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## 0.1 Lesaffre's paper.

- Lesaffre considers the case-weight perturbation approach.
- (Cook, 1986) describes a local approach wherein each case is given a weight  $w_i$  and the effect on the parameter estimation is measured by perturbing these weights. Choosing weights close to zero or one corresponds to the global case-deletion approach.
- Lesaffre describes the displacement in log-likelihood as a useful metric to evaluate local influence (Cook, 1986).
- ? describes a framework to detect outlying observations that matter in an LME model. Detection should be carried out by evaluating diagnostics  $C_i$ ,  $C_i(\alpha)$  and  $C_i(D, \sigma^2)$ .

Lesaffre defines the total local influence of individual  $i$  as

$$C_i = 2|\Delta_i' L^{-1} \Delta_i|. \quad (1)$$

The influence function of the MLEs evaluated at the  $i$ th point  $IF_i$ , given by

$$IF_i = -L^{-1} \Delta_i \quad (2)$$

can indicate how  $\hat{\theta}$  changes as the weight of the  $i$ th subject changes.

The manner by which influential observations distort the estimation process can be determined by inspecting the interpretable components in the decomposition of the above measures of local influence.

Lesaffre comments that there is no clear way of interpreting the information contained in the angles, but that this doesn't mean the information should be ignored.

## 0.2 Lesaffre's paper.

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Cook's 86 describes a local approach wherein each case is given a weight  $w_i$  and the effect on the parameter estimation is measured by perturbing these weights. Choosing weights close to zero or one corresponds to the global case-deletion approach.

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Lesaffre defines the total local influence of individual  $i$  as

$$C_i = 2|\Delta_i' L^{-1} \Delta_i|. \quad (3)$$

The influence function of the MLEs evaluated at the  $i$ th point  $IF_i$ , given by

$$IF_i = -L^{-1} \Delta_i \quad (4)$$

can indicate how  $\hat{\theta}$  changes as the weight of the  $i$ th subject changes.

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$$C_i = 2|\Delta_i' L^{-1} \Delta_i|. \quad (7)$$

The influence function of the MLEs evaluated at the  $i$ th point  $IF_i$ , given by

$$IF_i = -L^{-1} \Delta_i \quad (8)$$

can indicate how  $\hat{\theta}$  changes as the weight of the  $i$ th subject changes.

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# Bibliography

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Generalized linear models are a generalization of classical linear models.

## 0.5 Augmented GLMs

With the use of h-likelihood, a random effected model of the form can be viewed as an ‘augmented GLM’ with the response variables  $(y^t, \phi_m^t)^t$ , (with  $\mu = E(y), u = E(\phi)$ ,  $var(y) = \theta V(\mu)$ ). The augmented linear predictor is

$$\eta_{ma} = (\eta^t, \eta_m^t)^t = T\omega.$$

The subscript  $M$  is a label referring to the mean model.

$$\begin{pmatrix} Y \\ \psi_M \end{pmatrix} = \begin{pmatrix} X & Z \\ 0 & I \end{pmatrix} \begin{pmatrix} \beta \\ \nu \end{pmatrix} + e^* \quad (9)$$

The error term  $e^*$  is normal with mean zero. The variance matrix of the error term is given by

$$\Sigma_a = \begin{pmatrix} \Sigma & 0 \\ 0 & D \end{pmatrix}. \quad (10)$$

Weighted least squares equation

### 0.5.1 The Augmented Model Matrix

$$X = \begin{pmatrix} T & Z \\ 0 & I \end{pmatrix} \delta = \begin{pmatrix} \beta \\ \nu \end{pmatrix} \quad (11)$$

### 0.5.2 Importance-Weighted Least-Squares (IWLS)

### 0.5.3 H-Likelihood

## 0.6 Application to MCS

Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.



## 0.7 Grubbs' Data

For the Grubbs data the  $\hat{\beta}$  estimated are  $\hat{\beta}_0$  and  $\hat{\beta}_1$  respectively. Leaving the fourth case out, i.e.  $k = 4$  the corresponding estimates are  $\hat{\beta}_0^{-4}$  and  $\hat{\beta}_1^{-4}$

$$Y^{-Q} = \hat{\beta}^{-Q} X^{-Q} \tag{12}$$

When considering the regression of case-wise differences and averages, we write  $D^{-Q} = \hat{\beta}^{-Q} A^{-Q}$

	F	C	D	A
1	793.80	794.60	-0.80	794.20
2	793.10	793.90	-0.80	793.50
3	792.40	793.20	-0.80	792.80
4	794.00	794.00	0.00	794.00
5	791.40	792.20	-0.80	791.80
6	792.40	793.10	-0.70	792.75
7	791.70	792.40	-0.70	792.05
8	792.30	792.80	-0.50	792.55
9	789.60	790.20	-0.60	789.90
10	794.40	795.00	-0.60	794.70
11	790.90	791.60	-0.70	791.25
12	793.50	793.80	-0.30	793.65

$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (13)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of differences on averages can be fitted with the view to exploring some characteristics of the data.

When considering the regression of case-wise differences and averages, we write

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Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.

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$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (15)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of differences on averages can be fitted with the view to exploring some characteristics of the data.

