Multinomial logistic regression

- We use the `multinom` function from the **nnet** package to estimate a multinomial logistic regression model.
- Remark - There are other functions in other R packages capable of multinomial regression, such as the **mlogit** package.
- The multinom function does not require the data to be reshaped (as the mlogit package does) and to mirror the example code found in Hilbe's Logistic Regression Models.

Multinomial logistic regression

- We must choose the level of our outcome that we wish to use as our **baseline** and specify this in the `relevel` function.
  Let's choose "academic".
- Then, we run our model using *multinom*.
- The multinom command does not include p-value calculation for the regression coefficients, but we calculate p-values using Wald tests (here z-tests).

```
ml$prog2 <- relevel(ml$prog, ref = "academic")
test <- multinom(prog2 ~ ses + write, data = ml)

 # weights:  15 (8 variable)
 initial  value 219.722458
 iter  10 value 179.982880
 final  value 179.981726
 converged
```

Multinomial Regression with R

```
summary(test)

 Call:
 multinom(formula = prog2 ~ ses + write,
   data = ml)

 Coefficients:
          (Intercept) sesmiddle seshigh    write
 general        2.852   -0.5333 -1.1628 -0.05793
 vocation       5.218    0.2914 -0.9827 -0.11360
```

Multinomial Regression with R

```
....
 Std. Errors:
          (Intercept) sesmiddle seshigh   write
 general        1.166    0.4437  0.5142 0.02141
 vocation       1.164    0.4764  0.5956 0.02222

 Residual Deviance: 360
 AIC: 376
```

Multinomial Regression with R

```
# 2-tailed z test
# p.values

         (Intercept) sesmiddle seshigh      write
 general   1.448e-02    0.2294 0.02374 6.819e-03
 vocation  7.299e-06    0.5408 0.09895 3.176e-07
```

Multinomial Regression with R

- Some output is generated by running the model, even though we are assigning the model to a new R object.
- This model-running output includes some iteration history and includes the final negative log-likelihood 179.981726.
- This value multiplied by two is then seen in the model summary as the **Residual Deviance** and it can be used in comparisons of nested models.

Multinomial Regression with R

- As with many summary outputs, the output contains a column of coefficients and a column of standard errors.
- Each of these blocks has one row of values corresponding to a model equation.
- Focusing on the block of coefficients, we can look at the first row comparing `prog = "general"` to our baseline `prog = "academic"` and the second row comparing `prog = "vocation"` to our baseline `prog = "academic"`.

Multinomial Regression with R

- If we consider our coefficients from the first row to be b_1 and our coefficients from the second row to be b_2, we can write our model equations:

$$ln\left(\frac{P(prog = gen.)}{P(prog = acad.)}\right) = b_{10} + b_{11}(ses = 2) + b_{12}(ses = 3) + b_{13} write$$

$$ln\left(\frac{P(prog = voc.)}{P(prog = acad.)}\right) = b_{20} + b_{21}(ses = 2) + b_{22}(ses = 3) + b_{23} write$$

Multinomial Regression with R

- A one-unit increase in the variable **write** is associated with the decrease in the log odds of being in general program vs. academic program in the amount of 0.058 (b_{13}).
- A one-unit increase in the variable **write** is associated with the decrease in the log odds of being in vocation program vs. academic program in the amount of o.1136 (b_{23}).

Multinomial Regression with R

- The log odds of being in general program vs. in academic program will decrease by 1.163 if moving from ses="low" to ses="high"(b_12).
- The log odds of being in general program vs. in academic program will decrease by 0.533 if moving from ses="low" to ses="middle"(b_11), although this coefficient is not significant.

Multinomial Regression with R

- The log odds of being in vocation program vs. in academic program will decrease by 0.983 if moving from ses="low" to ses="high"(b_22).
- The log odds of being in vocation program vs. in academic program will increase by 0.291 if moving from ses="low" to ses="middle"(b_21), although this coefficient is not signficant.