The Poisson Distribution: Introduction

- The Poisson Distribution is a statistical distribution showing the frequency probability of specific events when the average probability of a single occurrence is known.
- The Poisson distribution is a discrete probability distribution.
- One of the most famous historical practical uses of the Poisson distribution was estimating the annual number of Prussian cavalry soldiers killed due to horse-kicks.
- Other modern examples include estimating the number of car crashes in a city of a given size; in physiology, this distribution is often used to calculate the probabilistic frequencies of different types of neurotransmitter secretions.

Poisson Random Variables

- A Poisson random variable is the number of successes that result from a Poisson experiment.
- The probability distribution of a Poisson random variable is called a Poisson distribution.
- **Very Important:** This distribution describes the number of occurrences in a ***unit period (or space)***
- **Very Important:** The expected number of occurrences is m per unit period (or unit space).

Characteristics of a Poisson Experiment

A Poisson experiment is a statistical experiment that has the following properties:

- The experiment results in outcomes that can be classified as successes or failures.
- The average number of successes (m) that occurs in a specified unit space is known.
- Note that the specified unit space could take many forms. For instance, it could be a length, an area, a volume, a period of time, etc.
- The probability that a success will occur is proportional to the size of the ***unit space***.
- The probability that a success will occur in an extremely small unit space is virtually zero.
- The **Poisson mean** λ (pronounced "m" or "lambda") is the expected number of occurrences per unit space / unit period.
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    - (Remark: Some texts will use the notation m rather than λ).

Important Assumptions

If the following two properties are satisfied, the number of occurrences is a random variable described by the Poisson probability distribution

(1) The probability of an occurrence is the same for any two intervals of equal length.

(2) The occurrence or non-occurrence in any interval is independent of the occurrence or non-occurrence in any other interval.

When To Use The Poisson Probability Distribution

Consider cars passing a point on a rarely used country road. Is this a Poisson Random Variable? Suppose that

1. Arrivals occur at an average rate of m cars per unit time.
2. The probability of an arrival in an interval of length k is constant.
3. The number of arrivals in two non-overlapping intervals of time are independent.

This would be an appropriate use of the Poisson Distribution.

Knowing which distribution to use

- For the end of semester examination, you will be required to know when it is appropriate to use the Poisson distribution, and when to use the binomial distribution.
- Recall the key parameters of each distribution.
- Binomial : number of ***successes*** in n ***independent trials***.
- Poisson : number of ***occurrences*** in a ***unit space***.