Supplementary Problems

8.17. Let $(X_1, \ldots, X_n)$ be a random sample of a Bernoulli r.v. X with pmf

$$f(x; p) = p^x(1-p)^{1-x} \qquad x = 0, 1$$

where it is known that $0 < p \le \frac{1}{2}$. Let

$$H_0: \ p = \tfrac{1}{2}$$
$$H_1: \ p = p_1 \ (<\tfrac{1}{2})$$

and $n = 20$. As a decision test, we use the rule to reject H_0 if $\sum_{i=1}^n x_i \le 6$.

(a) Find the power function $g(p)$ of the test.

(b) Find the probability of a Type I error α.

(c) Find the probability of a Type II error β (i) when $p_1 = \frac{1}{4}$ and (ii) when $p_1 = \frac{1}{10}$.

Ans. (a) $g(p) = \sum_{k=0}^{6} \binom{20}{k} p^k(1-p)^{20-k} \qquad 0 < p \le \frac{1}{2}$

(b) $\alpha = 0.0577$; (c) (i) $\beta = 0.2142$, (ii) $\beta = 0.0024$

8.18. Let $(X_1, \ldots, X_n)$ be a random sample of a normal r.v. X with mean μ and variance 36. Let

$$H_0: \ \mu = 50$$
$$H_1: \ \mu = 55$$

As a decision test, we use the rule to accept H_0 if $\bar{x} < 53$, where $\bar{x}$ is the value of the sample mean.

(a) Find the expression for the critical region R_1.

(b) Find α and β for $n = 16$.

Ans. (a) $R_1 = \{(x_1, \ldots, x_N); \bar{x} \ge 53\} \qquad$ where $\bar{x} = \frac{1}{n} \sum_{n=1}^{n} x_i$

(b) $\alpha = 0.0228$, $\beta = 0.0913$

8.19. Let $(X_1, \ldots, X_n)$ be a random sample of a normal r.v. X with mean μ and variance 100. Let

$$H_0: \ \mu = 50$$
$$H_1: \ \mu = 55$$

As a decision test, we use the rule that we reject H_0 if $\bar{x} \ge c$. Find the value of c and sample size n such that $\alpha = 0.025$ and $\beta = 0.05$.

Ans. $c = 52.718$, $n = 52$

8.20. Let X be a normal r.v. with zero mean and variance σ^2. Let

$$H_0: \ \sigma^2 = 1$$
$$H_1: \ \sigma^2 = 4$$

Determine the maximum likelihood test.

Ans. $|x| \underset{H_0}{\overset{H_1}{\gtrless}} 1.36$

8.21. Consider the binary decision problem of Prob. 8.20. Let $P(H_0) = \frac{2}{3}$ and $P(H_1) = \frac{1}{3}$. Determine the MAP test.

Ans. $|x| \underset{H_0}{\overset{H_1}{\gtrless}} 1.923$