Call: `lm(formula = D ~ A)`

Coefficients: (Intercept)	A
-37.51896	0.04656

When considering the regression of case-wise differences and averages, we write

$$D^{-Q} = \hat{\beta}^{-Q} A^{-Q} \quad (16)$$

### 0.7.1 Influence measures using R

R provides the following influence measures of each observation.

	dfb.1_	dfb.A	dffit	cov.r	cook.d	hat
1	0.42	-0.42	-0.56	1.13	0.15	0.18
2	0.17	-0.17	-0.34	1.14	0.06	0.11
3	0.01	-0.01	-0.24	1.17	0.03	0.08
4	-1.08	1.08	1.57	0.24	0.56	0.16
5	-0.14	0.14	-0.24	1.30	0.03	0.13
6	-0.00	0.00	-0.11	1.31	0.01	0.08
7	-0.04	0.04	-0.08	1.37	0.00	0.11
8	0.02	-0.02	0.15	1.28	0.01	0.09
9	0.69	-0.68	0.75	2.08	0.29	0.48
10	0.18	-0.18	-0.22	1.63	0.03	0.27
11	-0.03	0.03	-0.04	1.53	0.00	0.19
12	-0.25	0.25	0.44	1.05	0.09	0.12

# Chapter 1

## Augmented GLMs

Generalized linear models are a generalization of classical linear models.

### 1.1 Augmented GLMs

With the use of h-likelihood, a random effected model of the form can be viewed as an ‘augmented GLM’ with the response variables  $(y^t, \phi_m^t)^t$ , (with  $\mu = E(y), u = E(\phi)$ ,  $var(y) = \theta V(\mu)$ ). The augmented linear predictor is

$$\eta_{ma} = (\eta^t, \eta_m^t)^t = T\omega.$$

.

The subscript  $M$  is a label referring to the mean model.

$$\begin{pmatrix} Y \\ \psi_M \end{pmatrix} = \begin{pmatrix} X & Z \\ 0 & I \end{pmatrix} \begin{pmatrix} \beta \\ \nu \end{pmatrix} + e^* \quad (1.1)$$

The error term  $e^*$  is normal with mean zero. The variance matrix of the error term is given by

$$\Sigma_a = \begin{pmatrix} \Sigma & 0 \\ 0 & D \end{pmatrix}. \quad (1.2)$$

Weighted least squares equation

### 1.1.1 The Augmented Model Matrix

$$X = \begin{pmatrix} T & Z \\ 0 & I \end{pmatrix} \delta = \begin{pmatrix} \beta \\ \nu \end{pmatrix} \quad (1.3)$$

### 1.1.2 Importance-Weighted Least-Squares (IWLS)

### 1.1.3 H-Likelihood

# Chapter 2

## Application to Method Comparison Studies

### 2.1 Application to MCS

Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.

### 2.2 Grubbs' Data

For the Grubbs data the  $\hat{\beta}$  estimated are  $\hat{\beta}_0$  and  $\hat{\beta}_1$  respectively. Leaving the fourth case out, i.e.  $k = 4$  the corresponding estimates are  $\hat{\beta}_0^{-4}$  and  $\hat{\beta}_1^{-4}$

$$Y^{-Q} = \hat{\beta}^{-Q} X^{-Q} \tag{2.1}$$

When considering the regression of case-wise differences and averages, we write  $D^{-Q} = \hat{\beta}^{-Q} A^{-Q}$

	F	C	D	A
1	793.80	794.60	-0.80	794.20
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11	790.90	791.60	-0.70	791.25
12	793.50	793.80	-0.30	793.65



$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (2.2)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of differences on averages can be fitted with the view to exploring some characteristics of the data.

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$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (2.4)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of differences on averages can be fitted with the view to exploring some characteristics of the data.

Call: `lm(formula = D ~ A)`

Coefficients: (Intercept)	A
-37.51896	0.04656

When considering the regression of case-wise differences and averages, we write

$$D^{-Q} = \hat{\beta}^{-Q} A^{-Q} \quad (2.5)$$

### 2.2.1 Influence measures using R

R provides the following influence measures of each observation.

	dfb.1_	dfb.A	dffit	cov.r	cook.d	hat
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# Chapter 3

## Augmented GLMs

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### 3.1 Augmented GLMs

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$$\begin{pmatrix} Y \\ \psi_M \end{pmatrix} = \begin{pmatrix} X & Z \\ 0 & I \end{pmatrix} \begin{pmatrix} \beta \\ \nu \end{pmatrix} + e^* \quad (3.1)$$

The error term  $e^*$  is normal with mean zero. The variance matrix of the error term is given by

$$\Sigma_a = \begin{pmatrix} \Sigma & 0 \\ 0 & D \end{pmatrix}. \quad (3.2)$$

$$y_a = T\delta + e^* \quad (3.3)$$

Weighted least squares equation

### 3.1.1 The Augmented Model Matrix

$$X = \begin{pmatrix} T & Z \\ 0 & I \end{pmatrix} \delta = \begin{pmatrix} \beta \\ \nu \end{pmatrix} \quad (3.4)$$

### 3.1.2 Importance-Weighted Least-Squares (IWLS)

### 3.1.3 H-Likelihood

# Chapter 4

## Application to Method Comparison Studies

### 4.1 Application to MCS

Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.

### 4.2 Grubbs' Data

For the Grubbs data the  $\hat{\beta}$  estimated are  $\hat{\beta}_0$  and  $\hat{\beta}_1$  respectively. Leaving the fourth case out, i.e.  $k = 4$  the corresponding estimates are  $\hat{\beta}_0^{-4}$  and  $\hat{\beta}_1^{-4}$

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6	-0.00	0.00	-0.11	1.31	0.01	0.08
7	-0.04	0.04	-0.08	1.37	0.00	0.11
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