MA4702 2007/08

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Section A

1(a) $f(x) = \frac{1}{\sqrt{x-2}}$

(i) $f(4x^2+2) = \frac{1}{\sqrt{4x^2+2-2}}$

$= \frac{1}{\sqrt{4x^2}}$

$= \frac{1}{2x}$

(ii) Domain : $x-2 > 0$

$\Rightarrow x > 2$

$\Rightarrow (2,\infty)$

Range $\Rightarrow (0,\infty)$

(iii) $g(x) = x + \sin x$

$g(-x) = -x + \sin(-x)$

$= -x - \sin x$

$= -(x+\sin x)$

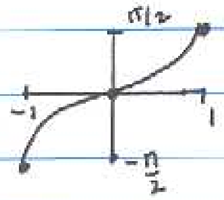
$= -g(x)$

$\Rightarrow$ odd

(b) $\sin^{-1}\left(\frac{1}{4}\right) = 14.47^\circ$

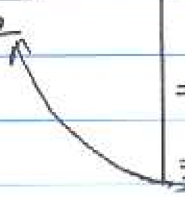
(i) or $.2526$ Radians

(ii)



(c) $\sinh 2x$

$\Rightarrow \frac{e^{2x} - e^{-2x}}{2}$



$2\sinh x \cosh x$

$2\frac{(e^x - e^{-x})}{2} \cdot \frac{(e^x + e^{-x})}{2}$

$\Rightarrow 2\frac{(e^{2x} - e^0 + e^0 - e^{-2x})}{4}$

$\Rightarrow \frac{e^{2x} - e^{-2x}}{2}$

$\Rightarrow \sinh 2x = 2\sinh x \cosh x$

2 $y = f(x) = x^4 - 6x^2 + 1$

(i) $x = 0 \Rightarrow y = 1 \quad \Rightarrow (0,1)$

(ii) $f'(x) = 4x^3 - 12x = 0$

$\Rightarrow x^3 - 3x = 0$

$= x(x^2-3) = 0$

$\Rightarrow x = 0,\ x^2 - 3 = 0$

$\Rightarrow x = 0 \quad x^2 = 3$

$\Rightarrow x = 0$, $y = 0$ | $x = \sqrt{3}$: $f(\sqrt{3}) = (\sqrt{3})^4 - 6(\sqrt{3})^2 + 1 = 9 - 18 + 1 = -8$ | $x = -\sqrt{3}$: $f(-\sqrt{3}) = (-\sqrt{3})^4 - 6(-\sqrt{3})^2 + 1 = 9 - 18 + 1 = -8$

$\Rightarrow (0,1)\ (\sqrt{3},-8)\ (-\sqrt{3},-8)$ critical points

classification:

$f''(x) = 12x^2 - 12$

$f''(0) = -12 < 0 \Rightarrow$ max. turning

$f''(\sqrt{3}) = 24 > 0 \Rightarrow$ min turning

$f''(-\sqrt{3}) = 24 > 0 \Rightarrow$ min turning

$\Rightarrow (0,1)$ max turning point $(\sqrt{3},-8)\ (-\sqrt{3},-8)$ min. turning points

(iii) $f''(x) = 12x^2 - 12 = 0$

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow (x-1)(x+1) = 0$

$\Rightarrow x = 1 \quad | \quad x = -1$

$\Rightarrow f(1) = -4 \quad | \quad f(-1) = -4$

$\Rightarrow (1,-4)\ (-1,-4)$

~~$\Rightarrow$ concave~~

(iv) $f''(x) = 12x^2 - 12$

$+ \quad -1 \quad - \quad 0 \quad 1 \quad +$

$f''(-2) = 36 > 0$

$f''(0) = -12 < 0$

$f''(2) = 36 > 0$

$\Rightarrow$ Concave up : $x < -1,\ x > 1$

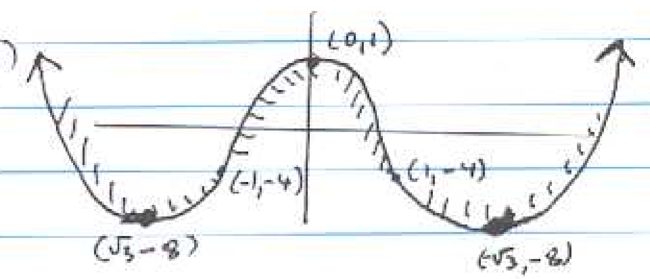
Concave down : $-1 < x < 1$

inflection points $(1,-4)\ (-1,-4)$

(v) $x \to +\infty,\ y \to +\infty$

$x \to -\infty,\ y \to +\infty$

(vi)



