Statistics for Computing

MA4413 Lecture 5A

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Exponential Distribution

- The Exponential Distribution
- The Normal Distribution
- Applied Normal Distribution

Exponential Distribution

The Exponential Distribution may be used to answer the following questions:

- How much time will elapse before an earthquake occurs in a given region?
- How long do we need to wait before a customer enters our shop?
- How long will it take before a call center receives the next phone call?
- How long will a piece of machinery work without breaking down?

Exponential Distribution

- All these questions concern the time we need to wait before a given event occurs. If this waiting time is unknown, it is often appropriate to think of it as a random variable having an exponential distribution.
- Roughly speaking, the time X we need to wait before an event occurs has an exponential distribution if the probability that the event occurs during a certain time interval is proportional to the length of that time interval.

Probability density function

The probability density function (PDF) of an exponential distribution is

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

The parameter λ is called ***rate*** parameter.

Cumulative density function

The cumulative distribution function (CDF) of an exponential distribution is

$$F(x;\lambda) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Expected Value and Variance

The expected value of an exponential random variable X is:

$$E[X] = \frac{1}{\lambda}$$

The variance of an exponential random variable X is:

$$V[X] = \frac{1}{\lambda^2}$$

Exponential Distribution: Example

Assume that the length of a phone call in minutes is an exponential random variable X with parameter $\lambda = 1/10$. If someone arrives at a phone booth just before you arrive, find the probability that you will have to wait

(a) less than 5 minutes,

(b) between 5 and 10 minutes.

Use the `R` code on the following slide to help answer these questions.

Exponential Distribution: Example

```
> dexp(0:10,rate=0.10)
 [1] 0.10000000 0.09048374 0.08187308 0.07408182 0.06703200 0.
 [7] 0.05488116 0.04965853 0.04493290 0.04065697 0.03678794
>
> pexp(0:10,rate=0.10)
 [1] 0.00000000 0.09516258 0.18126925 0.25918178 0.32967995 0.
 [7] 0.45118836 0.50341470 0.55067104 0.59343034 0.63212056
```

Exponential Distribution: Example

As it is CDF values that we are interested in, we use the output from the `pexp()` commands.

(a) $P(X \leq 5) = 0.39346934$

(b) $P(5 \leq X \leq 10)$
$= P(X \leq 10) - P(X \leq 5)$
$= 0.63212056 - 0.39346934$
$= 0.2386512$
$= 23.84\ \%$

Exponential Distribution

- The Exponential Rate
- Related to the Poisson mean (m)
- If we expect 12 occurrences per hour - what is the rate?
- We would expected to wait 5 minutes between occurrences.
-

Exponential Distribution

```
>
> pexp(0:9, rate = 0.25)
 [1] 0.0000000 0.2211992 0.3934693 0.5276334 0.6321206
 [6] 0.7134952 0.7768698 0.8262261 0.8646647 0.8946008
>
> pexp(0:9, rate = 0.20)
 [1] 0.0000000 0.1812692 0.3296800 0.4511884 0.5506710
 [6] 0.6321206 0.6988058 0.7534030 0.7981035 0.8347011
>
> pexp(0:9, rate = 0.50)
 [1] 0.0000000 0.3934693 0.6321206 0.7768698 0.8646647
 [6] 0.9179150 0.9502129 0.9698026 0.9816844 0.9888910
>
```

Today's Class

- Continuous Random Variables
- The Normal Distribution
- Characteristics of the Normal Distribution
- The Standard Normal (Z) Distribution
- Using Murdoch Barnes Table 3
- Standardization Formula
- Important Formulae

Continuous Random variables

- Previously we have been studying discrete random variables, such as the Binomial and the Poisson random variables.
- Now we turn our attention to continuous random variables.
- Recall that a continuous random variable is one which takes an infinite number of possible values, rather than just a countable number of distinct values.
- Continuous random variables are usually measurements.
- Examples include height, weight, the amount of sugar in an orange, the time required to run a mile.