Chapter 1

6. Discrete Probability Distributions

- Now consider an experiment with only two outcomes. Independent repeated trials of such an experiment are called Bernoulli trials, named after the Swiss mathematician Jacob Bernoulli (16541705).
- The term ***independent trials*** means that the outcome of any trial does not depend on the previous outcomes (such as tossing a coin).
- We will call one of the outcomes the "success" and the other outcome the "failure".
- Let p denote the probability of success in a Bernoulli trial, and so $q = 1 - p$ is the probability of failure. A binomial experiment consists of a fixed number of Bernoulli trials.
- A binomial experiment with n trials and probability p of success will be denoted by

$$B(n, p)$$

1.1 Discrete Probability Distributions

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- The first is the ***binomial*** probability distribution.
- The second is the Poisson probability distribution.
- In `R`, calculations are performed using the `binom` family of functions and `pois` family of functions respectively.

1.2 Discrete Probability Distributions

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  - Poisson
  - Binomial
  - Geometric
  - Hypergeometric

1.2.1 Discrete Random Variable

- A discrete random variable is one which may take on only a countable number of distinct values such as $\{0, 1, 2, 3, 4, ...\}$.
- Discrete random variables are usually (but not necessarily) counts.
- If a random variable can take only a finite number of distinct values, then it must be discrete.
- Examples of discrete random variables include the number of children in a family, the Friday night attendance at a cinema, the number of patients in a doctor's surgery, the number of defective light bulbs in a box of ten.

Discrete Random Variables

- For a discrete random variable observed values can occur only at isolated points along a scale of values. In other words, observed values must be integers.
- Consider a six sided die: the only possible observed values are 1, 2, 3, 4, 5 and 6.
- It is not possible to observe values that are real numbers, such as 2.091.
- *(Remark: it is possible for the average of a discrete random variable to be a real number.)*
- Therefore, it is possible that all numerical values for the variable can be listed in a table with accompanying probabilities.
- There are several standard probability distributions that can serve as models for a wide variety of discrete random variables involved in business applications.

1.2.2 Random Variables

There are two types of random variable - discrete and continuous. The distinction between both types will be important later on in the course.

Examples

- A coin is tossed ten times. The random variable X is the number of tails that are noted. X can only take the values $\{0, 1, ..., 10\}$, so X is a discrete random variable.
- A light bulb is burned until it burns out. The random variable Y is its lifetime in hours. Y can take any positive real value, so Y is a continuous random variable.

1.2.3 Random Variables

- The outcome of an experiment need not be a number, for example, the outcome when a coin is tossed can be 'heads' or 'tails'.
- However, we often want to represent outcomes as numbers.
- A ***random variable*** is a function that associates a unique numerical value with every outcome of an experiment.
- The value of the random variable will vary from trial to trial as the experiment is repeated.
- Numeric values can be assigned to outcomes that are not usually considered numeric.
- For example, we could assign a 'head' a value of 0, and a 'tail' a value of 1, or vice versa.

1.2.4 Random Variables

1. Random Variables
2. Expected Values of RVs

Hence, the random variable X should be thought of as the unknown numerical result of an experiment to be carried out (X is described by a distribution).

A realisation, denoted by a small letter, is a known result of an experiment already carried out.

X : random variable name (e.g Height)

x : realisation (e.g. 1.82 metres)

1.2.5 Random Experiments

- Typical examples of a random experiment are
  - a role of a die,
  - a toss of a coin,
  - drawing a card from a deck.

  If the experiment is yet to be performed we refer to possible outcomes or possibilities for short.

- If the experiment has been performed, we refer to realized outcomes or **realizations**
- A continuous random variable is one which takes an infinite number of possible values.
- Continuous random variables are usually measurements.
- Examples include height, weight, the amount of sugar in an orange, the time required to run a computer simulation.
- Consider an experiment in which each student in a class of 60 rolls a die 100 times.
- Each score is recorded, and a total score is calculated.
- As the expected value of rolled die is 3.5, the expected total is 350 for each student.
- At the end of the experiment the students reported their totals.
- The totals were put into ascending order, and tabulated as follows (next slide).

307	321	324	328	329	330	334	335	336	337
337	337	338	339	339	342	343	343	344	344
346	346	347	348	348	348	350	351	352	352
353	353	353	354	354	356	356	357	357	358
358	360	360	361	362	363	365	365	369	369
370	370	374	378	381	384	385	386	392	398

- What proportion of outcomes are less than or equal to 330?
  (Answer: 10%)
- What proportion of outcomes are greater than or equal to 370?
  (Answer: 16.66%)

For the die-throw experiment;

Constructing Histograms

- Compute an appropriate number of class intervals.
- As a rule of thumb, the number of class intervals is usually approximately the square root of the number of observations.
- As there are 60 observations, we would normally use 7 or 8 class intervals.
- To save time, we will just use 5 class intervals.
- Suppose that the experiment of throwing a die 100 times and recording the total was repeated 100,000 times.
- (If implemented on a computer, we would call this a simulation study)
- The histogram of data (with a class interval width of 2) is shown on the next slide.
- How should the shape of the histogram be described?
- "Bell-shaped" would be a suitable description.

A couple of remarks about the simulation study, some of which will be relevant later on.

- Approximately 68.7% of the values in the simulation study are between 332 and 367.
- Approximately 95% of the values are between 316 and 383.
- 2.5% of the values output are less than 316.
- 2.5% of the values study output are greater than 383.
- 175 values are greater than or equal to 400, whereas 198 values are less than or equal to 300.
- Results such as these are unusual, but they are not impossible.

Consider the following statistical experiment. You flip a coin 2 times and count the number of times the coin lands on heads. This is a binomial experiment because:

- The experiment consists of repeated trials. We flip a coin 2 times.
- Each trial can result in just two possible outcomes - heads or tails.
- The probability of success is constant - 0.5 on every trial.
- The trials are independent; that is, getting heads on one trial does not affect whether we get heads on other trials.