Overview

- Binary logistic regression, also called a logit model, is used to model **dichotomous outcome** variables. In the logit model the log odds of the outcome is modeled as a linear combination of the predictor variables.
- Multinomial logistic regression is used to model nominal outcome variables (categorical variables). Again the log odds of the outcomes are modeled as a linear combination of the predictor variables.

- Ordinal logistic regression is used to model nominal outcome variables, where a hierarchy within categories exists.
- Poisson regression is used to model count variables.
- Negative binomial regression is for modeling count variables, usually for over-dispersed count outcome variables.

Multinomial Logistic Regression

- In statistics, multinomial logistic regression is a classification method that generalizes logistic regression to multiclass problems, i.e. with more than two possible discrete outcomes.
- That is, it is a model that is used to predict the probabilities of the different possible outcomes of a categorically distributed dependent variable, given a set of independent variables (which may be real-valued, binary-valued, categorical-valued, etc.).

Multinomial Logistic Regression

- Multinomial logistic regression is used to model nominal outcome variables, in which the log odds of the outcomes are modeled as a linear combination of the predictor variables.

Multinomial Regression with R

The main package we will use is the **nnet** package. We will also use the **ggplot2** and **reshape2** package.

```
install.packages("nnet")
library(nnet)

library(ggplot2)
library(reshape2)
```

nnet

- Title : Feed-forward Neural Networks and Multinomial Log-Linear Models
- Description : Software for feed-forward neural networks with a single hidden layer, and for multinomial log-linear models.
- Authors : Brian Ripley ,William Venables
- URL : http://www.stats.ox.ac.uk/pub/MASS4/

Examples of multinomial logistic regression

- Which major will a college student choose, given their grades, stated likes and dislikes, etc.?
- Which blood type does a person have, given the results of various diagnostic tests?
- In a hands-free mobile phone dialing application, which person's name was spoken, given various properties of the speech signal?
- Which candidate will a person vote for, given particular demographic characteristics?
- Which country will a firm locate an office in, given the characteristics of the firm and of the various candidate countries?

Examples of multinomial logistic regression

Example 1. Entering high school students make program choices among general program, vocational program and academic program. Their choice might be modeled using their writing score and their social economic status.

Examples of multinomial logistic regression

Example 2. People's occupational choices might be influenced by their parents' occupations and their own education level. We can study the relationship of one's occupation choice with education level and father's occupation. The occupational choices will be the outcome variable which consists of categories of occupations.

Examples of multinomial logistic regression

Example 3. A biologist may be interested in food choices that alligators make. Adult alligators might have different preferences from young ones. The outcome variable here will be the types of food, and the predictor variables might be size of the alligators and other environmental variables.

Description of the Data

We will use the third example using the *multilog* data set (or *ml*).

```
ml <- read.csv("multilog.csv",header=T)
```

Description of the Data

- The outcome variable is **prog**, program type.
- The two predictor variables are

1. social economic status, **ses**, (a three-level categorical variable)
2. writing score, **write**, (a continuous variable).

- The data set contains variables on 200 students.

Multinomial Regression with R

```
table(ml$ses, ml$prog)

         general academic vocation
low           16       19       12
middle        20       44       31
high           9       42        7

              M    SD
 general   51.33 9.398
 academic  56.26 7.943
 vocation  46.76 9.319
```

Multinomial logistic regression

- We use the `multinom` function from the **nnet** package to estimate a multinomial logistic regression model.
- Remark - There are other functions in other R packages capable of multinomial regression, such as the **mlogit** package.
- The `multinom` function does not require the data to be reshaped (as the mlogit package does)
- (Similar format to example code found in Hilbe's *Logistic Regression Models*).

Multinomial logistic regression

- We must choose the level of our outcome that we wish to use as our **baseline** and specify this in the `relevel` function.
  Let's choose "academic".
- Then, we run our model using *multinom*.
- The `multinom` command does not include p-value calculation for the regression coefficients.
- (We can calculate p-values using Wald tests or z-tests).

Multinomial Logistic Regression with R

```
ml$prog2 <- relevel(ml$prog, ref = "academic")
test <- multinom(prog2 ~ ses + write, data = ml)

 # weights:  15 (8 variable)
 initial  value 219.722458
 iter  10 value 179.982880
 final    value 179.981726
 converged
```

Multinomial Logistic Regression with R

```
summary(test)

 Call:
 multinom(formula = prog2 ~ ses + write,
             data = ml)

 Coefficients:
          (Intercept) sesmiddle seshigh     write
 general        2.852   -0.5333 -1.1628 -0.05793
 vocation       5.218    0.2914 -0.9827 -0.11360
```

Multinomial Logistic Regression with R

```
....
 Std. Errors:
           (Intercept) sesmiddle seshigh   write
 general         1.166    0.4437  0.5142 0.02141
 vocation        1.164    0.4764  0.5956 0.02222

 Residual Deviance: 360
 AIC: 376
```

Multinomial Logistic Regression with R

Wald Test

- The Wald test in the context of logistic regression is used to determine whether a certain predictor variable X is significant or not. It rejects the null hypothesis of the corresponding coefficient being zero.
- The test consists of dividing the value of the coefficient by standard error

Multinomial Logistic Regression with R

```
# Coefficients Divided by Standard Errors
# Then Compute p-values.
# 2-tailed z test
# p.values

          (Intercept) sesmiddle seshigh       write
 general    1.448e-02    0.2294 0.02374 6.819e-03
 vocation   7.299e-06    0.5408 0.09895 3.176e-07
```

Multinomial Regression with R

- Remark: Some output is generated by running the model, even though we are assigning the model to a new R object.
- This model-running output includes some iteration history and includes the final **negative log-likelihood** (+ 179.981726).
- This value multiplied by two is then seen in the model summary as the **Residual Deviance** and it can be used in comparisons of nested models (360).

Multinomial Regression with R

- As with many summary outputs, the output contains a column of coefficients and a column of standard errors.
- Each of these blocks has one row of values corresponding to a model equation.
- Focusing on the block of coefficients, we can look at the first row comparing `prog = "general"` to our baseline `prog = "academic"` and the second row comparing `prog = "vocation"` to our baseline `prog = "academic"`.

Multinomial Regression with R

- If we consider our coefficients from the first row to be b_1 and our coefficients from the second row to be b_2, we can write our model equations:

$$ln\left(\frac{P(prog = gen.)}{P(prog = acad.)}\right) = b_{10} + b_{11}(ses = 2) + b_{12}(ses = 3) + b_{13}write$$

$$ln\left(\frac{P(prog = voc.)}{P(prog = acad.)}\right) = b_{20} + b_{21}(ses = 2) + b_{22}(ses = 3) + b_{23}write$$

Multinomial Regression with R

- A one-unit increase in the variable **write** is associated with the decrease in the log odds of being in general program vs. academic program in the amount of 0.058 (b_{13}).
- A one-unit increase in the variable **write** is associated with the decrease in the log odds of being in vocation program vs. academic program in the amount of 0.1136 (b_{23}).

Multinomial Regression with R

- The log odds of being in general program vs. in academic program will decrease by 1.163 if moving from ses="low" to ses="high" b_{12}.
- The log odds of being in general program vs. in academic program will decrease by 0.533 if moving from ses="low" to ses="middle" b_{11}, although this coefficient is not significant.

Multinomial Regression with R

- The log odds of being in vocation program vs. in academic program will decrease by 0.983 if moving from ses="low" to ses="high"(b_{22}).
- The log odds of being in vocation program vs. in academic program will increase by 0.291 if moving from ses="low" to ses="middle"(b_{21}), although this coefficient is not signficant.

Odds Ratios

- The ratio of the probability of choosing one outcome category over the probability of choosing the baseline category is often referred as **relative risk**
- It is also sometimes referred as **odds**.

Multinomial Logistic Regression with R

- The relative risk is the (right-hand side) linear equation exponentiated, leading to the fact that the exponentiated regression coefficients are relative risk ratios for a unit change in the predictor variable.
- We can exponentiate the coefficients from our model to see these odds ratios (next slide).

