

# Regression Techniques for MCS

- Deming ( and Orthonormal ) Regression
- The **mcr** package

# Deming Regression

- Conventional regression models are estimated using the ordinary least squares (OLS) technique, and are referred to as 'Model I regression' in some papers.
- A key feature of Model I models is that the independent variable( $X$ ) is assumed to be measured without error.
- As often pointed out in several papers, including Bland and Altman [1], this assumption invalidates simple linear regression for use in method comparison studies, as both methods must be assumed to be measured with error.

# Deming Regression

- The use of regression models that assumes the presence of error in both variables  $X$  and  $Y$  have been proposed for use instead.
- These methodologies are collectively known as 'Error in Variables Regression' or 'Model II regression'. They differ from OLS regression in the method used to estimate the parameters of the regression.

# Deming Regression

- Regression estimates depend on formulation of the model. A formulation with one method considered as the  $X$  variable will yield different estimates for a formulation where it is the  $Y$  variable.
- With Model I regression, the models fitted in both cases will entirely different and inconsistent. However with Model II regression, they will be consistent and complementary.
- The most commonly encountered approach is called "Deming Regression".

# Deming Regression

- Both Variables are assumed to have attendant measurement error.
- The ratio of the variances of measurement errors for the respective methods is known as the **Variance Ratio**, denoted  $\lambda$ .
- Orthonormal Regression is used to describe the case where Variance Ratio is equal to 1. (i.e. variances are assumed to be equal).
- Deming Regression describes the case where the Variance Ratio is specified at a value other than 1.
- Dunn [9] cautions against using the approach as there is no straightforward method for determining an estimate for  $\lambda$ .

# Deming Regression

- As with conventional regression methodologies, Deming's regression calculates an estimate for both the slope and intercept for the fitted line, and standard errors thereof.
- Therefore there is sufficient information to carry out hypothesis tests on both estimates, that are informative about presence of **fixed** and **proportional** bias.

# Deming Regression

- A 95% confidence interval for the intercept estimate can be used to test the intercept, and hence fixed bias, is equal to zero.
- This hypothesis is accepted if the confidence interval for the estimate contains the value 0 in its range. Should this be, it can be concluded that fixed bias is not present.
- Conversely, if the hypothesis is rejected, then it is concluded that the intercept is non zero, and that fixed bias is present.

# Deming Regression

- Testing for proportional bias is a very similar procedure. The 95% confidence interval for the slope estimate can be used to test the hypothesis that the slope is equal to 1.
- This hypothesis is accepted if the confidence interval for the estimate contains the value 1 in its range.
- If the hypothesis is rejected, then it is concluded that the slope is significant different from 1 and that a proportional bias exists.



# Deming Regression

- Deming's Regression suffers from some crucial drawback. Firstly it is computationally complex, and it requires specific software packages to perform calculations.
- Secondly it is uninformative about the comparative precision of two methods of measurement. Most importantly
- Carol andl Rupert [11] states that Deming's regression is acceptable only when the precision ratio ( $\lambda$ , in their paper as  $\eta$ ) is correctly specified ,but in practice this is often not the case, with the  $\lambda$  being underestimated.