THE MAP Test

* Let $P(H_i|x)$, $i = 0, 1$, denote the probability that H_i is true for a particular value of x.

* The conditional probability $P(H_i|x)$ is called a posterior probability (or "a posteriori"). That is to say, it is a probability computed after the observation has been made.

* The probability $P(H_i)$ $(i = 0, 1)$ is called the prior probability. As the name suggests, it is determined prior to the observation being made.

* In the maximum a posteriori (MAP) test, the decision regions are selected as

- $R_0 = \{x : P(H_0|x) > P(H_0|x)\}$
- $R_1 = \{x : P(H_1|x) > P(H_0|x)\}$

2/1

The MAP Test is given by

$$d(x) = \begin{cases} H_0 & \text{if } P(H_0|x) > P(H_1|x) \\ H_1 & \text{if } P(H_1|x) > P(H_0|x) \end{cases}.$$

We can rewrite this as

$$\boxed{\frac{P(H_1|x)}{P(H_0|x)} \underset{H_0}{\overset{H_1}{\gtrless}} 1.}$$

Decision operator

(i.e Accept H_1 if the Ratio is greater than 1
Accept H_0 if the ratio is less than 1)

Suppose $\eta = P(H_0)/P(H_1)$.

The MAP Test can be expressed as a Likelihood ratio test.

$$\boxed{\Lambda(x) \underset{H_0}{\overset{H_1}{\gtrless}} \eta}$$

η is called the threshold value for the MAP test.

MAP TEST – WORKED EXAMPLE

In a binary communication system, during every T seconds, one of two possible signals was sent: S_0 and S_1.

Our hypotheses are

H_0 : S_0 was transmitted

H_1 : S_1 was transmitted.

Suppose $P(H_0) = 2/3$ and $P(H_1) = 1/3$

Suppose the likelihood ratio test is specified as follows.

$$e^{(x-1/2)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

where $\eta = P(H_0) / P(H_1)$ here $\eta = 2$.

Determine what signal is transmitted if $x = 0.6$.

- $e^{(x-1/2)} \underset{H_0}{\overset{H_1}{\gtrless}} 2.$
- $\ln\left[e^{(x-1/2)}\right] \underset{H_0}{\overset{H_1}{\gtrless}} \ln(2)$

MAP TEST – WORKED EXAMPLE.

- $$X \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1}{2} + \ln(2)$$

- $$X \underset{H_0}{\overset{H_1}{\gtrless}} 1.193.$$

Since $x = 0.6$ we conclude that signal S_0 was transmitted.