0.1 Game Theory

If one player wins wwhat player loses, the game is called a zero-sum game. A two person game is a game with only two players.

Two person, Zero-sum games,also called matrix games,will be the focus here.

0.1.1 Stratgies

A pure strategy is a predetermined plan that prescribes for a player the sequence of moves and countermoves they will make during a complete game. In a matrix, either player has a finite set of pure strategies, although their number may be enormous

Thus a complete characterization of the game is provided by its pay-off matrix. Table 1.7 gives the amounta_{ij} won by player 1 from player II when I players their ith pure strategy.

		Player 2		
		Hawk	Dove	
~~Player 1~~		Hawk	(-2,-2)	(4,0)
	Dove	(0,4)		(2,2)
C	(2,6)	(4,7)	(0,8)	

The objective is game theory is to determine a best stategy for a given player under the assumption that the opponent is rational and will make intelligent countermoves. Consequently, if one player always chooses the same pure strategy or chooses pure strategies in a fixed order, their oppponent will in time recognize patterns and will move to defeat if possible.

Generally, therefore, the most effective strategy is a mixed staategy.

If the game in example 17.1 , if player 1 always shows three fingers, player II can defeat that pure strategy by always showing two fingers. If player I adopts the set sequence of pure strategies {3,3,2,3,3,2,3,3,2}, then player II can defeat it with the corresponding sequence {2,2,3,2,2,3,2,2,3, . . . }.

If player I adopts the mixed strategy $\boldsymbol{X} = \{1/6, 1/3, 1/2\}^T$, then player I plans to show 1 fnger one-sixth of the time, 2 fingers one-third of the time, and 3 fingers one-half of the time.

To implement the strategy, player i could roll one die before each play.

If the die showed a 1 (having probability of 1/6), they would show 1 finger, if the die showed a 2 or 3 (having combined probability of 1/3) they would show 2 fingers.

0.2 Stable Games

0.2.1 Unstable Games

$$E(\boldsymbol{X}.\boldsymbol{Y}) = \sum \sum a_{ij}$$

0.2.2 Minimax Theorem

For any game matrix, there exists optimal strategies X and Y such that

$$E(\boldsymbol{X}.\boldsymbol{Y}) = M = M = G^*$$

Solution by Linearr Programming

The optimal strategies guaranteed by the minimax theorem, as well as the value of the game, can be calculated via linear programming.

The optimal strategy for player II is incorporated in the solution of the following linear program.

0.3 Dominance

A pure strategy P is dominbated by a pure strategy Q if, for each pure strategy of the opponent's the pay-off asssociaed with P is no better than the payoff associated with Q. Since a dominated pure strategy can be never of an optimal strategy, the corresponding row or column of the game matrix may be deleted a priori.

0.3.1 Example 17.3

0.3.2 Example 17.4

Let G denoted the game matrix obtained from matrix G by eliminated dominated rows and columns. Show that G is stable if and only f G is stable.

0.3.3 Example 17.5

Is the game of Table 17.3 stable?

0.3.4 Example 17.6

Is the game of Table 17.4 stable?

0.3.5 Example 17.7

0.3.6 Example 17.8

0.3.7 Example 17.9

Supplementary Problems

matrix 17.11 Determine whether each matrix game, as defined by the payoff to the row player is stable

17.12 Solve problem 17.1 if chain I control's 70

17.17 Two ranchers have brought a dispute over a 6-yard strip of land that separates their

17.18 cigarette bootleggers use two routes for moving cigarettes out of North Carolina. Interstate 95 or back roads

17.19

With one day left before elections, both candidates for Governor have targeted the same three cities as crucial, and potential worth a last visit.

17.20

A game is fair if $G^* = 0$. A game is symmetric if both players have the same of pure strategies

17.21 In a well known gambling game, player I holds a red ace and a black deuce, while player II holds a red deuce and a black three. Simultaneously, both players show one card of their choice.

If the two cards match in terms of colour, player I wins