})\mathbf{W}_i\mathrm{D}(\mathbf{W}_i)\mathrm{D}^{-3}(\boldsymbol{\phi}).\end{aligned}$$

Appendix B: The Delta Matrix for Perturbation of the Multiplicative Bias

The $\boldsymbol{\Delta}$ matrix is given by

$$\boldsymbol{\Delta} = \sum_{i=1}^{n} \frac{\partial^2 l_i(\boldsymbol{\theta}/\boldsymbol{\omega})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\omega}^\top},$$

where the elements of the matrix $\dfrac{\partial^2 l_i(\boldsymbol{\theta}/\boldsymbol{\omega})}{\partial\boldsymbol{\theta}\partial\boldsymbol{\omega}^\top}$, are given by

$$\frac{\partial^2 l_i(\boldsymbol{\theta}|\boldsymbol{\omega})}{\partial\gamma\partial\boldsymbol{\omega}^\top} = -\frac{1}{2}\left[\frac{\partial^2\log|\boldsymbol{\Sigma}|}{\partial\gamma\partial\boldsymbol{\omega}^\top} + \frac{\partial^2\|\mathbf{T}_{\omega_i}\|^2}{\partial\gamma\partial\boldsymbol{\omega}^\top}\right], \quad \gamma = \mu_x, \boldsymbol{\alpha}, \phi_x, \ \boldsymbol{\phi},$$

with

$$\begin{aligned}
\frac{\partial^2\log|\boldsymbol{\Sigma}|}{\partial\gamma\partial\boldsymbol{\omega}^\top} &= 0, \quad \gamma = \mu_x, \boldsymbol{\alpha}, \\
\frac{\partial^2\log|\boldsymbol{\Sigma}|}{\partial\phi_x\partial\boldsymbol{\omega}^\top} &= 2c^{-2}\boldsymbol{\omega}^\top\boldsymbol{D}^{-1}(\boldsymbol{\psi}), \\
\frac{\partial^2\log|\boldsymbol{\Sigma}|}{\partial\boldsymbol{\phi}\partial\boldsymbol{\omega}^\top} &= -2c^{-1}\phi_x D^{-2}(\boldsymbol{\phi})[D_*(\boldsymbol{\omega}) - c^{-1}\phi_x D(\boldsymbol{b})\boldsymbol{b}\boldsymbol{\omega}^\top D^{-1}(\boldsymbol{\psi})],
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2\|\mathbf{T}_i\|^2}{\partial\mu_x\partial\boldsymbol{\omega}^\top} &= -2c^{-1}\boldsymbol{A}_i, \\
\frac{\partial^2\|\mathbf{T}_i\|^2}{\partial\phi_x\partial\boldsymbol{\omega}^\top} &= -2c^{-2}a_i(\boldsymbol{\omega})\boldsymbol{A}_i, \\
\frac{\partial^2\|\mathbf{T}_i\|^2}{\partial\boldsymbol{\alpha}\partial\boldsymbol{\omega}^\top} &= 2q_i[\boldsymbol{D}^{-1}(\boldsymbol{\psi}) - 2c^{-1}\phi_x\boldsymbol{D}^{-1}(\boldsymbol{\psi})\boldsymbol{\omega}\boldsymbol{\omega}^\top\boldsymbol{D}^{-1}(\boldsymbol{\psi})] \\
&\quad + 2c^{-1}\phi_x\boldsymbol{D}^{-1}(\boldsymbol{\psi})\boldsymbol{\omega}(\boldsymbol{Y}_{i2} - \boldsymbol{\alpha})^\top\boldsymbol{D}^{-1}(\boldsymbol{\psi}) \\
\frac{\partial^2\|\mathbf{T}_i\|^2}{\partial\boldsymbol{\phi}\partial\boldsymbol{\omega}^\top} &= 2\boldsymbol{D}^{-2}(\boldsymbol{\phi})[q_i(\boldsymbol{\omega})\boldsymbol{D}_*(\boldsymbol{Y}_{i2} - \boldsymbol{\alpha} - q_i(\boldsymbol{\omega})\boldsymbol{\omega}) \\
&\quad + c^{-1}\phi_x\boldsymbol{D}(\boldsymbol{b})(\boldsymbol{Y}_i - \boldsymbol{a} - \boldsymbol{b}q_i(\boldsymbol{\omega}))\boldsymbol{A}_i],
\end{aligned}$$