Multinomial Logistic Regression with R

Extract the coefficients from the model then and exponentiate

```
exp(coef(test))

         (Intercept) sesmiddle seshigh  write
 general       17.33    0.5867  0.3126 0.9437
 vocation     184.61    1.3383  0.3743 0.8926
```

Multinomial Logistic Regression with R

- The relative risk ratio for a one-unit increase in the variable **write** is 0.9437 for being in general program vs. academic program.
- The relative risk ratio switching from **ses** $= 1$ to 3 is 0.3126 for being in general program vs. academic program.

Multinomial Regression with R

- You can also use predicted probabilities to help you understand the model.
- You can calculate predicted probabilities for each of our outcome levels using the `fitted` function.
- We can start by generating the predicted probabilities for the observations in our dataset and viewing the first few rows

Multinomial Regression with R

```
head(pp <- fitted(test))
```

```
  academic general vocation
1   0.1483  0.3382   0.5135
2   0.1202  0.1806   0.6992
3   0.4187  0.2368   0.3445
4   0.1727  0.3508   0.4765
5   0.1001  0.1689   0.7309
6   0.3534  0.2378   0.4088
```

Prediction

- Suppose we want to examine the changes in predicted probability associated with one of our two variables, we can create small datasets varying one variable while holding the other constant.
- We will first do this holding **write** at its mean and examining the predicted probabilities for each level of **ses**.
- *(i.e. Three Cases to predict for)*

Multinomial Logistic Regression with R

```
dses <- data.frame(ses = c("low", "middle", "h
                   write = mean(ml$write))
predict(test, newdata = dses, "probs")

   academic general vocation
1    0.4397  0.3582   0.2021
2    0.4777  0.2283   0.2939
3    0.7009  0.1785   0.1206
```

- Another way to understand the model using the predicted probabilities is to look at the averaged predicted probabilities for different values of the continuous predictor variable **write** within each level of **ses**.

Multinomial Logistic Regression with R

Store the predicted probabilities for each value of ses and write

```
dwrite <- data.frame(
  ses = rep(c("low", "middle", "high"), each = 41),
  write = rep(c(30:70), 3))
```

```
pp.write <- cbind(dwrite,
     predict(test, newdata = dwrite,
     type = "probs", se = TRUE))
```

Multinomial Logistic Regression with R

Calculate the mean probabilities within each level of ses

```
by(pp.write[, 3:5], pp.write$ses, colMeans)
```

```
pp.write$ses: high
academic  general vocation
  0.6164   0.1808   0.2028
--------------------------------------
pp.write$ses: low
academic  general vocation
  0.3973   0.3278   0.2749
--------------------------------------
pp.write$ses: middle
academic  general vocation
  0.4256   0.2011   0.3733
```

Multinomial Regression with R

- A couple of plots can convey a good deal amount of information.
- Using the predictions we generated for the `pp.write` object above, we can plot the predicted probabilities against the writing score by the level of **ses** for different levels of the outcome variable.

Multinomial Logistic Regression with R

remark: melt data set to long for ggplot2

```
lpp <- melt(pp.write, id.vars = c("ses", "write"),
            value.name = "probability")

head(lpp)  # view first few rows

   ses write variable probability
 1 low    30 academic     0.09844
 2 low    31 academic     0.10717
 3 low    32 academic     0.11650
 4 low    33 academic     0.12646
 5 low    34 academic     0.13705
 6 low    35 academic     0.14828
```

Multinomial Regression with R

plot predicted probabilities across write values for each level of ses facetted by program type

```
ggplot(lpp, aes(x = write, y = probability, co
    ., scales = "free")
```

Ordered logistic regression with R

Ordered logit model

- The Ordered (or Ordinal) logit model (also ordered logistic regression or proportional odds model), is a regression model for ordinal dependent variables.
- For example, questions on a survey answered by a choice among "poor", "fair", "good", "very good", and "excellent".
- The purpose of the analysis is to see how well that response can be predicted by the responses to other questions, some of which may be quantitative

Ordered logistic regression with R

Ordered logit model

-
- It can be thought of as an extension of the logistic regression model that applies to dichotomous dependent variables, allowing for more than two (ordered) response categories.
- The model only applies to data that meet the proportional odds assumption

Ordered logistic regression with R

`polr`

- In this section we will use the `polr` command (from the MASS package) to estimate an ordered logistic regression model.
- The command name comes from **proportional odds logistic regression**, due to the the **proportional odds assumption** in the model.

Ordered logistic regression with R

`polr`

- `polr` uses the standard formula interface in R for specifying a regression model with outcome followed by predictors.
- We will also specify `Hess=TRUE` to have the model return the observed information matrix from optimization (called the Hessian) which is used to get standard errors.

Ordered logistic regression with R

```
## fit ordered logit model and store results 'm'
m <- polr(apply ~ pared +
          public + gpa, data = dat, Hess=TRUE)

## view a summary of the model
summary(m)
## Call:
## polr(formula = apply ~ pared +
             public + gpa, data = dat,
             Hess = TRUE)
```

Ordered logistic regression with R

```
Coefficients:
          Value Std. Error t value
pared   1.0477      0.266   3.942
public -0.0588      0.298  -0.197
gpa     0.6159      0.261   2.363

Intercepts:
                            Value  Std. Error t value
unlikely|somewhat likely     2.204  0.780       2.827
somewhat likely|very likely  4.299  0.804       5.345
```

Ordered logistic regression with R

1. The “Call”, what type of model we ran, what options we specified, etc.
2. The usual regression output coefficient table including the value of each coefficient, standard errors, and $t-$value, which is simply the ratio of the coefficient to its standard error.
   (Remark: There is no significance test by default.)

Ordered logistic regression with R

3. We then have the estimates for the two intercepts (which are sometimes called **cutpoints**).
4. The intercepts indicate where the latent variable is cut to make the three groups that we observe in our data.

Ordered logistic regression with R

In the ordered logit model, there is an observed ordinal variable, Y. Y, in turn, is a function of another latent variable, Y*, that is not measured.

a. In the ordered logit model, there is a continuous, unmeasured latent variable Y*, whose values determine what the observed ordinal variable Y equals.
b. The continuous latent variable Y* has various threshold (or cutoff) points.

Ordered logistic regression with R

Your value on the observed ordinal variable Y depends on whether or not you have crossed a particular threshold. For example, when $M = 3$

- $Yi = 1$ if Y^*i is $\leq$ CP1
- $Yi = 2$ if $CP1 \leq Y^*i \leq CP2$
- $Yi = 3$ id $Y^*i \geq CP2$

Ordered logistic regression with R

- Note that this latent variable is continuous. In general, these are not used in the interpretation of the results.
- The cutpoints are closely related to thresholds, which are reported by other statistical packages.

Model Diagnostics

- We see the residual deviance, -2 * Log Likelihood of the model as well as the AIC.
- Both the deviance and AIC are useful for model comparison.
- Of, course, some people are not satisfied without a $p-$value.
- One way to calculate a $p-$value in this case is by comparing the $t-$value against the standard normal distribution, like a $z-$test.

Ordered logistic regression R

- Of course this is only true with infinite degrees of freedom, but is reasonably approximated by large samples, becoming increasingly biased as sample size decreases.
- First we store the coefficient table, then calculate the $p-$values and combine back with the table.

Ordered logistic regression R

```
# store table
(ctable <- coef(summary(m)))
                              Value Std. Error t value
 pared                      1.04769     0.2658  3.9418
 public                    -0.05879     0.2979 -0.1974
 gpa                        0.61594     0.2606  2.3632
 unlikely|somewhat likely   2.20391     0.7795  2.8272
 somewhat likely|very likely 4.29936    0.8043  5.3453
```

Ordered Logistic regression R

```
# calculate and store p values
p <- pnorm(abs(ctable[, "t value"]),
      lower.tail = FALSE) * 2

# Combined table
(ctable <- cbind(ctable, "p value" = p))
                      Value Std. Error t value   p value
 pared              1.04769     0.2658  3.9418 8.087e-05
 public            -0.05879     0.2979 -0.1974 8.435e-01
 gpa                0.61594     0.2606  2.3632 1.812e-02
 unli..|some..      2.20391     0.7795  2.8272 4.696e-03
 some..|very..      4.29936     0.8043  5.3453 9.027e-08
```

Ordered Logistic Regression Examples of ordered logistic regression

- A marketing research firm wants to investigate what factors influence the size of soda (small, medium, large or extra large) that people order at a fast-food chain.
- These factors may include what type of sandwich is ordered (burger or chicken), whether or not fries are also ordered, and age of the consumer.
- While the outcome variable, size of soda, is obviously ordered, the difference between the various sizes is not consistent.
- The differece between small and medium is 10 ounces, between medium and large 8, and between large and extra large 12.

