Time Series – Tutorial 7

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Question 1

a) Assume that Y_t is a stationary process with $E(Y_t) = \mu^*$. Show that modelling $W_t = Y_t - \mu$ as an ARMA(p,q), i.e.,

$$\begin{aligned} Y_t - \mu = \phi_1(Y_{t-1} - \mu) + \cdots + \phi_p(Y_{t-p} - \mu) \\ + e_t + \theta_1 e_{t-1} + \cdots + \theta_q e_{t-q}. \end{aligned}$$

produces a model for Y_t. with $E(Y_t) = \mu$.

b) Show that the above model can be written as $\phi(B)\, Y_t = \beta_0 + \theta(B)\, e_t$ where $\beta_0 = (1 - \phi_1 - \cdots - \phi_p)\mu$

Question 2

Consider the `color` dataset.

a) Find the value of r_1 and $\hat{\gamma}_0$ using `acf` and `var`.

b) Estimate ϕ, β_0 and σ_e^2 for an AR(1) process using the method of moments.

c) Write down the equation of the estimated model.

d) Estimate the model using the `arima` function which uses maximum likelihood.

Question 3

Consider the `hare` dataset. It is easy to find that, for $W_t = \sqrt{Y_t}$, we have $r_1 = 0.736$, $r_2 = 0.304$, $\hat{\gamma}_0 = 5.88$ and $\bar{w} = 5.819$.

a) Estimate the parameters ϕ_1 and ϕ_2 for an AR(2) process (for W_t) using the method of moments. Also estimate the intercept and error variance.

b) Write out the equation of the fitted model.

c) Fit the AR(2) model using the `arima` function.

d) Assess the residuals for this model.

e) Fit AR(3) and AR(4) models using the `arima` function. Comment on the results in terms of parameter significance and AIC values.

Question 4

Consider the `oil.price` series.

a) Define the difference-log series $W_t = \nabla \log Y_t$ and, using `R`, calculate its mean, variance and lag-1 autocorrelation.

b) Use the results from part (a) to fit an MA(1) model with an intercept using the method of moments.

c) Write out the equation of the model in terms of Y_t.

Question 5

Consider an MA(1) process for which it is known that the process mean is zero, i.e., no intercept is needed. We observe the following data: $y_1 = 0$, $y_2 = -1$ and $y_3 = \frac{1}{2}$.

a) Derive expressions for e_1, e_2 and e_3 and, hence, show that the least squares estimate is $\hat{\theta} = \frac{1}{2}$.

b) Derive an estimate for $\hat{\sigma}_e^2$ using least squares theory.

Question 6

Show for an AR(1) process that the least squares estimate for σ_e^2 is $\hat{\gamma}_0\,(1 - r_1^2)$ when n is large, i.e., the same as the method of moments estimator.

Question 7

Consider the `color` series which contains the colour property from successive batches of an industrial process.

a) Fit an MA(1) model to this series using `R`.

b) Write down the equation of this fitted model.

c) Comment on the diagnostics produced by `tsdiag`.

Question 8

Consider the `oil.price` series which contains annual oil prices. We found in Lecture 6 that an IMA(1,1) or ARI(1,1) model may be appropriate for $\log Y_t$.

a) Fit both of these models using `R` and comment.

b) Write down the equations for these models.

c) Investigate the residuals.

Question 9

Consider the `robot` series. In Tutorial 6 we decided that the following models may be appropriate for this sereis: AR(3), ARMA(1,1), IMA(1,1).

Fit these three models and compare them. Which model would you choose?