(Prob. 8.8)

$$\Lambda(\mathbf{x}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta = \lambda \tag{8.17}$$

where the threshold value η of the test is equal to the Lagrange multiplier λ, which is chosen to satisfy the contraint $\alpha = \alpha_0$.

D. Bayes' Test:

Let C_{ij} be the cost associated with (D_i, H_j), which denotes the event that we accept H_i when H_j is true. Then the average cost, which is known as the *Bayes' risk*, can be written as

$$\bar{C} = C_{00}P(D_0, H_0) + C_{10}P(D_1, H_0) + C_{01}P(D_0, H_1) + C_{11}P(D_1, H_1) \tag{8.18}$$

where $P(D_i, H_j)$ denotes the probability that we accept H_i when H_j is true. By Bayes' rule (*1.42*), we have

$$\bar{C} = C_{00}P(D_0 \mid H_0)P(H_0) + C_{10}P(D_1 \mid H_0)P(H_0) + C_{01}P(D_0 \mid H_1)P(H_1) + C_{11}P(D_1 \mid H_1)P(H_1) \tag{8.19}$$

In general, we assume that

$$C_{10} > C_{00} \qquad \text{and} \qquad C_{01} > C_{11} \tag{8.20}$$

since it is reasonable to assume that the cost of making an incorrect decision is higher than the cost of making a correct decision. The test that minimizes the average cost $\bar{C}$ is called the Bayes' test, and it can be expressed in terms of the likelihood ratio test as (Prob. 8.10)

$$\Lambda(\mathbf{x}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta = \frac{(C_{10} - C_{00})P(H_0)}{(C_{01} - C_{11})P(H_1)} \tag{8.21}$$

Note that when $C_{10} - C_{00} = C_{01} - C_{11}$, the Bayes' test (*8.21*) and the MAP test (*8.15*) are identical.

E. Minimum Probability of Error Test:

If we set $C_{00} = C_{11} = 0$ and $C_{01} = C_{10} = 1$ in Eq. (*8.18*), we have

$$\bar{C} = P(D_1, H_0) + P(D_0, H_1) = P_e \tag{8.22}$$

which is just the probability of making an incorrect decision. Thus, in this case, the Bayes' test yields the minimum probability of error, and Eq. (*8.21*) becomes

$$\Lambda(\mathbf{x}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta = \frac{P(H_0)}{P(H_1)} \tag{8.23}$$

We see that the minimum probability of error test is the same as the MAP test.

F. Minimax Test:

We have seen that the Bayes' test requires the a priori probabilities $P(H_0)$ and $P(H_1)$. Frequently, these probabilities are not known. In such a case, the Bayes' test cannot be applied, and the following minimax (min-max) test may be used. In the minimax test, we use the Bayes' test which corresponds to the least favorable $P(H_0)$ (Prob. 8.12). In the minimax test, the critical region R_1^* is defined by

$$\max_{P(H_0)} \bar{C}[P(H_0), R_1^*] = \min_{R_1} \max_{P(H_0)} \bar{C}[P(H_0), R_1] < \max_{P(H_0)} \bar{C}[P(H_0), R_1] \tag{8.24}$$