

## Week 9 Question 6 - Difference in Means

- For a random sample of 10 light bulbs, the mean bulb life is 4,000 hr with a standard deviation of 200 hours.
- For another brand of bulbs whose useful life is also assumed to be normally distributed, a random sample of 8 has a sample mean of 4,300 hours and a sample standard deviation of 250 hours.
- Test the hypothesis that there is no difference between the mean operating life of the two brands of bulbs, using the 5 percent level of significance
- $n_1 = 10$  and  $n_2 = 8$ .
- $\bar{x}_1 = 4000$ ,  $\bar{x}_2 = 4,300$ , therefore  $\bar{x}_2 - \bar{x}_1 = 300$  hours
- $s_1 = 200$ ,  $s_2 = 250$  hours.
- Small sample - Degrees of freedom  $n_1 + n_2 - 2 = 10 + 8 - 2 = 16$

### Pooled variance estimate

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(9 \times 200^2) + (7 \times 250^2)}{16}$$
$$s_p^2 = 49843.75$$

### Computing the Standard Error

$$S.E.(x_1 - x_2) = \sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}$$
$$S.E.(x_1 - x_2) = \sqrt{49843.75 \left( \frac{1}{10} + \frac{1}{9} \right)}$$
$$S.E.(x_1 - x_2) = \sqrt{11214.84} = 105.9$$

### Test Statistic and Critical Value

- The Test Statistic is

$$TS = \frac{(-300) - 0}{105.9} = -2.83$$

- The Critical Value is determined using the following information:  $\alpha = 0.05$ ,  $k = 2$ ,  $df = 16$
- Therefore  $CV = 2.120$
- We can now apply the decision rule : Is the absolute value of the Test Statistic greater than the Critical Value?
- Is  $2.83 > 2.12$ ? Yes We reject  $H_0$ . There is evidence of a difference in means.

- Point estimate :  $\bar{x} - \bar{y} = 1.469$
- Standard Error: 0.5805

$$S.E.(\bar{x} - \bar{y}) = \sqrt{\frac{2.743}{17} + \frac{2.985}{17}}$$

- Quantile : 1.96 (Large sample, with confidence level of 95%.)

$$1.469 \pm (1.96 \times 0.5805) = (0.3321, 2.607)$$

This analysis provides evidence that the mean consumption level per hour for X-cel is higher than the mean consumption level per hour for Yellow, and that the difference between means in the population is likely to be between 0.332 and 2.607 units.