

- Let  $p$  denote the probability of success in a Bernoulli trial, and so  $q = 1 - p$  is the probability of failure. A binomial experiment consists of a fixed number of Bernoulli trials.
- A binomial experiment with  $n$  trials and probability  $p$  of success will be denoted by

$$B(n, p)$$

## 0.1 The Binomial Distribution

Binomial Probability Function

In general, if the random variable  $X$  follows the binomial distribution with parameters  $n$  and  $p \in [0, 1]$ , we write  $X \sim B(n, p)$ . The probability of getting exactly  $k$  successes in  $n$  trials is given by the probability mass function:

$$f(k; n, p) = \Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

for  $k = 0, 1, 2, \dots, n$ , where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

is the binomial coefficient, hence the name of the distribution.

[Remark ; Provided in exam formulae. Please see pg 142]

where

= the probability of successes in trials

= the number of trials

## 0.2 Probability Mass Function

(Formally defining something mentioned previously)

- a probability mass function (pmf) is a **function** that gives the probability that a discrete random variable is exactly equal to some value.

$$P(X = k)$$

- The probability mass function is often the primary means of defining a discrete probability distribution
- It is conventional to present the probability mass function in the form of a table.

# 1 Binomial Distribution

Number of independent trials

A coin is tossed eight times.

the number of trials is therefore 8.

A group of people or a batch of items can also be considered as a series of independent trials.

Probability of a success

A "success" is dependent on how the question is framed, or what is being estimated.

## 1.1 The Binomial Distribution

- The discrete random variable  $X$  is the number of successes in the  $n$  trials.  $X$  is modelled by the binomial distribution  $B(n, p)$ . You can write  $X \sim \text{Bin}(n, p)$ .
- The binomial distribution can be used to determine the probability of obtaining a designated number of successes in a Bernoulli process. Three values are required: the designated number of successes ( $X$ ); the number of trials, or observations ( $n$ ); and the probability of success in each trial ( $p$ ).
- $P(X = k)$  gives the probability of  $k$  successes in  $n$  binomial trials.

## 1.2 Binomial Distribution (1)

- Identify the event that can be considered the 'success'.
- (Remark : The success is usually the less likely of two complementary events.)
- Determine the probability of a success in a single trial  $p$ .
- Determine the number of independent trials  $n$ .