Time Series – Tutorial 2

Dr. Kevin Burke

Question 1

Let $E(X) = 2$, $\text{Var}(X) = 9$, $E(Y) = 0$, $\text{Var}(Y) = 4$, and $\text{Corr}(X, Y) = 0.25$. Find:

a) $\text{Var}(X + Y)$.

b) $\text{Cov}(X, X + Y)$.

c) $\text{Corr}(X + Y, X - Y)$.

Question 2

Assume X and Y are dependent but $\text{Var}(X) = \text{Var}(Y)$.

a) Find $\text{Cov}(X + Y, X - Y)$.

Question 3

Suppose $Y_t = 5 + 2t + X_t$ where $\{X_t\}$ is a zero-mean stationary series with auto-covariance function γ_k.

a) Find the mean function for $\{Y_t\}$.

b) Find the auto-covariance function for $\{Y_t\}$.

c) Is $\{Y_t\}$ stationary?

Question 4

Suppose $\text{Cov}(X_t, X_{t-k}) = \gamma_k$ is free of t but that $E(X_t) = 3t$.

a) Is $\{X_t\}$ stationary?

b) Let $Y_t = 7 - 3t + X_t$. Is $\{Y_t\}$ stationary?

Question 5

Let $Y_1 = \mu_0 + e_1$ where μ_0 is a constant. Now define Y_t recursively by $Y_t = \mu_0 + Y_{t-1} + e_t$ for $t > 1$. The process $\{Y_t\}$ is called a random walk with drift.

a) Show that Y_t may be rewritten as

$$Y_t = t\,\theta_0 + e_1 + e_2 + \cdots + e_t.$$

b) Using part (a), find the mean function and autocovariance function for Y_t.

c) Is $W_t = \nabla Y_t$ stationary?

d) Simulate a random walk with drift using `R` where $\mu_0 = 1$ and e_t is normally distributed with $\mu_e = 0$ and $\sigma_e^2 = 25$.

Question 6

Evaluate the mean and autocovariance function for each of the following processes. In each case, determine whether or not the process is stationary.

a) $Y_t = \theta_0 + t e_t$.

b) $W_t = \nabla Y_t$, where Y_t is as given in part (a).

c) $Y_t = (-1)^t U_t$ where $\{U_t\}$ a zero-mean stationary series.

d) $S_t = X_t + Y_t$ where $\{X_t\}$ and $\{Y_t\}$ are independent stationary series.

e) $X_t = e_t\, e_{t-1}$. Hint: use $\text{Cov}(X, Y) = E(XY) - E(X)E(Y)$ for the covariance.

Question 7

Suppose that $\{Y_t\}$ is stationary with acvf γ_k.

a) Show that $W_t = \nabla Y_t = Y_t - Y_{t-1}$ is stationary by finding the mean and auto-covariance function for $\{W_t\}$.

b) Show that $U_t = \nabla^2 Y_t$ is stationary. Hint: use your answer to part (a).

c) Suppose $Y_t = \beta_0 + \beta_1 t + X_t$, where $\{X_t\}$ is a zero-mean stationary series with auto-covariance function γ_k and β_0 and β_1 are constants. Show that $\{Y_t\}$ is not stationary but that $W_t = \nabla Y_t$ is.

Question 8

a) Plot the ACF for the `hare` series and comment. Now difference this data and plot its ACF.

b) Plot the ACF for the `tempdub` series and comment. Now *seasonal* difference this data and plot its ACF.

Question 9

Download monthly FTSE100 data as follows:

```
F100m <- get.hist.quote("^ftse", start="2000-01-01",
            end="2015-12-31", quote=c("Close"),
            provider=c("yahoo"), compression="m")
```

a) Plot the ACF and comment.

b) Carry out a first order difference and plot its ACF.