

Bayesian analysis is the branch of statistics based on the idea that we have some knowledge in advance about the probabilities that we are interested in, so called a priori probabilities. This might be your degree of belief in a particular event, the results from previous studies, or a general agreed-upon starting value for a probability. The terminology "Bayesian" comes from the Bayesian rule or law, a law about conditional probabilities. The opposite of "Bayesian" is sometimes referred to as "Classical Statistics."

Example

Consider a box with 3 coins, with probabilities of showing heads respectively $1/4$, $1/2$ and $3/4$. We choose arbitrarily one of the coins. Hence we take $1/3$ as the a priori probability $P(C_1)$ of having chosen coin number 1. After 5 throws, in which $X=4$ times heads came up, it seems less likely that the coin is coin number 1.

Example

We calculate the a posteriori probability that the coin is coin number 1, as:

In words:

The probability that the Coin is the first Coin, given that we know heads came up 4 times... Is equal to the probability that heads came up 4 times given we know it's the first coin, times the probability that the coin is the first coin. All divided by the probability that heads comes up 4 times (ignoring which of the three Coins is chosen).

Example

The binomial coefficients cancel out as well as all denominators when expanding $1/2$ to $2/4$. This results in

$$\frac{3}{3 + 32 + 81} = \frac{3}{116}$$

In the same way we find:

$$P(C_2|X = 4) = \frac{32}{3 + 32 + 81} = \frac{32}{116}$$

and

$$P(C_3|X = 4) = \frac{81}{3 + 32 + 81} = \frac{81}{116}.$$

This shows us that after examining the outcome of the five throws, it is most likely we did choose coin number 3.

Actually for a given result the denominator does not matter, only the relative Probabilities $p(C_i) = P(C_i|X = 4)/P(X = 4)$ When the result is 3 times heads the Probabilities change in favor of Coin 2 and further as the following table shows:

Heads	$p(C_1)$	$p(C_2)$	$p(C_3)$
5	1	32	243
4	3	32	81
3	9	32	27
2	27	32	9
1	81	32	3
0	243	32	1