The Continuous Uniform Distributions

IMAGE : 5A uniform

Parameters

The continuous uniform distribution is characterised by the following parameters

- The lower limit a
- The upper limit b

It is not possible to have an outcome that is lower than a or larger than b.

$$P(X < a) = P(X > b) = 0$$

- The only possible outcomes are between a and b. Suppose $a = 3$ and $b = 6$.
- The following values are possible outcomes: 3.14, 3.78, 4.66, 5.8, 5.9999.
- The probability of being exactly equal to 3 or 6 can be assumed to be zero.
- The following outcomes are not possible, either because they are too high or too low. 1.67, 2, 67, 7.14, 8.78.

Continuous Random Variables

- Probability Density Function
- Cumulative Density Function

If X is a continuous random variable then we can say that the probability of obtaining a **precise** value x is infinitely small, i.e. close to zero.

$$P(X = x) \approx 0$$

Consequently, for continuous random variables (only), $P(X \leq x)$ and $P(X < x)$ can be used interchangeably.

$$P(X \leq x) \approx P(X < x)$$

- L :lower bound of an interval
- U: upper bound of an interval

Probability of an outcome being between lower bound L and upper bound U

$$P(L \leq X \leq U) = \frac{U - L}{b - a}$$

Reminder "$\leq$" is less than or equal to.
"$\geq$" is greater than or equal to.
$L \leq X \leq U$ xan be verbalized as X between L and U. simply states that X is between L and U inclusively. ("inclusively" mean that X could be exactly L or U also, although the probability of this is extremely low)

Continuous Uniform Distribution

- The Uniform distributions model (some) continuous random variables and (some) discrete random variables.
- The values of a uniform random variable are uniformly distributed over an interval.
- For example, if buses arrive at a given bus stop every 15 minutes, and you arrive at the bus stop at a random time, the time you wait for the next bus to arrive could be described by a uniform distribution over the interval from 0 to 15.

Continuous Uniform Distribution

A random variable X is called a continuous uniform random variable over the interval (a,b) if it's probability density function is given by

$$f_X(x) = \frac{1}{b-a} \qquad \text{when } a \leq x \leq b$$

The corresponding cumulative density function is

$$F_x(x) = \frac{x-a}{b-a} \qquad \text{when } a \leq x \leq b$$

Continuous Uniform Distribution

The mean of the continuous uniform distribution is

$$E(X) = \frac{a+b}{2}$$

$$V(X) = \frac{(b-a)^2}{12}$$

Uniform Distribution: Variance

The variance of the continuous uniform distribution, denoted $\mathrm{Var}[X]$, is computed using the following formula

$$\mathrm{Var}[X] = \frac{(b-a)^2}{12}$$

For our previous example this is

$$\mathrm{Var}[X] = \frac{(3-0)^2}{12} = \frac{3^2}{12} = \frac{9}{12} = 0.75$$

The Exponential Distribution

A continuous random variable having p.d.f. f(x), where: $f(x) = \lambda x e^{-\lambda x}$ is said to have an exponential distribution, with parameter λ. The cumulative distribution is given by: $F(x) = 1e^{\lambda x}$
Expectation and Variance $E(X) = 1/\lambda \; V(X) = 1/\lambda^2$

Example

Suppose that the service time for a customer at a fast-food outlet has an exponential distribution with mean 3 minutes. What is the probability that a customer waits more than 4 minutes?

$$P(X \leq 4) = 1 - e^{-4/3}$$

$$P(X \leq 4) = e^{-4/3} = 0.2636$$

Exponential Distribution Lifetimes

The average lifespan of a laptop is 5 years. You may assume that the lifespan of computers follows an exponential probability distribution.

- (3 marks) What is the probability that the lifespan of the laptop will be at least 6 years?
- (3 marks) What is the probability that the lifespan of the laptop will not exceed 4 years?
- (3 marks) What is the probability of the lifespan being between 5 years and 6 years?

Suppose the lifetime of a PC is exponentially distributed with mean $\mu = 5$ We should be told the average lifetime μ.

$$P(X \geq x_o) = e^{\frac{-x_o}{\mu}}$$

The Memoryless property

The most interesting property of the exponential distribution is the ***memoryless*** property. By this , we mean that if the lifetime of a component is exponentially distributed, then an item which has been in use for some time is a good as a brand new item with regards to the likelihood of failure. The exponential distribution is the only distribution that has this property.