Chapter 1

Model Diagnostics

1.1 Introduction

In classical linear models model diagnostics have been become a required part of any statistical analysis, and the methods are commonly available in statistical packages and standard textbooks on applied regression. However it has been noted by several papers that model diagnostics do not often accompany LME model analyses. Model diagnostic techniques determine whether or not the distributional assumptions are satisfied, and to assess the influence of unusual observations.

1.1.1 Model Data Agreement

? describes the examination of model-data agreement as comprising several elements; residual analysis, goodness of fit, collinearity diagnostics and influence analysis.

$$r_{mi} = x_i^T \hat{\beta} \tag{1.1}$$

1.1.2 Marginal Residuals

$$\hat{\beta} \;\; = \;\; (X^T R^{-1} X)^{-1} X^T R^{-1} Y$$

$$= BY$$

1.2 Standardized and studentized residuals

To alleviate the problem caused by inconstant variance, the residuals are scaled (i.e. divided) by their standard deviations. This results in a 'standardized residual'. Because true standard deviations are frequently unknown, one can instead divide a residual by the estimated standard deviation to obtain the 'studentized residual.

1.2.1 Standardization

A random variable is said to be standardized if the difference from its mean is scaled by its standard deviation. The residuals above have mean zero but their variance is unknown, it depends on the true values of θ. Standardization is thus not possible in practice.

1.2.2 Studentization

Instead, you can compute studentized residuals by dividing a residual by an estimate of its standard deviation.

1.2.3 Internal and External Studentization

If that estimate is independent of the $i-$th observation, the process is termed 'external studentization'. This is usually accomplished by excluding the $i-$th observation when computing the estimate of its standard error. If the observation contributes to the standard error computation, the residual is said to be internally studentized.

Externally studentized residual require iterative influence analysis or a profiled residuals variance.

1.2.4 Computation

The computation of internally studentized residuals relies on the diagonal entries of $\boldsymbol{V}(\hat{\theta})$ - $\boldsymbol{Q}(\hat{\theta})$, where $\boldsymbol{Q}(\hat{\theta})$ is computed as

$$Q(\hat{\theta}) = X(X'Q(\hat{\theta})^{-1}X)X^{-1}$$

1.2.5 Pearson Residual

Another possible scaled residual is the 'Pearson residual', whereby a residual is divided by the standard deviation of the dependent variable. The Pearson residual can be used when the variability of $\hat{\beta}$ is disregarded in the underlying assumptions.

1.3 Covariance Parameters

The unknown variance elements are referred to as the covariance parameters and collected in the vector θ.

1.3.1 Methods and Measures

The key to making deletion diagnostics useable is the development of efficient computational formulas, allowing one to obtain the case deletion diagnostics by making use of basic building blocks, computed only once for the full model.

? lists several established methods of analyzing influence in LME models. These methods include

- Cook's distance for LME models,
- likelihood distance,
- the variance (information) ration,
- the Cook-Weisberg statistic,
- the Andrews-Prebigon statistic.

1.4 Iterative and non-iterative influence analysis

? highlights some of the issue regarding implementing mixed model diagnostics.

A measure of total influence requires updates of all model parameters.

however, this doesnt increase the procedures execution time by the same degree.

1.4.1 Iterative Influence Analysis

For linear models, the implementation of influence analysis is straightforward. However, for LME models, the process is more complex. Update formulas for the fixed effects are available only when the covariance parameters are assumed to be known. A measure of total influence requires updates of all model parameters. This can only be achieved in general is by omitting observations, then refitting the model.

? describes the choice between iterative influence analysis and non-iterative influence analysis.

1.5 The CPJ Paper

1.5.1 Case-Deletion results for Variance components

? examines case deletion results for estimates of the variance components, proposing the use of one-step estimates of variance components for examining case influence. The method describes focuses on REML estimation, but can easily be adapted to ML or other methods.

This paper develops their global influences for the deletion of single observations in two steps: a one-step estimate for the REML (or ML) estimate of the variance components, and an ordinary case-deletion diagnostic for a weighted regression problem (conditional on the estimated covariance matrix) for fixed effects.