where

$$\boldsymbol{D}_*(\boldsymbol{Y}_{i2} - \boldsymbol{\alpha} - q_i(\boldsymbol{\omega})) = \begin{pmatrix} \mathbf{0}^\top_{(p-1)\times 1} \\ D(\boldsymbol{Y}_{i2} - \boldsymbol{\alpha} - q_i(\boldsymbol{\omega})\boldsymbol{\omega}) \end{pmatrix}, \quad \boldsymbol{D}_*(\boldsymbol{\omega}) = \begin{pmatrix} \mathbf{0}^\top_{(p-1)\times 1} \\ D(\boldsymbol{\omega}) \end{pmatrix},$$

$\boldsymbol{A}_i = (\boldsymbol{Y}_{i2} - \boldsymbol{\alpha} - 2q_i(\boldsymbol{\omega})\boldsymbol{\omega})^\top D^{-1}(\boldsymbol{\psi})$, $a_i(\boldsymbol{\omega}) = (\boldsymbol{Y}_i - \boldsymbol{a} - \boldsymbol{b}\mu_x)_i^\top \mathrm{D}^{-1}(\boldsymbol{\phi})\mathbf{b}$, $q_i(\boldsymbol{\omega}) = \mu_x + c^{-1}\phi_x a_i(\boldsymbol{\omega})$ and $\boldsymbol{Y}_{i2} = (y_{i2}, \ldots, y_{ip})^\top$.

Acknowledgements

The authors acknowledges the partial financial support from CAPES and Fapesp, 03/02747-4, Brasil and Project Fondecyt 1000424, Chile. The authors are grateful to one referee for helpful comments and suggestions.

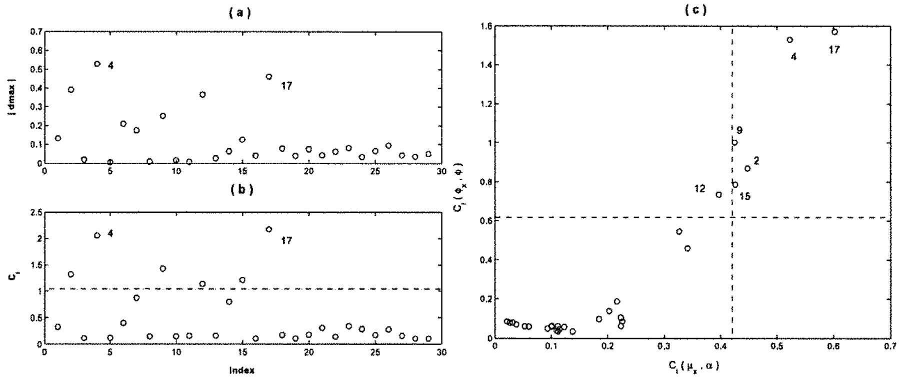

Fig. 1 Grubbs data set. Index plots for perturbation of cases of (a) $|d_{max}|$, (b) C_i and (c) $C_i(\mu_x, \boldsymbol{\alpha})$ versus $C_i(\phi_x, \boldsymbol{\phi})$

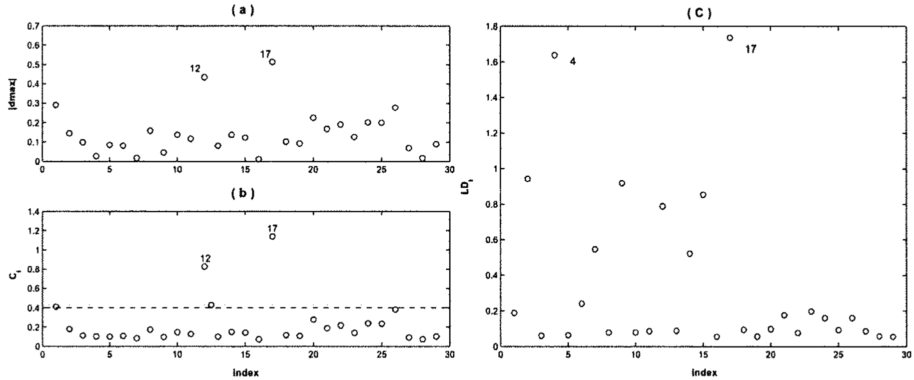

Fig. 2 Grubbs data set. Index plot for perturbation of the response of (a) $|d_{max}|$, (b) C_i (c) Likelihood Displacement LD_i

