Taking the natural logarithm of both sides of the last expression yields

$$|x| \underset{H_0}{\overset{H_1}{\lessgtr}} -\ln(\tfrac{1}{2}) = 0.693$$

Thus, the decision regions are given by

$$R_0 = \{x: |x| > 0.693\} \qquad R_1 = \{x: |x| < 0.693\}$$

Then
$$P_I = P(D_1 | H_0) = \int_{-0.693}^{0.693} \tfrac{1}{2}e^{-|x|}\,dx = 2\int_0^{0.693} \tfrac{1}{2}e^{-x}\,dx = 0.5$$

$$P_{II} = P(D_0 | H_1) = \int_{-\infty}^{-0.693} e^{2x}\,dx + \int_{0.693}^{\infty} e^{-2x}\,dx = 2\int_{0.693}^{\infty} e^{-2x}\,dx = 0.25$$

and by Eq. (*8.19*), the Bayes' risk is

$$\bar{C} = P(D_1 | H_0)P(H_0) + 2P(D_0 | H_1)P(H_1) = (0.5)(\tfrac{2}{3}) + 2(0.25)(\tfrac{1}{3}) = 0.5$$

(*b*) The Bayes' test is

$$2e^{-|x|} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{(1-0)\frac{1}{2}}{(2-0)\frac{1}{2}} = \frac{1}{2} \qquad \text{or} \qquad e^{-|x|} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1}{4}$$

Again, taking the natural logarithm of both sides of the last expression yields

$$|x| \underset{H_0}{\overset{H_1}{\lessgtr}} -\ln(\tfrac{1}{4}) = 1.386$$

Thus, the decision regions are given by

$$R_0 = \{x: |x| > 1.386\} \qquad R_1 = \{x: |x| < 1.386\}$$

Then
$$P_I = P(D_1 | H_0) = 2\int_0^{1.386} \tfrac{1}{2}e^{-x}\,dx = 0.75$$

$$P_{II} = P(D_0 | H_1) = 2\int_{1.386}^{\infty} e^{-2x}\,dx = 0.0625$$

and by Eq. (*8.19*), the Bayes' risk is

$$\bar{C} = (0.75)(\tfrac{1}{2}) + 2(0.0625)(\tfrac{1}{2}) = 0.4375$$

**8.12.** Consider the binary decision problem of Prob. 8.11 with the same Bayes' costs. Determine the minimax test.

From Eq. (*8.33*), the likelihood ratio is

$$\Lambda(x) = \frac{f(x | H_1)}{f(x | H_0)} = 2e^{-|x|}$$

In terms of $P(H_0)$, the Bayes' test [Eq. (*8.21*)] becomes

$$2e^{-|x|} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1}{2}\frac{P(H_0)}{1-P(H_0)} \qquad \text{or} \qquad e^{-|x|} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1}{4}\frac{P(H_0)}{1-P(H_0)}$$

Taking the natural logarithm of both sides of the last expression yields

$$|x| \underset{H_0}{\overset{H_1}{\lessgtr}} \ln\frac{4[1-P(H_0)]}{P(H_0)} = \delta \tag{8.34}$$

For $P(H_0) > 0.8$, $\delta$ becomes negative, and we always decide $H_0$. For $P(H_0) \le 0.8$, the decision regions are

$$R_0 = \{x: |x| > \delta\} \qquad R_1 = \{x: |x| < \delta\}$$