Technology Mathematics 4 (Statistics)

MA4704 Lecture 3C

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Spring Semester 2013

Characteristics of a Poisson Experiment

A Poisson experiment is a statistical experiment that has the following properties:

- The experiment results in outcomes that can be classified as successes or failures.
- The average number of successes (m) that occurs in a specified region is known.
- The probability that a success will occur is proportional to the size of the ***region***.
- The probability that a success will occur in an extremely small region is virtually zero.
- The `pois` family of functions are used to compute probabilities and quantiles.

Note that the specified region could take many forms. For instance, it could be a length, an area, a volume, a period of time, etc.

The Poisson Probability Distribution

- A Poisson random variable is the number of successes that result from a Poisson experiment.
- The probability distribution of a Poisson random variable is called a Poisson distribution.
- This distribution describes the number of occurrences in a unit period (or space)
- The expected number of occurrences is m.
- R refers to the mean number of occurrences as `lambda` rather than `m`.

Poisson Formulae

The probability that there will be k occurrences in a unit time period is denoted $P(X = k)$, and is computed as below.

Remark: This is known as the probability density function. The corresponding `R` command is `dpois()`.

$$P(X = k) = \frac{m^k e^{-m}}{k!}$$

The Poisson Probability Distribution

- The number of occurrences in a unit period (or space)
- The expected number of occurrences is m
- Given the mean number of successes (m) that occur in a specified region, we can compute the Poisson probability based on the following formula.

Poisson Formulae

Given that there is on average 2 occurrences per hour, what is the probability of no occurrences in the next hour?

i.e. Compute $P(X = 0)$ given that $m = 2$

$$P(X = 0) = \frac{2^0 e^{-2}}{0!}$$

- $2^0 = 1$
- $0! = 1$

The equation reduces to

$$P(X = 0) = e^{-2} = 0.1353$$

Poisson Formulae

What is the probability of one occurrences in the next hour?

i.e. Compute $P(X = 1)$ given that $m = 2$

$$P(X = 1) = \frac{2^1 e^{-2}}{1!}$$

- $2^1 = 2$
- $1! = 1$

The equation reduces to

$$P(X = 1) = 2 \times e^{-2} = 0.2706$$

Poisson Distribution (Example)

- Suppose that electricity power failures occur according to a Poisson distribution with an average of 2 outages every twenty weeks.
- Calculate the probability that there will not be more than one power outage during a particular week.

Solution:

- The average number of failures per week is:
  $m = 2/20 = 0.10$
- "Not more than one power outage" means we need to compute and add the probabilities for "0 outages" plus "1 outage".

Poisson Distribution (Example)

Recall:

$$P(X=k)=e^{-m}\frac{m^k}{k!}$$

- $P(X=0)$

$$P(X=0)=e^{-0.10}\frac{0.10^0}{0!}=e^{-0.10}=0.9048$$

- $P(X=1)$

$$P(X=1)=e^{-0.10}\frac{0.10^1}{1!}=e^{-0.10}\times 0.1=0.0905$$

- $P(X\leq 1)$

$$P(X\leq 1)=P(X=0)+P(X=1)=0.9048+0.0905=0.995$$

Implementation using R

- Probability Density Function $P(X = k)$
  - For a given poisson mean m, which in R is specified as `lambda`
  - `dpois(k,lambda = ...)`
- Cumulative Density Function $P(X \leq k)$
  - `ppois(k,lambda = ...)`

Implementation using `R`

From before: $P(X = 0)$ given than the mean number of occurrences is 2.

```
> dpois(0,lambda=2)
[1] 0.1353353
> dpois(1,lambda=2)
[1] 0.2706706
> dpois(2,lambda=2)
[1] 0.2706706
```

Implementation using R

Compute the cumulative distribution functions for the values $k = \{0, 1, 2\}$, given that the mean number of occurrences is 2

```
> ppois(0,lambda=2)
[1] 0.1353353
> ppois(1,lambda=2)
[1] 0.4060058
> ppois(2,lambda=2)
[1] 0.6766764
```