Graph Theory: Tutorial Sheet

1. Draw a graph with degree sequence 4,3,2,2. If it is not possible to draw this graph, explain why.
2. Draw a graph with degree sequence 4,3,3,2,2. If it is not possible to draw this graph, explain why.
3. Explain what is meant by a complete graph. How is a complete graph, with n vertices denoted?
4. How many edges does a complete graph with 8 vertices contain?
5. A graph is called k-regular if each of its vertices has degree k. Construct an example of:
   (i) a 2-regular graph with 5 vertices;
   (ii) a 3-regular graph with 6 vertices
   (iii) a 4-regular graph with 8 vertices.
6. Is it possible to construct an 8 vertex graph where each vertex is connected to exactly 5 vertices? Is it possible to do so for a 9 vertex graph?
7. Consider a d-regular graph on 7 vertices. What are the possible values for d. For each viable value for d, how many edges would there be?
8. Given the following definitions for simple, connected graphs:
   (a) K_n is a graph on n vertices where each pair of vertices is connected by an edge;
   (b) C_n is the graph with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_n, v_1\}$;
   (c) W_n is the graph obtained from C_n by adding an extra vertex,v_{n+1}, and edges from this to each of the original vertices in C_n.

   Draw K_4, C_4, and W_4.
9. Let G be a simple graph with vertex set $V(G) = \{v1, v2, v3, v4, v5\}$ and adjacency lists as follows:

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v1 : v2 v3 v4
v2 : v1 v3 v4 v5
v3 : v1 v2 v4
v4 : v1 v2 v3.
v5 : v2
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   (i) List the degree sequence of G. Draw the graph of G.
   (ii) Find two distinct paths of length 3, starting at v3 and ending at v4. Find a 4 cycle in G.
   (iii) Let G be a graph and let v be a vertex of G. Say what is meant by the degree of v.
   (iv) State, without proving, a result connecting the degrees of the vertices of a graph G with the number of its edges.
10. (i) State, without proving, a result connecting the degrees of the vertices of a graph G with the number of its edges.
    (ii) Use this result to find the number of edges of a 3-regular graph with 10 vertices.

(iii) Explain why it is not possible to construct a 3-regular graph with 9 vertices.

(iv) A simple, connected graph has 7 vertices, all having the same degree d. State the possible values of d and for each value also give the number of edges in the corresponding graph.

(v) Another simple, connected graph has 6 vertices, all having the same degree; n. Draw such a graph when $n = 3$ and state the other possible values of n.