Statistics and Probability

The Geometric Distribution

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The Geometric Distribution

- The geometric distribution is used for Bernouilli Trials, where the outcome are classified as either failures or successes (either one or the other).
- The success event is usually the less probable of the two outcomes, although the definitions can be switched if makes the calculation easier.

The Geometric Distribution

In probability theory, the geometric distribution is either of two discrete probability distributions:

- The probability distribution of the number of trials needed to get the first success, supported on the set $\{1, 2, 3, \ldots\}$,
- The probability distribution of the number of failures before the first success, supported on the set $\{0, 1, 2, 3, \ldots\}$

The Geometric Distribution

- Which of these one calls "the" geometric distribution is a matter of convention and convenience.
- A solution for one type can quickly be surmised from the other.
- These two different geometric distributions should not be confused with each other.
- Often, the name **shifted geometric distribution** is adopted for the former one (distribution of the number X);
- However, to avoid ambiguity, it is good practice to indicate which is intended, by mentioning what type is used explicitly.

The Geometric Distribution

- The following geometric distribution equation is used for modeling the number of trials until the first success.
- each with success probability p in each of the independent trials
- Its the probability that the $k-$th trial is the first success, after $k-1$ failures in the previous trials,

The Geometric Distribution

- If the probability of success on each trial is p, then the probability that the $k-$th trial (out of k trials) is the first success is

$$P(X = k) = (1 - p)^{k-1} p \qquad \text{for k} = 1, 2, 3, \ldots$$

The Geometric Distribution

- By contrast, the following form of geometric distribution is used for modeling number of failures until the first success:

$$P(X = k) = (1 - p)^k \, p \qquad \text{for k} = 1, 2, 3, \ldots$$

The Geometric Distribution

- In either case, the sequence of probabilities is a geometric sequence.
- For example, suppose an ordinary die is thrown repeatedly until the first time a "1" appears.
- The probability distribution of the number of times it is thrown is supported on the infinite set $1, 2, 3, \ldots$ and is a geometric distribution with p $= 1/6$.

The Geometric Distribution

- The expected value of a geometrically distributed random variable X is 1/p and the variance is $(1-p)/p^2$:

$$\mathrm{E}(X) = \frac{1}{p}, \qquad \mathrm{var}(X) = \frac{1-p}{p^2}.$$

The Geometric Distribution

- Similarly, the expected value of the geometrically distributed random variable Y (where Y corresponds to the pmf listed in the right column) is (1 - p)/p, and its variance is (1 - p)/p2:

$$\mathrm{E}(Y) = \frac{1-p}{p}, \qquad \mathrm{var}(Y) = \frac{1-p}{p^2}.$$