Two-Way Worked Example

Review Question No 22.

Three varieties of potatoes are being compared for yield. The experiment was carried out by assigning each variety at random to four of twelve equal size plots, one being chosen in each of four locations. The following yields in bushels per plot resulted:

	Potato		
Location	A	B	C
1	18	13	12
2	20	23	21
3	14	12	9
4	11	17	10

Additional Information

Expect to be given S_R^2 and S_C^2 in an exam standard question, as well as the variance for the response variable. You will also be given the formula for MS_A and MS_B.
*(**Important**: Partially Completed table, you would not need this information.)*

- Location is Factor A - it is arranged along the rows
- Potato Type is Factor B - it is arranged along the columns.
- The variance of the Row means is : $S_R^2 = 19.037$. Therefore
$$\text{MS}_A = c \times S_R^2 = 3 \times 19.037 = 57.111$$
- the variance of the Column means is : $S_C^2 = 3.0625$. Therefore
$$\text{MS}_B = r \times S_C^2 = 4 \times 3.0625 = 12.25$$
- Also the overall variance of the 12 observations is
$$\text{Var}(Y) = 21.6363$$

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Solution

Step 1 : We have already be given information that we can incorporate into our solution, once we have done the necessary calculations.

(Remark: The following calculation for SS_{Tot} features in each ANOVA procedures on the course).

$$\text{Var}(Y) = \frac{SS_{tot}}{n-1}$$
$$21.6363 = \frac{SS_{tot}}{12-1}$$
$$SS_{tot} = 21.6362 \times 11 = 238$$

Source	df	SS	MS	F	p.values
Treatment			57.111		
Block			12.25		
Total					
Total	11	238			

Step 2 : We will now determine the Degrees of Freedom for Blocks, Treatments and Error.

- There are 4 types of treatment. The degrees of freedom for treament is therefore 3 $(r-1)$.
- There are 3 blocks. The degrees of freedom for blocks is therefore 2 (i.e. $c-1$).
- The total degrees of freedom should add up to 11. Therefore the degrees of freedom for error is 6. (i.e. $(r-1)\times(c-1)$.)
- **Remark:** Do not confuse $r-1$ with c. Here both are equal to 3, but this is a co-incidence.

Source	df	SS	MS	F	p.values
Treatment	3		57.111		
Block	2		12.25		
Error	6				
Total	11	238			

Step 3 : In general, Mean Square Terms are computed by dividing the relevant SS terms by the corresponding degrees of freedom.

$$MS = \frac{SS}{df}$$

Rearranging this , we can say : $SS = MS \times df$. Therefore

- $\text{SS}_{Trt} = 3 \times 57.111 = 171.33$
- $\text{SS}_{Block} = 2 \times 12.25 = 24.50$

Importantly

$$\boxed{SS_{tot} = SS_{trt} + SS_{block} + SS_{error}}$$

We can compute SS_{error}

$$238 = 171.33 + 24.5 + SS_{error}$$

Necessarily $SS_{error} = 42.166$

Source	df	SS	MS	F	p.values
Treatment	3	171.333	57.111		
Block	2	24.50	12.25		
Error	6	42.166			
Total	11	238			

Step 4 : We can now complete the table as follows

- MS_{error}

$$MS_{error} = \frac{SS_{error}}{df_{error}} - \frac{42.166}{6} = 7.033$$

- Test Statistic 1

$$F_{Trt} = \frac{MS_{Trt}}{MS_error} = \frac{57.111}{7.033} = 8.12$$

- Test Statistic 1

$$F_{Block} = \frac{MS_{Block}}{MS_error} = \frac{12.15}{7.033} = 1.74$$

Source	df	SS	MS	F	p.values
Treatment	3	171.333	57.111	8.12	
Block	2	24.50	12.250	1.74	
Error	6	42.166	7.033		
Total	11	238			