Ordered Logistic Regression Examples of ordered logistic regression

- A researcher is interested in what factors influence medaling in Olympic swimming.
- Relevant predictors include at training hours, diet, age, and popularity of swimming in the athlete's home country.
- The researcher believes that the distance between gold and silver is larger than the distance between silver and bronze.

Ordered Logistic Regression

- A study looks at factors that influence the decision of whether to apply to graduate school.
- College juniors are asked if they are unlikely, somewhat likely, or very likely to apply to graduate school.
- Hence, our outcome variable has three categories. Data on parental educational status, whether the undergraduate institution is public or private, and current GPA is also collected.
- The researchers have reason to believe that the "distances" between these three points are not equal.
- For example, the "distance" between *"unlikely"* and *"somewhat likely"* may be shorter than the distance between *"somewhat likely"* and *"very likely"*.

Ordered Logistic Regression Data Set - Graduate School Entry (ologit.csv)

- This hypothetical data set has a three level variable called **apply**, with levels *"unlikely"*, *"somewhat likely"*, and *"very likely"*, coded 1, 2, and 3, respectively, that we will use as our outcome variable.

Ordered Logistic Regression Predictors:

- pared , which is a 0/1 variable indicating whether at least one parent has a graduate degree;
- public , which is a 0/1 variable where 1 indicates that the undergraduate institution is public and 0 private,
- gpa , which is the student's grade point average.

	apply	pared	public	gpa
1	very likely	0	0	3.26
2	somewhat likely	1	0	3.21
3	unlikely	1	1	3.94
4	somewhat likely	0	0	2.81
5	somewhat likely	0	0	2.53
6	unlikely	0	1	2.59

Confidence Intervals

- We can also get confidence intervals for the parameter estimates.
- These can be obtained either by profiling the likelihood function or by using the standard errors and assuming a normal distribution.
- Note that profiled CIs are not symmetric (although they are usually close to symmetric).
- If the 95% CI does not cross 0, the parameter estimate is statistically significant.

Ordered Logistic Regression with R

```
(ci <- confint(m))
# default method gives profiled CIs

 Waiting for profiling to be done...
            2.5 %   97.5 %
 pared    0.5282   1.5722
 public -0.6522   0.5191
 gpa      0.1076   1.1309
```

Ordered Logistic Regression with R

```
# CIs assuming normality

confint.default(m)

             2.5 %   97.5 %
 pared     0.5268    1.569
 public   -0.6426    0.525
 gpa       0.1051    1.127
```

Confidence Intervals

- The CIs for both pared and gpa do not include 0; but the CI for public does.
- The estimates in the output are given in units of ordered logits, or ordered log odds.
- For pared, we would say that for a one unit increase in pared (i.e., going from 0 to 1), we expect a 1.05 increase in the expect value of apply on the log odds scale, given all of the other variables in the model are held constant.

Confidence Intervals

- For gpa, we would say that for a one unit increase in gpa, we would expect a 0.62 increase in the expected value of apply in the log odds scale, given that all of the other variables in the model are held constant.

Ordered Logistic Regression with R

- The coefficients from the model can be somewhat difficult to interpret because they are scaled in terms of logs.
- Another way to interpret logistic regression models is to convert the coefficients into odds ratios.
- To get the Odds Ratios and confidence intervals, we just exponentiate the estimates and confidence intervals.

Ordered Logistic Regression with R

Odds Ratios

```
exp(coef(m))
  pared public    gpa
 2.8511 0.9429 1.8514
```

```
 # Odds Ratios and CIs
exp(cbind(OR = coef(m), ci))
            OR  2.5 % 97.5 %
 pared  2.8511 1.6958  4.817
 public 0.9429 0.5209  1.681
 gpa    1.8514 1.1136  3.098
```

Ordered Logistic Regression with R

- These coefficients are called **proportional odds ratios** and we would interpret these pretty much as we would odds ratios from a binary logistic regression.
- For pared, we would say that for a one unit increase in parental education, i.e., going from 0 (*Low*) to 1 (*High*), the odds of "*very likely*" applying versus "*somewhat likely*" or "*unlikely*" applying combined are 2.85 greater, given that all of the other variables in the model are held constant.

Ordered Logistic Regression with R

- Similarly, the odds "*very likely*" or "*somewhat likely*" applying versus "*unlikely*" applying is 2.85 times greater, given that all of the other variables in the model are held constant.
- For gpa (and other continuous variables), the interpretation is that when a student's gpa moves 1 unit, the odds of moving from "*unlikely*" applying to "*somewhat likely*" or "*very likely*" applying (or from the lower and middle categories to the high category) are multiplied by 1.85.

Ordinal Logistic Regression with R

Assumption of Proportional Odds

- One of the assumptions underlying ordinal logistic regression is that the relationship between each pair of outcome groups is the same.
- In other words, ordinal logistic regression assumes that the coefficients that describe the relationship between, say, the lowest versus all higher categories of the response variable are the same as those that describe the relationship between the next lowest category and all higher categories, etc.

Testing the Assumption

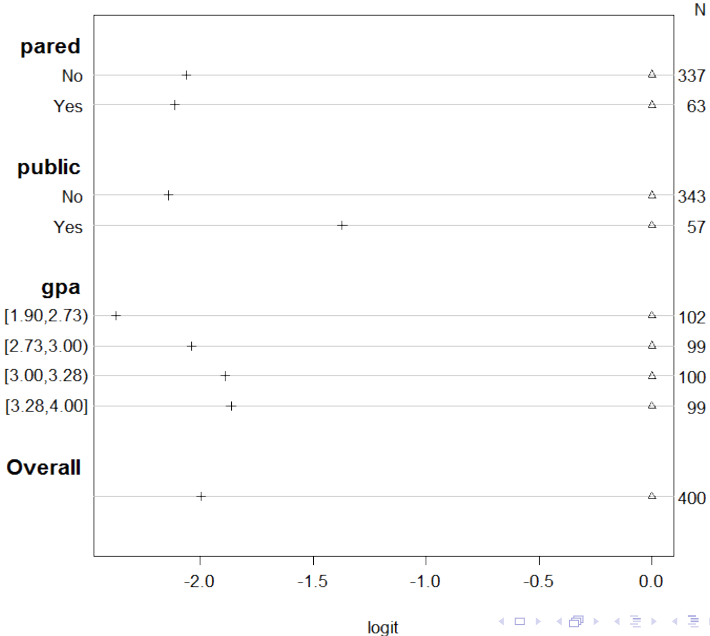
- Because the relationship between all pairs of groups is the same, there is only one set of coefficients.
- If this was not the case, we would need different sets of coefficients in the model to describe the relationship between each pair of outcome groups.
- Thus, in order to asses the appropriateness of our model, we need to evaluate whether the proportional odds assumption is tenable.

Testing the Assumption

- Statistical tests to do this are available in some software packages.
- However, these tests have been criticized for having a tendency to reject the null hypothesis (that the sets of coefficients are the same), and hence, indicate that there the parallel slopes assumption does not hold, in cases where the assumption does hold *(Harrell 2001 p. 335)*.
- Currently R to perform any of the tests commonly used to test the parallel slopes assumption.

Ordinal Logistic Regression with R

- Harrell does recommend a graphical method for assessing the parallel slopes assumption.
- The values displayed in this graph are essentially (linear) predictions from a logit model, used to model the probability that y is greater than or equal to a given value (for each level of y), using one predictor (x) variable at a time.


