Cournot duopoly, also called Cournot competition, is a model of imperfect competition in which two firms with identical cost functions compete with homogeneous products in a static setting. It was developed by Antoine A. Cournot in his Researches Into the Mathematical principles of the Theory of Wealth, 1838.

0.0.1 Cournot duopoly

Cournot duopoly, also called Cournot competition, is a model of imperfect competition in which two firms with identical cost functions compete with homogeneous products in a static setting. It was developed by Antoine A. Cournot in his Researches Into the Mathematical principles of the Theory of Wealth, 1838. Cournots duopoly represented the creation of the study of oligopolies, more particularly duopolies, and expanded the analysis of market structures which, until then, had concentrated on the extremes: perfect competition and monopolies.

Cournot really invented the concept of game theory almost 100 years before John Nash, when he looked at the case of how businesses might behave in a duopoly. There are two firms operating in a limited market. Market production is: P(Q)=a-bQ, where Q=q1+q2 for two firms. Both companies will receive profits derived from a simultaneous decision made by both on how much to produce, and also based on their cost functions: TCi=C-qi.

0.1 Cournot-Nash Equilibrium in Duopoly

This is a homework question, but resources online are exceedingly complicated, so I was hoping there was a fast, efficient way of solving the following question:

There are 2 firms in an industry, which have the following total cost functions and inverse demand functions.

$$Firm1: C1 = 50Q_1 P1 = 1000.5(Q_1 + Q_2) Firm2: C2 = 24Q_2 P2 = 1000.5(Q_1 + Q_2)$$

What is the Cournot-Nash equilibrium for this industry?

I've tried to solve this dozens of times. My idea was to find the profit equation for both, take the derivative, set equal to zero, and then solve for Q_1 and Q_2.

Doing this, I get:

$$Q_1 = 5Q_2 + 500Q_2 = 5Q_1 + 760$$

There is a standard way of solving for Q_1 and Q_2.

Determine the profit functions. Determine the best response function for the firms. Substitute Q_1 or Q_2 in the other profit function and solve. All these steps are already mentioned, so you know what to do. Below you can search for your mistake.

- The profit function for firm 1 equals

$$1 = P1Q_1C1 = Q_1(1000.5(Q_1 + Q_2))50Q_1 1 = P1Q_1C1 = Q_1(1000.5(Q_1 + Q_2))50Q_1$$

- The profit function for firm 2 equals

$$2 = P2Q_2C2 = Q_2(1000.5(Q_1 + Q_2))24Q_2 2 = P2Q_2C2 = Q_2(1000.5(Q_1 + Q_2))24Q_2$$

The best response function can be determined by deriving the profit function of firm 1 w.r.t. Q_1 and for firm 2 w.r.t. Q_2 and set them equal to zero

$$\frac{\partial \pi_1}{\partial Q_1} = 100Q_1 0.5Q_2 50 = 50Q_1 0.5Q_2 = 0$$

$$Q_1 = 500.5Q_2$$

$$\frac{\partial \Pi_2}{\partial Q_2} = 100Q_2 0.5Q_1 24 = 76Q_2 0.5Q_1 = 0$$

Now we can make the substitution

$$76Q_2 0.5(500.5Q_2) = 0$$

$$51Q_2 + 0.25Q_2 = 0 \rightarrow 0.75Q_2 = 51$$

And thus we find $Q_2 = 68$ and can solve easily for Q_1 $Q_2 = 68$ and $Q_1 = 500.568 = 16$

0.2 Cournot Nash Equilibrium Between Two Firms

Suppose we have two firms with specialized, but similar products. Suppose market demand for the two products is:

$$p1(Q_1, Q_2) = abQ_1dQ_2$$

$$p2(Q_1, Q_2) = abQ_2dQ_1$$

where $d \in (b, b)$. Suppose that both firms have cost $c(q) = q$ What does d mean intuitively? Is the Cournot Nash Equilibrium for this

$$Q_1 = 2baad + dc(Q_2)c(Q_1)2b1d2$$
$$Q_2 = 2baad + dc(Q_1)c(Q_2)2b1d2$$

0.2.1 What does d mean intuitively?

To answer this question, think about the "vanilla" Cournot competition case, where products p1p1 and p2p2 are identical; they're perfect substitutes. In this case, increases in production from your competitor (i.e. Q_2) displaces your own production, so d=bd=b and

$$p1(Q_1, Q_2) = ab(Q_1 + Q_2)p1(Q_1, Q_2) = ab(Q_1 + Q_2).$$

On the other hand, if an increase in production of Q_2 increases demand for your own product Q_1, then these products are compliments. Be careful about stating they are perfect compliments, because without looking at consumer indifference curves, we can't determine this.