1.5.2 CPJ Notation

$$\boldsymbol{C} = \boldsymbol{H}^{-1} = \begin{bmatrix} c_{ii} & \boldsymbol{c}_i' \\ \boldsymbol{c}_i & \boldsymbol{C}_{[i]} \end{bmatrix}$$

? noted the following identity:

$$\boldsymbol{H}_{[i]}^{-1} = \boldsymbol{C}_{[i]} - \frac{1}{c_{ii}} \boldsymbol{c}_{[i]} \boldsymbol{c}_{[i]}'$$

? use the following as building blocks for case deletion statistics.

- $\breve{x}_i$
- $\breve{z}_i$
- $\breve{z}_i j$
- $\breve{y}_i$
- $p_i i$
- m_i

All of these terms are a function of a row (or column) of $\boldsymbol{H}$ and $\boldsymbol{H}_{[i]}^{-1}$

1.6 Matrix Notation for Case Deletion

1.6.1 Case deletion notation

For notational simplicity, $\boldsymbol{A}(i)$ denotes an $n \times m$ matrix $\boldsymbol{A}$ with the i-th row removed, a_i denotes the i-th row of $\boldsymbol{A}$, and a_{ij} denotes the $(i, j)-$th element of $\boldsymbol{A}$.

1.6.2 Partitioning Matrices

Without loss of generality, matrices can be partitioned as if the $i-$th omitted observation is the first row; i.e. $i = 1$.

1.7 CPJ's Three Propositions

Proposition 1

$$\boldsymbol{V}^{-1} = \begin{bmatrix} \nu^{ii} & \lambda_i' \\ \lambda_i & \Lambda_{[i]} \end{bmatrix}$$

$$\boldsymbol{V}_{[i]}^{-1} = \boldsymbol{\Lambda}_{[i]} - \frac{\lambda_i \lambda_i'}{\lambda_i}$$

1.7.1 Proposition 2

(i) $\boldsymbol{X}_{[i]}^T \boldsymbol{V}_{[i]}^{-1} \boldsymbol{X}_{[i]} = \boldsymbol{X}' \boldsymbol{V}^{-1} \boldsymbol{X}$

(ii) $= (\boldsymbol{X}' \boldsymbol{V}^{-1} \boldsymbol{Y})^{-1}$

(iii) $\boldsymbol{X}_{[i]}^T \boldsymbol{V}_{[i]}^{-1} \boldsymbol{Y}_{[i]} = \boldsymbol{X}' \boldsymbol{V}^{-1} \boldsymbol{Y}$

1.7.2 Proposition 3

This proposition is similar to the formula for the one-step Newtown Raphson estimate of the logistic regression coefficients given by Pregibon (1981) and discussed in Cook Weisberg.

1.8 Measures of Influence

The impact of an observation on a regression fitting can be determined by the difference between the estimated regression coefficient of a model with all observations and the estimated coefficient when the particular observation is deleted. The measure DFBETA is the studentized value of this difference.

Influence arises at two stages of the LME model. Firstly when V is estimated by $\hat{V}$, and subsequent estimations of the fixed and random regression coefficients β and u, given $\hat{V}$.

1.8.1 DFFITS

DFFITS is a statistical measured designed to a show how influential an observation is in a statistical model. It is closely related to the studentized residual.

$$DFFITS = \frac{\widehat{y_i} - \widehat{y_{i(k)}}}{s_{(k)}\sqrt{h_{ii}}}$$

1.8.2 PRESS

The prediction residual sum of squares (PRESS) is an value associated with this calculation. When fitting linear models, PRESS can be used as a criterion for model selection, with smaller values indicating better model fits.

$$PRESS = \sum (y - y^{(k)})^2 \tag{1.2}$$

- $e_{-Q} = y_Q - x_Q \hat{\beta}^{-Q}$
- $PRESS_{(U)} = y_i - x\hat{\beta}_{(U)}$

1.8.3 DFBETA

$$\begin{aligned} DFBETA_a &= \hat{\beta} - \hat{\beta}_{(a)} && (1.3) \\ &= B(Y - Y_{\bar{a}} && (1.4) \end{aligned}$$

Chapter 2

Zewotir's Paper

2.1 Efficient Updating Theorem

? describes the basic theorem of efficient updating.