N
pared
No
Yes
public
No
Yes
gpa
[1.90,2.73)
[2.73,3.00)
[3.00,3.28)
[3.28,4.00]
Overall
337
63
343
57
102
99
100
99
400
-2.0
-1.5
-1.0
-0.5
0.0
logit

Ordered Logistic regression R

- Turning our attention to the predictions with public as a predictor variable, we see that when public is set to "no" the difference in predictions for apply greater than or equal to two, versus apply greater than or equal to three is about 2.14 (-0.204 - -2.345 = 2.141).
- When public is set to "yes" the difference between the coefficients is about 1.37 (-0.175 - -1.547 = 1.372).

Ordered Logistic regression R

- The differences in the distance between the two sets of coefficients (2.14 vs. 1.37) may suggest that the parallel slopes assumption does not hold for the predictor public.
- That would indicate that the effect of attending a public versus private school is different for the transition from "unlikely" to "somewhat likely" and "somewhat likely" to "very likely."

Ordered logistic regression R

- If the proportional odds assumption holds, for each predictor variable, distance between the symbols for each set of categories of the dependent variable, should remain similar.
- To help demonstrate this, we normalized all the first set of coefficients to be zero so there is a common reference point.

Ordered logistic regression R

- Looking at the coefficients for the variable pared we see that the distance between the two sets of coefficients is similar.
- In contrast, the distances between the estimates for public are different (i.e. the markers are much further apart on the second line than on the first), suggesting that the proportional odds assumption may not hold.

Ordered logistic regression R

```
plot(s, which=1:3, pch=1:3,
    xlab='logit', main=' ',
    xlim=range(s[,3:4]))
```

Ordered logistic regression R

- Once we are done assessing whether the assumptions of our model hold, we can obtain predicted probabilities, which are usually easier to understand than either the coefficients or the odds ratios.
- For example, we can vary gpa for each level of pared and public and calculate the probability of being in each category of apply.
- We do this by creating a new dataset of all the values to use for prediction.

Ordered logistic regression R

```
newdat <- data.frame(
  pared = rep(0:1, 200),
  public = rep(0:1, each = 200),
  gpa = rep(seq(from = 1.9, to = 4,
      length.out = 100), 4))

newdat <- cbind(newdat, predict(m,
   newdat, type = "probs"))
```

Ordered logistic regression R

```
# Show first few rows
head(newdat)
  pared public   gpa unlikely somewhat likely very likely
1     0      0 1.900   0.7376          0.2205     0.04192
2     1      0 1.921   0.4932          0.3946     0.11221
3     0      0 1.942   0.7325          0.2245     0.04299
4     1      0 1.964   0.4867          0.3985     0.11484
5     0      0 1.985   0.7274          0.2285     0.04407
6     1      0 2.006   0.4802          0.4023     0.11753
```

Ordered logistic regression R

- Now we can reshape the data long with the reshape2 package and plot all of the predicted probabilities for the different conditions.
- We plot the predicted probilities, connected with a line, coloured by level of the outcome, apply, and facetted by level of pared and public.
- We also use a custom label function, to add clearer labels showing what each column and row of the plot represent.

Ordered logistic regression R

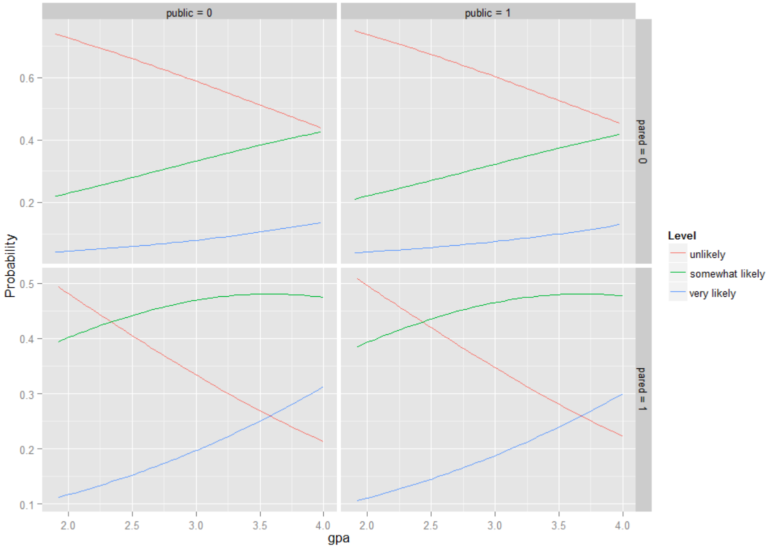
```
library(reshape2)

lnewdat <- melt(newdat,
  id.vars = c("pared", "public", "gpa"),
  variable.name = "Level",
  value.name="Probability")
```

Ordered logistic regression R

```
%## view first few rows
%head(lnewdat)
%##   pared public   gpa    Level Probability
%## 1     0      0 1.900 unlikely      0.7376
%## 2     1      0 1.921 unlikely      0.4932
%## 3     0      0 1.942 unlikely      0.7325
%## 4     1      0 1.964 unlikely      0.4867
%## 5     0      0 1.985 unlikely      0.7274
%## 6     1      0 2.006 unlikely      0.4802
```

Ordered logistic regression R

```
ggplot(lnewdat, aes(x = gpa, y = Probability,
  colour = Level)) +
  geom_line() +
  facet_grid(pared ~ public, scales="free",
    labeller=function(x, y) sprintf("%s = %d", x, y))
```


public = 0
public = 1
pared = 0
pared = 1
Probability
gpa
0.6
0.4
0.2
0.5
0.4
0.3
0.2
0.1
2.0
2.5
3.0
3.5
4.0
Level
unlikely
somewhat likely
very likely

Things to consider **Perfect prediction:** Perfect prediction means that one value of a predictor variable is associated with only one value of the response variable.

Ordered logistic regression R

Sample size: Both ordered logistic and ordered probit, using maximum likelihood estimates, require sufficient sample size.

Empty cells or small cells: You should check for empty or small cells by doing a crosstab between categorical predictors and the outcome variable. If a cell has very few cases, the model may become unstable or it might not run at all.

Pseudo-R-squared: There is no exact analog of the R-squared found in OLS. There are many versions of pseudo-R-squares. Please see Long and Freese 2005 for more details and explanations of various pseudo-R-squares.
Diagnostics: Doing diagnostics for non-linear models is difficult, and ordered logit/probit models are even more difficult than binary models.

Poisson Regression

- Poisson regression is used to model count variables.
- Poisson regression has a number of extensions useful for count models.

Negative Binomial regression

- Negative binomial regression can be used for over-dispersed count data, that is when the conditional variance exceeds the conditional mean.
- It can be considered as a generalization of Poisson regression since it has the same mean structure as Poisson regression and it has an extra parameter to model the over-dispersion.
- If the conditional distribution of the outcome variable is over-dispersed, the confidence intervals for Negative binomial regression are likely to be narrower as compared to those from a Poisson regression.

Zero-inflated Regression models

- Zero-inflated models attempt to account for excess zeros.
- In other words, two kinds of zeros are thought to exist in the data, "true zeros" and "excess zeros".
- Zero-inflated models estimate two equations simultaneously, one for the count model and one for the excess zeros.

OLS regression

- Count outcome variables are sometimes log-transformed and analyzed using OLS regression.
- Many issues arise with this approach, including loss of data due to undefined values generated by taking the log of zero (which is undefined) and biased estimates.

Examples of Poisson regression

- The number of persons killed by mule or horse kicks in the Prussian army per year.
- Ladislaus Bortkiewicz collected data from 20 volumes of Preussischen Statistik.
- These data were collected on 10 corps of the Prussian army in the late 1800s over the course of 20 years.

Examples of Poisson regression

- The number of people in line in front of you at the grocery store.
- Predictors may include the number of items currently offered at a special discounted price and whether a special event (e.g., a holiday, a big sporting event) is three or fewer days away.

Poisson Regression with R

Examples of Poisson regression

- The number of awards earned by students at one high school.
- Predictors of the number of awards earned include the type of program in which the student was enrolled (e.g., vocational, general or academic) and the score on their final exam in math.

