Introduction to Statistics and Probability Calculus For Random Variables

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The random variables X has the probability density function f(X) given by:

$$f(x) = kx^2(1-x), \qquad 0 \le x \le 1$$

- 1. Compute the value for *k*,
- 2. Compute the mean and variance for *X*,
- 3. Determine the cumulative distribution function F(x),
- 4. Compute the probability that X lies within one standard deviation of its mean.



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Part 1

The definite integral of f(x) between 0 and 1 must equal 1.

$$\int_0^1 f(x) \, dx = \int_0^1 kx^2 (1 - x) \, dx = \int_0^1 kx^2 - kx^3 \, dx$$

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Part 1

The definite integral of f(x) between 0 and 1 must equal 1.

$$\int_0^1 f(x) \, dx = \int_0^1 kx^2 (1 - x) \, dx = \int_0^1 kx^2 - kx^3 \, dx$$

$$= \left[\frac{kx^3}{3}\right]_0^1 - \left[\frac{kx^4}{4}\right]_0^1 = \frac{k}{3} - \frac{k}{4} = \frac{k}{12} \quad (=1)$$

$$k = 12$$



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Part 2: Compute the Mean and Variance

$$E(x) = \int_0^1 x f(x) \, dx$$

$$Var(x) = E(x^2) - [E(x)]^2$$

$$E(x^2) = \int_0^1 x^2 f(x) dx$$

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Part 2: Compute the Mean and Variance

$$E(x) = \int_0^1 x f(x) dx$$

$$E(x) = \int_0^1 x (12x^2 - 12x^3) dx = \int_0^1 12x^3 - 12x^4 dx$$

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Part 2: Compute the Mean and Variance

$$E(x) = \int_0^1 x f(x) dx$$

$$E(x) = \int_0^1 x (12x^2 - 12x^3) dx = \int_0^1 12x^3 - 12x^4 dx$$

$$E(x) = \left[\frac{12x^4}{4} - \frac{12x^5}{5} \right]_0^1 = \frac{6}{5}$$

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Part 2 : Compute the Mean and Variance

$$E(x^{2}) = \int_{0}^{1} x^{2} f(x) dx$$

$$E(x^{2}) = \int_{0}^{1} x^{2} (12x^{2} - 12x^{3}) dx = \int_{0}^{1} 12x^{4} - 12x^{5} dx$$

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Part 2: Compute the Mean and Variance

$$E(x^{2}) = \int_{0}^{1} x^{2} f(x) dx$$

$$E(x^{2}) = \int_{0}^{1} x^{2} (12x^{2} - 12x^{3}) dx = \int_{0}^{1} 12x^{4} - 12x^{5} dx$$

$$E(x) = \left[\frac{12x^{5}}{5} - \frac{12x^{6}}{6} \right]_{0}^{1} = \frac{2}{5}$$



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Part 2: Compute the Mean and Variance

$$Var(x) = E(x^2) - [E(x)]^2$$

$$Var(x) = \frac{2}{5} - \left(\frac{3}{5}\right)^2$$

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Part 3 : Determine the cumulative distribution function F(x).

$$F(x) = \int_{o}^{x} f(u) \ du = \int_{o}^{x} 12u^{2} - 12u^{3} \ du$$

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Part 3 : Determine the cumulative distribution function F(x).

$$F(x) = \int_{o}^{x} f(u) du = \int_{o}^{x} 12u^{2} - 12u^{3} du$$

$$F(x) = \left[\frac{12u^{3}}{3} - \frac{12u^{4}}{4} \right]_{0}^{x}$$

$$F(x) = \left[\frac{12x^{3}}{3} - \frac{12x^{4}}{4} \right] = 4x^{3}$$

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