Technology Mathematics 4 (Statistics)

MA4704 Lecture 3C

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Today's Class

- Definition of Cumulative Distribution Function.
- Binomial Example
- Using cumulative tables.
- Poisson distribution - example

The Cumulative Distribution Function

- The Cumulative Distribution Function, denoted $F(x)$, is a common way that the probabilities of a random variable (both discrete and continuous) can be summarized.
- The Cumulative Distribution Function, which also can be described by a formula or summarized in a table, is defined as:

$$F(x) = P(X \leq x)$$

- The notation for a cumulative distribution function, F(x), entails using a capital "F". (The notation for a probability mass or density function, f(x), i.e. using a lowercase "f". The notation is not interchangeable.

Useful Results

(Demonstration on the blackboard re: partitioning of the sample space, using examples on next slide)

- $P(X \leq 1) = P(X = 0) + P(X = 1)$
- $P(X \leq r) = P(X = 0) + P(X = 1) + \ldots P(X = r)$
- $P(X \leq 0) = P(X = 0)$
- $P(X = r) = P(X \geq r) - P(X \geq r + 1)$
- **Complement Rule**:
  $P(X \leq r - 1) = P(X < r) = 1 - P(X \geq r)$
- **Interval Rule**:$P(a \leq X \leq b) = P(X \geq a) - P(X \geq b + 1)$.

For the binomial distribution, if the probability of success is greater than 0.5, instead of considering the number of successes, to use the table we consider the number of failures.

Binomial Example 1

Suppose a signal of 100 bits is transmitted and the probability of sending a bit correctly is 0.9. What is the probability of

1. at least 10 errors
2. exactly 7 errors
3. Between 5 and 15 errors (inclusively).

Binomial Example 1

- Since the probability of success is 0.9. We consider the distribution of the number of failures (errors).
- We reverse the definition of 'success' and 'failure'. Success is now defined as an error.
- The probability that a bit is sent incorrectly is 0.1.
- Let X be the total number of errors. $X \sim B(100, 0.1)$.
- Answer : $P(X \geq 10) = 0.5487$.
- $P(X = 7) = P(X \geq 7) - P(X \geq 8) = 0.8828 - 0.7939 = 0.0889$.
- $P(5 \leq X \leq 15) = P(X \geq 5) - P(X \geq 16) = 0.9763 - 0.0399 = 0.9364$

The Poisson Probability Distribution

- A Poisson random variable is the number of successes that result from a Poisson experiment.
- The probability distribution of a Poisson random variable is called a Poisson distribution.
- Very Important: This distribution describes the number of occurrences in a ***unit period (or space)***
- Very Important: The expected number of occurrences is m

The Poisson Probability Distribution

We use the following notation.

$$X \sim Poisson(m)$$

Note the expected number of occurrences per unit time is conventionally denoted λ rather than m. As the Murdoch

Barnes cumulative Poisson Tables (Table 2) use m, so shall we. Recall that Tables 2 gives values of the probability $P(X \geq r)$, when X has a Poisson distribution with parameter m.

The Poisson Probability Distribution

Consider cars passing a point on a rarely used country road. Is this a Poisson Random Variable? Suppose

1. Arrivals occur at an average rate of m cars per unit time.
2. The probability of an arrival in an interval of length k is constant.
3. The number of arrivals in two non-overlapping intervals of time are independent.

This would be an appropriate use of the Poisson Distribution.

Changing the unit time.

- The number of arrivals, X, in an interval of length t has a Poisson distribution with parameter $\mu = mt$.
- m is the expected number of arrivals in a unit time period.
- μ is the expected number of arrivals in a time period t, that is different from the unit time period.
- Put simply : if we change the time period in question, we adjust the Poisson mean accordingly.
- If 10 occurrences are expected in 1 hour, then 5 are expected in 30 minutes. Likewise, 20 occurrences are expected in 2 hours, and so on.
- (Remark : we will not use μ in this context anymore).

Poisson Example

A motor dealership which specializes in agricultural machinery sells one vehicle every 2 days, on average. Answer the following questions.

1. What is the probability that the dealership sells at least one vehicle in one particular day?
2. What is the probability that the dealership will sell exactly one vehicle in one particular day?
3. What is the probability that the dealership will sell 4 vehicles or more in a six day working week?

Poisson Example

1. Expected Occurrences per Day: m = 0.5
2. Probability that the dealership sells at least one vehicle in one particular day?

$$P(X \geq 1) = 0.3935$$

3. Probability that the dealership will sell exactly one vehicle in one particular day?

$$P(X = 1) = P(X \geq 1) - P(X \geq 2) = 0.3935 - 0.0902 = 0.3031$$

4. Probability that the dealership will sell 4 vehicles or more in a six day working week?
   - For a 6 day week, m=3
   - $P(X \geq 4) = 0.3528$

Knowing which distribution to use

- For the end of semester examination, you will be required to know when it is appropriate to use the Poisson distribution, and when to use the binomial distribution.
- Recall the key parameters of each distribution.
- Binomial : number of ***successes*** in n ***independent trials***.
- Poisson : number of ***occurrences*** in a ***unit space***.