To maximize J by selecting the critical region R_1, we select $\mathbf{x} \in R_1$ such that the integrand in Eq. (8.30) is positive. Thus R_1 is given by

$$R_1 = \{\mathbf{x}: [f(\mathbf{x} \mid H_1) - \lambda f(\mathbf{x} \mid H_0)] > 0\}$$

and the Neyman-Pearson test is given by

$$\Lambda(\mathbf{x}) = \frac{f(\mathbf{x} \mid H_1)}{f(\mathbf{x} \mid H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

and λ is determined such that the constraint

$$\alpha = P_1 = P(D_1 \mid H_0) = \int_{R_1} f(\mathbf{x} \mid H_0)\, d\mathbf{x} = \alpha_0$$

is satisfied.

8.9. Consider the binary communication system of Prob. 8.6 and suppose that we require that $\alpha = P_1 = 0.25$.

(a) Using the Neyman-Pearson test, determine which signal is transmitted when $x = 0.6$.

(b) Find P_{II}.

(a) Using the result of Prob. 8.6 and Eq. (8.17), the Neyman-Pearson test is given by

$$e^{(x-1/2)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

Taking the natural logarithm of the above expression, we get

$$x - \tfrac{1}{2} \underset{H_0}{\overset{H_1}{\gtrless}} \ln \lambda \qquad \text{or} \qquad x \underset{H_0}{\overset{H_1}{\gtrless}} \tfrac{1}{2} + \ln \lambda$$

The critical region R_1 is thus

$$R_1 = \{x: x > \tfrac{1}{2} + \ln \lambda\}$$

Now we must determine λ such that $\alpha = P_1 = P(D_1 \mid H_0) = 0.25$. By Eq. (8.1), we have

$$P_1 = P(D_1 \mid H_0) = \int_{R_1} f(x \mid H_0)\, dx = \frac{1}{\sqrt{2\pi}} \int_{1/2+\ln\lambda}^{\infty} e^{-x^2/2}\, dx = 1 - \Phi\left(\frac{1}{2} + \ln \lambda\right)$$

Thus $\qquad 1 - \Phi(\tfrac{1}{2} + \ln \lambda) = 0.25 \qquad$ or $\qquad \Phi(\tfrac{1}{2} + \ln \lambda) = 0.75$

From Table A (Appendix A), we find that $\Phi(0.674) = 0.75$. Thus

$$\tfrac{1}{2} + \ln \lambda = 0.674 \rightarrow \lambda = 1.19$$

Then the Neyman-Pearson test is

$$x \underset{H_0}{\overset{H_1}{\gtrless}} 0.674$$

Since $x = 0.6 < 0.674$, we determine that signal $s_0(t)$ was transmitted.

(b) By Eq. (8.2), we have

$$P_{II} = P(D_0 \mid H_1) = \int_{R_0} f(x \mid H_1)\, dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.674} e^{-(x-1)^2/2}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-0.326} e^{-y^2/2}\, dy = \Phi(-0.326) = 0.3722$$

8.10. Derive Eq. (8.21).