Description of the data

- For the purpose of illustration, we have simulated a data set for the last example.
- The data set is called *poissonreg.csv*
- In this example, **num_awards** is the outcome variable and indicates the number of awards earned by students at a high school in a year

Predictor Variables

- **math** is a continuous predictor variable and represents students' scores on their math final exam,
- **prog** is a categorical predictor variable with three levels indicating the type of program in which the students were enrolled.
- **prog** is coded as 1 = "*General*", 2 = "*Academic*" and 3 = "*Vocational*".

Poisson Regression with R

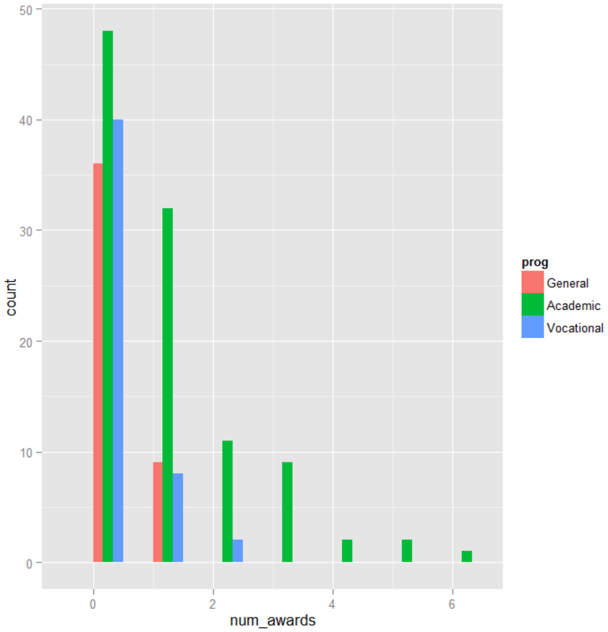
```
       id          num_awards              prog            math    
 1       :  1   Min.   :0.00   General   : 45   Min.   :33.0  
 2       :  1   1st Qu.:0.00   Academic  :105   1st Qu.:45.0  
 3       :  1   Median :0.00   Vocational: 50   Median :52.0  
 4       :  1   Mean   :0.63                    Mean   :52.6  
 5       :  1   3rd Qu.:1.00                    3rd Qu.:59.0  
 6       :  1   Max.   :6.00                    Max.   :75.0  
 (Other):194                                                  
```

Poisson Regression with R

- Each variable has 200 valid observations and their distributions seem quite reasonable.
- The unconditional mean and variance of our outcome variable are not extremely different.
- Our model assumes that these values, conditioned on the predictor variables, will be equal (or at least roughly so).

Poisson Regression with R

- Additionally, the means and variances within each level of prog–*the conditional means and variances*–are similar.
- A conditional histogram separated out by program type is plotted to show the distribution.



50
40
30
20
10
0
count
0
2
4
6
num_awards
prog
General
Academic
Vocational

Figure:

Poisson regression

- At this point, we are ready to perform our Poisson model analysis using the `glm` function.
- We fit the model and store it in the object `m1` and get a summary of the model.

Poisson Regression with R

```
m1 <- glm(num_awards ~ prog + math,
  family="poisson", data=p)

summary(m1)
```

Poisson Regression with R

```
Call:
glm(formula = num_awards ~ prog + math, family = "po
 Deviance Residuals:
    Min       1Q   Median       3Q      Max
-2.204   -0.844   -0.511    0.256    2.680
```

Poisson Regression with R

```
Coefficients:
                Estimate Std. Error z value Pr(>|z|)
(Intercept)      -5.2471     0.6585   -7.97  1.6e-15
progAcademic      1.0839     0.3583    3.03   0.0025
progVocational    0.3698     0.4411    0.84   0.4018
math              0.0702     0.0106    6.62  3.6e-11
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.'
```

Poisson Regression with R

```
(Dispersion parameter for poisson family taken to be

    Null deviance: 287.67  on 199  degrees of freedo
Residual deviance: 189.45  on 196  degrees of freedo
AIC: 373.5

Number of Fisher Scoring iterations: 6
```

Poisson Regression with R

- It is recommended using robust standard errors for the parameter estimates to control for mild violation of the distribution assumption that the variance equals the mean.
- The R package **sandwich** can be used to obtain the robust standard errors and calculated the p-values accordingly.
- Together with the p-values, we have also calculated the 95% confidence interval using the parameter estimates and their robust standard errors.

Poisson Regression with R

sandwich R Package

- Robust Covariance Matrix Estimators
- Model-robust standard error estimators for cross-sectional, time series, and longitudinal data.

Poisson Regression with R

Robust Standard Errors

```
cov.m1 <- vcovHC(m1, type="HC0")
std.err <- sqrt(diag(cov.m1))

r.est <- cbind(Estimate= coef(m1),
  "Robust SE" = std.err,
  "Pr(>|z|)" = 2 * pnorm(abs(coef(m1)/std.err),
  lower.tail=FALSE),
LL = coef(m1) - 1.96 * std.err,
UL = coef(m1) + 1.96 * std.err)
```

Poisson Regression with R

```
r.est

                  Estimate Robust SE  Pr(>|z|)       LL
 (Intercept)      -5.24712   0.64600 4.567e-16 -6.5133
 progAcademic      1.08386   0.32105 7.355e-04  0.4546
 progVocational    0.36981   0.40042 3.557e-01 -0.4150
 math              0.07015   0.01044 1.784e-11  0.0497
```

Poisson Regression with R

glm function output

- The output begins with echoing the function call. The information on deviance residuals is displayed next.
- Deviance residuals are approximately normally distributed if the model is specified correctly.
- Here it shows a little bit of skeweness since median is not quite zero.

glm function output

- The Poisson regression coefficients for each of the variables along with the standard errors, z-scores, p-values and 95% confidence intervals for the coefficients.
- The coefficient for math is 0.07.
- This means that the expected log count for a one-unit increase in math is 0.07.

glm function output

- The indicator variable **progAcademic** compares between **prog = Academic** and **prog = "General"** , the expected log count for **prog = Academic** increases by about 1.1.
- The indicator variable **prog.Vocational** is the expected difference in log count (≈ 0.37) between **prog = "Vocational"** and the reference group (**prog = "General"**).

Deviance

- In statistics, deviance is a quality of fit statistic for a model that is often used for statistical hypothesis testing.
- It is a generalization of the idea of using the sum of squares of residuals in ordinary least squares to cases where model-fitting is achieved by maximum likelihood.

glm function output

- The information on deviance is also provided.
- We can use the residual deviance to perform a goodness of fit test for the overall model.
- The residual deviance is the difference between the deviance of the current model and the maximum deviance of the ideal model where the predicted values are identical to the observed.

Poisson Regression with R

glm function output

- Therefore, if the residual difference is small enough, the goodness of fit test will not be significant, indicating that the model fits the data. We conclude that the model fits reasonably well because the goodness-of-fit chi-squared test is not statistically significant.
- If the test had been statistically significant, it would indicate that the data do not fit the model well.
- We could try to determine if there are omitted predictor variables, if our linearity assumption holds and/or if there is an issue of over-dispersion.

Poisson Regression with R

```
with(m1, cbind(res.deviance = deviance,
df = df.residual,
        p = pchisq(deviance, df.residual,
            lower.tail=FALSE)))

      res.deviance  df      p
 [1,]        189.4 196 0.6182
```

Poisson Regression with R

- We can also test the overall effect of prog by comparing the deviance of the full model with the deviance of the model excluding prog.
- The two degree-of-freedom chi-square test indicates that prog, taken together, is a statistically significant predictor of num_awards.

Poisson Regression with R

```
# update m1 model dropping prog
m2 <- update(m1, . ~ . - prog)
# test model differences with chi square test
anova(m2, m1, test="Chisq")
```

Poisson Regression with R

```
Analysis of Deviance Table

Model 1: num_awards ~ math
Model 2: num_awards ~ prog + math
  Resid. Df Resid. Dev Df Deviance Pr(>Chi)
1       198        204
2       196        189  2     14.6  0.00069 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.'
```

Incident Rate Ratios

- Sometimes, we might want to present the regression results as **incident rate ratios** (IRRs) and their standard errors, together with the confidence interval.
- To compute the standard error for the incident rate ratios, we will use the **Delta method** (Numerical Computation Method).
- To this end, we make use the function `deltamethod` implemented in R package **msm**.