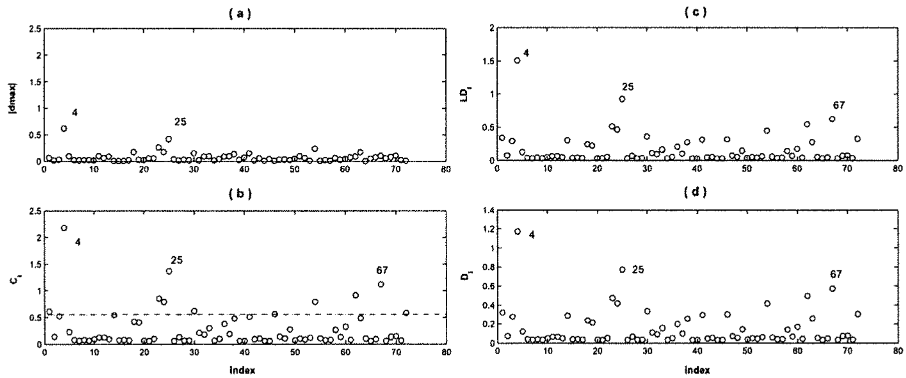

Fig. 3 Barnett data set. Index plots for perturbation of cases of (a) $|d_{max}|$ (b) C_i for $\boldsymbol{\theta}$ and (c) Likelihood Displacement LD_i (d) Cook's distance D_i.

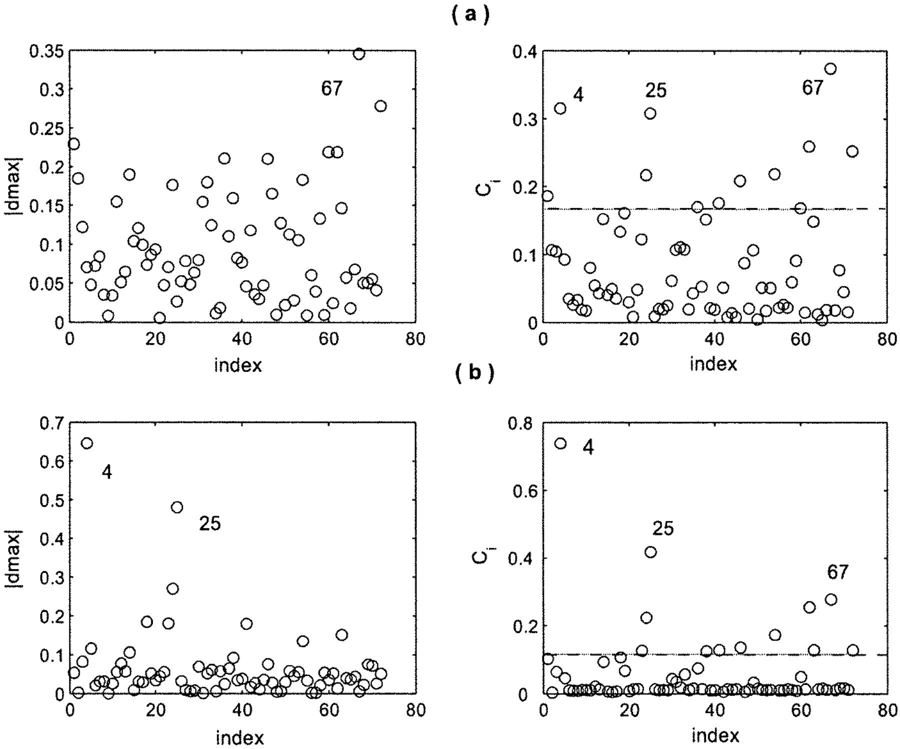

Fig. 4 Barnett data set. Index plots of $|d_{max}|$ and C_i for perturbation of cases (a) for $\boldsymbol{\alpha}$ (b) for $\boldsymbol{\phi}$.

