

Number Theory : Tutorial Sheet A

1. State which of the following sets the following numbers belong to.

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|------------------|-------------------|-----------|-----------------|
| 1) 18 | 3) π | 5) $17/4$ | 7) $\sqrt{\pi}$ |
| 2) $8.2347\dots$ | 4) $1.33333\dots$ | 6) 4.25 | 8) $\sqrt{25}$ |

The possible answers are

- a) Natural number : $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
 - b) Integer : $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
 - c) Rational Number : $\mathbb{Q} \subseteq \mathbb{R}$
 - d) Real Number \mathbb{R}
2. Express the recurring decimal $0.4242424\dots$ as a rational number in its simplest form.
3. Perform the binary multiplications
- $(1101)_2 \times (101)_2$
 - $(1101)_2 \times (1101)_2$
4. Express 42900 as a product of its prime factors, using index notation for repeated factors.
5. Express the recurring decimals
- (i) $0.727272\dots$
 - (ii) $0.126126126\dots$
 - (iii) $0.7545454545\dots$
- as rational numbers in its simplest form.
6. Given that π is an irrational number, can you say whether $\frac{\pi}{2}$ is rational or irrational. or is it impossible to tell?
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8. (i) Write down the numbers 0.0000526 in floating point form.
(ii) How is the number 1 expressed in floating point form.
9. (i) Given x is the irrational positive number $\sqrt{2}$, express x^8 in binary notation
(ii) From part (i), is x^8 a rational number?

10. (i) $5/7$ lies between 0.714 and 0.715.
(ii) $\sqrt{2}$ is at least 1.41.
(iii) $\sqrt{3}$ 9s at lrastr 1.732 and at most 1.7322.
11. (i) Deduce that every composite integer n has a prime factor such that $p \leq \sqrt{n}$.
(ii) Decide whether 899 is a prime.
12. (i) What would be the maximum numbber of digits that a decimal fraction with denominator 13 could have in a recurring block in theory?
(ii) Can you predict which other fractions with denominator 13 will have the same digits as $1/13$ in their recurring block?
13. Suppose 2341 is a base-5 number Compute the equivalent in each of the following forms:
 - (i) decimal number
 - (ii) hexadecimal number
 - (iii) binary number
14. A number is expressed in base 5 as $(234)_5$. What is it as decimal number? Suppose you multiply $(234)_5$ by 5. what would be the answer in base 5.
15. Express 42900 as a product of its prime factors, using index notation for repeated factors.
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