- $$m_i = \frac{1}{c_{ii}}$$

2.2 Zewotir Measures of Influence in LME Models

? describes a number of approaches to model diagnostics, investigating each of the following;

- Variance components
- Fixed effects parameters
- Prediction of the response variable and of random effects
- likelihood function

2.2.1 Cook's Distance

- For variance components γ: $CD(\gamma)_i$,

- For fixed effect parameters β: $CD(\beta)_i$,
- For random effect parameters $\boldsymbol{u}$: $CD(u)_i$,
- For linear functions of $\hat{beta}$: $CD(\psi)_i$

Random Effects

A large value for $CD(u)_i$ indicates that the $i-$th observation is influential in predicting random effects.

linear functions

$CD(\psi)_i$ does not have to be calculated unless $CD(\beta)_i$ is large.

2.2.2 Information Ratio

2.3 Computation and Notation

with $\boldsymbol{V}$ unknown, a standard practice for estimating $\boldsymbol{X\beta}$ is the estime the variance components σ_j^2, compute an estimate for $\boldsymbol{V}$ and then compute the projector matrix A, $\boldsymbol{X}\hat{\boldsymbol{\beta}} = \boldsymbol{AY}$.

? remarks that $\boldsymbol{D}$ is a block diagonal with the $i-$th block being $u\boldsymbol{I}$

2.4 Measures 2

2.4.1 Cook's Distance

- For variance components γ

Diagnostic tool for variance components

$$C_{\theta i} = ((\hat{\theta})_{[i]} - (\hat{\theta}))^T \mathrm{cov}((\hat{\theta}))^{-1}((\hat{\theta})_{[i]} - (\hat{\theta}))$$

2.4.2 Variance Ratio

- For fixed effect parameters β.

2.4.3 Cook-Weisberg statistic

- For fixed effect parameters β.

2.4.4 Andrews-Pregibon statistic

- For fixed effect parameters β.

The Andrews-Pregibon statistic AP_i is a measure of influence based on the volume of the confidence ellipsoid. The larger this statistic is for observation i, the stronger the influence that observation will have on the model fit.

2.5 Haslett's Analysis

For fixed effect linear models with correlated error structure Haslett (1999) showed that the effects on the fixed effects estimate of deleting each observation in turn could be cheaply computed from the fixed effects model predicted residuals.

Chapter 3

Augmented GLMs

Generalized linear models are a generalization of classical linear models.

3.1 Augmented GLMs

With the use of h-likihood, a random effected model of the form can be viewed as an 'augmented GLM' with the response varaibkes $(y^t, \phi_m^t)^t$, (with $\mu = E(y)$,$u = E(\phi)$, $var(y) = \theta V(\mu)$. The augmented linear predictor is

$$\eta_{ma} = (\eta^t, \eta_m^t)^t) = T\omega.$$

.

The subscript M is a label referring to the mean model.

$$\begin{pmatrix} Y \\ \psi_M \end{pmatrix} = \begin{pmatrix} X & Z \\ 0 & I \end{pmatrix} \begin{pmatrix} \beta \\ \nu \end{pmatrix} + e^* \tag{3.1}$$

The error term e^* is normal with mean zero. The variance matrix of the error term is given by

$$\Sigma_a = \begin{pmatrix} \Sigma & 0 \\ 0 & D \end{pmatrix}. \tag{3.2}$$

y$_a = T\delta + e^*(3.2)$

Weighted least squares equation

3.1.1 The Augmented Model Matrix

$$X = \begin{pmatrix} T & Z \\ 0 & I \end{pmatrix} \delta = \begin{pmatrix} \beta \\ \nu \end{pmatrix} \tag{3.3}$$

3.1.2 Importance-Weighted Least-Squares (IWLS)

3.1.3 H-Likelihood

Chapter 4

Application to Method Comparison Studies

4.1 Application to MCS

Let $\hat{\beta}$ denote the least square estimate of β based upon the full set of observations, and let $\hat{\beta}^{(k)}$ denoted the estimate with the k^{th} case excluded.

4.2 Grubbs' Data

For the Grubbs data the $\hat{\beta}$ estimated are $\hat{\beta}_0$ and $\hat{\beta}_1$ respectively. Leaving the fourth case out, i.e. $k = 4$ the corresponding estimates are $\hat{\beta}_0^{-4}$ and $\hat{\beta}_1^{-4}$

$$Y^{-Q} = \hat{\beta}^{-Q} X^{-Q} \tag{4.1}$$

When considering the regression of case-wise differences and averages, we write $D^{-Q} = \hat{\beta}^{-Q} A^{-Q}$

	F	C	D	A
1	793.80	794.60	-0.80	794.20
2	793.10	793.90	-0.80	793.50
3	792.40	793.20	-0.80	792.80
4	794.00	794.00	0.00	794.00
5	791.40	792.20	-0.80	791.80
6	792.40	793.10	-0.70	792.75
7	791.70	792.40	-0.70	792.05
8	792.30	792.80	-0.50	792.55
9	789.60	790.20	-0.60	789.90
10	794.40	795.00	-0.60	794.70
11	790.90	791.60	-0.70	791.25
12	793.50	793.80	-0.30	793.65

$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \tag{4.2}$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages A and case-wise differences D respectively. A regression model of differences on averages can be fitted with the view to exploring some characteristics of the data.