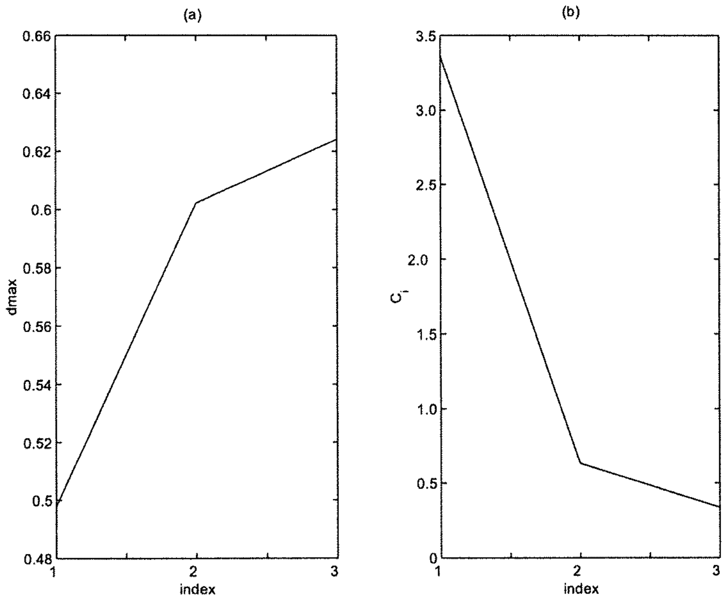

Fig. 5 Barnett data set. Index plots of (a) d_{max} and (b) C_i for perturbation of the multiplicative bias.

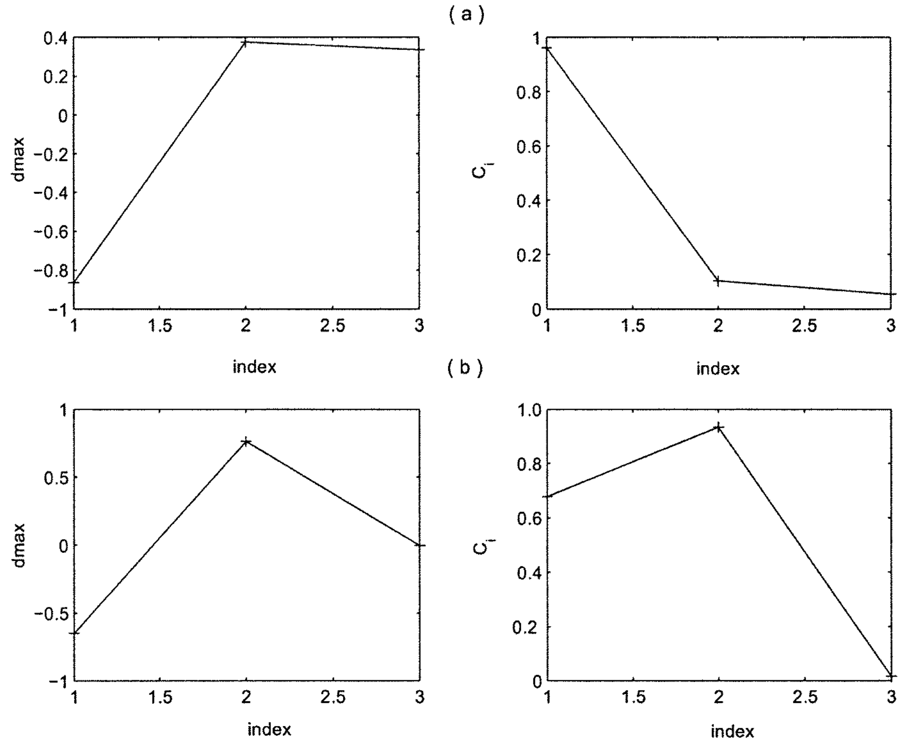

Fig. 6 Barnett data set. Index plots of d_{max} and C_i for perturbation of the multiplicative bias for (a) $\boldsymbol{\alpha}$ and (b) $\boldsymbol{\phi}$.

Table 1 Grubbs data set. Maximum likelihood estimative and its asymptotic standard error when some observations are removed.

