- Any value X from a normally distributed population can be converted into the equivalent standard normal value Z (i.e. a 'Z value') by the formula

$$Z = \frac{X - \mu}{\sigma}$$

- The standard normal distribution has been tabulated (usually in the form of value of the cumulative distribution function F), and the other normal distributions are the simple transformations, as described above, of the standard one.
- Therefore, one can use tabulated values of the cdf of the standard normal distribution to find values of the cdf of a general normal distribution.
- For some particular value x_o of the normal distribution X, there is a corresponding ***z-score*** z_o.
- The z-score is the distance, in terms of standard deviations, that x_o is from the mean μ.

0.0.1 Exact Probability

Remarks: This is for continuous distributions only.

- The probability that a continuous random variable will take an exact value is infinitely small. We will usually treat it as if it was zero.
- When we write probabilities for continuous random variables in mathematical notation, we often retain the equality component (i.e. the "...or equal to.."). For example, we would write expressions $P(X \leq 2)$ or $P(X \geq 5)$.
- Because the probability of an exact value is almost zero, these two expression are equivalent to $P(X < 2)$ or $P(X > 5)$.
- The complement of $P(X \geq k)$ can be written as $P(X \leq k)$.

Normal Distribution The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1. Normal distributions can be transformed to standard normal distributions by the formula:

$$Z = \frac{X - \mu}{\sigma}$$

where X is a score from the original normal distribution, μ is the mean of the original normal distribution, and σ is the standard deviation of original normal distribution. The standard normal distribution is sometimes called the Z distribution.

The Standardized Value

- Suppose that mean $\mu = 80$ and that standard deviation $\sigma = 8$.
- What is the Z value for $X = 100$?

$$Z_{100} = \frac{X_0 - \mu}{\sigma} = \frac{100 - 80}{8} = \frac{20}{8} = 2.5$$

- Therefore $Z_{100} = 2.5$

- We can find a probability associated with a value, that is from a normally distribution, by computing the Z value.

$$z_0 = \frac{x_0 - \mu}{\sigma}$$

  - X_o - Some random value from the population of X values.
  - μ - The mean of the population of X values.
  - σ - The variance of the population of X values.
  - Z_o - The Z value that corresponds to X_o

All normally distributed random variables have corresponding Z values, called Z-scores.
For normally distributed random variables, the z-score can be found using the ***standardization formula***;

$$z_o = \frac{x_o - \mu}{\sigma}$$

where x_o is a score from the original normal ("X") distribution, μ is the mean of the original normal distribution, and σ is the standard deviation of original normal distribution.
Therefore z_o is the z-score that corresponds to x_o.

- Terms with subscripts mean particular values, and are not variable names.
- The z distribution will only be a normal distribution if the original distribution (X) is normal.

The Standardized Value

- The first step in solving the problem is to compute the standardized value, also known as the 'Z' value.
- We must know the value of the mean μ and the standard deviation σ.
- To find the 'Z' value Z_0 for a particular quantity X_0.

$$Z_0 = \frac{X_0 - \mu}{\sigma}$$

0.0.2 Z-scores

- A Z-score always reflects the number of standard deviations above or below the mean a particular score is.
- Suppose the scores of a test are normally distributed with a mean of 50 and a standard deviation of 9
- For instance, if a person scored a 68 on a test, then they scored 2 standard deviations above the mean.
- Converting the test scores to z scores, an X value of 68 would yield:
$$Z = \frac{68 - 50}{9} = 2$$
- So, a Z score of 2 means the original score was 2 standard deviations above the mean.
- Note that the z distribution will only be a normal distribution if the original distribution (X) is normal.

0.0.3 Solving using the Z distribution

When we have a normal distribution with any mean μ and any standard deviation σ , we answer probability questions about the distribution by first converting all values to corresponding values of the standard normal ("z") distribution.

Z is the standard normal random variable with a mean of zero and a standard deviation of 1.

It can be thought of as a measure of how many standard deviations that a value "x" is from mean μ .

0.0.4 Computing the Z-score

The normal distribution has the following paramters

- μ the mean of the normal distribution
- σ the standard deviation of the distribution

$$z = \frac{x - \mu}{\sigma}$$

Suppose μ =1000 σ = 400

$$X \sim N(1000, 400)$$

0.0.5 The Standard Normal Distribution

- The standard normal distribution (commonly called the Z distribution) is a special case of the ***normal distribution***.
- It is characterized by the following
  - The mean μ is always equal to 0.
  - The standard deviation σ is always equal to 1.
  - The variance σ^2 is therefore equal to 1 also .