From first principles, show that

$$\mathcal{L}\left[t.e^{-2t}\right] = \frac{1}{(s+2)^2}$$

$$\boxed{\mathcal{L}\left[f(t)\right] = \int_0^{+\infty} f(t).e^{-st}\,dt}$$

Solution

$$\int_0^{+\infty}\left[t.e^{-2t}\right].e^{-st}\,dt$$

$$= \int_0^{+\infty} t.e^{-(s+2)t}\,dt$$

Integration by parts

let • $u = t$ • let $dv = e^{-(s+2)t}\,dt$

• $du/dt = 1$ • $v = \int dv = \dfrac{e^{-(s+2)t}}{-(s+2)}$

• $du = dt$

$$I = u.v - \int v\,du$$

$$= \frac{t.e^{-(s+2)t}}{-(s+2)} - \int \frac{e^{-(s+2)t}}{-(s+2)}\,dt$$

$$= \frac{t.e^{-(s+2)t}}{-(s+2)} + \frac{1}{(s+2)} \int e^{-(s+2)t}\,dt$$

$$= \frac{te^{-(s+2)t}}{-(s+2)} + \frac{1}{(s+2)} \left[\frac{e^{-(s+2)t}}{-(s+2)}\right]$$

Evaluate the definite integral

$$\left[\frac{te^{-(s+2)t}}{-(s+2)}\right]_0^\infty - \left[\frac{1}{(s+2)^2}\left[e^{-(s+2)t}\right]\right]_0^\infty$$

$$= \left[0 - 0\right] - \left[\frac{1}{(s+2)^2}(0-1)\right]$$

$$\mathcal{L}[f(t) = \frac{1}{(s+2)^2}$$