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```
Call: lm(formula = D ~ A)

Coefficients: (Intercept)            A
  -37.51896      0.04656
```

When considering the regression of case-wise differences and averages, we write

$$D^{-Q} = \hat{\beta}^{-Q} A^{-Q} \tag{4.5}$$

4.2.1 Influence measures using R

R provides the following influence measures of each observation.

	dfb.1_	dfb.A	dffit	cov.r	cook.d	hat
1	0.42	-0.42	-0.56	1.13	0.15	0.18
2	0.17	-0.17	-0.34	1.14	0.06	0.11
3	0.01	-0.01	-0.24	1.17	0.03	0.08
4	-1.08	1.08	1.57	0.24	0.56	0.16
5	-0.14	0.14	-0.24	1.30	0.03	0.13
6	-0.00	0.00	-0.11	1.31	0.01	0.08
7	-0.04	0.04	-0.08	1.37	0.00	0.11
8	0.02	-0.02	0.15	1.28	0.01	0.09
9	0.69	-0.68	0.75	2.08	0.29	0.48
10	0.18	-0.18	-0.22	1.63	0.03	0.27
11	-0.03	0.03	-0.04	1.53	0.00	0.19
12	-0.25	0.25	0.44	1.05	0.09	0.12

Chapter 5

Appendices

5.1 The Hat Matrix

The projection matrix H (also known as the hat matrix), is a well known identity that maps the fitted values $\hat{Y}$ to the observed values Y, i.e. $\hat{Y} = HY$.

$$H = \quad X(X^TX)^{-1}X^T \tag{5.1}$$

H describes the influence each observed value has on each fitted value. The diagonal elements of the H are the 'leverages', which describe the influence each observed value has on the fitted value for that same observation. The residuals (R) are related to the observed values by the following formula:

$$R = (I - H)Y \tag{5.2}$$

The variances of Y and R can be expressed as:

$$\begin{aligned} \text{var}(Y) &= H\sigma^2 \\ \text{var}(R) &= (I - H)\sigma^2 \end{aligned} \tag{5.3}$$

Updating techniques allow an economic approach to recalculating the projection matrix, H, by removing the necessity to refit the model each time it is updated. However this approach is known for numerical instability in the case of down-dating.

5.2 Sherman Morrison Woodbury Formula

The 'Sherman Morrison Woodbury' Formula is a well known result in linear algebra;

$$(A + a^TB)^{-1} \quad = \quad A^{-1} - A^{-1}a^T(I - bA^{-1}a^T)^{-1}bA^{-1} \tag{5.4}$$

This result is highly useful for analyzing regression diagnostics, and for matrices inverses in general. Consider a $p \times p$ matrix X, from which a row x_i^T is to be added or deleted. **?** sets $A = X^TX$, $a = -x_i^T$ and $b = x_i^T$, and writes the above equation as

$$(X^TX \pm x_ix_i^T)^{-1} = \quad (X^TX)^{-1} \mp \quad \frac{(X^TX)^{-1}(x_ix_i^T(X^TX)^{-1}}{1 - x_i^T(X^TX)^{-1}x_i} \tag{5.5}$$

The projection matrix H (also known as the hat matrix), is a well known identity that maps the fitted values $\hat{Y}$ to the observed values Y, i.e. $\hat{Y} = HY$.

$$H = \quad X(X^T X)^{-1} X^T \tag{5.6}$$

H describes the influence each observed value has on each fitted value. The diagonal elements of the H are the 'leverages', which describe the influence each observed value has on the fitted value for that same observation. The residuals (R) are related to the observed values by the following formula:

$$R = (I - H)Y \tag{5.7}$$

The variances of Y and R can be expressed as:

$$\begin{aligned} \text{var}(Y) &= H\sigma^2 \\ \text{var}(R) &= (I - H)\sigma^2 \end{aligned} \tag{5.8}$$

Updating techniques allow an economic approach to recalculating the projection matrix, H, by removing the necessity to refit the model each time it is updated. However this approach is known for numerical instability in the case of down-dating.

5.2.1 Hat Values for MCS regression

With A as the averages and D as the casewise differences.

