maker's choice; he or she is restricted to one or another "pure strategy" $D_1, \ldots, D_m$. Because of these differences, optimal game strategies tend, for decision processes, to be too conservative.

NAIVE DECISION CRITERIA

The *minimax* (or *pessimistic*) *criterion* is to select the decision that minimizes the maximum possible loss to the decision maker. In terms of a gain matrix, it is the decision that maximizes the minimum possible gain. The *optimistic criterion* is to choose the decision that maximizes the possible gain. The *middle-of-the road criterion* is to select that decision for which the average of the maximum and minimum gains is greatest. (See Problems 18.1 and 18.2.) As none of these three criteria is based on the *probable* state of nature, they are considered inferior to other criteria that are so based. Two probabilistic criteria will now be given.

A PRIORI CRITERION

The *a priori* (or *Bayes'*) *criterion* is to select the decision that maximizes the expected gain. (See Problems 18.3 and 18.4.)

A POSTERIORI CRITERION

If an imperfect experiment can be conducted that provides information on the true state of nature, then data from this experiment may be combined with the initial probabilities of the various states to yield an updated probability distribution. Designate the outcome of the experiment by θ and assume that the reliability of the experiment is given by the conditional probabilities $P(\theta|S_1)$, $P(\theta|S_2), \ldots, P(\theta|S_n)$. The updated (or *a posteriori*) probabilities of the states—$P(S_1|\theta), P(S_2|\theta), \ldots, P(S_n|\theta)$—are determined from Bayes' theorem (Problem 18.5). The *a posteriori* criterion is to select the decision that maximizes the expected gain with respect to the updated probability distribution. (See Problems 18.6 and 18.7.)

DECISION TREES

A *decision tree* is an oriented tree (see Chapter 13) that represents a decision process. The nodes designate points in time where (i) one or another decision must be made by the decision maker, or (ii) the decision maker is faced with one or another state of nature, or (iii) the process terminates. Directed out of a node (i) is a branch for each possible decision; directed out of a node (ii) is a branch for each possible state of nature. Under each branch the probability of the corresponding event is written, when defined. (See Problems 18.3 and 18.6.)

Decision trees are useful in determining optimal decisions for complicated processes. The technique is to begin with the terminal nodes and sequentially to move backwards through the network, calculating the expected gains at the intermediate nodes. Each gain is written above its corresponding node. A recommended decision is one that leads to a maximum expected gain. Decisions that turn out to be nonrecommended have their corresponding branches crossed out. (See Problems 18.8 and 18.9.)

UTILITY

The *utility* of a payoff is its numerical value to a decision maker. Since no decision criterion is applicable unless all payoffs are quantified in identical units, the first step in analyzing any decision process is to determine the utility of all nonnumeric payoffs. (See Problem 18.12.)