Poisson Probability Distribution: Tutorial Sheet

1. Suppose that a telephone help-line receives 4 calls per hour during offices hours.

   a. (1 Mark) Compute the value of m for a 30 minute period during office hours.

   b. (1 Mark) Compute the probability of the help-line getting exactly one call in a 30 minute period during office hours.

2. The Binomial Distribution

   (a) State the four conditions to be satisfied for the Binomial probability distribution to apply.

   (b) When can the Poisson distribution be used as an approximation to the Binomial distribution?

3. It is assumed that claims arising on an industrial policy can be modelled as a Poisson process at a rate of 0.5 per year.

   (a) Determine the probability that no claims arise in a single year.

   (b) Determine the probability that, in three consecutive years, there is one or more claims in one of the years and no claims in each of the other two years.

   (c) Suppose a claim has just occurred. Determine the probability that more than two years will elapse before the next claim occurs.

$$P(X > 115.5) = 1 - \theta(-2.26) = \theta(2.26) = 0.99$$

4. Flaws occur in an LCD display at the rate of 0.5 per square mm. Calculate the probability that:

   (a) exactly 2 flaws will occur in a square mm section

   (b) exactly 3 flaws will occur in a 5 square mm section

   (c) 5 or more flaws will occur in a 10 square mm section

5. There is a constant probability of 0.05 that the power supply in a server network will fail. You are required to calculate the probability that the power supply will fail the 4th time it is switched on.

6. A motor dealership which specializes in agricultural machinery sells on vehicle every 2 days, on average

   In this question the unit period is one day. The company expects to sell, on average, 0.5 vehicles every day. The Poisson mean m is therefore 0.5.

   $P(X \geq 1)$

> Go to your Poisson tables, and search for the $m = 0.5$ column. We are interested in the probability of **exactly** one vehicle sold on a particular day. From the tables we can easily work out P($X \geq 1$), but this is probability of one or more vehicles being sold. This is not the same thing.
>
> $$P(X \geq 1) = P(X = 1) + P(X = 2) + P(X = 3) + \ldots P(X \geq 1) = P(X = 1) + P(X \geq 2)$$
>
> From tables $P(X \geq 1)$ $P(X \geq 2)$

Six day working week? our unit period is now six days. How many vehicles do we expect to sell in 6 days? answer = 3 $m = 3$

P($X \geq 4$)

$$e^{-6/5} = 0.3011942e^{-4/5} = 0.449329e^{-5/5} = 0.3678794 \tag{1}$$

7. XYZ Ltd supplies motherboards to Dell. You are a production manager with Dell. There is a constant probability of 0.4 that a board will be defective. You select 100 boards at random. Using the log tables for the binomial distribution what is the probability that

   (i) 0 boards will be defective?

   (ii) 2 or more boards will be defective?

   (iii) 5 or less boards will be defective?

   (d) Use the normal approximation to the binomial to answer (i), (ii) and (iii) in part (c) above.

8. The number of particles emitted by a radioactive source in 1 minute has a Poisson distribution with parameter m=1.2, called the emission rate. Calculate the probability that

   (a) No particle is emitted

   (b) At least 4 particles are emitted

9. Phone calls arrive at a help desk at the rate of 48 per hour.

   (a) Find the probability of receiving three calls in a five minute period of time.

   (b) What is the probability of receiving 5 or fewer calls in a 15 minute interval of time.

10. Cars enter a car wash at an average rate of 4 per half hour.

    (a) Compute the probability that the 3 cars arrive in a half hour period.

    (b) Compute the probability that the 6 cars arrive in a one hour period.

11. A particular electronic component is produced by a manufacturing process for which 0.1% of components are produced are defective. Consider a consignment of 5,000 of these times. Let X denote the number of defective components in the consignment. Use the Poisson Approximation of the binomial distribution to calculate $P(X \leq 5)$

12. A computer server breaks down on average once every three months.

    (a) What is the probability that the server breaks down three times in a quarter?

    (b) What is the probability that a server breaks down exactly five times in one year?