Parameters

The continuous uniform distribution is characterised by the following parameters

- The lower limit a
- The upper limit b

It is not possible to have an outcome that is lower than a or larger than b.

$$P(X < a) = P(X > b) = 0$$

- The only possible outcomes are between a and b. Suppose $a = 3$ and $b = 6$.
- The following values are possible outcomes: 3.14, 3.78, 4.66, 5.8, 5.9999.
- The probability of being exactly equal to 3 or 6 can be assumed to be zero.
- The following outcomes are not possible, either because they are too high or too low. 1.67, 2, 67, 7.14, 8.78.

Continuous Uniform Distribution

A random variable X is called a continuous uniform random variable over the interval (a,b) if it's probability density function is given by

$$f_X(x) = \frac{1}{b-a} \qquad \text{when } a \leq x \leq b$$

The corresponding cumulative density function is

$$F_x(x) = \frac{x-a}{b-a} \qquad \text{when } a \leq x \leq b$$

The mean of the continuous uniform distribution is

$$E(X) = \frac{a+b}{2}$$

$$V(X) = \frac{(b-a)^2}{12}$$

Continuous Random Variables

- Probability Density Function
- Cumulative Density Function

If X is a continuous random variable then we can say that the probability of obtaining a **precise** value x is infinitely small, i.e. close to zero.

$$P(X = x) \approx 0$$

Consequently, for continuous random variables (only), $P(X \leq x)$ and $P(X < x)$ can be used interchangeably.

$$P(X \leq x) \approx P(X < x)$$

- L :lower bound of an interval
- U: upper bound of an interval

Probability of an outcome being between lower bound L and upper bound U

$$P(L \leq X \leq U) = \frac{U-L}{b-a}$$

Reminder "$\leq$" is less than or equal to.
"$\geq$" is greater than or equal to.
$L \leq X \leq U$ xan be verbalized as X between L and U. simply states that X is between L and U inclusively. ("inclusively" mean that X could be exactly L or U also, although the probability of this is extremely low)

Continuous Uniform Distribution

- The Uniform distributions model (some) continuous random variables and (some) discrete random variables.
- The values of a uniform random variable are uniformly distributed over an interval.
- For example, if buses arrive at a given bus stop every 15 minutes, and you arrive at the bus stop at a random time, the time you wait for the next bus to arrive could be described by a uniform distribution over the interval from 0 to 15.

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Continuous Uniform Distribution

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Uniform Distribution: Variance

The variance of the continuous uniform distribution, denoted $\mathrm{Var}[X]$, is computed using the following formula

$$\mathrm{Var}[X] = \frac{(b-a)^2}{12}$$

For our previous example this is

$$\mathrm{Var}[X] = \frac{(3-0)^2}{12} = \frac{3^2}{12} = \frac{9}{12} = 0.75$$

The Uniform Distribution

In the last class, we had a look at the continuous uniform distribution. It is very useful in constructing simulations. Briefly we will look at some relevant `R` function. The distribution has two parameters: i.e `min` and `max`. (Here chosen as 5 and 10 respectively)

```
># Generate Four Random Number
> runif(4, min=5,max=10)
[1] 9.709372 7.884805 5.571331 5.017549
>
># Compute Density
> dunif(4:11,min=5,max=10)
[1] 0.0 0.2 0.2 0.2 0.2 0.2 0.2 0.0
>
> #Compute distribution of
> punif(4:11,min=5,max=10)
[1] 0.0 0.0 0.2 0.4 0.6 0.8 1.0 1.0
```