$\widehat{\boldsymbol{\theta}}$	None	#4	#12	#17	#4, 12	#4, 17	#12, 17	#4, 12, 17
$\widehat{\mu}_x$	9.7414 (0.0387)	9.7425 (0.0401)	9.7239 (0.0360)	9.7207 (0.0342)	9.7244 (0.0374)	9.7211 (0.0355)	9.7019 (0.0298)	9.7015 (0.0310)
$\widehat{\alpha}_2$	0.0238 (0.0048)	0.0218 (0.0046)	0.0243 (0.0050)	0.0246 (0.0050)	0.0222 (0.0047)	0.0226 (0.0047)	0.0252 (0.0051)	0.0231 (0.0049)
$\widehat{\alpha}_3$	0.0141 (0.0032)	0.0150 (0.0032)	0.0143 (0.0033)	0.0139 (0.0033)	0.0152 (0.0033)	0.0148 (0.0033)	0.0141 (0.0034)	0.0150 (0.0034)
$\widehat{\phi}_x$	0.0434 (0.0114)	0.0449 (0.0120)	0.0362 (0.0097)	0.0326 (0.0087)	0.0376 (0.0102)	0.0338 (0.0092)	0.0240 (0.0065)	0.0249 (0.0069)
$\widehat{\phi}_1$	0.0001 (0.0001)	0.0001 (0.0001)	0.0001 (0.0001)	0.0001 (0.0001)	0.0001 (0.0001)	0.0001 (0.0001)	0.0001 (0.0001)	0.0001 (0.0001)
$\widehat{\phi}_2$	0.0006 (0.0002)	0.0005 (0.0001)	0.0006 (0.0002)	0.0006 (0.0002)	0.0005 (0.0002)	0.0005 (0.0002)	0.0006 (0.0002)	0.0005 (0.0002)
$\widehat{\phi}_3$	0.0002 (0.0001)	0.0002 (0.0001)	0.0002 (0.0001)	0.0002 (0.0001)	0.0002 (0.0001)	0.0002 (0.0001)	0.0003 (0.0001)	0.0002 (0.0001)

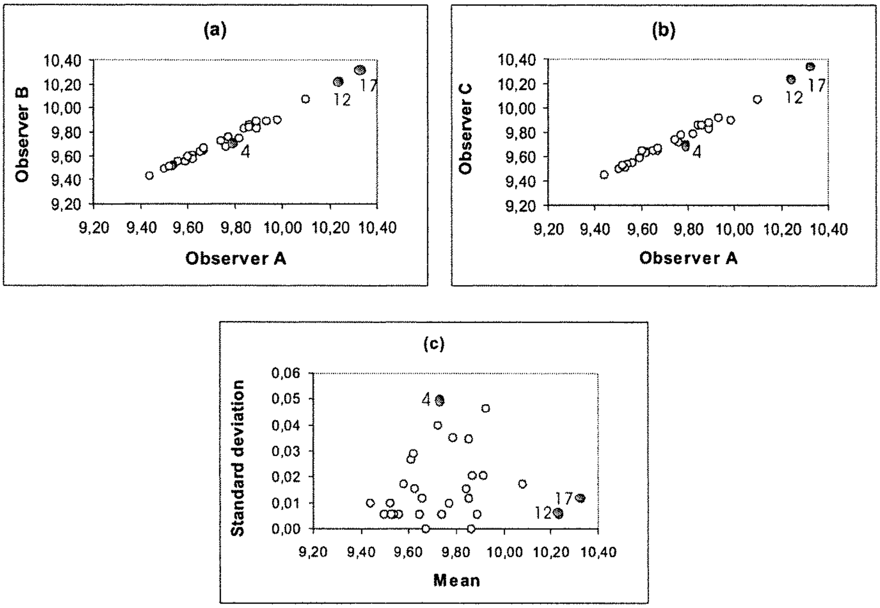

Fig. 7 Grubbs data set. Scatter plots for (a) Measurements of observer A and B, (b) Measurements of observer A and C and (c) Means and standard deviations of the measurements of the observers.

