Time Series – Tutorial 4

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Question 1

Consider the process $Y_t = 5 + e_t - \frac{1}{2}e_{t-1} + \frac{1}{4}e_{t-2}$

a) What type of process is this?

b) Calculate the mean and autocovariance functions for this process directly.

c) Calculate the autocovariance function using the fact that, for a general linear process,

$$\gamma_k = \sigma_e^2 \sum_{j=0}^{\infty} \psi_j \, \psi_{j+k}.$$

d) Calculate the autocorrelation function.

Question 2

Using the formulae in Section 3.2 of Lecture 4, calculate ρ_1 and ρ_2 for MA(2) processes with parameters:

a) $\theta_1 = 0.5$, $\theta_2 = 0.4$.

b) $\theta_1 = 1.2$, $\theta_2 = 0.4$.

c) $\theta_1 = -1$, $\theta_2 = 0.6$.

d) Verify your answers to the above using the `ARMAacf` function in `R`.

Question 3

Consider the following MA(1) processes:

$$Y_t = e_t - \theta e_{t-1}$$
$$Y_t = e_t - \frac{1}{\theta} e_{t-1}$$

and assume that $|\theta| < 1$.

a) Using the formula derived in Section 3.1, show that both of these have the same autocorrelation function.

b) By considering the MA characteristic equation, explain which model is preferred.

Question 4

Consider two MA(2) processes: one with $\theta_1 = \theta_2 = \frac{1}{6}$ and the other with $\theta_1 = -1$ and $\theta_2 = 6$.

a) Show that these processes have the same autocorrelation function.

b) Compare the roots of the MA characteristic equations.

Question 5

Suppose that $\{Y_t\}$ is an AR(1) process with $|\phi| < 1$ which has $\gamma_k = \phi^k \frac{\sigma_e^2}{1-\phi^2}$ for $k \geq 0$.

a) Let $W_t = \nabla Y_t$ and show that, for $k \geq 1$,

$$\text{Cov}(W_t, W_{t-k}) = -\left(\frac{1-\phi}{1+\phi}\right) \phi^{k-1} \sigma_e^2.$$

b) Show that $\text{Var}(W_t) = \frac{2\sigma_e^2}{1+\phi}$.

Question 6

Let $\{Y_t\}$ be an AR(2) process of the special form

$$Y_t = \phi Y_{t-2} + e_t.$$

a) Derive the ψ weights using $\phi(B)\,\psi(B) = 1$ and, hence, find the range of ϕ values over which this process is stationary.

b) As an alternative to part (a) determine the conditions for stationarity by considering the roots of the AR characteristic equation.

c) Using the ψ weights from part (a), derive the autocovariance function:

$$\gamma_k = \begin{cases} \phi^{k/2} \frac{\sigma_e^2}{1-\phi^2} & k \text{ even} \\ 0 & k \text{ odd} \end{cases}$$

d) Derive the autocovariance function directly (without using general linear process theory).

Question 7

For the ARMA(1,2) model

$$Y_t = 0.8Y_{t-1} + e_t + 0.7e_{t-1} + 0.6e_{t-2},$$

show that

a) $\rho_k = 0.8\rho_{k-1}$ for $k > 2$.

b) $\rho_2 = 0.8\rho_1 + 0.6\frac{\sigma_e^2}{\gamma_0}$.

Question 8

a) Consider an MA(6) model with $\theta_1 = 0.5$, $\theta_2 = -0.25$, $\theta_3 = 0.125$, $\theta_4 = -0.0625$, $\theta_5 = 0.03125$, $\theta_6 = -0.015625$. Find an AR(1) process which has almost the same ψ weights.

b) Consider an MA(7) model with $\theta_1 = 1$, $\theta_2 = -0.5$, $\theta_3 = 0.25$, $\theta_4 = -0.125$, $\theta_5 = 0.0625$, $\theta_6 = -0.03125$, $\theta_7 = 0.015625$. Find an ARMA(1,1) process which has almost the same ψ weights.

Question 9

Consider the "ARMA(2,2)" process

$$Y_t = 0.4Y_{t-1} + 0.45Y_{t-2} + e_t + e_{t-1} + 0.25e_{t-2}.$$

a) Write this model in the form $\phi(B)\, Y_t = \theta(B)\, e_t$.

b) Find the roots of the AR and MA polynomials. Hence, write the model in the form

$$(1 - \tfrac{1}{a_1}B)(1 - \tfrac{1}{a_2}B)\, Y_t = (1 - \tfrac{1}{m_1}B)(1 - \tfrac{1}{m_2}B)\, e_t$$

where a_1 and a_2 are the roots of the AR equation and m_1 and m_2 are the roots of the MA equation. Show that the model is in fact an ARMA(1,1) model.

c) Is the model invertible and stationary?