

Chapter 18

Decision Theory

DECISION PROCESSES

A *decision process* is a process requiring either a single or sequential set of decisions for its completion. Each allowable decision has a gain or loss associated with it which is codetermined by *external* circumstances surrounding the process, a feature which distinguishes these processes from the processes treated in Chapter 19. The set of possible circumstances, known as the *states of nature*, and a probability distribution governing the occurrence of each state are presumed known. Both the set of allowable decisions and the set of states of nature will be assumed finite (an assumption not made in the more elaborate theory).

We denote the allowable decisions by D_1, D_2, \dots, D_m ; the states of nature by S_1, S_2, \dots, S_n ; and the return associated with decision D_i and state S_j by g_{ij} ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$). A process requiring the implementation of just one decision is defined completely by Table 18-1. This payoff table is known as a *gain matrix* whenever the entries g_{ij} are in terms of gains to the decision maker. Losses are then represented as negative gains.

Table 18-1

| | | States of Nature | | | |
|-----------|---------|------------------|----------|---------|----------|
| | | S_1 | S_2 | \dots | S_n |
| Decisions | D_1 | g_{11} | g_{12} | \dots | g_{1n} |
| | D_2 | g_{21} | g_{22} | \dots | g_{2n} |
| | \dots | \dots | \dots | \dots | \dots |
| | D_m | g_{m1} | g_{m2} | \dots | g_{mn} |

Table 18-2

| | States of Nature | |
|-----------|------------------|----------------|
| | S_1 | S_2 |
| Decisions | D_1 | 60 660 |
| | D_2 | -100 2000 |

Example 18.1 A major energy company offers a landowner \$60 000 for the exploration rights to natural gas on a certain site and the option for future development. The option, if exercised, is worth an additional \$600 000 to the landowner, but this will occur only if natural gas is discovered during the exploration phase. The landowner, believing that the energy company's interest is a good indication that gas is present, is tempted to develop the field herself. To do so, she must contract with local outfits with expertise in exploration and development. The initial cost is \$100 000, which is lost if no gas is found. If gas is discovered, however, the landowner estimates a net profit of 2 million dollars.

The decisions for the landowner are D_1 (to accept the energy company's offer) and D_2 (to explore and develop on her own). The states of nature are S_1 (there is no gas on the land) and S_2 (there is gas on the land). The gains (in thousands of dollars) to the landowner for each combination of events are given in Table 18-2.

It remains to specify or estimate the probabilities attached to the two states of nature, $P(S_1)$ and $P(S_2)$.

Although Table 18-1 is identical in form to Table 17-1, there are significant differences between decision processes and matrix games. In a decision process, only the decision maker is capable of making rational decisions; nature is not. The actual state of nature in existence at any given time is a random event, but the underlying probability distribution cannot be considered a "mixed strategy," designed to inflict losses on the decision maker. Furthermore, we generally rule out any randomness in the decision