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Estimation of the fixed parameters

The vector y has marginal density $y \sim N(X\beta, V)$, where $V = \Sigma + ZDZ'$ is specified through the variance component parameters θ . The log-likelihood of the fixed parameters (β, θ) is

$$\ell(\beta, \theta | y) = -\frac{1}{2} \log |V| - \frac{1}{2} (y - X\beta)' V^{-1} (y - X\beta), \quad (1)$$

and for fixed θ the estimate $\hat{\beta}$ of β is obtained as the solution of

$$(X'V^{-1}X)\beta = X'V^{-1}y. \quad (2)$$

Maximum likelihood and restricted maximum likelihood have become the most common strategies for estimating the variance component parameter θ . Substituting $\hat{\beta}$ from (2) into $\ell(\beta, \theta | y)$ from (1) returns the *profile* log-likelihood

$$\begin{aligned} \ell_P(\theta | y) &= \ell(\hat{\beta}, \theta | y) \\ &= -\frac{1}{2} \log |V| - \frac{1}{2} (y - X\hat{\beta})' V^{-1} (y - X\hat{\beta}) \end{aligned}$$

of the variance parameter θ . Estimates of the parameters θ specifying V can be found by maximizing $\ell_P(\theta | y)$ over θ . In practice the *restricted* log-likelihood

$$\ell_R(\theta | y) = \ell_P(\theta | y) - \frac{1}{2} \log |X'VX|$$

is preferred. This approach is based on maximizing the likelihood of linear combinations of y that do not depend on β , and in this way takes into account the estimation of β .

Estimation of the random effects

The established approach for estimating the random effects is to use the best linear predictor of b from y , which for a given β equals $DZ'V^{-1}(y - X\beta)$. In practice β is replaced by an estimator such as $\hat{\beta}$ from (2) so that $\hat{b} = DZ'V^{-1}(y - X\hat{\beta})$. Pre-multiplying by

the appropriate matrices it is straightforward to show that these estimates $\hat{\beta}$ and \hat{b} satisfy the equations in (??).

References