Incident Rate Ratios

A rate ratio (sometimes called an incidence density ratio) in epidemiology, is a relative difference measure used to compare the incidence rates of events occurring at any given point in time. A common application for this measure in analytic epidemiologic studies is in the search for a causal association between a certain risk factor and an outcome.[1]

$$\text{Incidence Rate Ratio} = \frac{\text{Incidence Rate 1}}{\text{Incidence Rate 2}}$$

Incident Rate Ratios

Incidence rate is the occurrence of an event over person-time, for example person-years.

$$\text{Incidence Rate} = \frac{\text{events}}{\text{Person Time}}$$

Note: the same time intervals must be used for both incidence rates.

Poisson Regression with R

```
s <- deltamethod(list(~ exp(x1), ~ exp(x2),
#exponentiate old estimates dropping the p v
rexp.est <- exp(r.est[, -3])

# replace SEs with estimates for exponentiat
rexp.est[, "Robust SE"] <- s
```

Poisson Regression with R

```
rexp.est

                 Estimate Robust SE       LL       UL
 (Intercept)     0.005263   0.00340 0.001484 0.01867
 progAcademic    2.956065   0.94904 1.575551 5.54620
 progVocational  1.447458   0.57959 0.660335 3.17284
 math            1.072672   0.01119 1.050955 1.09484
```

Poisson Regression with R

- The output above indicates that the incident rate for prog = "Academic" is 2.96 times the incident rate for the reference group (prog = "General").
- Likewise, the incident rate for prog = "Vocational" is 1.45 times the incident rate for the reference group holding the other variables at constant.

Poisson Regression with R

- The percent change in the incident rate of num_awards is by 7% for every unit increase in math.

Poisson Regression with R

- Sometimes, we might want to look at the expected marginal means.
- For example, what are the expected counts for each program type holding math score at its overall mean?
- To answer this question, we can make use of the predict function.
- First off, we will make a small data set to apply the predict function to it.

Poisson Regression with R

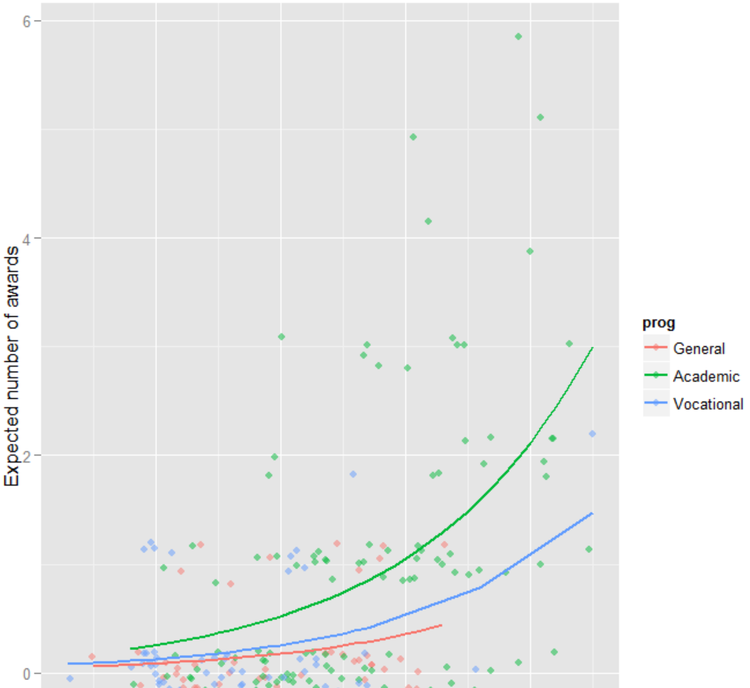
```
(s1 <- data.frame(math = mean(p$math),
  prog = factor(1:3, levels = 1:3,
  labels = levels(p$prog))))

    math       prog
 1 52.65    General
 2 52.65   Academic
 3 52.65 Vocational
```

Poisson Regression with R

```
predict(m1, s1, type="response", se.fit=TRUE)

 $fit
      1      2      3
 0.2114 0.6249 0.3060

 $se.fit
       1       2       3
 0.07050 0.08628 0.08834

 $residual.scale
 [1] 1
```

Poisson Regression with R

- In the output above, we see that the predicted number of events for level 1 of prog is about 0.21, holding math at its mean.
- The predicted number of events for level 2 of prog is higher at 0.62, and the predicted number of events for level 3 of prog is about .31.
- The ratios of these predicted counts ($\frac{0.625}{0.211} = 2.96$, $\frac{0.306}{0.211} = 1.45$) match what we saw looking at the IRR.



Expected number of awards
6
4
2
0
prog
General
Academic
Vocational

Poisson Regression with R

- We can also graph the predicted number of events with the commands below.
- The graph indicates that the most awards are predicted for those in the academic program (prog $= 2$), especially if the student has a high math score.
- The lowest number of predicted awards is for those students in the general program (prog $=$ 1).
- The graph overlays the lines of expected values onto the actual points, although a small amount of random noise was added vertically to lessen overplotting.

Poisson Regression with R

```
# Calculate and store predicted values
p$phat <- predict(m1, type="response")

# order by program and then by math
p <- p[with(p, order(prog, math)), ]
```

Poisson Regression with R

```
ggplot(p, aes(x = math, y = phat, colour = prog)) +
  geom_point(aes(y = num_awards), alpha=.5, position=
  geom_line(size = 1) +
  labs(x = "Math Score", y = "Expected number of awar
```

Over-Dispersion Overdispersion is the presence of greater variability (statistical dispersion) in a data set than would be expected based on a given simple statistical model.

Over-Dispersion

- When there seems to be an issue of dispersion, we should first check if our model is appropriately specified, such as omitted variables and functional forms.
- For example, if we omitted the predictor variable prog in the example above, our model would seem to have a problem with over-dispersion.
- In other words, a misspecified model could present a symptom like an over-dispersion problem.

Poisson Regression with R

- Assuming that the model is correctly specified, the assumption that the conditional variance is equal to the conditional mean should be checked.
- There are several tests including the likelihood ratio test of over-dispersion parameter alpha by running the same model using negative binomial distribution.
- The R package pscl (Political Science Computational Laboratory, Stanford University) provides many functions for binomial and count data including odTest for testing over-dispersion.

Poisson Regression with R

- One common cause of over-dispersion is excess zeros, which in turn are generated by an additional data generating process.
- In this situation, zero-inflated model should be considered.
- If the data generating process does not allow for any 0s (such as the number of days spent in the hospital), then a zero-truncated model may be more appropriate.

Poisson Regression with R

- Count data often have an exposure variable, which indicates the number of times the event could have happened.
- This variable should be incorporated into a Poisson model with the use of the offset option.
- The outcome variable in a Poisson regression cannot have negative numbers, and the exposure cannot have 0s.

Poisson Regression with R

- Many different measures of pseudo-R-squared exist. They all attempt to provide information similar to that provided by R-squared in OLS regression, even though none of them can be interpreted exactly as R-squared in OLS regression is interpreted.
- Poisson regression is estimated via maximum likelihood estimation. It usually requires a large sample size.

Introduction Negative binomial regression is for modeling count variables, usually for over-dispersed count outcome variables.

Examples of negative binomial regression

- **Example 1** School administrators study the attendance behavior of high school juniors at two schools.
  Predictors of the number of days of absence include the type of program in which the student is enrolled and a standardized test in math.
- **Example 2** A health-related researcher is studying the number of hospital visits in past 12 months by senior citizens in a community based on the characteristics of the individuals and the types of health plans under which each one is covered.