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Section B

3(a)
(i) $\int_0^1 2x(1+x^2)^5\,dx$

let $u = 1+x^2$

$\frac{du}{dx} = 2x$

$\Rightarrow du = 2x\,dx$

$\Rightarrow \frac{du}{2x} = dx$

$\Rightarrow \int \cancel{2x}.\,u^5.\,\frac{du}{\cancel{2x}}$

$\Rightarrow \int u^5\,du$

$\Rightarrow \frac{u^6}{6}$

$\Rightarrow \left.\frac{(1+x^2)^6}{6}\right|_{x=0}^{x=1}$

$\Rightarrow \frac{2^6}{6} - \frac{1^6}{6}$

$\Rightarrow \frac{64}{6} - \frac{1}{6}$

$\Rightarrow \frac{63}{6} = \boxed{10\tfrac{1}{2}}$

(ii) $\int \cos x \sin^3 x\,dx$

let $u = \sin x$

$\frac{du}{dx} = \cos x$

$\Rightarrow du = \cos x\,dx$

$\Rightarrow \frac{du}{\cos x} = dx$

$\Rightarrow \int \cancel{\cos x}.\,u^3.\,\frac{du}{\cancel{\cos x}}$

$\Rightarrow \int u^3\,du$

$\Rightarrow \frac{u^4}{4}$

$\Rightarrow \boxed{\frac{\sin^4 x}{4} + C}$

(iii) $\int x\cos x\,dx$

let $u = x$ | $dv = \cos x\,dx$

$\Rightarrow \frac{du}{dx} = 1$ | $\Rightarrow v = \int \cos x\,dx$

$\Rightarrow du = dx$ | $\Rightarrow v = \sin x$

$\int u\,dv = uv - \int v\,du$

$\Rightarrow \int x\cos x\,dx = x\sin x - \int \sin x\,dx$

$= x\sin x - (-\cos x) + c$

$= x\sin x + \cos x + c$

(b) $i(t) = 3 + \sin 2t$

$q(t) = \int 3 + \sin 2t\,dt$

$\Rightarrow q(t) = 3t - \frac{\cos 2t}{2} + c$

given $q = 0$ when $t = 0$

$\Rightarrow 0 = 3(0) - \frac{\cos 0}{2} + c$

$\Rightarrow 0 = 0 - \frac{1}{2} + c$

$\Rightarrow \frac{1}{2} = c$

$\Rightarrow q(t) = 3t - \frac{\cos 2t}{2} + \frac{1}{2}$

4 (a) $y = x^2 - 1$

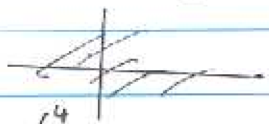
$y = 4x - 1$

$\Rightarrow x^2 - 1 = 4x - 1$

$\Rightarrow x^2 - 4x = 0$

$\Rightarrow x(x-4) = 0$

$\Rightarrow x = 0,\ x = 4.$



$A = \int_0^4 f(x) - g(x)\,dx$

$= \int_0^4 4x - 1 - (x^2 - 1)\,dx$

$= \int_0^4 4x - 1 - x^2 + 1\,dx$

$\Rightarrow \int_0^4 4x - x^2\,dx$

$\Rightarrow A = \left.\frac{4x^2}{2} - \frac{x^3}{3}\right|_{x=0}^{x=4}$

$= \left.2x^2 - \frac{x^3}{3}\right|_{x=0}^{x=4}$

$= \left(32 - \frac{64}{3}\right) - 0$

$= \frac{32}{3} = \boxed{10\tfrac{2}{3}}$

4(b) $y = \cosh(x^2+1)$

$h = \frac{2-1}{4} = \frac{1}{4} = 0.25$

$y = \cosh(x^2+1)$

$x =$	1	1.25	1.5	1.75	2
$y =$	3.762	6.522	12.914	29.068	74.21

~~$A = \frac{.25}{3}[(3.762 + 74.21)$~~

$A =$

$\frac{.25}{3}\left[(3.762 + 74.21) + 4(6.522 + 29.068) + 2(12.914)\right]$

$\Rightarrow \frac{1}{12}(77.972 + 142.34 + 25.828)$

$\Rightarrow \frac{1}{12}(246.14)$

$\Rightarrow 20.511$

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Section C

5 (a) $\sum_{n=1}^{\infty} \frac{5}{(5n+1)(5n+6)}$

$\Rightarrow a_n = \frac{1}{5n+1} - \frac{1}{5n+6}$

$\Rightarrow a_1 = \frac{1}{6} - \cancel{\frac{1}{11}}$

$a_2 = \cancel{\frac{1}{11}} - \cancel{\frac{1}{16}}$

$a_3 = \cancel{\frac{1}{16}} - \frac{1}{21}$

$\vdots$

$a_n = \cancel{\frac{1}{5n+1}} - \frac{1}{5n+6}$

$\Rightarrow S_n = \frac{1}{6} - \frac{1}{5n+6}$

$\Rightarrow \lim_{n\to\infty} S_n = \frac{1}{6} - \frac{1}{\infty}$

$= \frac{1}{6} - 0$

$= \boxed{\frac{1}{6}}$

(b)(i) $\sum_{n=1}^{\infty} \frac{3n+1}{n+5}$

$\Rightarrow \lim_{n\to\infty} \frac{3n+1}{n+5} = \frac{\infty}{\infty}$

$\Rightarrow \lim_{n\to\infty} \frac{3+\frac{1}{n}}{1+\frac{5}{n}} = \frac{3+0}{1+0}$

