Probability Distributions

Question 1

Computing Z-Scores : Preparatory work for Week 6.

$$\text{Important Formula: } z_0 = \frac{x_o - \mu}{\sigma} \text{ where } X \sim N(\mu, \sigma^2)$$

(a) Suppose $X \sim N(100, 225)$. Compute the z-scores for X=118,91,120 and 124.

(b) Suppose $X \sim N(100, 256)$. Compute the z-scores for X=128, 124, and 126.

(c) Suppose $X \sim N(100, 100)$. Determine the values for X that would yield z-scores of 1.24 and -1.63 respectively.

Question 2

Telephone calls arrive at a switchboard at the rate of 12 per hour. Telecentre operators typically take 3 minutes to deal with a customer query. Calculate the probability of :

(a) 2 or more calls arriving in any 15 minute period.

(b) No phone calls arriving in a 15 minute period,

(c) Exactly one phone call arriving in any 15 minute period,

(d) What is the expected value and standard deviation of the number of phone calls arriving in a 30 minute period.

Question 3

Flaws occur in an LCD display at the rate of 0.5 per square mm. Calculate the probability that:

(a) exactly 2 flaws will occur in a square mm section,

(b) exactly 3 flaws will occur in a 5 square mm section,

(c) 1 flaw will occur in a 10 square mm section.

Question 4

Suppose X is a binomial variable specified as $X \sim \text{Bin}(1000, 0.001)$. Using an appropriate approximation method, compute the following probabilities

(a) The probability that X is zero: $P(X = 0)$

(b) The probability that X is equal to one: $P(X = 1)$

(c) The probability that X is equal to two: $P(X = 2)$

Question 5

The average lifespan of a PC monitor is 6 years. You may assume that the lifespan of monitors follows an exponential probability distribution.

(a) What is the probability that the lifespan of the monitor will be at least 5 years?

(b) What is the probability that the lifespan of the monitor will not exceed 4 years?

(c) What is the probability of the lifespan being between 5 years and 7 years?

Question 6

A power supply unit for a computer component is assumed to follow an exponential distribution with a mean life of 1,400 hours. What is the probability that the component will:

(a) fail in the first 700 hours?

(b) survive more than 1,750 hours?

(c) last between 1,050 hours and 1,750 hours?

Question 7

On average, six people per hour use an electronic teller machine during the prime shopping hours in a department store. Therefore it is assumed that the expected time until the next customer will arrive will be 10 minutes. You may assume that the distributions of waiting times can be described by the exponential probability distribution.

(a) What is the probability that at least 10 minutes will pass between the arrival of two customers?

(b) What is the probability that after a customer leaves, another customer does not arrive for at least 20 minutes?