A common utility is monetary worth, whereby each payoff (e.g., a new house) is replaced in the gain matrix by its dollar value. Monetary worth, however, is not always appropriate. A payoff of 2 million dollars is twice that of 1 million dollars, yet the former may not be worth twice the latter to a decision maker. The first million may be more valuable than the second million. In cases where dollars do not reflect the true worth of one payoff relative to another payoff, or where dollars are not a convenient quantification unit, other utilities must be used.

LOTTERIES

A *lottery* $\mathscr{L}(A, B; p)$ is a random event having two outcomes, A and B, occurring with probabilities p and $1 - p$, respectively.

VON NEUMANN UTILITIES

The following four-step procedure is used to determine *von Neumann utilities* for a finite number of payoffs.

STEP 1: List the payoffs in decreasing order of desirability: $e_1, e_2, \ldots, e_p$. Here, e_i is at least as desirable as e_j if $i < j$.

STEP 2: Arbitrarily assign finite numerical values $u(e_1)$ and $u(e_p)$ to payoffs e_1 and e_p, respectively, such that $u(e_1) > u(e_p)$.

STEP 3: For each payoff e_j ranked between e_1 and e_p in desirability, determine an *equivalence probability* p_j having the property that the decision maker is indifferent between obtaining e_j with certainty and participating in the lottery $\mathscr{L}(e_1, e_p; p_j)$.

STEP 4: Let $u(e_j) \equiv p_j u(e_1) + (1 - p_j)u(e_p)$ be the utility of payoff e_j.

Step 3 is highly subjective. The value of p_j for each payoff e_j ($j = 2, 3, \ldots, p - 1$) is an individual determination that may change drastically from one person to another or even for the same person at two different times. The resulting utilities, therefore, quantify the relative worths of payoffs to a particular decision maker at a particular moment. However, for a rational individual, it may always be expected that the *order* of the p's, and therefore of the u's, will be the same as the order of the e's. (See Problems 18.10 and 18.12.)

A utility is *normalized* if $u(e_1) = 1$ and $u(e_p) = 0$, making the utilities identical to the equivalence probabilities.

Solved Problems

18.1 Determine recommended decisions under each naive criterion for the process described in Example 18.1.

The gain matrix for this process is Table 18-2. The minimum gain for decision D_1 is 60, while that for D_2 is -100. Since $\max\{60, -100\} = 60$ is the gain associated with D_1, D_1 is the recommended decision under the minimax criterion.

The largest entry in the matrix is 2000, the gain associated with D_2. Therefore D_2 is the recommended decision under the optimistic criterion.

The averages of the maximum and minimum gains for D_1 and D_2 are, respectively,

$$\frac{660 + 60}{2} = 360 \quad \text{and} \quad \frac{2000 + (-100)}{2} = 950$$