Integration : Tutorial Sheet 2

1. Evaluate the following indefinite integrals using partial fractions:

   (i) $$\int \frac{x}{x^2-9}dx$$

   (ii) $$\int \frac{x-2}{x^2-4x+3}dx$$

   (iii) $$\int \frac{2x-4}{x^2-4x+8}dx$$

2. Evaluate the following using integration by parts.

   (i) $$\int -4\ln(x)\,dx$$

   (ii) $$\int (-7x+38)\cos(x)\,dx$$

   (iii) $$\int_0^{\frac{\pi}{2}} (-6x+45)\cos(x)\,dx$$

   (iv) $$\int (5x+1)(x-6)^4\,dx$$

   (v) $$\int_{-1}^{1} (2x+8)^3(-x+2)\,dx$$

   (vi) $$\int \sin(x)\,e^x\,dx$$

Formula:
If u and v are functions of x that have continuous derivatives, then

$$\int udv = uv - \int vdu$$

It is considered a rule of thumb to remember the acronym **LIPET** when performing integration by parts. This acronym will help you to determine what to use as u.

L -logarithms,

I -inverse trigonometric functions,

P -polynomials (i.e. x, x^2) ,

E -exponentials (i.e. e^x, e^{3x}),

T -trigonometric functions.

3. Evaluate the following:

(i) $\int x^2 - (2x)^2\, dx$

(ii) $\int 8x^3\, dx$

(iii) $\int (4x^2 + 11x^3)\, dx$

(iv) $\int (31x^{32} + 4x^3 - 9x^4)\, dx$

(v) $\int 5x^{-2}\, dx$

4. The following questions are from previous past papers. Please be advised of the notes below.

   (i) Use integration by parts to find $\int xe^x dx$

   (ii) Use integration by parts to find $\int xln(x)dx$

   (iii) Use integration by parts to find $\int xsinh(x)dx$

   (iv) Use integration by parts to find $\int xcos(x)dx$

   (v) Use integration by parts to find $\int xcosh(x)dx$

   (vi) Use integration by parts to find $\int xe^x dx$

- $\cosh(x)$ is both the derivative and integral of $\sinh(x)$
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5. Evaluate the following definite integrals

(i) $\int_1^2 (x^2 - 1)dx$

(ii) $\int_0^{\frac{\pi}{2}} \cos x dx$

(iii) $\int_0^{\pi} \cos x dx$

(iv) $\int_1^2 (y^2 - y^{-2})dy$

(v) $\int_{-3}^1 (6x^2 - 5x + 2)dx$

(vi) $\int_4^0 \sqrt{t}(t - 2)dt$

Hint:

$$\int \sqrt{t}(t-2)dt$$

$$\sqrt{t}(t-2) = t^{1/2} \times (t-2) = t^{3/2} - 2t^{1/2}$$

6. Evaluate the following definite integral

$$\int_1^3 \frac{x}{3} dx$$

Solution

$$\int_1^3 \frac{x}{3} dx = \left[\frac{x^4}{4}\right]_1^3 = \frac{81}{4} - \frac{1}{4} = 20$$

7. Evaluate the following definite integral

$$\int_1^3 \frac{x^2 - 4x + 3}{x - 3} dx$$

Solution

Factorize the numerator $x^2 - 4x + 3 = (x-1)(x-3)$

Treat it as an indefinite integral for time being.

$$\int \frac{x^2 - 4x + 3}{x - 3} dx = \int \frac{(x-1)(x-3)}{x-3} dx = \int (x-1) dx = \frac{x^2}{2} - x + c$$

$$\left[\frac{x^2}{2} - x\right]_1^3 = (4.5 - 3) - (0.5 - 1) = 2$$