Normal Distribution : Tutorial Sheet 2

1. Assume that a character in a game is programmed to have an attack power according to $X \sim \text{Normal}(\mu = 40, \sigma = 3)$.

   (a) What is the probability that the attack is greater than 45?

   (b) What is the probability that the attack is between 32 and 42?

   (c) Let X_1 and X_2 be the first and second attacks. What is the probability that the *sum* of these two attacks is greater than 85 units?

   (d) Calculate 99% limits for the sum of two attacks.

   (e) What is the probability that the *difference* in attacks is more than 5 units? Note that attack 2 can be 5 units more than attack 1 or attack 1 can be 5 units more than attack 2, i.e., $\Pr(|D| > 5) = \Pr(D < -5) + \Pr(D > 5)$.

2. A character in a game deals a standard attack 75% of the time and a critical attack the rest of the time (call these events S and S^c). Given that it is a standard attack, the attack power is $X \,|\, S \sim \text{Normal}(\mu = 40, \sigma = 3)$. When the character deals a critical attack, a random fluctuation is added to this according to a $\text{Normal}(\mu = 5, \sigma = 1)$ distribution.

   (a) What is the distribution of $X \,|\, S^c$?

   (b) Calculate $\Pr(X < 43 \,|\, S)$ and $\Pr(X < 43 \,|\, S^c)$.

   (c) Calculate $\Pr(X < 43)$. (hint: law of total probability)

   (d) If the character deals less than 43 damage points, what is the probability that the attack was a critical attack?

3. The income of a technician (in thousands) is $X_1 \sim \text{Normal}(\mu = 30, \sigma = 2)$. The income of an engineer is $X_2 \sim \text{Normal}(\mu = 40, \sigma = 3.5)$.

   (a) Calculate the probability that an engineer earns more than a technician.

   (b) Calculate 90% limits for the difference in their income.

   (c) For a group of 25 technicians, calculate the probability that the average wage is less than 30500, i.e., $\Pr(\overline{X}_1 < 30.5)$.

   (d) In a group of 10 engineers, what is the probability that *at least two* of them earn more than 45000? (hint: binomial with $p = \Pr(X_2 > 45)$)

   (e) For a sample of 30 technicians and 35 engineers, calculate the 80% limits for the difference in their average wages.

4. Let $X \sim \text{Normal}(\mu = 10, \sigma = 2)$. Calculate the following:

   (a) $\Pr(X > 10)$.

   (b) $\Pr(X < 3)$.

   (c) $\Pr(X > 8.4)$.

   (d) $\Pr(6 < X < 14)$.

(e) The value of x such that $\Pr(X > x) = 0.3$.

(f) The value of x such that $\Pr(X > x) = 0.8$.

5. Assume that speeds of cars on a motorway have a normal distribution with mean 115km/hr and standard deviation 4km/hr.

   (a) Draw a rough sketch of the distribution.

   (b) $\Pr(X > 120) = ?$

   (c) $\Pr(X < 100) = ?$

   (d) $\Pr(100 < X < 110) = ?$

   (e) 1% of drivers travel above what speed?

6. Assume that Z scores are normally distributed with a mean of Zero and a standard deviation of 1

   (a) $P(0 < Z < a) = 0.1685$ Find a

   (b) $P(-b \leq Z < b) = 0.95$ Find b

   (c) P$(Z \leq c) = 0.3015$ Find c

7. Assume that z scores are normally distributed with a mean of zero and standard deviation of 1.

   (a) If $P(0 < Z < a) = 0.4778$ find a

   (b) If $P(-bZb) = 0.7814$ find b

   (c) If $P(Z < c) = 0.0062$ find c.

8. Assume that the number of weekly study hours for students at a certain university is approximately normally distributed with a mean of 22 and a standard deviation of 6.

   (a) Find the probability that a randomly chosen student studies less than 12 hours.

   (b) Estimate the percentage of students that study more than 37 hours.

9. For *any* normal variable $X \sim \text{Normal}(\mu, \sigma)$:

   (a) Show that $\Pr(\mu - 3\,\sigma < X < \mu + 3\,\sigma) = 0.997$.

   (b) Find a value for k such that $\Pr(\mu - k\,\sigma < X < \mu + k\,\sigma) = 0.95$.

   (c) Find k such that $\Pr(\mu - k\,\sigma < X < \mu + k\,\sigma) = 0.99$.

   (d) Show that $\Pr(X > \mu + 1.64\,\sigma) = 0.05$.

10. The height of Dutchmen is normally distributed with mean 180cm and variance 144cm2. Calculate

    (a) the probability that a Dutchman is between 171cm and 195cm tall.

    (b) the height that 15% of Dutchmen exceed.

11. The lengths of Padraig Harrington's drives are normally distributed with mean of 250m and standard deviation of 15m. The lengths of Rory McIlroy's drives are normally distributed with a mean of 245m and a standard deviation of 20m. Calculate the probability that Rory drives further than Padraig.

12. IQ is defined to have a normal distribution with mean 100 and standard deviation 15.

    (a) Calculate the probability that a persons IQ is greater than 130

    (b) Calculate the probability that a persons IQ is less than 110

    (c) Calculate the probability that a persons IQ is between 82 and 120

    (d) Calculate the IQ that is exceeded by 15% of the population.

13. Assume that readings on thermometers are normally distributed with a mean of zero, a standard deviation of one degree. In each of the following readings draw a sketch and find the probability of the following readings occurring.

    (a) Between 0 and 1.50 degrees.

    (b) Less than 2.17 degrees.

    (c) Between -1.96 and + 1.96 degrees.

    (d) Between -2.00 and -1.50 degrees.