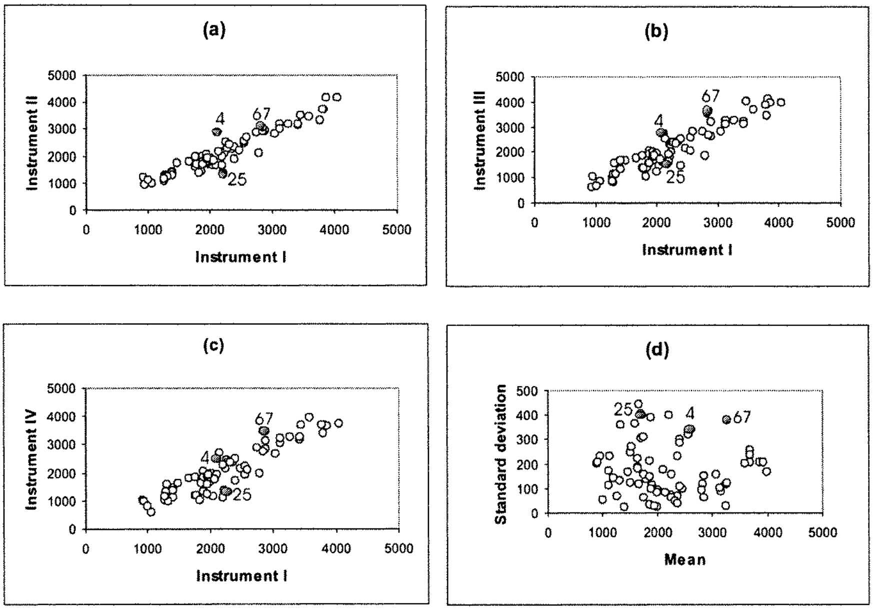

Fig. 8 Barnett data set. Scatter plots for (a) Measurements of instrument I and II, (b) Measurements of instrument I and III, (c) Measurements of instrument I and IV and (d) Means and standard deviations of the measurements of the instruments.

Table 2 Barnett data set. Maximum likelihood estimative and its asymptotic standard errors when some observations are removed.

$\widehat{\theta}$	None	#4	#25	#67	#4, 25	#4, 67	#25,67	#4, 25, 67
$\widehat{\mu}_x$	0.0225 (0.0010)	0.0225 (0.0010)	0.0225 (0.0010)	0.0224 (0.0010)	0.0225 (0.0010)	0.0224 (0.0010)	0.0224 (0.0010)	0.0224 (0.0010)
$\widehat{\alpha}_2$	-0.0007 (0.0003)	-0.0008 (0.0003)	-0.0006 (0.0003)	-0.0007 (0.0003)	-0.0007 (0.0002)	-0.0009 (0.0003)	-0.0006 (0.0003)	-0.0008 (0.0002)
$\widehat{\alpha}_3$	-0.0010 (0.0004)	-0.0011 (0.0004)	-0.0009 (0.0004)	-0.0011 (0.0004)	-0.0010 (0.0004)	-0.0012 (0.0003)	-0.0011 (0.0003)	-0.0012 (0.0003)
$\widehat{\alpha}_4$	-0.0014 (0.0004)	-0.0015 (0.0004)	-0.0013 (0.0004)	-0.0016 (0.0004)	-0.0014 (0.0004)	-0.0016 (0.0004)	-0.0015 (0.0004)	-0.0015 (0.0004)
$\widehat{\phi}_x$	6.2906 (1.0609)	6.2720 (1.0633)	6.2783 (1.0646)	6.2542 (1.0620)	6.2600 (1.0665)	6.2446 (1.0665)	6.2556 (1.0684)	6.2429 (1.0719)
$\widehat{\phi}_1$	0.4998 (0.0993)	0.4170 (0.0851)	0.4419 (0.0891)	0.4805 (0.0963)	0.3602 (0.0745)	0.3982 (0.0823)	0.4269 (0.0869)	0.3439 (0.0723)
$\widehat{\phi}_2$	0.1413 (0.0518)	0.1057 (0.0466)	0.1090 (0.0471)	0.1476 (0.0511)	0.0716 (0.0413)	0.1166 (0.0464)	0.1179 (0.0466)	0.0839 (0.0412)
$\widehat{\phi}_3$	0.4383 (0.0898)	0.4807 (0.0949)	0.4604 (0.0920)	0.3928 (0.0827)	0.5075 (0.0974)	0.4265 (0.0867)	0.4081 (0.0839)	0.4477 (0.0885)
$\widehat{\phi}_4$	0.4633 (0.0936)	0.5087 (0.0949)	0.4989 (0.0980)	0.4328 (0.0888)	0.5522 (0.1046)	0.4704 (0.0936)	0.4610 (0.0922)	0.5069 (0.0980)

