Titration experiment

Four Students, A,B,C and D performing the same experiment five times, hence each yield 5 results.

	Results (ml)					Comment
A	10.08	10.11	10.09	10.10	10.12	Precise, biased
B	9.88	10.14	10.02	9.80	10.21	Imprecise unbiased
C	10.19	9.79	9.69	10.05	9.78	Imprecise, biased
D	10.04	9.98	10.02	9.97	10.04	Precise, unbiased

Consider the 4 students performing the titration experiment 5 times each. The outcome of each trial was expected to be 10. The first student A obtained the following values:1 0.08 , 10.11 , 10.09, 10.10, 10.12 (which we described as precise, but biased) Let us implement this test in R, using the following null and alternative hypotheses:

H_0: $\mu = 10$

H_1: μ not equal to 10

The code is as follows:

```
> X.a= c(10.08 ,10.11 ,10.09,10.10,10.12)
>
> t.test(X.a,mu=10)
The output is as follows:
One Sample t-test
data: X.a
t = 14.1421, df = 4, p-value = 0.0001451
alternative hypothesis: true mean is not equal to 10
95 percent confidence interval:
10.08037 10.11963
sample estimates:
mean of x
```

Titration experiment

(As an aside - not part of overall lesson plan)

- Two criteria were used to compare these results, the average value (technically know as a measure of location and the degree of spread (or dispersion).
- The average value used was the arithmetic mean (usually abbreviated to *the mean*), which is the sum of all the measurements divided by the number of measurements.

Titration experiment

The mean, $\bar{X}$, of n measurements is given by

$$\bar{X} = \frac{\sum x}{n}$$

In Chapter 1 the spread was measured by the difference between the highest and lowest values (i.e. the range). A more useful measure, which utilizes all the values, is the sample standard deviation, s, which is defined as follows:
The standard deviation, s, of n measurements is given by

$$s = \sqrt{\frac{\sum(x - \bar{X})^2}{n-1}} (2.2)$$

Review of Inference Procedures

A	10.08	10.11	10.09	10.10	10.12
B	9.88	10.14	10.02	9.80	10.21
C	10.19	9.79	9.69	10.05	9.78
D	10.04	9.98	10.02	9.97	10.04

We shall perform a series of one sample and two sample tests. Recall the true value of the titration experiment in each case is supposed to be 10. First we will consider the case of As measurements (as we did in the previous lecture).

Review of Inference Procedures

The mean and standard deviation of As measurements are as follows:

```
> mean(X.A)
[1] 10.1
> sd(X.A)
[1] 0.01581139
```

We will perform two one-tailed tests. To recap, we performed a two tailed test in the previous lecture (i.e. the true mean is equal to zero). To contrast with the one-tailed tests, here is it again, with the alternative specified.

Review of Inference Procedures

```
> t.test(X.A, mu=10, alternative = "two.sided")
One Sample t-test
data: X.A
t = 14.1421, df = 4, p-value = 0.0001451
alternative hypothesis:
true mean is not equal to 10

95 percent confidence interval:
10.08037 10.11963
sample estimates:
mean of x
10.1
```

Review of Inference Procedures

We will perform a greater than test. The null and alternative are specified as follows:

- H0: $A \leq 10$ True value of population mean is no more than 10
- H1: $A > 10$ True value of population mean is greater than 10.

Review of Inference Procedures

```
> t.test(X.A, mu=10, alternative = "greater")
One Sample t-test
data: X.A
t = 14.1421, df = 4, p-value = 7.256e-05
alternative hypothesis: true mean is greater than 10
95 percent confidence interval:
10.08493 Inf
sample estimates:
mean of x
10.1
```

In this case, we would reject the null hypothesis, based on the extremely low p-value. There is very convincing evidence to say that the true mean of As measurements is greater than 10. (Furthermore there is a systematic upward bias in As measurements)

Review of Inference Procedures

Now we will perform a less than test. The null and alternative are specified as follows:

- H0: μ_A *geq* 10 True value of population mean is at least 10
- H1: $\mu_A < 10$ True value of population mean is less than 10.

Review of Inference Procedures

```
> t.test(X.A, mu=10, alternative = "less")
One Sample t-test
data: X.A
t = 14.1421, df = 4, p-value = 0.9999

alternative hypothesis: true mean is less than 10
95 percent confidence interval:
-Inf 10.11507
sample estimates:
mean of x
10.1
```

In this case, we would fail to reject the null hypothesis, based on the extremely high p-value.