Poisson Distribution A Poisson random variable is the number of successes that result from a Poisson experiment.

The probability distribution of a Poisson random variable is called a Poisson distribution.

Given the mean number of successes (m) that occur in a specified region, we can compute the Poisson probability based on the following formula:

The Poisson Probability Distribution

- The number of occurrences in a unit period (or space)
- The expected number of occurrences is m

Poisson Formulae The probability that there will be k occurrences in a unit time period is denoted $P(X = k)$, and is computed as follows.

$$P(X = k) = \frac{m^k e^{-m}}{k!}$$

Poisson Formulae Given that there is on average 2 occurrences per hour, what is the probability of no occurences in the next hour?

i.e. Compute $P(X = 0)$ given that $m = 2$

$$P(X = 0) = \frac{2^0 e^{-2}}{0!}$$

- $2^0 = 1$
- $0! = 1$

The equation reduces to

$$P(X = 0) = e^{-2} = 0.1353$$

Poisson Formulae What is the probability of one occurrences in the next hour?

i.e. Compute $P(X = 1)$ given that $m = 2$

$$P(X = 1) = \frac{2^1 e^{-2}}{1!}$$

- $2^1 = 2$
- $1! = 1$

The equation reduces to

$$P(X = 1) = 2 \times e^{-2} = 0.2706$$

Probability of events for a Poisson distribution

An event can occur 0, 1, 2, times in an interval. The average number of events in an interval is designated λ (lambda). Lambda is the event rate, also called the rate parameter. The probability of observing k events in an interval is given by the equation

$$P(k \text{ events in interval}) = e^{-\lambda} \frac{\lambda^k}{k!}$$

where

- λ is the average number of events per interval
- e is the number 2.71828... (Euler's number) the base of the natural logarithms
- k takes values 0, 1, 2,
- $k! = k \times (k1) \times (k2) \times \ldots \times 2 \times 1$ is the factorial of k.

This equation is the probability mass function (PMF) for a Poisson distribution.

Poisson Expected Value and Variance If the random variable X has a Poisson distribution with para

Poisson Distribution : ExampleThe number of faults in a fibre optic cable were recorded for each ki

Poisson Approximation of the Binomial

- The Poisson distribution can sometimes be used to approximate the binomial distribution
- When the number of observations n is large, and the success probability p is small, the $B(n, p)$ distribution approaches the Poisson distribution with the parameter given by $m = np$.
- This is useful since the computations involved in calculating binomial probabilities are greatly reduced.
- As a rule of thumb, n should be greater than 50 with p very small, such that np should be less than 5.
- If the value of p is very high, the definition of what constitutes a "success" or "failure" can be switched.

Poisson Approximation: Example

- Suppose we sample 1000 items from a production line that is producing, on average, 0.1% defective components.
- Using the binomial distribution, the probability of exactly 3 defective items in our sample is

$$P(X = 3) =^{1000} C_3 \times 0.001^3 \times 0.999^{997}$$

Poisson Approximation: Example Lets compute each of the component terms individually.

- ${}^{1000}C_3$

$${}^{1000}C_3 = \frac{1000 \times 999 \times 998}{3 \times 2 \times 1} = 166,167,000$$

- 0.001^3

$$0.001^3 = 0.000000001$$

- 0.999^{997}

$$0.999^{997} = 0.36880$$

Multiply these three values to compute the binomial probability $P(X = 3) = 0.06128$

- Lets use the Poisson distribution to approximate a solution.
- First check that $n \geq 50$ and $np < 5$ (Yes to both).
- We choose as our parameter value $m = np = 1000 \times 0.001 = 1$

$$P(X = 3) = \frac{e^{-1} \times 1^3}{3!} = \frac{e^{-1}}{6} = \frac{0.36787}{6} = 0.06131$$

Compare this answer with the Binomial probability P(X = 3) = 0.06128. Very good approximation, with much less computation effort.