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## The extended likelihood

The desire to have an entirely likelihood-based justification for estimates of random effects, in contrast to Henderson's equation, has motivated ?, page 429 to define the *extended likelihood*. He remarks "In mixed effects modelling the extended likelihood has been called *h-likelihood* (for hierarchical likelihood) by ?, while in smoothing literature it is known as the *penalized likelihood* (e.g. ? ?)." The extended likelihood can be written  $L(\beta, \theta, b|y) = p(y|b; \beta, \theta)p(b; \theta)$  and adopting the same distributional assumptions used by ? yields the log-likelihood function

$$\begin{aligned} \ell_h(\beta, \theta, b|y) = & -\frac{1}{2} \{ \log |\Sigma| + (y - X\beta - Zb)' \Sigma^{-1} (y - X\beta - Zb) \\ & + \log |D| + b' D^{-1} b \} . \end{aligned}$$

Given  $\theta$ , differentiating with respect to  $\beta$  and  $b$  returns Henderson's equations in (??).

## The LME model as a general linear model

Henderson's equations in (??) can be rewritten  $(T'W^{-1}T)\delta = T'W^{-1}y_a$  using

$$\delta = \begin{pmatrix} \beta \\ b \end{pmatrix}, \quad y_a = \begin{pmatrix} y \\ \psi \end{pmatrix}, \quad T = \begin{pmatrix} X & Z \\ 0 & I \end{pmatrix}, \quad \text{and } W = \begin{pmatrix} \Sigma & 0 \\ 0 & D \end{pmatrix},$$

where ? describe  $\psi = 0$  as quasi-data with mean  $E(\psi) = b$ . Their formulation suggests that the joint estimation of the coefficients  $\beta$  and  $b$  of the linear mixed effects model can be derived via a classical augmented general linear model  $y_a = T\delta + \varepsilon$  where  $E(\varepsilon) = 0$  and  $\text{var}(\varepsilon) = W$ , with *both*  $\beta$  and  $b$  appearing as fixed parameters. The usefulness of this reformulation of an LME as a general linear model will be revisited.

## 0.1 Repeated measurements in LME models

In many statistical analyzes, the need to determine parameter estimates where multiple measurements are available on each of a set of variables often arises. Further to ?, ? performs an analysis of the correlation of replicate measurements, for two variables of interest, using LME models.

Let  $y_{Aij}$  and  $y_{Bij}$  be the  $j$ th repeated observations of the variables of interest  $A$  and  $B$  taken on the  $i$ th subject. The number of repeated measurements for each variable may differ for each individual. Both variables are measured on each time points. Let  $n_i$  be the number of observations for each variable, hence  $2 \times n_i$  observations in total.

It is assumed that the pair  $y_{Aij}$  and  $y_{Bij}$  follow a bivariate normal distribution.

$$\begin{pmatrix} y_{Aij} \\ y_{Bij} \end{pmatrix} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \text{ where } \boldsymbol{\mu} = \begin{pmatrix} \mu_A \\ \mu_B \end{pmatrix}$$

The matrix  $\boldsymbol{\Sigma}$  represents the variance component matrix between response variables at a given time point  $j$ .

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_A^2 & \sigma_{AB} \\ \sigma_{AB} & \sigma_B^2 \end{pmatrix}$$

$\sigma_A^2$  is the variance of variable  $A$ ,  $\sigma_B^2$  is the variance of variable  $B$  and  $\sigma_{AB}$  is the covariance of the two variable. It is assumed that  $\boldsymbol{\Sigma}$  does not depend on a particular time point, and is the same over all time points.

## 0.2 Lesaffre's paper.

- Lesaffre considers the case-weight perturbation approach.
- (Cook, 1986) describes a local approach wherein each case is given a weight  $w_i$  and the effect on the parameter estimation is measured by perturbing these weights. Choosing weights close to zero or one corresponds to the global case-deletion approach.
- Lesaffre describes the displacement in log-likelihood as a useful metric to evaluate local influence (Cook, 1986).
- ? describes a framework to detect outlying observations that matter in an LME model. Detection should be carried out by evaluating diagnostics  $C_i$ ,  $C_i(\alpha)$  and  $C_i(D, \sigma^2)$ .

