1 Poisson Approximation of the Binomial

- The Poisson distribution can sometimes be used to approximate the binomial distribution
- When the number of observations n is large, and the success probability p is small, the $Bin(n,p)$ distribution approaches the Poisson distribution with the parameter given by $m = np$.
- This is useful since the computations involved in calculating binomial probabilities are greatly reduced.
- As a rule of thumb, n should be greater than 50 with p very small, such that np should be less than 5.
- We set = np (other notation m = np) and use the Poisson tables.
- If the value of p is very high, the definition of what constitutes a “success” or “failure” can be switched.

1.1 Poisson Approximation: Example

- Suppose we sample 1000 items from a production line that is producing, on average, 0.1% defective components.
- Using the binomial distribution, the probability of exactly 3 defective items in our sample is

$$P(X=3) =^{1000} C_3 \times 0.001^3 \times 0.999^{997}$$

- Lets compute each of the component terms individually.

  * ${}^{1000}C_3$

$$^{1000}C_3 = \frac{1000 \times 999 \times 998}{3 \times 2 \times 1} = 166,167,000$$

  * 0.001^3

$$0.001^3 = 0.000000001$$

  * 0.999^{997}

$$0.999^{997} = 0.36880$$

- Multiply these three values to compute the binomial probability $P(X=3) = 0.06128$

1.2 Using the Poisson Approximation (Same Example As previous)

- Lets use the Poisson distribution to approximate a solution.
- First check that $n \geq 50$ and $np < 5$ (Yes to both).
- We choose as our parameter value $m = np = 1000 \times 0.001 = 1$

$$P(X=3) = \frac{e^{-1} \times 1^3}{3!} = \frac{e^{-1}}{6} = \frac{0.36787}{6} = 0.06131$$

Compare this answer with the Binomial probability P(X = 3) = 0.06128. Very good approximation, with much less computation effort.

2 Poisson Approximation

The Poisson Approximation of the binomial ditribution

Example

$$P(X \geq 2) = 1 - (0.134 + 0.27) = 0.596$$

$$P(X = 1) = 2000.010.99199$$

$$P(X = 1) = 0.270$$

Poisson Approximaitions

$$X \sim \text{Binomial}(200, 0.01)$$

P(X= k)=e-kk!

$$P(X \geq 2) = 1 - (0.135 + 0.27) = 0.595$$

The Poisson Distribution

- The **Poisson mean** λ (pronounced 'lambda') is the expected number of occurrences per unit space / unit period.
- (Remark: Some texts will use the notation m rather than λ)

Probability Density Function

$$P(X = k) = \frac{e^{m!}}{k!}$$

Question 6 : Poisson approximation of the Binomial Distribution
binomial parameters number of trialsn =100 probability of success p = 0.01
from binomial tables (MB1)
complement P (X $\geq$ 2) = 0.2642
therefore answer is P ($X \leq 1$) = 0.7358

2.1 Poisson

- M=15 (1/2 hour or 30 minutes)
- 5 minute period m=2.5
- X : No of arrivals
- P(X=0) when M = 2.5

$$\begin{aligned} P(X = 0) &= 1 - P(X \geq 1)(Complement) \\ &= 1 - 0.9179 \\ &= 0.0821 \end{aligned}$$

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