MathsCast Presentations

MA4413 Lecture 5B

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Normal Distribution : Solving problems

In today's class, we will continue with the Normal Distribution.

- We must know the normal mean μ and the normal standard deviation σ.
- The normal random variable is $X \sim \mathrm{N}(\mu, \sigma^2)$.
- (If we don't, we usually have to determine them, given the information in the question.)
- The standard normal random variable is $Z \sim \mathrm{N}(0, 1^2)$.
- The standard normal distribution is well described in Murdoch Barnes Table 3, which tabulates $P(Z \geq z_o)$ for a range of Z values.

Normal Distribution : Solving problems

- For the given value x_o from the variable X, we compute the corresponding z-score z_o.

$$z_o = \frac{x_o - \mu}{\sigma}$$

- When z_o corresponds to x_o, the following identity applies:

$$P(X \geq x_o) = P(Z \geq z_o)$$

- Alternatively $P(X \leq x_o) = P(Z \leq z_o)$

Normal Distribution : Solving problems

- **Complement Rule**:

$$P(Z \leq k) = 1 - P(Z \geq k)$$

  for some value k

- Alternatively $P(Z \geq k) = 1 - P(Z \leq k)$
- **Symmetry Rule**:

$$P(Z \leq -k) = P(Z \geq k)$$

  for some value k

- Alternatively $P(Z \geq -k) = P(Z \leq k)$

Normal Distribution : Solving problems

- **Intervals**:

$$P(L \leq Z \leq U) = 1 - [P(Z \leq L) + P(Z \geq U)]$$

  where L and U are the lower and upper bounds of an interval.
- Probability of having a value too low for the interval : $P(Z \leq L)$
- Probability of having a value too high for the interval : $P(Z \geq U)$

Working Backwards

- Suppose we wish to find a value (lets call it A) from the normal distribution, such that a certain proportion of values is greater than A (e.g. 10%)
- Find A such that $P(X \geq A) = 0.10$. (with $\mu = 350$ and $\sigma = 17$)
- In general, our first step is to use the standardization equation to find the corresponding Z-score z_A.
- Because we don't know what value A has, we can't use this approach.
- However, we can say the following

$$P(X \geq A) = P(Z \geq z_A) = 0.10$$

- From the tables, we can approximate a value for z_A, by finding the closest probability value, and determining the corresponding Z-score.

Find z_A such that $P(Z \geq z_a) = 0.10$

- The closest probability value in the tables is 0.1003.
- The Z-score that corresponds to 0.1003 is 1.28.
- (Row : 1.2 , Column : 0.08)
- Therefore $z_A \approx 1.28$

	...	...	0.006	0.07	0.08	0.09
...	...	...	...	...	...	...
1.0	...	...	0.1446	0.1423	0.1401	0.1379
1.1	...	...	0.1230	0.1210	0.1190	0.1170
1.2	...	...	0.1038	0.1020	0.1003	0.0985
1.3	...	...	0.0869	0.0853	0.0838	0.0823
...	...	...	...	...	...	...

Working Backwards

- We can now use the standardization formula.
- We have only one unknown in the formula: A.

$$1.28 = \frac{A - 350}{17}$$

- Re-arranging (multiply both sides by 17):
  $21.76 = A - 350$
- Re-arranging (add 350 to both sides):
  $A = 371.76$
- $P(X \geq 371.76) \approx 0.10$
- (Remark: for sums of die-throws, round it to nearest value)

Working Backwards: Another Example

- Find B such that $P(X \geq B) = 0.90$. (with $\mu = 350$ and $\sigma = 17$)
- Necessarily $P(X \leq B) = 0.10$
- Find some value Z_B such that $P(Z \leq z_B) = 0.10$
- z_B could be negative.
- Use the symmetry rule $P(Z \leq z_B) = P(Z \geq -z_B)$
- $-z_B$ could be positive.
- Based on last example $-z_B = 1.28$. Therefore $z_B = -1.28$

Working Backwards

- Again ,we can now use the standardization formula
- We have only one unknown in the formula: B.

$$-1.28 = \frac{B-350}{17}$$

- Re-arranging (multiply both sides by 17):
  $-21.76 = B - 350$
- Re-arranging (add 350 to both sides):
  $x_o = 350 - 21.76 = 328.24$
- $P(X \leq 328.24) \approx 0.10$