Negative Binomial Regression with R

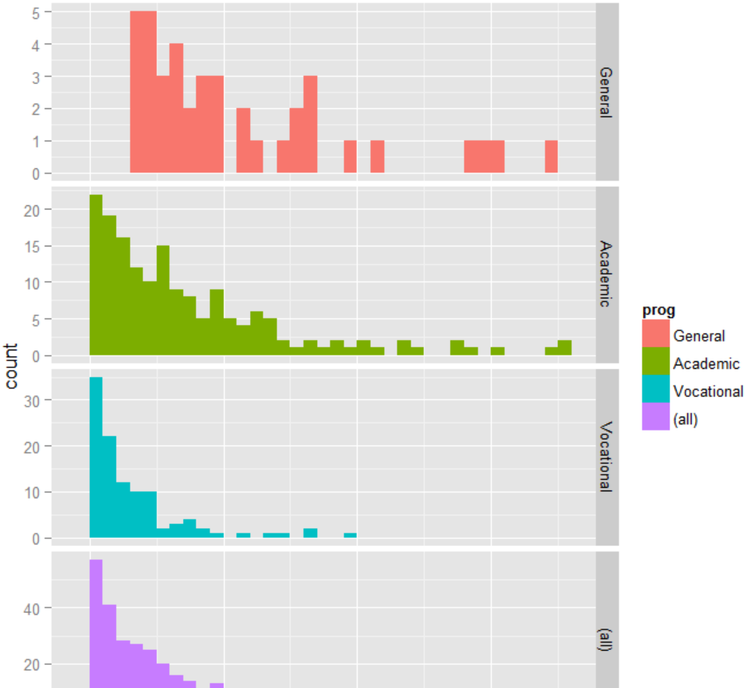
Description of the data Let's pursue Example 1 from above.

- We have attendance data on 314 high school juniors from two urban high schools in the file **negbin.csv** .
- The response variable of interest is days absent, **daysabs**.
- The variable **math** gives the standardized math score for each student.
- The variable **prog** is a three-level nominal variable indicating the type of instructional program in which the student is enrolled.

Negative Binomial Regression with R

Exploratory Data Analysis

```
summary(dat)
       id          gender          math          daysabs
 1001   :  1   female:160   Min.   : 1.0   Min.   : 0.00
 1002   :  1   male  :154   1st Qu.:28.0   1st Qu.: 1.00
 1003   :  1                Median :48.0   Median : 4.00
 1004   :  1                Mean   :48.3   Mean   : 5.96
 1005   :  1                3rd Qu.:70.0   3rd Qu.: 8.00
 1006   :  1                Max.   :99.0   Max.   :35.00
 (Other):308
         prog
 General   : 40
 Academic  :167
 Vocational:107
```



count
General
Academic
Vocational
(all)
prog
General
Academic
Vocational
(all)

Negative Binomial Regression with R

```
ggplot(dat, aes(daysabs, fill = prog)) + geom_
., margins = TRUE, scales = "free")
```

Histogram plots showing distribution of the data
Each variable has 314 valid observations and their distributions seem quite reasonable. The unconditional mean of our outcome variable is much lower than its variance.

Negative Binomial Regression with R

Data Set

- The table below shows the average numbers of days absent by program type and seems to suggest that program type is a good candidate for predicting the number of days absent, our outcome variable, because the mean value of the outcome appears to vary by prog.

Data Set

- The variances within each level of prog are higher than the means within some of the levels.
- These are the conditional means and variances. These differences suggest that over-dispersion is present and that a Negative Binomial model would be appropriate.

Negative Binomial Regression with R

```
with(dat, tapply(daysabs, prog, function(x) {
sprintf("M (SD) = %1.2f (%1.2f)", mean(x), sd(x))
}))
##                   General
## "M (SD) = 10.65 (8.20)"
##                  Academic
## "M (SD) = 6.93 (7.45)"
##                Vocational
## "M (SD) = 2.67 (3.73)"
```

Negative Binomial Regression with R

Negative binomial regression analysis

We will use the `glm.nb` function from the MASS package to estimate a negative binomial regression.

```
summary(m1 <- glm.nb(daysabs ~ math + prog, data = da
##
## Call:
## glm.nb(formula = daysabs ~ math + prog,
        data = dat, init.theta = 1.032713156,
##     link = log)
##
## Deviance Residuals:
##    Min       1Q   Median       3Q      Max
## -2.155  -1.019  -0.369   0.229   2.527
```

Negative Binomial Regression with R

```
Coefficients:
                 Estimate Std. Error z value Pr(>|z|)
(Intercept)       2.61527    0.19746   13.24  < 2e-16
math             -0.00599    0.00251   -2.39    0.017
progAcademic     -0.44076    0.18261   -2.41    0.016
progVocational   -1.27865    0.20072   -6.37  1.9e-10
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.'
```

Negative Binomial Regression with R

```
## (Dispersion parameter for Negative Binomi
##
##     Null deviance: 427.54  on 313  degree
## Residual deviance: 358.52  on 310  degree
## AIC: 1741
##
## Number of Fisher Scoring iterations: 1
##
##
##               Theta:  1.033
##           Std. Err.:  0.106
##
```

Negative Binomial Regression with R

- R first displays the call and the deviance residuals.
- Next, we see the regression coefficients for each of the variables, along with standard errors, z-scores, and p-values.

Negative Binomial Regression with R

- The variable math has a coefficient of -0.006, which is statistically significant.
- This means that for each one-unit increase in math, the expected log count of the number of days absent decreases by 0.006. The indicator variable shown as progAcademic is the expected difference in log count between group 2 and the reference group (prog=1).

Negative Binomial Regression with R

- The expected log count for level 2 of prog is 0.44 lower than the expected log count for level 1.
- The indicator variable for progVocational is the expected difference in log count between group 3 and the reference group.

Negative Binomial Regression with R

- The expected log count for level 3 of prog is 1.28 lower than the expected log count for level 1.
- To determine if prog itself, overall, is statistically significant, we can compare a model with and without prog. The reason it is important to fit separate models, is that unless we do, the overdispersion parameter is held constant.

Negative Binomial Regression with R

```
m2 <- update(m1, . ~ . - prog)
anova(m1, m2)
## Likelihood ratio tests of Negative Binomial Models
##
## Response: daysabs
##            Model   theta Resid. df    2 x log-lik.   Test
## 1           math 0.8559       312          -1776
## 2 math + prog 1.0327       310          -1731 1 vs 2
##      Pr(Chi)
## 1
## 2 1.652e-10
```

Negative Binomial Regression with R

- The two degree-of-freedom chi-square test indicates that prog is a statistically significant predictor of daysabs.
- The null deviance is calculated from an intercept-only model with 313 degrees of freedom. Then we see the residual deviance, the deviance from the full model. We are also shown the AIC and 2*log likelihood.

Negative Binomial Regression with R

- The theta parameter shown is the dispersion parameter.
- Note that R parameterizes this differently from SAS, Stata, and SPSS.
- The R parameter (theta) is equal to the inverse of the dispersion parameter (alpha) estimated in these other software packages. Thus, the theta value of 1.033 seen here is equivalent to the 0.968 value seen in the Stata Negative Binomial Data Analysis Example because $1/0.968 = 1.033$.

Checking model assumption

- As we mentioned earlier, negative binomial models assume the conditional means are not equal to the conditional variances.
- This inequality is captured by estimating a dispersion parameter (not shown in the output) that is held constant in a Poisson model.
- Thus, the Poisson model is actually nested in the negative binomial model.
- We can then use a likelihood ratio test to compare these two and test this model assumption.
- To do this, we will run our model as a Poisson.

Negative Binomial Regression with R

```
m3 <- glm(daysabs ~ math + prog,
        family = "poisson", data = dat)
pchisq(2 * (logLik(m1) - logLik(m3)),
        df = 1, lower.tail = FALSE)
## 'log Lik.' 2.157e-203 (df=5)
```

Negative Binomial Regression with R

- In this example the associated chi-squared value is 926.03 with one degree of freedom.
- This strongly suggests the negative binomial model, estimating the dispersion parameter, is more appropriate than the Poisson model.

