Linear Algebra

Cramer's Rule

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Cramer's rule

Suppose we have the following matrix M

$$\mathrm{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The determinant of M is denoted as

$$\det(\mathrm{M}) = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

and is computed as

$$\det(\mathrm{M}) = (a \times d) - (b \times c)$$

Cramer's rule

Consider the linear system

$$\begin{cases} ax + by &= e \\ cx + dy &= f \end{cases}$$

which in matrix format is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}.$$

Cramer's rule

- We require **ad - bc** to be some value other than zero. If the value is zero, we have a "*Divide by Zero*" problem.
- Then, x and y can be found with Cramer's rule as

$$x = \begin{vmatrix} e & b \\ f & d \end{vmatrix} / \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \frac{ed - bf}{ad - bc}$$

$$y = \begin{vmatrix} a & e \\ c & f \end{vmatrix} / \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \frac{af - ec}{ad - bc}.$$

Cramer's rule

Using Cramer's Rule, solve the equations

$$2x - 3y - 1 = 0$$

$$5x + 2y - 12 = 0$$

Cramer's rule

$$\begin{cases} 2x - 3y &= 1 \\ 5x + 2y &= 12 \end{cases}$$

in matrix format is

$$\begin{bmatrix} 2 & -3 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 12 \end{bmatrix}.$$

Cramer's rule

$$\begin{bmatrix} 2 & -3 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 12 \end{bmatrix}.$$

$$x = \begin{vmatrix} e & b \\ f & d \end{vmatrix} / \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \frac{ed - bf}{ad - bc}$$

Cramer's rule

$$\begin{bmatrix} 2 & -3 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 12 \end{bmatrix}.$$

$$y = \begin{vmatrix} a & e \\ c & f \end{vmatrix} / \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \frac{af - ec}{ad - bc}.$$