which is called the *likelihood ratio test*, and 1 is called the *threshold value* of the test.

Note that the likelihood ratio $\Lambda(\mathbf{x})$ is also often expressed as

$$\Lambda(\mathbf{x}) = \frac{f(\mathbf{x} \mid H_1)}{f(\mathbf{x} \mid H_0)} \tag{8.10}$$

B. MAP Test:

Let $P(H_i \mid \mathbf{x})$, $i = 0, 1$, denote the probability that H_i was true given a particular value of $\mathbf{x}$. The conditional probability $P(H_i \mid \mathbf{x})$ is called *a posteriori* (or posterior) probability, that is, a probability that is computed after an observation has been made. The probability $P(H_i)$, $i = 0, 1$, is called *a priori* (or prior) probability. In the *maximum a posteriori* (MAP) test, the decision regions R_0 and R_1 are selected as

$$\begin{aligned} R_0 &= \{\mathbf{x}: P(H_0 \mid \mathbf{x}) > P(H_1 \mid \mathbf{x})\} \\ R_1 &= \{\mathbf{x}: P(H_0 \mid \mathbf{x}) < P(H_1 \mid \mathbf{x})\} \end{aligned} \tag{8.11}$$

Thus, the MAP test is given by

$$d(\mathbf{x}) = \begin{cases} H_0 & \text{if } P(H_0 \mid \mathbf{x}) > P(H_1 \mid \mathbf{x}) \\ H_1 & \text{if } P(H_0 \mid \mathbf{x}) < P(H_1 \mid \mathbf{x}) \end{cases} \tag{8.12}$$

which can be rewritten as

$$\frac{P(H_1 \mid \mathbf{x})}{P(H_0 \mid \mathbf{x})} \underset{H_0}{\overset{H_1}{\gtrless}} 1 \tag{8.13}$$

Using Bayes' rule [Eq. (*1.42*)], Eq. (*8.13*) reduces to

$$\frac{P(\mathbf{x} \mid H_1)P(H_1)}{P(\mathbf{x} \mid H_0)P(H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} 1 \tag{8.14}$$

Using the likelihood ratio $\Lambda(\mathbf{x})$ defined in Eq. (*8.8*), the MAP test can be expressed in the following likelihood ratio test as

$$\Lambda(\mathbf{x}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta = \frac{P(H_0)}{P(H_1)} \tag{8.15}$$

where $\eta = P(H_0)/P(H_1)$ is the threshold value for the MAP test. Note that when $P(H_0) = P(H_1)$, the maximum-likelihood test is also the MAP test.

C. Neyman-Pearson Test:

As we mentioned before, it is not possible to simultaneously minimize both $\alpha(=P_{\mathrm{I}})$ and $\beta(=P_{\mathrm{II}})$. The Neyman-Pearson test provides a workable solution to this problem in that the test minimizes β for a given level of α. Hence, the Neyman-Pearson test is the test which maximizes the power of the test $1 - \beta$ for a given level of significance α. In the Neyman-Pearson test, the critical (or rejection) region R_1 is selected such that $1 - \beta = 1 - P(D_0 \mid H_1) = P(D_1 \mid H_1)$ is maximum subject to the constraint $\alpha = P(D_1 \mid H_0) = \alpha_0$. This is a classical problem in optimization: maximizing a function subject to a constraint, which can be solved by the use of Lagrange multiplier method. We thus construct the objective function

$$J = (1 - \beta) - \lambda(\alpha - \alpha_0) \tag{8.16}$$

where $\lambda \geq 0$ is a Lagrange multiplier. Then the critical region R_1 is chosen to maximize J. It can be shown that the Neyman-Pearson test can be expressed in terms of the likelihood ratio test as