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Consider a binary decision problem with the following conditional PDFs

$$f(x|H_0) = \frac{1}{3} e^{-|x|}$$

$$f(x|H_1) = e^{-2|x|}$$

$$C_{00} = C_{11} = 0 \quad C_{01} = 3 \quad C_{10} = 1$$

Determine the Bayes test if  $P(H_0) = 3/4$ .

Compute the associated Bayes risk

Likelihood Ratio Test

$$\Lambda(x) = \frac{f(x|H_1)}{f(x|H_0)} = 3e^{-|x|}$$

$$\eta = \frac{(1-0)\frac{3}{4}}{(3-0)\frac{1}{4}} = \frac{3/4}{3/4} = 1.$$

$$\therefore e^{-|x|} \underset{H_0}{\overset{H_1}{>}} \pm 1.$$

$$|x| \underset{H_0}{\overset{H_1}{>}} 0.3678$$

$$e^{-1} = 0.3678$$

2) The decision regions are therefore

$$\bullet R_0 = \{x : |x| > 0.3678\}$$

$$\bullet R_1 = \{x : |x| < 0.3678\}$$

ANS

Compute  $P(D_1 | H_0)$  and  $P(D_0 | H_1)$

$$\bullet P(D_1 | H_0) = \int_{-0.3678}^{0.3678} \frac{1}{3} e^{-|x|} dx$$

$$= 2 \int_0^{0.3678} \frac{1}{3} e^{-x} dx$$

$$= \frac{2}{3} \left[ -e^{-0.3678} + 1 \right]$$

$$= \frac{2}{3} [0.3077] = \underline{\underline{0.2051}}$$

$$\bullet P(D_0 | H_1) = 2 \int_{0.3678}^{\infty} e^{-2x} dx$$

$$= 2 \left[ \frac{e^{-\infty}}{2} + \frac{e^{-(2 \times 0.3678)}}{2} \right] = \underline{\underline{0.4792}}$$

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Bayes cost

$$\begin{aligned}\bar{C} &= 3 \times 0.4792 + 1 \times 0.2051 \\ &= \underline{\underline{1.6427}}.\end{aligned}$$