Then by Eqs. (8.1) and (8.2) and using Table A (Appendix A), we obtain

$$P_{I} = P(D_1 \mid H_0) = \int_{R_1} f(x \mid H_0)\,dx = \frac{1}{\sqrt{2\pi}} \int_{1/2}^{\infty} e^{-x^2/2}\,dx = 1 - \Phi(\tfrac{1}{2}) = 0.3085$$

$$P_{II} = P(D_0 \mid H_1) = \int_{R_0} f(x \mid H_1)\,dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{1/2} e^{-(x-1)^2/2}\,dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-1/2} e^{-y^2/2}\,dy = \Phi(-\tfrac{1}{2}) = 0.3085$$

8.7. In the binary communication system of Prob. 8.6, suppose that $P(H_0) = \frac{2}{3}$ and $P(H_1) = \frac{1}{3}$.

(a) Using the MAP test, determine which signal is transmitted when $x = 0.6$.

(b) Find P_I and P_{II}.

(a) Using the result of Prob. 8.6 and Eq. (8.15), the MAP test is given by

$$e^{(x-1/2)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{P(H_0)}{P(H_1)} = 2$$

Taking the natural logarithm of the above expression, we get

$$x - \tfrac{1}{2} \underset{H_0}{\overset{H_1}{\gtrless}} \ln 2 \qquad \text{or} \qquad x \underset{H_0}{\overset{H_1}{\gtrless}} \tfrac{1}{2} + \ln 2 = 1.193$$

Since $x = 0.6 < 1.193$, we determine that signal $s_0(t)$ was transmitted.

(b) The decision regions are given by

$$R_0 = \{x: x < 1.193\} = (-\infty, 1.193)$$
$$R_1 = \{x: x > 1.193\} = (1.193, \infty)$$

Thus, by Eqs. (8.1) and (8.2) and using Table A (Appendix A), we obtain

$$P_I = P(D_1 \mid H_0) = \int_{R_1} f(x \mid H_0)\,dx = \frac{1}{\sqrt{2\pi}} \int_{1.193}^{\infty} e^{-x^2/2}\,dx = 1 - \Phi(1.193) = 0.1164$$

$$P_{II} = P(D_0 \mid H_1) = \int_{R_0} f(x \mid H_1)\,dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{1.193} e^{-(x-1)^2/2}\,dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.193} e^{-y^2/2}\,dy = \Phi(0.193) = 0.5765$$

8.8. Derive the Neyman-Pearson test, Eq. (8.17).

From Eq. (8.16), the objective function is

$$J = (1 - \beta) - \lambda(\alpha - \alpha_0) = P(D_1 \mid H_1) - \lambda[P(D_1 \mid H_0) - \alpha_0] \tag{8.29}$$

where λ is an undetermined Lagrange multiplier which is chosen to satisfy the constraint $\alpha = \alpha_0$. Now, we wish to choose the critical region R_1 to maximize J. Using Eqs. (8.1) and (8.2), we have

$$J = \int_{R_1} f(\mathbf{x} \mid H_1)\,d\mathbf{x} - \lambda\left[\int_{R_1} f(\mathbf{x} \mid H_0)\,d\mathbf{x} - \alpha_0\right]$$

$$= \int_{R_1} [f(\mathbf{x} \mid H_1) - \lambda f(\mathbf{x} \mid H_0)]\,d\mathbf{x} + \lambda\alpha_0 \tag{8.30}$$