0.0.1 Example 3.8.1

Consider the co-ordination game given above. Since there are only 2 possible actions, we need only to consider the evolution of the proportion of individuals using A. Let pn be the proportion of individuals using A in generation n (the remaining individuals use B). The proportion of individuals using A at the start of the process is p0. 27 / 46

0.0.2 Example 3.8.1

The average reward of A players in generation n is given by RA,n = R(A, pnA + (1 pn)B) = 5pn. The average reward of B players in generation n is given by RB,n = R(B, pnA + (1 pn)B) = 1 pn. 28 / 46

0.0.3 Example 3.8.1

The average reward of the population as a whole is given by Rn = pnRA,n + (1 pn)RB,n = 5p 2 n + (1 pn) 2 = 1 2pn + 6p 2 n . Hence, the equation governing the replicator dynamics is pn+1 = pnRA,n Rn = 5p 2 n 1 2pn + 6p 2 n . 29 / 46

0.0.4 Example 3.8.1

A fixed point of these dynamics satisfies pn+1 = pn = p. We have p = 5p 2 1 2p + 6p 2 p(1 2p + 6p 2) = 5p 2 . p = 0 is clearly a root of this equation. Otherwise, dividing by p, we obtain (1 2p + 6p 2) = 5p 1 7p + 6p 2 = 0 p = 1 or p = 1 6 . Hence, there are 3 fixed points of the replicator dynamics 0, 1 6 and 1. 30 / 46

0.0.5 Example 3.8.1

Note that 0 and 1 will always be fixed points of the replicator dynamics in 22 games. This is due to the fact that the replicator dynamics assume that there is no mutation. Hence, if one of the actions is not played, then there is no way of introducing it into the population. We now check which of these fixed points are attractors. 31 / 46

0.0.6 Example 3.8.1

To check whether p = 0 is an attractor (this corresponds to the strategy B), we assume that pn is close to 0 and see whether pn+1 is closer to 0, i.e. let pn = where is small (¿ 0). We have pn+1 0 = pn+1 = 5 2 1 2 + 6 2 = 5 1 2 + 6 2 . For small , pn+1 will be of the order 5 times pn, i.e. less than pn. Hence, p = 0 is an attractor. 32 / 46 Example 3.8.1 To check whether p = 1 is an attractor (this corresponds to the strategy B), we assume that pn is close to 1, i.e. pn = 1 , where is small and positive. We then look at the distance between 1 and pn+1 (normally as a multiple of). Setting pn = 1 . pn+1 = 5(1) 2 1 2(1) + 6(1) 2 33 / 46

0.0.7 Example 3.8.1

Hence, 1 pn+1 = 1 5(1) 2 1 2(1) + 6(1) 2 =   5 10 + 6 2 . It follows that when pn is away from 1, then pn+1 will be of order 5 times further away, i.e. closer to 1. Hence, p = 1 is an attractor. 34 / 46

0.0.8 Example 3.8.1

Finally, we check whether p = 1 6 is an attractor. Let pn+1 = 1 6 + and look at the distance between pn+1 and 1 6 as a multiple of . We have pn+1 1 6 = 5(1/6 +) 2 1 2(1/6 +) + 6(1/6 +) 2 1 6 This gives pn+1 1 6 = 5/3 + 4 5/6 + 6 2 35 / 46

0.0.9 Example 3.8.1

It follows that the difference between pn+1 and 1 6 is of order 2, i.e. twice as far away from 1 6 . It follows that p = 1 6 is not an attractor. Hence, the only ESSs in this game are p = 0 (corresponding to B) and p = 1 (corresponding to A). 36 / 46

0.0.10 Relation Between the Evolution of the Population and the Risk Factor

It should be noted that if p0 ¿ 1 6 , then the population will evolve to everyone using A. If p0 ¡ 1 6 , then the population will evolve to everyone using B. It should be noted that 1 6 is the risk factor associated with the ESS strategy A. When the probability of an opponent playing A is greater than 1 6 , then a player should play A. 37 / 46

0.0.11 3.9 Asymmetric Evolutionary Games

In general, a Hawk-Dove game will not be symmetric, e.g. 1. One individual may be bigger (stronger) than the other and hence more likely to win a fight. For simplicity assume that one player is always bigger and wins a fight with probability p, where p ¿ 1 2 . 2. Two individuals will not find a resource at the same time, hence one can be treated as an owner and the other as an intruder. In such cases the strategy of an individual should take into account the role of a player. 38 / 46

0.0.12 Asymmetric Evolutionary Games

Suppose that in the Hawk-Dove game one player is an owner and the other player is an intruder. Strictly speaking, in such a game since each player can play either role, the strategy of an individual should be given by a rule defining how an individual behaves when she is an intruder and a rule defining how she behaves when she is an owner. However, assuming that it is clear which role is being played by each individual, in order to define ESSs for such a game, we only need to consider the game in which Player 1 is the owner and Player 2 is the intruder.

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Asymmetric Evolutionary Games The payoff matrix is given by H D H (0.5[v c], 0.5[v c]) (v, 0) D (0, v) 0.5(v, v) 40 / 46 Asymmetric Evolutionary Games In this case an ESS profile (1 , 2) satisfies the condition that if the vast majority of the population play 1 when in Role 1 and 2 when in Role 2, then an individual who plays a different strategy in either role will be selected against. It follows that (1 , 2) is an ESS profile if R1(1 , 2)¿R1(1, 2), 1 6= 1 R2(1 , 2)¿R2(1 , 2) 2 6= 2 41 / 46

Asymmetric Evolutionary Games It follows from this definition that any strong Nash equilibrium of an asymmetric game is an ESS profile. Note that it is not clear at this stage that an ESS profile has to satisfy this condition, e.g. no profile with a mixed strategy can satisfy this condition. It will be argued that no such profile can be an ESS in an asymmetric game. 42 / 46

Example 3.9.1 Consider the asymmetric Hawk-Dove game in which Player 1 is the owner and Player 2 is an intruder. Since (H, D) and (D, H) are strong Nash equilibria of this game, they are ESS profiles. The first

ESS profile is the intuitively appealing be aggressive when you are the owner, but avoid fights when you are an intruder. The second ESS profile is the less intuitive avoid fights when you are the owner, but be aggressive when you are an intruder. 43 / 46

Example 3.9.1 The only other possible ESS profile corresponds to the mixed Nash equilibrium in which both players play H with probability p = v c . Suppose that occupiers play Hawk with probability p and intruders play H with probability q. The expected payoff of an occupier who plays H is R1(H, qH + (1 q)D) = 0.5q(v c) + (1 q)v. 44 / 46

0.0.13 Example 3.9.1

The expected payoff of an occupier who plays D is R1(D, qH + (1 q)D) = 0.5(1 q)v. Comparing these two expected payoffs, occupiers playing Hawk obtain a higher payoff than occupiers playing Dove if q ¡ v c . i.e. if the level of aggression in intruders is low, then owners should be aggressive. It follows from the symmetry of the payoffs that intruders playing Hawk obtain a higher payoff than intruders playing Dove if p ¡ v c (i.e. the level of aggression in owners is low). Suppose occupiers play H with probability v c + and intruders play H with probability v It follows that selection will favour occupiers playing H and intruders playing D. It can be seen that evolution will take the population away from the mixed Nash equilibrium. Hence, the mixed Nash equilibrium is not an ESS. 46 / 46