Probability Distributions
The Z-score

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Standardization formula

- All normally distributed random variables have corresponding Z values, called Z-scores.
- The Z-score is simply the number of standard deviations away from the mean that a particular score is.

Standardization formula

- For normally distributed random variables, the z-score can be found using the ***standardization formula***;

$$z_o = \frac{x_o - \mu}{\sigma}$$

  where x_o is a observed value from the underlying normal ("X") distribution, μ is the mean of that distribution, and σ is the standard deviation of that distribution.

Standardization formula

- Z-scores are typically given to 2 decimal places only.
- Terms with subscripts mean particular observed values, and are not variable names (Not usual, but useful for sake of clarity.)
- The distribution of Z-values will only be a normal distribution if the original distribution (X) is normal.

The Z-score

- Suppose that X is a normal distribution with mean $\mu = 80$ and that standard deviation $\sigma = 8$.

$$X \sim N(80, 8^2)$$

- What is the Z-score for $x_o = 100$?

$$z_{100} = \frac{x_0 - \mu}{\sigma} = \frac{100 - 80}{8} = \frac{20}{8} = 2.5$$

- Therefore $z_{100} = 2.5$

The Z-score

Again suppose X is a normal distribution with mean $\mu = 80$ and that standard deviation $\sigma = 8$.

Compute the Z-score for each of the following:

(i) $x_a = 96$

(ii) $x_b = 72$

(iii) $x_c = 86$

The Z-score

Again suppose X is a normal distribution with mean $\mu = 80$ and that standard deviation $\sigma = 8$.

Compute the Z-score for each of the following:

(i) $x_a = 96$ $\quad z_a = 2$

(ii) $x_b = 72$

(iii) $x_c = 86$

The Z-score

Again suppose X is a normal distribution with mean $\mu = 80$ and that standard deviation $\sigma = 8$.

Compute the Z-score for each of the following:

(i) $x_a = 96$ $\quad$ $z_a = 2$

(ii) $x_b = 72$ $\quad$ $z_b = -1$

(iii) $x_c = 86$

The Z-score

Again suppose X is a normal distribution with mean $\mu = 80$ and that standard deviation $\sigma = 8$.

Compute the Z-score for each of the following:

(i) $x_a = 96$ $\qquad z_a = 2$

(ii) $x_b = 72$ $\qquad z_b = -1$

(iii) $x_c = 86$ $\qquad z_c = 0.75$