

Faculty of Science and Engineering Department of Mathematics & Statistics

MID TERM ASSESSMENT PAPER

MODULE CODE: MA4003 SEMESTER: Autumn 2010/11

MODULE TITLE: Engineering Mathematics 3 DURATION OF EXAMINATION: 45 minutes

LECTURER: Dr. M. Burke PERCENTAGE OF TOTAL MARKS: 20 %

Colour: Green

INSTRUCTIONS TO CANDIDATES: Answer all questions. All questions carry equal marks. Use the Answer Sheet below.

Answer Sheet

STUDENT'S NAME: STUDENT'S ID NUMBER:

For each question, place an "X" in the box of your choice.

Question	a	b	c	d	e	Do not write in this column
1		X				
2					X	
3	X					
4					X	
5		X				
6					X	
7					X	
8		X				
9	X					
10		X				

Table of Laplace Transforms

$f(t), t \ge 0$	$F(s) = \mathcal{L}[f(t)]$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$\frac{1}{a-b}(e^{at} - e^{bt})$	$\frac{1}{(s-a)(s-b)}$
$\frac{a}{a-b}e^{at} - \frac{b}{a-b}e^{bt}$	$\frac{s}{(s-a)(s-b)}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
f'(t)	sF(s) - f(0)
f''(t)	$s^2 F(s) - s f(0) - f'(0)$
$\int_0^t f(\tau) d\tau$	$\frac{1}{s}F(s)$
$e^{at}f(t)$	F(s-a)
Heaviside $u_a(t)$	$\frac{e^{-as}}{s}$
$f(t-a)u_a(t)$	$e^{-as}F(s)$
Ramp $R(t-a)$	$\frac{e^{-as}}{s^2}$
tf(t)	-F'(s)
$rac{f(t)}{t}$	$\int_{s}^{\infty} F(\sigma) d\sigma$
$(f * g)(t) \equiv \int_0^t f(t - \tau)g(\tau) d\tau$	F(s)G(s)
f(t) = f(t+p)	$\frac{1}{1 - e^{-sp}} \int_0^p f(t)e^{-st} dt$

All f(t) are defined for $t \geq 0$.

1. The Laplace Transform of $\cosh 2t + \sinh 2t$ is

(a)
$$\frac{2s-2}{(s^2-1)^2}$$
 (b) $\frac{1}{s-2}$ (c) $\frac{1}{s}$ (d) $\frac{1}{s+2}$ (e) $\frac{s-2}{(s^2-4)^2}$

2. The Laplace Transform of $e^{-2t}(t+2)$ is

$$(a) \ \frac{2s-5}{(s-2)^2} \ (b) \ \frac{2s+1}{s^2(s-2)} \ (c) \ \frac{2s+1}{s^2(s+2)} \ (d) \ \frac{-2s+1}{s^2(s+2)} \ (e) \ \frac{2s+5}{(s+2)^2}$$

3. The Laplace Transform of $f(t) = \sin(2t - 2)u_1(t)$ is

(a)
$$\frac{2}{s^2+4}e^{-s}$$
 (b) $\frac{e^{-2}}{s(s^2+4)}$ (c) $\frac{s}{s^2+1}e^{-s}$ (d) $\frac{s}{s^2+4}e^{-1}$ (e) $\frac{2}{s^2+1}e^{-s}$

4. The inverse *Laplace* transform of $\frac{s}{s^2-2s+1}$ is

(a)
$$(1-t)e^{-t}$$
 (b) te^{t} (c) $e^{-t}\cos t$ (d) $e^{t}\cos t$ (e) $(1+t)e^{t}$

5. The inverse *Laplace* transform of $\frac{s-2}{s^2-s-6}$ is

(a)
$$e^{-3t}$$
 (b) $\frac{4}{5}e^{-2t} + \frac{1}{5}e^{3t}$ (c) $2e^{-2t} - e^{3t}$ (d) $\frac{3}{7}e^{-t} + \frac{4}{7}e^{6t}$ (e) e^{3t}

6. The convolution of e^t with e^{-t} (also denoted by $e^t * e^{-t}$) is given by

(a)
$$t$$
 (b) te^t (c) $-1 + e^{-t}$ (d) $1 - e^{-t}$ (e) $\frac{e^t - e^{-t}}{2}$

7. The period of $\sin\left(\frac{x}{2}\right)$ is

(a) 1 (b) 2 (c)
$$2\pi$$
 (d) 4 (e) 4π

8. The functions $f(x) = x - x^5$ and $g(x) = x^2 \sin x$ defined on -1 < x < 1 have the property that

- (a) both are even (b) both are odd (c) f is odd and g is even (d) f is even and g is odd (e) neither is even nor odd
- 9. The function f(x) = -x for -1 < x < 1 is periodic with period 2. It has a *Fourier Series* $\sum_{n=1}^{\infty} b_n \sin(nx)$ where b_n is given by

(a)
$$\frac{2}{n\pi}(-1)^n$$
 (b) $\frac{1}{n\pi}(-1)^n$ (c) $\frac{1}{n}(-1)^n$ (d) 0 (e) $\frac{2}{n}$

10. The coefficient a_0 in the *Fourier* Series for the periodic function $f(x) = -x^3$ if -1 < x < 1 with period 2 has the value

(a)
$$-\frac{1}{4}$$
 (b) 0 (c) $\frac{1}{4}$ (d) $\frac{1}{3}$ (e) $\frac{2}{3}\pi^2$