Introduction to Statistics and Probability

Calculus For Random Variables

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The random variables X has the probability density function $f(X)$ given by:

$$f(x) = kx^2(1-x), \qquad 0 \leq x \leq 1$$

1. Compute the value for k,
2. Compute the mean and variance for X,
3. Determine the cumulative distribution function $F(x)$,
4. Compute the probability that X lies within one standard deviation of its mean.

Part 1

The definite integral of $f(x)$ between 0 and 1 must equal 1.

$$\int_0^1 f(x)\,dx = \int_0^1 kx^2(1-x)\,dx = \int_0^1 kx^2 - kx^3\,dx$$

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$$\int_0^1 f(x)\,dx = \int_0^1 kx^2(1-x)\,dx = \int_0^1 kx^2 - kx^3\,dx$$

$$= \left[\frac{kx^3}{3}\right]_0^1 - \left[\frac{kx^4}{4}\right]_0^1 \quad = \frac{k}{3} - \frac{k}{4} \quad = \frac{k}{12} \quad (= 1)$$

$$\boldsymbol{k = 12}$$

Part 2 : Compute the Mean and Variance

$$\mathrm{E}(x) = \int_0^1 x\,f(x)\,dx$$

$$\mathrm{Var}(x) = \mathrm{E}(x^2) - [\mathrm{E}(x)]^2$$

$$\mathrm{E}(x^2) = \int_0^1 x^2 f(x)\,dx$$

Part 2 : Compute the Mean and Variance

$$\mathrm{E}(x) = \int_0^1 x f(x)\, dx$$

$$\mathrm{E}(x) = \int_0^1 x(12x^2 - 12x^3)\, dx = \int_0^1 12x^3 - 12x^4\, dx$$

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$$\mathrm{E}(x) = \left[\frac{12x^4}{4} - \frac{12x^5}{5}\right]_0^1 = \frac{6}{5}$$

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$$E(x) = \left[\frac{12x^5}{5} - \frac{12x^6}{6}\right]_0^1 = \frac{2}{5}$$

Part 2 : Compute the Mean and Variance

$$\mathrm{Var}(x) = \mathrm{E}(x^2) - [\mathrm{E}(x)]^2$$

$$\mathrm{Var}(x) = \frac{2}{5} - \left(\frac{3}{5}\right)^2$$

Part 3 : Determine the cumulative distribution function $F(x)$.

$$F(x) = \int_{o}^{x} f(u)\, du = \int_{o}^{x} 12u^2 - 12u^3\, du$$

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$$F(x) = \left[\frac{12x^3}{3} - \frac{12x^4}{4}\right] = \mathbf{4x^3}$$