Technology Mathematics 4 (Statistics)

MA4704 Lecture 4A

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Binomial Expected Value and Variance

- Lecture is called off for Thursday of Week 4.
- The first midterm is to take place Thursday of Week 5.
- The first midterm will cover:
  - Basic Probability
  - Descriptive statistics (mean, median variance etc)
  - Discrete probability distributions (binomial and Poisson)
  - The exponential distribution
  - (The normal distribution will be not be included).

Binomial Expected Value and Variance

If the random variable X has a binomial distribution with parameters n and p, we write

$$X \sim B(n, p)$$

Expectation and Variance If $X \sim B(n, p)$, then:

- Expected Value of X : E(X) = np
- Variance of X : Var(X) = np(1-p)

Binomial Distribution: Example 1

- Diagrams of the probability mass functions of the two binomial distributions $B(10, 0.5)$ and $B(10, 0.25)$ are shown in the bar-plots (next slide).
- Which is which? Give a reason for your answer.

Binomial Distribution

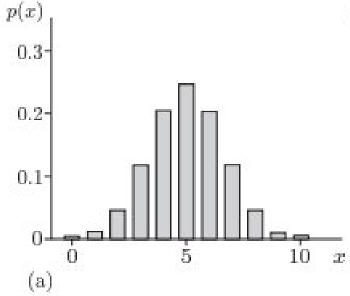

Figure: Bar Charts

Binomial Distribution: Example 1

- Figure A is $B(10, 0.5)$ and Figure B is $B(10, 0.25)$.
- The mean of B(10, 0.5) is 5, and the mean of B(10, 0.25) is 2.5.
- Also the variance of a binomial distribution corresponding to $B(10, 0.25)$ is 1.875 ,while for $B(10, 0.25)$ it is 2.500.
- A visual inspection of the two bar-charts indicates that Figure A has the higher variance.

Binomial Distribution: Example 2

- Components are placed into containers containing 100 items.
- After an inspection of a large number of containers the average number of defective items was found to be 10 with a standard deviation of three.
- Is the binomial distribution a good useful distribution, given the observed data?

Binomial Distribution: Example 2

- Let the number of containers be the number of independent trials is $n = 100$.
- A success may be defined as a defective component.
- The probability of a success is approximate $p = 0.10$. (The probability of "failure" is $1 - p = 0.9$).
- The expected number of defective components is $np = 10$, which concurs with our observed data.
- The variance is computed as

$$np(1 - p) = 100 \times 0.1 \times 0.9 = 9$$

- The observed standard deviation is 3 units, i.e. a variance of 9 square units.
- Yes the binomial distribution is useful in this case.

Poisson Expected Value and Variance

If the random variable X has a Poisson distribution with parameter m, we write

$$X \sim Poisson(m)$$

- Expected Value of X : E(X) = m
- Variance of X : $Var(X) = m$
- Standard Deviation of X : $SD(X) = \sqrt{m}$

Poisson Distribution : Example

- The number of faults in a fibre optic cable were recorded for each kilometre length of cable.
- The mean number of faults was found to be 4 faults per kilometre.
- The standard deviation of the number of faults was found to be 2 faults per kilometre.
- Is the Poisson Distribution is a useful technique for modelling the number of faults in fibre optic cable?
- (Looking at the last slide, the answer is yes).

Poisson Approximation of the Binomial

- The Poisson distribution can sometimes be used to approximate the binomial distribution
- When the number of observations n is large, and the success probability p is small, the $B(n,p)$ distribution approaches the Poisson distribution with the parameter given by $m = np$.
- This is useful since the computations involved in calculating binomial probabilities are greatly reduced.
- As a rule of thumb, n should be greater than 50 with p very small, such that np should be less than 5.
- If the value of p is very high, the definition of what constitutes a “success” or “failure” can be switched.

Poisson Approximation: Example

- Suppose we sample 1000 items from a production line that is producing, on average, 0.1% defective components.
- Using the binomial distribution, the probability of exactly 3 defective items in our sample is

$$P(X = 3) = {}^{1000}C_3 \times 0.001^3 \times 0.999^{997}$$

Poisson Approximation: Example

Lets compute each of the component terms individually.

- ${}^{1000}C_3$

$$ {}^{1000}C_3 = \frac{1000 \times 999 \times 998}{3 \times 2 \times 1} = 166,167,000 $$

- 0.001^3

$$ 0.001^3 = 0.000000001 $$

- 0.999^{997}

$$ 0.999^{997} = 0.36880 $$

Multiply these three values to compute the binomial probability $P(X = 3) = 0.06128$

Poisson Approximation: Example

- Lets use the Poisson distribution to approximate a solution.
- First check that $n \geq 50$ and $np < 5$ (Yes to both).
- We choose as our parameter value
  $m = np = 1000 \times 0.001 = 1$

$$P(X = 3) = \frac{e^{-1} \times 1^3}{3!} = \frac{e^{-1}}{6} = \frac{0.36787}{6} = 0.06131$$

Compare this answer with the Binomial probability P(X = 3) = 0.06128. Very good approximation, with much less computation effort.