```
fit = lm(D~A)
```

$$H = A\left(A^\top A\right)^{-1} A^\top,$$

5.3 Cross Validation

Cross validation techniques for linear regression employ the use 'leave one out' re-calculations. In such procedures the regression coefficients are estimated for $n-1$ covariates, with the Q^{th} observation omitted.

Let $\hat{\beta}$ denote the least square estimate of β based upon the full set of observations, and let $\hat{\beta}^{-Q}$ denoted the estimate with the Q^{th} case excluded.

In leave-one-out cross validation, each observation is omitted in turn, and a regression model is fitted on the rest of the data. Cross validation is used to estimate the generalization error of a given model. alternatively it can be used for model selection by determining the candidate model that has the smallest generalization error.

Evidently leave-one-out cross validation has similarities with 'jackknifing', a well known statistical technique. However cross validation is used to estimate generalization error, whereas the jackknife technique is used to estimate bias.

5.3.1 Cross Validation: Updating standard deviation

The variance of a data set can be calculated using the following formula.

$$S^2 = \frac{\sum_{i=1}^{n}(x_i^2) - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}{n-1} \tag{5.9}$$

While using bivariate data, the notation Sxx and Syy shall apply to the variance of x and of y respectively. The covariance term Sxy is given by

$$Sxy = \frac{\sum_{i=1}^{n}(x_i y_i) - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n}}{n-1} \tag{5.10}$$

Let the observation j be omitted from the data set. The estimates for the variance identities can be updating using minor adjustments to the full sample estimates. Where (j) denotes that the jth has been omitted, these identities are

$$Sxx^{(j)} = \frac{\sum_{i=1}^{n}(x_i^2) - (x_j)^2 - \frac{((\sum_{i=1}^{n} x_i) - x_j)^2}{n-1}}{n-2} \tag{5.11}$$

$$Syy^{(j)} = \frac{\sum_{i=1}^{n}(y_i^2) - (y_j)^2 - \frac{((\sum_{i=1}^{n} y_i) - y_j)^2}{n-1}}{n-2} \tag{5.12}$$

$$Sxy^{(j)} = \frac{\sum_{i=1}^{n}(x_i y_i) - (y_j x_j) - \frac{((\sum_{i=1}^{n} x_i) - x_j)(\sum_{i=1}^{n} y_i) - y_k)}{n-1}}{n-2} \tag{5.13}$$

The updated estimate for the slope is therefore

$$\hat{\beta}_1^{(j)} = \frac{Sxy^{(j)}}{Sxx^{(j)}} \tag{5.14}$$

It is necessary to determine the mean for x and y of the remaining $n-1$ terms

$$\bar{x}^{(j)} = \frac{(\sum_{i=1}^{n} x_i) - (x_j)}{n-1}, \tag{5.15}$$

$$\bar{y}^{(j)} = \frac{(\sum_{i=1}^{n} y_i) - (y_j)}{n-1}. \tag{5.16}$$

The updated intercept estimate is therefore

$$\hat{\beta}_0^{(j)} = \bar{y}^{(j)} - \hat{\beta}_1^{(j)} \bar{x}^{(j)}. \tag{5.17}$$

5.4 Updating Estimates

5.4.1 Updating of Regression Estimates

Updating techniques are used in regression analysis to add or delete rows from a model, allowing the analyst the effect of the observation associated with that row. In time series problems, there will be scientific interest in the changing relationship between variables. In cases where there a single row is to be added or deleted, the procedure used is equivalent to a geometric rotation of a plane.

Updating techniques are used in regression analysis to add or delete rows from a model, allowing the analyst the effect of the observation associated with that row.

5.4.2 Updating Standard deviation

A simple, but useful, example of updating is the updating of the standard deviation when an observation is omitted, as practised in statistical process control analyzes. From first principles, the variance of a data set can be calculated using the following formula.

$$S^2 = \frac{\sum_{i=1}^{n}(x_i^2) - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}{n-1} \tag{5.18}$$

