

## 0.1 The CPJ Paper

### 0.1.1 Case-Deletion results for Variance components

Christensen et al. (1992) examines case deletion results for estimates of the variance components, proposing the use of one-step estimates of variance components for examining case influence. The method describes focuses on REML estimation, but can easily be adapted to ML or other methods.

This paper develops their global influences for the deletion of single observations in two steps: a one-step estimate for the REML (or ML) estimate of the variance components, and an ordinary case-deletion diagnostic for a weighted regression problem ( conditional on the estimated covariance matrix) for fixed effects.

### 0.1.2 CPJ Notation

$$\mathbf{C} = \mathbf{H}^{-1} = \begin{bmatrix} c_{ii} & \mathbf{c}'_i \\ \mathbf{c}_i & \mathbf{C}_{[i]} \end{bmatrix}$$

Christensen et al. (1992) noted the following identity:

$$\mathbf{H}^{-1}_{[i]} = \mathbf{C}_{[i]} - \frac{1}{c_{ii}} \mathbf{c}_{[i]} \mathbf{c}'_{[i]}$$

Christensen et al. (1992) use the following as building blocks for case deletion statistics.

- $\check{x}_i$
- $\check{z}_i$
- $\check{z}_i j$
- $\check{y}_i$
- $p_i i$

- $m_i$

All of these terms are a function of a row (or column) of  $\mathbf{H}$  and  $\mathbf{H}_{[i]}^{-1}$

## 0.2 Matrix Notation for Case Deletion

### 0.2.1 Case deletion notation

For notational simplicity,  $\mathbf{A}(i)$  denotes an  $n \times m$  matrix  $\mathbf{A}$  with the  $i$ -th row removed,  $a_i$  denotes the  $i$ -th row of  $\mathbf{A}$ , and  $a_{ij}$  denotes the  $(i, j)$ -th element of  $\mathbf{A}$ .

### 0.2.2 Partitioning Matrices

Without loss of generality, matrices can be partitioned as if the  $i$ -th omitted observation is the first row; i.e.  $i = 1$ .

## 0.3 The CPJ Paper

### 0.3.1 Case-Deletion results for Variance components

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### 0.3.2 CPJ Notation

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All of these terms are a function of a row (or column) of  $\mathbf{H}$  and  $\mathbf{H}_{[i]}^{-1}$

## 0.4 Matrix Notation for Case Deletion

### 0.4.1 Case deletion notation

For notational simplicity,  $\mathbf{A}(i)$  denotes an  $n \times m$  matrix  $\mathbf{A}$  with the  $i$ -th row removed,  $a_i$  denotes the  $i$ -th row of  $\mathbf{A}$ , and  $a_{ij}$  denotes the  $(i, j)$ -th element of  $\mathbf{A}$ .

### 0.4.2 Partitioning Matrices

Without loss of generality, matrices can be partitioned as if the  $i$ -th omitted observation is the first row; i.e.  $i = 1$ .

## 0.5 CPJ's Three Propositions

### Proposition 1

$$\mathbf{V}^{-1} = \begin{bmatrix} \nu^{ii} & \lambda'_i \\ \lambda_i & \Lambda_{[i]} \end{bmatrix}$$
$$\mathbf{V}_{[i]}^{-1} = \Lambda_{[i]} - \frac{\lambda_i \lambda'_i}{\lambda_i}$$

### 0.5.1 Proposition 2

- (i)  $\mathbf{X}_{[i]}^T \mathbf{V}_{[i]}^{-1} \mathbf{X}_{[i]} = \mathbf{X}' \mathbf{V}^{-1} \mathbf{X}$
- (ii)  $= (\mathbf{X}' \mathbf{V}^{-1} \mathbf{Y})^{-1}$
- (iii)  $\mathbf{X}_{[i]}^T \mathbf{V}_{[i]}^{-1} \mathbf{Y}_{[i]} = \mathbf{X}' \mathbf{V}^{-1} \mathbf{Y}$

### 0.5.2 Proposition 3

This proposition is similar to the formula for the one-step Newtown Raphson estimate of the logistic regression coefficients given by Pregibon (1981) and discussed in Cook Weisberg.

## 0.6 CPJ's Three Propositions

**Proposition 1**

$$\mathbf{V}^{-1} = \begin{bmatrix} \nu^{ii} & \lambda'_i \\ \lambda_i & \Lambda_{[i]} \end{bmatrix}$$

$$\mathbf{V}_{[i]}^{-1} = \Lambda_{[i]} - \frac{\lambda_i \lambda'_i}{\lambda_i}$$

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(iii)  $\mathbf{X}_{[i]}^T \mathbf{V}_{[i]}^{-1} \mathbf{Y}_{[i]} = \mathbf{X}' \mathbf{V}^{-1} \mathbf{Y}$

## 0.7 CPJ's Three Propositions

### 0.7.1 Proposition 2

$$(i) \quad \mathbf{X}_{[i]}^T \mathbf{V}_{[i]}^{-1} \mathbf{X}_{[i]} = \mathbf{X}' \mathbf{V}^{-1} \mathbf{X}$$

$$(ii) \quad = (\mathbf{X}' \mathbf{V}^{-1} \mathbf{Y})^{-1}$$

$$(iii) \quad \mathbf{X}_{[i]}^T \mathbf{V}_{[i]}^{-1} \mathbf{Y}_{[i]} = \mathbf{X}' \mathbf{V}^{-1} \mathbf{Y}$$

# Bibliography

Christensen, R., L. M. Pearson, and W. Johnson (1992). Case-deletion diagnostics for mixed models. *Technometrics* 34(1), 38–45.