Financial Mathematics

The Pareto Distribution

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The Pareto Distribution

Suppose the distribution of monthly salaries of full-time workers in the UK has a Pareto distribution with minimum monthly salary $x_m = 1000$ and concentration factor $\alpha = 3$.

The Pareto Distribution

1. Calculate the mean monthly salary of UK full-time workers.
2. Calculate the probability that a UK full-time worker earns more than 2000 per month.
3. Calculate the median monthly salary of UK full-time workers.

The Pareto Distribution

The expected value of a random variable following a Pareto distribution is

$$E(X) = \begin{cases} \infty & \text{if } \alpha \leq 1, \\ \frac{\alpha x_{\mathrm{m}}}{\alpha - 1} & \text{if } \alpha > 1. \end{cases}$$

The Pareto Distribution

Because $\alpha = 3$, we will use this

$$E(X) = \frac{\alpha x_{\mathrm{m}}}{\alpha - 1}$$

Recall that $X_m = 1000$.

The Pareto Distribution

The cumulative distribution function of a Pareto random variable with parameters α and x_m is

$$F_X(x) = \begin{cases} 1 - \left(\frac{x_{\mathrm{m}}}{x}\right)^{\alpha} & \text{for } x \geq x_{\mathrm{m}}, \\ 0 & \text{for } x < x_{\mathrm{m}}. \end{cases}$$

Using values for this example:

$$F_X(x) = \begin{cases} 1 - \left(\frac{1000}{x}\right)^{3} & \text{for } x \geq 1000, \\ 0 & \text{for } x < 1000. \end{cases}$$

The Pareto Distribution

Calculate the probability that a UK full-time worker earns more than 2000 per month.

$$F_X(x) = \begin{cases} 1 - \left(\frac{1000}{x}\right)^3 & \text{for } x \geq 1000, \\ 0 & \text{for } x < 1000. \end{cases}$$

The Pareto Distribution

Calculate the probability that a UK full-time worker earns **more than** 2000 per month.

$$F_X(x) = \begin{cases} 1 - \left(\frac{1000}{x}\right)^3 & \text{for } x \geq 1000, \\ 0 & \text{for } x < 1000. \end{cases}$$

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Calculate the median monthly salary of UK full-time workers.

$$\text{Median} : F_X(x) = 0.50$$

$$F_X(x) = \begin{cases} 1 - \left(\frac{1000}{x}\right)^3 & \text{for } x \geq 1000, \\ 0 & \text{for } x < 1000. \end{cases}$$

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$$F_X(x) = 0.5 \quad \rightarrow \quad 1 - \left(\frac{1000}{x}\right)^3 = 0.50$$

$$\sqrt[3]{0.5} = 0.7937$$

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$$\frac{1000}{0.7937} = 1259.92$$

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