Lesaffre defines the total local influence of individual  $i$  as

$$C_i = 2|\Delta_i' L^{-1} \Delta_i|. \quad (1)$$

The influence function of the MLEs evaluated at the  $i$ th point  $IF_i$ , given by

$$IF_i = -L^{-1} \Delta_i \quad (2)$$

can indicate how  $\hat{\theta}$  changes as the weight of the  $i$ th subject changes.

The manner by which influential observations distort the estimation process can be determined by inspecting the interpretable components in the decomposition of the above measures of local influence.

Lesaffre comments that there is no clear way of interpreting the information contained in the angles, but that this doesn't mean the information should be ignored.

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Lesaffre defines the total local influence of individual  $i$  as

$$C_i = 2|\Delta_i' L^{-1} \Delta_i|. \quad (3)$$

The influence function of the MLEs evaluated at the  $i$ th point  $IF_i$ , given by

$$IF_i = -L^{-1} \Delta_i \quad (4)$$

can indicate how  $\hat{\theta}$  changes as the weight of the  $i$ th subject changes.

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The influence function of the MLEs evaluated at the  $i$ th point  $IF_i$ , given by

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Lesaffre defines the total local influence of individual  $i$  as

$$C_i = 2|\Delta_i' L^{-1} \Delta_i|. \quad (7)$$

The influence function of the MLEs evaluated at the  $i$ th point  $IF_i$ , given by

$$IF_i = -L^{-1} \Delta_i \quad (8)$$

can indicate how  $\hat{\theta}$  changes as the weight of the  $i$ th subject changes.

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# Bibliography

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Generalized linear models are a generalization of classical linear models.

## 0.6 Augmented GLMs

With the use of h-likelihood, a random effected model of the form can be viewed as an ‘augmented GLM’ with the response variables  $(y^t, \phi_m^t)^t$ , (with  $\mu = E(y), u = E(\phi)$ ,  $var(y) = \theta V(\mu)$ ). The augmented linear predictor is

$$\eta_{ma} = (\eta^t, \eta_m^t)^t = T\omega.$$

The subscript  $M$  is a label referring to the mean model.

$$\begin{pmatrix} Y \\ \psi_M \end{pmatrix} = \begin{pmatrix} X & Z \\ 0 & I \end{pmatrix} \begin{pmatrix} \beta \\ \nu \end{pmatrix} + e^* \quad (9)$$

The error term  $e^*$  is normal with mean zero. The variance matrix of the error term is given by

$$\Sigma_a = \begin{pmatrix} \Sigma & 0 \\ 0 & D \end{pmatrix}. \quad (10)$$

Weighted least squares equation

### 0.6.1 The Augmented Model Matrix

$$X = \begin{pmatrix} T & Z \\ 0 & I \end{pmatrix} \delta = \begin{pmatrix} \beta \\ \nu \end{pmatrix} \quad (11)$$

### 0.6.2 Importance-Weighted Least-Squares (IWLS)

### 0.6.3 H-Likelihood

## 0.7 Application to MCS

Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.

## 0.8 Grubbs' Data

For the Grubbs data the  $\hat{\beta}$  estimated are  $\hat{\beta}_0$  and  $\hat{\beta}_1$  respectively. Leaving the fourth case out, i.e.  $k = 4$  the corresponding estimates are  $\hat{\beta}_0^{-4}$  and  $\hat{\beta}_1^{-4}$

$$Y^{-Q} = \hat{\beta}^{-Q} X^{-Q} \tag{12}$$

When considering the regression of case-wise differences and averages, we write  $D^{-Q} = \hat{\beta}^{-Q} A^{-Q}$

	F	C	D	A
1	793.80	794.60	-0.80	794.20
2	793.10	793.90	-0.80	793.50
3	792.40	793.20	-0.80	792.80
4	794.00	794.00	0.00	794.00
5	791.40	792.20	-0.80	791.80
6	792.40	793.10	-0.70	792.75
7	791.70	792.40	-0.70	792.05
8	792.30	792.80	-0.50	792.55
9	789.60	790.20	-0.60	789.90
10	794.40	795.00	-0.60	794.70
11	790.90	791.60	-0.70	791.25
12	793.50	793.80	-0.30	793.65

$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (13)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of differences on averages can be fitted with the view to exploring some characteristics of the data.

When considering the regression of case-wise differences and averages, we write

$$D^{-Q} = \hat{\beta}^{-Q} A^{-Q} \quad (14)$$

Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.

