The Exponential Distribution

- The exponential distribution is a continuous probability distribution commonly used to model durations or "lifetimes".
- A lifetime could mean
  - the lifespan of a component
  - the time it takes to complete a task
  - the amount of time between two successive occurrences, such as withdrawals from a bank machine.
- The average lifetime is denoted $E(X) = \mu$.
- The variance of lifetimes is computed as $V(X) = \mu^2$

Important Formulae

The probability that a lifetime X will be less than a period of k time units is given by

$$P(X \leq k) = 1 - e^{\frac{-k}{\mu}}.$$

Similarly, the probability that a lifetime X will be greater than a period of k time units is given by

$$P(X \geq k) = e^{\frac{-k}{\mu}}.$$

The Exponential Distribution

A continuous random variable having p.d.f. f(x), where: $f(x) = \lambda x e^{-\lambda x}$ is said to have an exponential distribution, with parameter λ. The cumulative distribution is given by: $F(x) = 1 - e^{\lambda x}$
Expectation and Variance $E(X) = 1/\lambda$
$V(X) = 1/\lambda^2$

Example

Suppose that the service time for a customer at a fast-food outlet has an exponential distribution with mean 3 minutes. What is the probability that a customer waits more than 4 minutes?

$$P(X \leq 4) = 1 - e^{-4/3}$$

$$P(X \leq 4) = e^{-4/3} = 0.2636$$

Exponential Distribution Lifetimes

The average lifespan of a laptop is 5 years. You may assume that the lifespan of computers follows an exponential probability distribution.

- (3 marks) What is the probability that the lifespan of the laptop will be at least 6 years?
- What is the probability that the lifespan of the laptop will not exceed 4 years?
- What is the probability of the lifespan being between 5 years and 6 years?

Suppose the lifetime of a PC is exponentially distributed with mean $\mu = 5$ We should be told the average lifetime μ.

$$P(X \geq x_o) = e^{\frac{-x_o}{\mu}}$$

Exponential Distribution Lifetimes

- What is the probability that the lifespan of the laptop will be at least 6 years?
- What is the probability that the lifespan of the laptop will not exceed 4 years?
- What is the probability of the lifespan being between 5 years and 6 years?