where D_i ($i = 0, 1$) denotes the event that the decision is made to accept H_i. P_I is often denoted by α and is known as the *level of significance*, and P_{II} is denoted by β and $(1 - \beta)$ is known as the *power of the test*. Note that since α and β represent probabilities of events from the same decision problem, they are not independent of each other or of the sample size n. It would be desirable to have a decision process such that both α and β will be small. However, in general, a decrease in one type of error leads to an increase in the other type for a fixed sample size (Prob. 8.4). The only way to simultaneously reduce both type of errors is to increase the sample size (Prob. 8.5). One might also attach some relative importance (or cost) to the four possible courses of action and minimize the total cost of the decision (see Sec. 8.3D).

The probabilities of correct decisions (actions 1 and 3) may be expressed as

$$P(D_0 \mid H_0) = P(\mathbf{x} \in R_0;\, H_0) \tag{8.3}$$

$$P(D_1 \mid H_1) = P(\mathbf{x} \in R_1;\, H_1) \tag{8.4}$$

In radar signal detection, the two hypotheses are

H_0: No target exists

H_1: Target is present

In this case, the probability of a Type I error $P_I = P(D_1 \mid H_0)$ is often referred to as the *false-alarm* probability (denoted by P_F), the probability of a Type II error $P_{II} = P(D_0 \mid H_1)$ as the *miss* probability (denoted by P_M), and $P(D_1 \mid H_1)$ as the *detection* probability (denoted by P_D). The cost of failing to detect a target cannot be easily determined. In general we set a value of P_F which is acceptable and seek a decision test that constrains P_F to this value while maximizing P_D (or equivalently minimizing P_M). This test is known as the *Neyman-Pearson* test (see Sec. 8.3C).

8.3 DECISION TESTS

A. Maximum-Likelihood Test:

Let $\mathbf{x}$ be the observation vector and $P(\mathbf{x} \mid H_i)$, $i = 0.1$, denote the probability of observing $\mathbf{x}$ given that H_i was true. In the *maximum-likelihood* test, the decision regions R_0 and R_1 are selected as

$$\begin{aligned} R_0 &= \{\mathbf{x}\colon P(\mathbf{x} \mid H_0) > P(\mathbf{x} \mid H_1)\} \\ R_1 &= \{\mathbf{x}\colon P(\mathbf{x} \mid H_0) < P(\mathbf{x} \mid H_1)\} \end{aligned} \tag{8.5}$$

Thus, the maximum-likelihood test can be expressed as

$$d(\mathbf{x}) = \begin{cases} H_0 & \text{if } P(\mathbf{x} \mid H_0) > P(\mathbf{x} \mid H_1) \\ H_1 & \text{if } P(\mathbf{x} \mid H_0) < P(\mathbf{x} \mid H_1) \end{cases} \tag{8.6}$$

The above decision test can be rewritten as

$$\frac{P(\mathbf{x} \mid H_1)}{P(\mathbf{x} \mid H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} 1 \tag{8.7}$$

If we define the likelihood ratio $\Lambda(\mathbf{x})$ as

$$\Lambda(\mathbf{x}) = \frac{P(\mathbf{x} \mid H_1)}{P(\mathbf{x} \mid H_0)} \tag{8.8}$$

then the maximum-likelihood test (8.7) can be expressed as

$$\Lambda(\mathbf{x}) \underset{H_0}{\overset{H_1}{\gtrless}} 1 \tag{8.9}$$