Computing Laplace transforms from first principles.

$$\mathcal{L}[f(t)] = \int_0^{+\infty} e^{-st} f(t)\, dt$$

From first principles, show that

$$\mathcal{L}[t+2] = \frac{1}{s^2} + \frac{2}{s}$$

$$\mathcal{L}[f(t)] = \int_0^{+\infty} e^{-st} . [2+t]\, dt$$

$$= \underbrace{\int_0^{+\infty} 2e^{-st}\, dt}_{1} + \underbrace{\int_0^{\infty} t . e^{-st}\, dt}_{2}$$

1 $$2\int_0^{+\infty} e^{-st}\, dt = 2\left[\frac{e^{-st}}{-s}\right]_0^{\infty}$$

Recall $e^{-\infty} = 0$

$e^{0} = 1$

$$2\left[\frac{e^{-st}}{-s}\right]_0^{\infty} = 2\left[\frac{e^{-\infty}}{-s} - \frac{e^{0}}{-s}\right]$$

$$= 2\left[0 - \frac{1}{-s}\right]$$

$$= \frac{2}{s}.$$

Ans

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(2) $$\int_0^{+\infty} t \cdot e^{-st}\,dt$$

use Integration by Parts

let $u = t$

$du/dt = 1$

$du = dt$

let $dv = e^{-st}dt$

$v = \int dv = \int e^{-st}dt$

$$= \frac{e^{-st}}{-s}$$

$$I = u.v - \int v\,du$$

$$= t.\frac{e^{-st}}{-s} - \frac{1}{-s}\left[\int e^{-st}\,dt\right]$$

$$= \frac{te^{-st}}{-s} + \frac{1}{s}\left[\frac{e^{-st}}{-s}\right]$$

$$= \left[\frac{t.e^{-st}}{-s} - \frac{1}{s^2}\left[e^{-st}\right]\right]_0^\infty$$

$$= \left[0 - 0\right] - \left[0 - \frac{1}{s^2}\left[1\right]\right]$$

$$= \frac{1}{s^2}.$$ ANSWER TO 2

$$L\left[f(t)\right] = \underset{1}{\frac{2}{s}} + \underset{2}{\frac{1}{s^2}}$$