Negative Binomial Regression with R

We can get the confidence intervals for the coefficients by profiling the likelihood function.

```
(est <- cbind(Estimate = coef(m1), confint(m1
## Waiting for profiling to be done...
##                    Estimate   2.5 %     97.5 %
## (Intercept)        2.615265  2.2421   3.012936
## math              -0.005993 -0.0109  -0.001067
## progAcademic      -0.440760 -0.8101  -0.092643
## progVocational    -1.278651 -1.6835  -0.890078
```

Incidence Rate Ratios

- We might be interested in looking at incident rate ratios rather than coefficients.
- To do this, we can exponentiate our model coefficients. The same applies to the confidence intervals.

Negative Binomial Regression with R

```
exp(est)
##                   Estimate  2.5 %  97.5 %
## (Intercept)        13.6708 9.4127 20.3470
## math                0.9940 0.9892  0.9989
## progAcademic        0.6435 0.4448  0.9115
## progVocational      0.2784 0.1857  0.4106
```

Negative Binomial Regression with R

The output above indicates that the incident rate for prog $= 2$ is 0.64 times the incident rate for the reference group (prog $= 1$). Likewise, the incident rate for prog $= 3$ is 0.28 times the incident rate for the reference group holding the other variables constant. The percent change in the incident rate of daysabs is a 1

Negative Binomial Regression with R

The form of the model equation for negative binomial regression is the same as that for Poisson regression. The log of the outcome is predicted with a linear combination of the predictors:

Negative Binomial Regression with R

$$ln(\widehat{daysabs_i}) = Intercept + b_1(prog_i = 2) + b_2(prog_i = 3) + $$

$$\therefore$$

$$\widehat{daysabs_i} = e^{Intercept + b_1(prog_i=2) + b_2(prog_i=3) + b_3 math_i} = e^{Intercept}$$

The coefficients have an additive effect in the $ln(y)$ scale and the IRR have a multiplicative effect in the y scale. The dispersion parameter in negative binomial regression does not effect the expected counts, but it does effect the estimated variance of the expected counts.

Negative Binomial Regression with R

- More details can be found in the Modern Applied Statistics with S by W.N. Venables and B.D. Ripley (the book companion of the MASS package).
- For additional information on the various metrics in which the results can be presented, and the interpretation of such, please see Regression Models for Categorical Dependent Variables Using Stata, Second Edition by J. Scott Long and Jeremy Freese (2006).

Predicted values

- For assistance in further understanding the model, we can look at predicted counts for various levels of our predictors. Below we create new datasets with values of math and prog and then use the predict command to calculate the predicted number of events.

Negative Binomial Regression with R

- First, we can look at predicted counts for each value of prog while holding math at its mean. To do this, we create a new dataset with the combinations of prog and math for which we would like to find predicted values, then use the predict command.

Negative Binomial Regression with R

```
newdata1 <- data.frame(math = mean(dat$math), prog =
labels = levels(dat$prog)))
newdata1$phat <- predict(m1, newdata1, type = "respon
newdata1
   math       prog   phat
 1 48.27    General 10.237
 2 48.27   Academic  6.588
 3 48.27 Vocational  2.850
```

Negative Binomial Regression with R

- In the output above, we see that the predicted number of events (e.g., days absent) for a general program is about 10.24, holding math at its mean.
- The predicted number of events for an academic program is lower at 6.59, and the predicted number of events for a vocational program is about 2.85.

Negative Binomial Regression with R

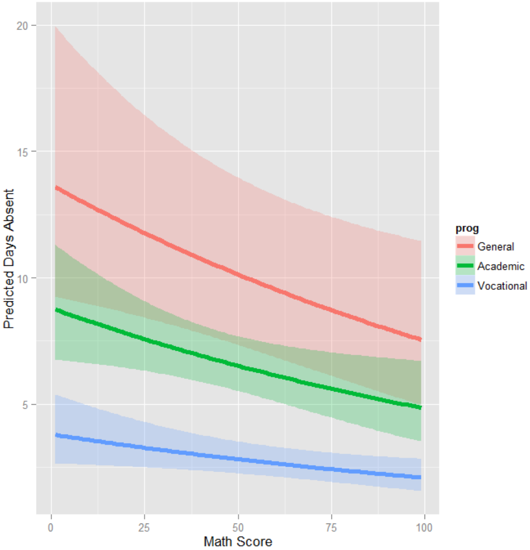
Below we will obtain the mean predicted number of events for values of math across its entire range for each level of prog and graph these.

Negative Binomial Regression with R

```
newdata2 <- data.frame(
math = rep(seq(from = min(dat$math),
       to = max(dat$math), length.out = 100), 3),
prog = factor(rep(1:3, each = 100), levels = 1:3, lab
levels(dat$prog)))
```

Negative Binomial Regression with R

```
newdata2 <- cbind(newdata2, predict(m1, newdata2, type
newdata2 <- within(newdata2, {
DaysAbsent <- exp(fit)
LL <- exp(fit - 1.96 * se.fit)
UL <- exp(fit + 1.96 * se.fit)
})
```



Predicted Days Absent
20
15
10
5
0
25
50
75
100
Math Score
prog
General
Academic
Vocational

Figure:

Negative Binomial Regression with R

```
ggplot(newdata2, aes(math, DaysAbsent)) +
geom_ribbon(aes(ymin = LL, ymax = UL, fill = prog), a
geom_line(aes(colour = prog), size = 2) +
labs(x = "Math Score", y = "Predicted Days Absent")
```

Negative Binomial Regression with R

The graph shows the expected count across the range of math scores, for each type of program along with 95 percent confidence intervals. Note that the lines are not straight because this is a log linear model, and what is plotted are the expected values, not the log of the expected values.

Things to consider

- It is not recommended that negative binomial models be applied to small samples.
- One common cause of over-dispersion is excess zeros by an additional data generating process.
- In this situation, zero-inflated model should be considered.

Negative Binomial Regression with R

- If the data generating process does not allow for any 0s (such as the number of days spent in the hospital), then a zero-truncated model may be more appropriate.

Negative Binomial Regression with R

- Count data often have an exposure variable, which indicates the number of times the event could have happened.
- This variable should be incorporated into your negative binomial regression model with the use of the offset option.

Negative Binomial Regression with R

- The outcome variable in a negative binomial regression cannot have negative numbers.
- You will need to use the `m1$resid` command to obtain the residuals from our model to check other assumptions of the negative binomial model

Multinomial Logistic Regression with R

```
## melt data set to long for ggplot2
lpp <- melt(pp.write, id.vars = c("ses", "write"), value.
head(lpp)  # view first few rows
```

```
##    ses write variable probability
## 1 low    30 academic     0.09844
## 2 low    31 academic     0.10717
## 3 low    32 academic     0.11650
## 4 low    33 academic     0.12646
## 5 low    34 academic     0.13705
## 6 low    35 academic     0.14828
```

Multinomial Logistic Regression with R

```
## plot predicted probabilities across write values for e
## facetted by program type
ggplot(lpp, aes(x = write, y = probability, colour = ses)
    ., scales = "free")
```

Ordinal Logistic Regression with R

```
ggplot(dat, aes(x = apply, y = gpa)) +
  geom_boxplot(size = .75) +
  geom_jitter(alpha = .5) +
  facet_grid(pared ~ public, margins = TRUE) +
  theme(axis.text.x = element_text(angle = 45, hjust = 1,
```

Poisson Regression with R

```
with(p, tapply(num_awards, prog, function(x) {
  sprintf("M (SD) = %1.2f (%1.2f)", mean(x), sd(x))
}))
```

Poisson Regression with R

```
##                  General                   Academic
## "M (SD) = 0.20 (0.40)" "M (SD) = 1.00 (1.28)" "M (SD)

ggplot(p, aes(num_awards, fill = prog)) +
  geom_histogram(binwidth=.5, position="dodge")
```

Negative Binomial Regression with R

```
ggplot(dat, aes(daysabs, fill = prog)) + geom_histogram(b
    ., margins = TRUE, scales = "free")
```

```
with(dat, tapply(daysabs, prog, function(x) {
    sprintf("M (SD) = %1.2f (%1.2f)", mean(x), sd(x))
}))
```

```
##                    General                     Academic
## "M (SD) = 10.65 (8.20)"  "M (SD) = 6.93 (7.45)"  "M (S
```