Remark : must show workings.

The Binomial distribution is a discrete distribution used.

The outcome of interest is known as a 'success'. If we are interested in how many times we get a six when a dice is rolled.

The probability of success is denoted p.

binomial coefficients

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

binomial probability

$$y = \frac{n!}{k!\,(n-k)!} p^k q^{n-k} = \binom{n}{k} p^k q^{n-k}$$

mean and variance of Binomial distribution

$M_{bin} = np$ and $\sigma^2_{bin} = np(1-p)$

For example, if the sample size is 12 and the probability of success is 0.25, the mean is $12 \times 0.25 = 3$ and the variance is $\sigma^2_{bin} = 12 \times 0.25 \times 0.75 = 2.25$.

$$E(X) = ? = np$$

The variance of the binomial distribution is

$$\text{Var}(X) = ?2 = npq$$

Note: In a binomial distribution, only 2 parameters, namely n and p, are needed to determine the probability.

P(X) gives the probability of successes in n binomial trials.

1 Binomial Distribution

A Quick Review of the Binomial Distribution

- The number of independent trials is denoted n.
- The outcome of interest is known as a "Success".
- The other outcome is known as a "failure".
- Often the applications of these names is counter-intuitive, i.e. defective components being the "success".
- The probability of a 'success' is p
- The expected number of 'successes' from n trials is $E(X) = np$
- The `binom` family of commands in `R` are what we use to compute necessary values.
- The formula can be understood as follows: we want exactly k successes (p^k) and $n - k$ failures ($(1-p)^{n-k}$).
- However, the k successes can occur anywhere among the n trials, and there are $\binom{n}{k}$ different ways of distributing k successes in a sequence of n trials.

2 Binomial Distribution : Worked Example

- A manufacturer of hospital equipment knows from experience that 5% of the production will have some type of minor default, and will require adjustment.
  - Number of independent trials n
  - Probability of a "success" p

3 Binomial Distribution

MCQ questions - 25% chance of getting a single question right at random.

number of questions is 10

Binomial parameter values n=10, p = 0.25

X number of correct answers

P(X7) = 0.0035 [00.35%]

The Binomial Distribution

- The formula can be understood as follows: we want exactly k successes (p^k) and n ? k failures $(1?p)^{n?k}$.
- However, the k successes can occur anywhere among the n trials, and there are $\binom{n}{k}$ different ways of distributing k successes in a sequence of n trials.

The Binomial Distribution

The number of ways of choosing x items from n different items with no concern for order.

It is how we calculate the all the numbers of ways we can get x successes from n trials

Remark : How many ways are there of getting two heads when a coin is tossed three times?

$$\{HHT, HTH, THH\}$$

3 different ways With a larger number of trials or successes, this is difficult to compute without using the above formula.

The Binomial Distribution

Binomial Distribution: Example 2 Example

The Binomial distribution is a discrete distribution used.

The outcome of interest is known as a 'success'. If we are interested in how many times we get a six when a dice is rolled.

The probability of success is denoted p.

binomial coefficients

$$\left(\begin{array}{c} n \\ k \end{array} \right) = \frac{n!}{k!\,(n-k)!}$$

binomial probability

$$y = \frac{n!}{k!\,(n-k)!} p^k q^{n-k} = \left(\begin{array}{c} n \\ k \end{array} \right) p^k q^{n-k}$$

mean and variance of Binomial distribution

$M_{bin} = np$ and $\sigma^2_{bin} = np(1-p)$

3.1 The Binomial Probability Distribution

P(X) gives the probability of successes in n binomial trials.

The word "success" means that the outcome is the outcome of interest.

If the outcome of interest is something like a flat tire, using the word "success" is coutner intuituive.

4 Binomial Distribution : Example

- A manufacturer of hospital equipment knows from experience that 5% of the production will have some type of minor default, and will require adjustment.
  - Number of independent trials n
  - Probability of a "success" p

5 Binomial Example 4

Using recent data provided by the low-cost arriving on time is estimated to be 0.9.
On four different occasions I am taking a flight with Brianair.

(i) What is the probability that I arrive on time on all four flights?

(ii) What is the probability that I arrive on time on exactly two occasions?

The Binomial Distribution
Solution

- Firstly, identify the probability distribution to be used?

  Answer: the binomial distribution

- We are given the number of trials (" choose 10 employees")
- We are given a definition of a "success", which is finding an employee that did NOT reads the WSJ
- We are given the probability of such a success : 30% or 0.30
- So our binomial parameters are n= 10 and p = 0.30
- Open Murdoch Barnes Table 1 and find the relevant section (Page 62)

The Binomial Distribution
Let denote the number of employees in the sample of 10 who did not read the WSJ. part i
Here our value of r is 5
Our answer is 0.1503
Part ii What is the probability of there being between 4 and 8 successes?
We can find out the probability of four or more successes, and then exclude the probability of 9 or more success to find the answer we are looking for.
part iii

- Probability of no more than 7 successes?
- So, we are interested in the probability of between 0 and 7 successes.
- The complement of this is the probability of 8 or more successes.

part iv
mean and variance
[Page 134 for extra question?] [Finish here]