References

1. Barnett, V.D. (1969). Simultaneous pairwise linear structural relationships. *Biometrics*, 25, 129-142.
2. Bechman, R., Nachtsheim, C. and Cook, R. (1987). Diagnostics for mixed-model analysis of variance. *Technometrics*, **29**, 413-426.
3. Bedrick, E. J. (2001). An efficient scores test for comparing several measuring devices. *Journal of Quality Technololy*, **33**, 96-103.
4. Bolfarine, H. and Galea, M.(1995). Structural comparative calibration using the EM algorithm. *Journal of Applied Statistics*, **22**, 277-292.
5. Chatterjee, S. and Hadi, A.S. (1988). *Sensivity Analysis in Linear Regression*, John Wiley, New York.
6. Christensen, R. and Blackwood, L.(1993). Test for precision and acurracy of multiple measuring devices. *Technometrics*, **35**, 411-420.
7. Cook, R.D. (1986). Assessment of local influence. *Journal of the Royal Statistical Society*, B, **48**, 133-169.
8. Cook, R.D. and Weisberg, S. (1982). *Residuals and Influence in Regression*, Chapman and Hall, London.
9. Escobar, E. and Meeker, W. (1992). Assessing influence in regression analysis with censored data. *Biometrics*, **48**, 507-528.
10. Galea, M., Bolfarine, H. and de Castro, M. (2002). Local influence in comparative calibration models. *Biometrical Journal*, **44**, 59-81.
11. Galea, M., Paula, G.A. and Bolfarine, H. (1997). Local influence in elliptical linear regression models. *The Statistician*, **46**, 71-79.
12. Grubbs, F.E. (1948). On estimating precision of measuring instruments and product variability. *Journal of the American Statistical Association*, **43**, 243-264.
13. Grubbs, F.E. (1973). Errors of measurement, precision, accuracy and the statistical comparison of measuring instruments. *Technometrics*, **15**, 53-66.
14. Grubbs, F.E. (1983). Grubbs's estimator. *Encyclopedia of Statistical Sciences*, **3**, 542-549.

15. Jaech, J.L. (1985). Statistical analysis of measurement errors. *Exxon Monographs.* John Wiley, New York.
16. Kelly, G. (1984).The influence function in the errors in variables problem. *The Annals of Statistics*, **12**, 87-100.
17. Kwan, C. and Fung, W. (1998). Assessing local influence for specific restricted likelihood: Application to factor analysis. *Psychometrika*, **63**, 35-46.
18. Lawrance, A.J. (1988). Regression transformation diagnostic using local influence. *Journal of the American Statistical Association*, **83**, 1067-1072.
19. Lesaffre, E. and Verbeke, G. (1998). Local influence in linear mixed models. *Biometrics*, **54**, 570-583.
20. Paula, G.A. (1993). Assessing local influence in restricted regressions models. *Computational Statistics* and *Data Analysis*, **16**, 63-79.
21. Tanaka, Y., Watadani, S. and Moon, S. (1991). Influence in covariance structure analysis with an application to confirmatory factor analysis. *Communitations in Statistics-Theory and Methods*, **20**, 3805-3821.
22. Thomas, W. and Cook, R.D. (1990). Assessing influence on predictions from generalized linear models. *Technometrics*, **32**, 59-65.
23. Tsai, C.L. and Wu, X. (1992). Assessing local influence in linear regression models with first-order autoregressive or heteroscedastic error structure. *Statistics and Probability Letters*, **14**, 247-252.
24. Verbeke, G. and Molenberghs, G. (2000). *Linear mixed models for longitudinal data.* Springer, New York.
25. Zhao, Y. and Lee, A. (1998). Influence diagnostics for simultaneous equations models. *Australian and New Zealand Journal Statistics*, **40**, 345-357.