USE LAPLACE TRANSFORMS to solve the integral equation

$$y(t) = e^t + \int_0^t e^{-(t-u)} y(u)\,du$$

Solution

$$y(t) = e^t + \left[e^{-t} * y(t)\right]$$

Convolution

LHS $\mathcal{L}[y(t)] = Y(s)$

RHS $\mathcal{L}[e^t] = \frac{1}{s-1}$

$$\mathcal{L}\left[e^{-t} * y(t)\right] = \frac{1}{s+1} \cdot Y(s)$$

$$Y(s) = \frac{1}{s-1} + \left(\frac{Y(s)}{s+1}\right)$$

$$\left(\frac{S+1}{S+1}\right)Y(S) = \frac{1}{S-1} + \frac{Y(S)}{S+1}$$

$$\frac{S}{S+1}Y(S) = \frac{1}{S-1}$$

$$Y(s) = \frac{S+1}{S(S-1)}$$

USE PARTIAL FRACTION Expansion to Solve

$$Y(S) = \frac{S+1}{S(S-1)} = \frac{A}{S} + \frac{B}{S-1}$$

$$= \frac{A(S)-A \ + B(S)}{S(S-1)}$$

$$= \frac{(A+B)S - A}{S(S-1)}$$

$\therefore A = -1 \qquad A+B = 1 \qquad \therefore B=2$

$$Y(s) = \frac{-1}{s} + \frac{2}{s-1}$$

$$y(t) = -1 + 2e^{t}. \quad (t \geq 0)$$