MA4604: Science Maths 4: Homework Week 12

1. Find the volume of the parallelepiped spanned by the given vectors:

(a) $\langle 5, 3, 1 \rangle$, $\langle 4, 2, 2 \rangle$, $\langle 1, 1, 0 \rangle$;

(b) $\langle 0, -1, 1 \rangle$, $\langle 1, 1, 1 \rangle$, $\langle 1, 1, 0 \rangle$.

2. Three vectors are *coplanar* or *linearly dependent* (that is, lie in the same plane) if the volume of the parallelepiped spanned by them is zero (that is, 'the box is flat'). Which of the following sets of vectors are coplanar?

(a) $\langle 2, 2, 1 \rangle$, $\langle 3, 1, 5 \rangle$, $\langle 1, 3, -3 \rangle$;

(b) $\langle 0, 5, 1 \rangle$, $\langle 1, 1, 2 \rangle$, $\langle 3, 1, -1 \rangle$.

3. Find the eigenvalues and eigenvectors of each matrix A:

(a) $\begin{pmatrix} 2 & 4 \\ 2 & 0 \end{pmatrix}$,   (b) $\begin{pmatrix} 2 & -1 \\ 3 & 6 \end{pmatrix}$,   (c) $\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$.

4. Write each matrix in question 3 as $A = PDP^{-1}$, where D is a diagonal matrix.

5. Each matrix M below can be written as $M = PDP^{-1}$. Given M, D and P, find M^9.

(a) $M = \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix}$, $D = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ and $P = \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}$;

(b) $M = \begin{pmatrix} 15 & -24 \\ 8 & -13 \end{pmatrix}$, $D = \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix}$ and $P = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$.

6. Find $\begin{pmatrix} 4 & -6 \\ 2 & -3 \end{pmatrix}^{1234567}$. (Hint: first find M^2).