Chapter 1

A Simplified LME Framework for Method Comparison

The computation of the limits of agreement require that the variance of the difference of measurements. This variance is easily computable from the estimate of the Block - $\boldsymbol{\Omega}_i$ matrix. Lack of agreement can arise if there is a disagreement in overall variabilities. This may be due to due to the disagreement in either between-item variabilities or within-item variabilities, or both. Roy (2009) allows for a formal test of each.

1.1 Model Terms for Roy's Techniques

$\boldsymbol{b}_i$ is a $m-$dimensional vector comprised of the random effects.

$$\boldsymbol{b}_i = \begin{pmatrix} b_{1i} \\ b_{21} \end{pmatrix} \tag{1.1}$$

$\boldsymbol{V}$ represents the correlation matrix of the replicated measurements on a given method. $\boldsymbol{\Sigma}$ is the within-subject VC matrix.

$\boldsymbol{V}$ and $\boldsymbol{\Sigma}$ are positive definite matrices. The dimensions of $\boldsymbol{V}$ and $\boldsymbol{\Sigma}$ are $3 \times 3 (= p \times p)$ and $2 \times 2 (= k \times k)$.

It is assumed that $\boldsymbol{V}$ is the same for both methods and $\boldsymbol{\Sigma}$ is the same for all replications. $\boldsymbol{V} \bigotimes \boldsymbol{\Sigma}$ creates a $6 \times 6 (= kp \times kp)$ matrix. $\boldsymbol{R}_i$ is a sub-matrix of this.

1.2 Model terms

It is important to note the following characteristics of this model. Let the number of replicate measurements on each item i for both methods be n_i, hence $2 \times n_i$ responses. However, it is assumed that there may be a different number of replicates made for different items. Let the maximum number of replicates be p. An item will have up to $2p$ measurements, i.e. $\max(n_i) = 2p$.

Later on $\boldsymbol{X}_i$ will be reduced to a 2×1 matrix, to allow estimation of terms. This is due to a shortage of rank. The fixed effects vector can be modified accordingly. $\boldsymbol{Z}_i$ is the $2n_i \times 2$ model matrix for the random effects for measurement methods on item i. $\boldsymbol{b}_i$ is the 2×1 vector of random-effect coefficients on item i, one for each method. $\boldsymbol{\epsilon}$ is the $2n_i \times 1$ vector of residuals for measurements on item i. $\boldsymbol{G}$ is the 2×2 covariance matrix for the random effects. $\boldsymbol{R}_i$ is the $2n_i \times 2n_i$ covariance matrix for the residuals on item i. The expected value is given as $\mathrm{E}(\boldsymbol{y}_i) = \boldsymbol{X}_i\boldsymbol{\beta}$. (Hamlett et al., 2004) The variance of the response vector is given by $\mathrm{Var}(\boldsymbol{y}_i) = \boldsymbol{Z}_i\boldsymbol{G}\boldsymbol{Z}_i' + \boldsymbol{R}_i$ (Hamlett et al., 2004).

Bibliography

Hamlett, A., L. Ryan, and R. Wolfinger (2004). On the use of PROC MIXED to estimate correlation in the presence of repeated measures. *Proceedings of the Statistics and Data Analysis Section, SAS Users Group International 198-229*, 1–7.

Roy, A. (2009). An application of the linear mixed effects model to ass the agreement between two methods with replicated observations. *Journal of Biopharmaceutical Statistics 19*, 150–173.