$\Rightarrow 3 \neq 0$

$\Rightarrow$ divergent series

(ii) $\sum_{n=1}^{\infty} \frac{2n^2+1}{n^4+2n^2-1}$ Compare with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$\frac{\left(\frac{2n^2+1}{n^4+2n^2-1}\right)}{\left(\frac{1}{n^2}\right)} = \frac{2n^2+1}{n^4+2n^2-1} \cdot \frac{n^2}{1}$

$= \frac{2n^4+n^2}{n^4+2n^2-1}$

$\Rightarrow \lim_{n\to\infty} \frac{2n^4+n^2}{n^4+2n^2-1} = \frac{\infty}{\infty}$

Divide by n^4

$\Rightarrow \lim_{n\to\infty} \frac{2+\frac{1}{n^2}}{1+\frac{2}{n^2}-\frac{1}{n^4}}$

$\Rightarrow \frac{2+0}{1+0-0}$

$\Rightarrow 2$

$\Rightarrow$ as $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent

$\Rightarrow \sum_{n=1}^{\infty} \frac{2n^2+1}{n^4+2n^2-1}$ is convergent

(iii) $\sum_{n=1}^{\infty} \frac{x^n}{n+2}$

$a_{n+1} = \frac{x^{n+1}}{n+3}$

$a_n = \frac{x^n}{n+2}$

$\frac{a_{n+1}}{a_n} = \frac{x^{n+1}}{n+3} \cdot \frac{(n+2)}{x^n}$

$= \frac{x(n+2)}{(n+3)}$

$\Rightarrow \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{x(n+2)}{(n+3)}\right|$

$= \frac{\infty}{\infty}$

$= \lim_{n\to\infty} \left|\frac{x(1+\frac{2}{n})}{(1+\frac{3}{n})}\right|$

$= \left|\frac{x(1+0)}{(1+0)}\right|$

$= \left|\frac{x}{1}\right|$

$= |x|$

Series is convergent for $|x| < 1$

6 (a) $f(x) = e^x$

$f(0) = 1$

$f'(x) = e^x$

$f'(0) = 1$

$f''(x) = e^x$

$f''(0) = 1$

$f'''(x) = e^x$

$f'''(0) = 1$

$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!}$

$\Rightarrow \boxed{e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}}$

$\Rightarrow e^{-x} = 1 + (-x) + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!}$ (i)

$\Rightarrow \boxed{e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}}$

(ii) $e^{\cdot 3} = 1 + (\cdot 3) + \frac{(\cdot 3)^2}{2!} + \frac{(\cdot 3)^3}{3!}$

$= 1 + \cdot 3 + \cdot 045 + \cdot 0045 = \boxed{1\cdot 3495}$

6 (b) $z = 4x^3 - 5xy + 3y^7$

(i) $\frac{\partial z}{\partial x} = 12x^2 - 5y$

$\frac{\partial^2 z}{\partial x^2} = 24x$

$\frac{\partial^2 z}{\partial y \partial x} = -5$

(ii) $z = \sin(3x+2y)$

$\frac{\partial z}{\partial x} = \cos(3x+2y).(3)$

$= 3\cos(3x+2y)$

$\frac{\partial^2 z}{\partial x^2} = 3(-\sin(3x+2y)).(3)$

$\boxed{\frac{\partial^2 z}{\partial x^2} = -9\sin(3x+2y)}$

$\frac{\partial z}{\partial y} = \cos(3x+2y).(2)$

$= 2\cos(3x+2y)$

$\frac{\partial^2 z}{\partial y^2} = 2(-\sin(3x+2y))(2)$

$\Rightarrow \boxed{\frac{\partial^2 z}{\partial y^2} = -4\sin(3x+2y)}$

$3\frac{\partial^2 z}{\partial y^2} - 2\frac{\partial^2 z}{\partial x^2}$

$3(-4\sin(3x+2y)) - 2(-9\sin(3x+2y))$

$\Rightarrow -12\sin(3x+2y) + 18\sin(3x+2y)$

$\Rightarrow 6\sin(3x+2y)$

$\Rightarrow 6z.$

Section D

7 (a)
```
Evalf((10+ sqrt(20) +4^3)/sqrt(5), 10);
```

(b)
```
Subs(x = (4+sqrt(6))/2, x^2 + 4*sqrt(x));
```

(c)
```
Factor(5*x^2 - 2*x^3 - 8 + 2*x);
```

(d)
```
plot(cos(x), x = -1..1);
```

(e)
```
Diff(x^3 * ln(x)/(2 - sin(x)), x);
    Simplify(%);
```

(f)
```
Diff(2*x^3 * ln(x), x $ 2);
    Simplify(%);
```

(g)
```
Int((sin(x))^2 * cos(2*x), x);
```

8 (i) x intercepts $x = -\frac{1}{2}, 2, -3$

y intercepts $y = -6$

(ii) $(\cdot 9433\text{---}, -12\cdot 028\text{---})$ min. turning point

$(-1\cdot 9433\text{----}, 12\cdot 028\text{----})$ max turning point

(iii) $(-\cdot 5, 0)$

(iv) $x \to +\infty, y \to +\infty$

$x \to -\infty, y \to -\infty$