Exercise: Combinations

A committee of 4 must be chosen from 3 women and 4 men.

(1) In how many ways can the committee be chosen, regardless of sex.

(2) In how many cans 2 men and 2 women be chosen.

(3) Compute the probability of a committee of 2 men and 2 women are chosen.

(4) Compute the probability of at least two women.

Part 1

We need to choose 4 people from 7:

This can be done in

$$^7C_4 = \frac{7!}{4! \times 3!} = \frac{7 \times 6 \times 5 \times 4!}{4! \times 3!} = 35 \text{ ways.}$$

Part 2

With 4 men to choose from, 2 men can be selected in

$$^4C_2 = \frac{4!}{2! \times 2!} = \frac{4 \times 3 \times 2!}{2! \times 2!} = 6 \text{ ways.}$$

Similarly 2 women can be selected from 3 in

$$^3C_2 = \frac{3!}{2! \times 1!} = \frac{3 \times 2!}{2! \times 1!} = 3 \text{ ways.}$$

Thus a committee of 2 men and 2 women can be selected in $6 \times 3 = 18$ ways.

Part 3

The probability of two men and two women on a committee is

$$\frac{\text{Number of ways of selecting 2 men and 2 women}}{\text{Number of ways of selecting 4 from 7}} = \frac{18}{35}$$

Part 4

- The probability of at least two women is the probability of 2 women or 3 women being selected.
- We can use the addition rule, noting that these are two mutually exclusive events.
- From before we know that probability of 2 women being selected is 18/35.
- We have to compute the number of ways of selecting 1 man from 4 (4 ways) and the number of ways of selecting three women from 3 (only 1 way)

- The probability of selecting three women is therefore $\frac{4\times 1}{35} = 4/35$
- So using the addition rule

$$Pr(\text{ at least 2 women }) = Pr(\text{ 2 women }) + Pr(\text{ 3 women })$$

$$Pr(\text{ at least 2 women }) = 18/35 + 4/35 = 22/35$$