In this case, dd is negative, and is bounded by bb.

In short, dd is a measure of the degree to which these two goods are complements or substitutes. Another approach would be to take the derivative of demand with respect to production of the other good, like this:

$$p1Q_2 = dp1Q_2 = d.$$

If d¿0d¿0, $p1Q_2 < 0$ and Q_2 is a complement to Q_1. Likewise, if $d < 0$, $p1Q_2 > 0$ and Q_2 is a substitute for Q_1. Because of the symmetry of the problem, both will either be complements or substitutes. However, in the real world this is not always the case.

0.2.2 What is the Cournot-Nash equilibrium?

The Cournot-Nash equilibrium is the output $\{Q_1, Q_2\}$ from which neither firm can profitably deviate. To answer this, you need to find the best response function for each firm by solving for the optimal output, given the production of the other firm. This is accomplished by equating Marginal Revenue = Marginal Cost. Note that the marginal cost of production is zero; i.e.$c(Q_1) = c(Q_2) = 0c(Q_1) = c(Q_2) = 0$.

$$BR1(Q_2) = adQ_22bBR1(Q_2) = adQ_22b$$

and

$$BR2(Q_1) = adQ_12bBR2(Q_1) = adQ_12b.$$

The Cournot-Nash equilibrium is located where these two Best Response functions intersect. Solving the system of two equations and two unknowns, I get:

$$q_1^* = q_2^* = a(12d4b)bd24b$$
$$Q_1^* = Q_2^* = a(12d4b)bd24b.$$

0.2.3 2012

Question 6

Consider the (symmetric) Cournot duopoly game: Firm i, i = 1, 2 produces xi items at a cost of C(xi) = x 2 i 1000 + 3xi + 20 . The items sell at a price of p(x1, x2) = 5 x1 + x2 500 each.

(a) Find the equilibrium of this game, and prove that it is a Nash equilibrium. 8 %

(b) Investigate this game if a collusive strategy is used. Contrast its solution with that of part (a). 8 %

(c) If the game is to be played repeatedly, does it ever pay to defect from the collusive strategy? In particular, consider the stern strategy: a firm produces the collusive number of items until the other firm defects, after which it reverts to producing the Cournot number of items. Using the discount factor ω per period, when is this stern strategy a Nash equilibrium ? 9 %

0.2.4 2013

Consider the asymmetric duopoly game: Firm i, i = 1, 2 produces xi items at a cost of C(xi) = 1 i xi + 20. The items sell at a price of p(x1, x2) = 5 x1 + x2 500 each.

(a) Find the equilibrium of the game if it is played as a Cournot game, and prove that it is a Nash equilibrium. 8

(b) Find the equilibrium if it played as a Stackelberg game with Firm 1 as leader. 8

(c) Contrast and comment on the two solutions. 4

(d) Firm 1 receives an injection of capital and initiates a leader strategy. However Firm 2 persists with its Cournot strategy. What happens and what should each firm do in future interactions? 5

0.2.5 2015

3 Consider the following asymmetric duopoly game with isoelastic demand: Two firms sell equivalent items at a price of p(x1, x2) = 600 x1 + x2 per item, where Firm i,(i = 1, 2), produces xi items at a cost of C(xi) = mixi . The marginal costs are given by mi = i + 1 respectively.

(a) If the game is played as a Cournot game, show that the best response xi = B(xj) of Firm i to Firm j is given by B(xj) = r 600xj mi xj and hence find the Nash equilibrium. 8 (b) Find the equilibrium if it played as a Stackelberg game with Firm 1 as leader. 8 (c) Contrast and comment on the two solutions. 4 (d) Firm 1 decides it will stick with the leader strategy in further production cycles. However Firm 2 decides to ignore this and persist with its Cournot strategy of part(a). What happens and what should each firm do in future interactions?