1 Ordinary Differential Equations

1.1 Euler integration method

The Euler method is a first-order numerical procedure for solving ordinary differential equations (ODEs) with a given initial value. It is the most basic explicit method for numerical integration of ordinary differential equations and is the simplest RungeKutta method.

Question

Consider the following ODE

$$\frac{df(t)}{dt} = tf(t),$$

with $f(t_0) = f_0$, and suppose you use Euler integration with time step ΔT, so that $t_n = t_0 + n\Delta T$. What is the value of the approximation at time t_n?

(a) $f(t_n) \approx f_0 \prod_{k=1}^{n}(1 + t_k \Delta T)$

(b) $f(t_n) \approx f_0 \prod_{k=0}^{n1}(1 + t_k \Delta T)$

(c) $f(t_n) \approx f_0 + f_0 \sum_{k=0}^{n1} t_k \Delta T$

(d) $f(t_n) \approx f_0 + f_0 \sum_{k=1}^{n} t_k \Delta T$

1.2 Reversion to the Mean

Consider the following ODE for $\sigma(t)$,

$$\frac{d\sigma(t)}{dt} = -0.5(\sigma - 0.20)$$

with $\sigma(0) = 0.30$. How will $\sigma(t)$ evolve with time?

(a) It will increase.

(b) It will oscillate around the value 0.30.

(c) It will decrease, reaching the value 0.20 in a finite time.

(d) It will decrease, approaching the value 0.20, but never reaching it.(correct)