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$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (15)$$

Consider two sets of measurements , in this case F and C , with the vectors of case-wise averages  $A$  and case-wise differences  $D$  respectively. A regression model of differences on averages can be fitted with the view to exploring some characteristics of the data.

Call: `lm(formula = D ~ A)`

Coefficients: (Intercept)	A
-37.51896	0.04656

When considering the regression of case-wise differences and averages, we write

$$D^{-Q} = \hat{\beta}^{-Q} A^{-Q} \quad (16)$$

### 0.8.1 Influence measures using R

R provides the following influence measures of each observation.

	dfb.1_	dfb.A	dffit	cov.r	cook.d	hat
1	0.42	-0.42	-0.56	1.13	0.15	0.18
2	0.17	-0.17	-0.34	1.14	0.06	0.11
3	0.01	-0.01	-0.24	1.17	0.03	0.08
4	-1.08	1.08	1.57	0.24	0.56	0.16
5	-0.14	0.14	-0.24	1.30	0.03	0.13
6	-0.00	0.00	-0.11	1.31	0.01	0.08
7	-0.04	0.04	-0.08	1.37	0.00	0.11
8	0.02	-0.02	0.15	1.28	0.01	0.09
9	0.69	-0.68	0.75	2.08	0.29	0.48
10	0.18	-0.18	-0.22	1.63	0.03	0.27
11	-0.03	0.03	-0.04	1.53	0.00	0.19
12	-0.25	0.25	0.44	1.05	0.09	0.12

# Chapter 1

## Augmented GLMs

Generalized linear models are a generalization of classical linear models.

### 1.1 Augmented GLMs

With the use of h-likelihood, a random effected model of the form can be viewed as an ‘augmented GLM’ with the response variables  $(y^t, \phi_m^t)^t$ , (with  $\mu = E(y), u = E(\phi)$ ,  $var(y) = \theta V(\mu)$ ). The augmented linear predictor is

$$\eta_{ma} = (\eta^t, \eta_m^t)^t = T\omega.$$

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The subscript  $M$  is a label referring to the mean model.

$$\begin{pmatrix} Y \\ \psi_M \end{pmatrix} = \begin{pmatrix} X & Z \\ 0 & I \end{pmatrix} \begin{pmatrix} \beta \\ \nu \end{pmatrix} + e^* \quad (1.1)$$

The error term  $e^*$  is normal with mean zero. The variance matrix of the error term is given by

$$\Sigma_a = \begin{pmatrix} \Sigma & 0 \\ 0 & D \end{pmatrix}. \quad (1.2)$$

Weighted least squares equation

### 1.1.1 The Augmented Model Matrix

$$X = \begin{pmatrix} T & Z \\ 0 & I \end{pmatrix} \delta = \begin{pmatrix} \beta \\ \nu \end{pmatrix} \quad (1.3)$$

### 1.1.2 Importance-Weighted Least-Squares (IWLS)

### 1.1.3 H-Likelihood



## Chapter 2

# Application to Method Comparison Studies

### 2.1 Application to MCS

Let  $\hat{\beta}$  denote the least square estimate of  $\beta$  based upon the full set of observations, and let  $\hat{\beta}^{(k)}$  denoted the estimate with the  $k^{th}$  case excluded.

### 2.2 Grubbs' Data

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$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (2.2)$$

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$$Y^{(k)} = \hat{\beta}^{(k)} X^{(k)} \quad (2.4)$$

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# Chapter 3

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$$\begin{pmatrix} Y \\ \psi_M \end{pmatrix} = \begin{pmatrix} X & Z \\ 0 & I \end{pmatrix} \begin{pmatrix} \beta \\ \nu \end{pmatrix} + e^* \quad (3.1)$$

The error term  $e^*$  is normal with mean zero. The variance matrix of the error term is given by

$$\Sigma_a = \begin{pmatrix} \Sigma & 0 \\ 0 & D \end{pmatrix}. \quad (3.2)$$

$$y_a = T\delta + e^* \quad (3.3)$$

Weighted least squares equation

### 3.1.1 The Augmented Model Matrix

$$X = \begin{pmatrix} T & Z \\ 0 & I \end{pmatrix} \delta = \begin{pmatrix} \beta \\ \nu \end{pmatrix} \quad (3.4)$$

### 3.1.2 Importance-Weighted Least-Squares (IWLS)

### 3.1.3 H-Likelihood

# Chapter 4

## Application to Method Comparison Studies

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