for all $R_1 \neq R_1^*$. In other words, R_1^* is the critical region which yields the minimum Bayes' risk for the least favorable $P(H_0)$. Assuming that the minimization and maximization operations are interchangeable, then we have

$$\min_{R_1} \max_{P(H_0)} \bar{C}[P(H_0), R_1] = \max_{P(H_0)} \min_{R_1} \bar{C}[P(H_0), R_1] \tag{8.25}$$

The minimization of $\bar{C}[P(H_0), R_1]$ with respect to R_1 is simply the Bayes' test, so that

$$\min_{R_1} \bar{C}[P(H_0), R_1] = C^*[P(H_0)] \tag{8.26}$$

where $C^*[P(H_0)]$ is the minimum Bayes' risk associated with the a priori probability $P(H_0)$. Thus, Eq. (*8.25*) states that we may find the minimax test by finding the Bayes' test for the least favorable $P(H_0)$, that is, the $P(H_0)$ which maximizes $\bar{C}[P(H_0)]$.

Solved Problems

HYPOTHESIS TESTING

8.1. Suppose a manufacturer of memory chips observes that the probability of chip failure is $p = 0.05$. A new procedure is introduced to improve the design of chips. To test this new procedure, 200 chips could be produced using this new procedure and tested. Let r.v. X denote the number of these 200 chips that fail. We set the test rule that we would accept the new procedure if $X \leq 5$. Let

$$\begin{aligned} H_0&: \ p = 0.05 \qquad \text{(No change hypothesis)} \\ H_1&: \ p < 0.05 \qquad \text{(Improvement hypothesis)} \end{aligned}$$

Find the probability of a Type I error.

If we assume that these tests using the new procedure are independent and have the same probability of failure on each test, then X is a binomial r.v. with parameters $(n, p) = (200, p)$. We make a Type I error if $X \leq 5$ when in fact $p = 0.05$. Thus, using Eq. (*2.37*), we have

$$\begin{aligned} P_1 = P(D_1 \mid H_0) &= P(X \leq 5;\ p = 0.05) \\ &= \sum_{k=0}^{5} \binom{200}{k} (0.05)^k (0.95)^{200-k} \end{aligned}$$

Since n is rather large and p is small, these binomial probabilities can be approximated by Poisson probabilities with $\lambda = np = 200(0.05) = 10$ (see Prob. 2.40). Thus, using Eq. (*2.100*), we obtain

$$P_1 \approx \sum_{k=0}^{5} e^{-10} \frac{10^k}{k!} = 0.067$$

Note that H_0 is a simple hypothesis but H_1 is a composite hypothesis.

8.2. Consider again the memory chip manufacturing problem of Prob. 8.1. Now let

$$\begin{aligned} H_0&: \ p = 0.05 \qquad \text{(No change hypothesis)} \\ H_1&: \ p = 0.02 \qquad \text{(Improvement hypothesis)} \end{aligned}$$

Again our rule is, we would reject the new procedure if $X > 5$. Find the probability of a Type II error.