While using bivariate data, the notation Sxx and Syy shall apply hither to the variance of x and of y respectively. The covariance term Sxy is given by

$$Sxy = \frac{\sum_{i=1}^{n}(x_i y_i) - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n}}{n-1}. \tag{5.19}$$

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Consider a $p \times p$ matrix X, from which a row x_i^T is to be added or deleted. **?** sets $A = X^T X$, $a = -x_i^T$ and $b = x_i^T$, and writes the above equation as

$$(X^T X \pm x_i x_i^T)^{-1} = \quad (X^T X)^{-1} \mp \quad \frac{(X^T X)^{-1}(x_i x_i^T (X^T X)^{-1}}{1 - x_i^T (X^T X)^{-1} x_i} \tag{5.20}$$

5.4.4 Updating Regression Estimates

Let the observation j be omitted from the data set. The estimates for the variance identities can be updating using minor adjustments to the full sample estimates. Where (j) denotes that the jth has been omitted, these identities are

$$Sxx^{(j)} = \frac{\sum_{i=1}^{n}(x_i^2) - (x_j)^2 - \frac{((\sum_{i=1}^{n} x_i) - x_j)^2}{n-1}}{n-2} \tag{5.21}$$

$$Syy^{(j)} = \frac{\sum_{i=1}^{n}(y_i^2) - (y_j)^2 - \frac{((\sum_{i=1}^{n} y_i) - y_j)^2}{n-1}}{n-2} \tag{5.22}$$

$$Sxy^{(j)} = \frac{\sum_{i=1}^{n}(x_i y_i) - (y_j x_j) - \frac{((\sum_{i=1}^{n} x_i) - x_j)(\sum_{i=1}^{n} y_i) - y_k)}{n-1}}{n-2} \tag{5.23}$$

The updated estimate for the slope is therefore

$$\hat{\beta}_1^{(j)} = \frac{Sxy^{(j)}}{Sxx^{(j)}} \tag{5.24}$$

It is necessary to determine the mean for x and y of the remaining $n-1$ terms

$$\bar{x}^{(j)} = \frac{(\sum_{i=1}^{n} x_i) - (x_j)}{n-1}, \tag{5.25}$$

$$\bar{y}^{(j)} = \frac{(\sum_{i=1}^{n} y_i) - (y_j)}{n-1}. \tag{5.26}$$

The updated intercept estimate is therefore

$$\hat{\beta}_0^{(j)} = \bar{y}^{(j)} - \hat{\beta}_1^{(j)} \bar{x}^{(j)}. \tag{5.27}$$

5.4.5 Inference on intercept and slope

$$\hat{\beta}_1 \pm t_{(\alpha,n-2)}\sqrt{\frac{S^2}{(n-1)S_x^2}} \tag{5.28}$$

$$\frac{\hat{\beta}_0 - \beta_0}{SE(\hat{\beta}_0)} \tag{5.29}$$

$$\frac{\hat{\beta}_1 - \beta_1}{SE(\hat{\beta}_0)} \tag{5.30}$$

Inference on correlation coefficient

This test of the slope is coincidentally the equivalent of a test of the correlation of the n observations of X and Y.

$$\begin{aligned} H_0 &: \rho_{XY} = 0 \\ H_A &: \rho_{XY} \neq 0 \end{aligned} \tag{5.31}$$

5.5 Lesaffre's paper.

Lesaffre considers the case-weight perturbation approach.

Cook's 86 describes a local approach wherein each case is given a weight w_i and the effect on the parameter estimation is measured by perturbing these weights. Choosing weights close to zero or one corresponds to the global case-deletion approach.

Lesaffre describes the displacement in log-likelihood as a useful metric to evaluate local influence

Lesaffre describes a framework to detect outlying observations that matter in an LME model. Detection should be carried out by evaluating diagnostics C_i , $C_i(\alpha)$ and $C_i(D, \sigma^2)$.

Lesaffre defines the total local influence of individual i as

$$C_i = 2|\triangle\prime_i L^{-1} \triangle_i|. \tag{5.32}$$

The influence function of the MLEs evaluated at the ith point IF_i, given by

$$IF_i = -L^{-1} \triangle_i \tag{5.33}$$

can indicate how $th\hat{e}ta$ changes as the weight of the ith subject changes.

The manner by which influential observations distort the estimation process can be determined by inspecting the interpretable components in the decomposition of the above measures of local influence.

Lesaffre comments that there is no clear way of interpreting the information contained in the angles, but that this doesn't mean the information should be ignored.