UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

Faculty of Science and Engineering
Department of Mathematics & Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4702

SEMESTER: Annual Repeats 10/11

MODULE TITLE: Technological Mathematics 2

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: J. O'Shea

PERCENTAGE OF TOTAL MARKS: 100 %

EXTERNAL EXAMINER: Prof. T. Myers

INSTRUCTIONS TO CANDIDATES: **Answer 5 questions, one each from sections A, B, C, D, and any other question.**

N.B. There are some useful formulae at the end of the paper.

University of Limerick approved calculators may be used.

SECTION A

1. (a) (i) $f(x) = \frac{1}{\sqrt{1-x^2}}$, find $f(\cos x)$ and simplify answer.

(ii) Prove that the function $f(x) = \frac{e^x - e^{-x}}{2}$ is odd.

(iii) Find $g^{-1}(x)$ the inverse of the function $g(x) = \log_e 2x$. 10

(b) (i) Evaluate $\tan^{-1}(-2)$.

(ii) Sketch the graph of $\sin^{-1} x$ (the principal value of the inverse sine curve) indicating clearly the domain and range of the function. 5

(c) Using their definition in terms of exponentials, prove the following hyperbolic identity:

$\cosh 2x \ = \ \cosh^2 x + \sinh^2 x.$ 5

2. Consider the function $y = f(x) = \frac{x}{x+4} \qquad (x \neq -4).$

(i) Find the x and y intercepts of $f(x)$. 2

(ii) Show that the function has no local maximum or local minimum turning point. 5

(iii) Explain why the function is increasing for all values of x. 2

(iv) Find the equation of the vertical asymptote. 3

(v) Find the equation of the horizontal asymptote. 3

(vi) Sketch the graph of $y = f(x)$ indicating clearly the features of the curve obtained in parts (i) - (v). 5

SECTION B

3. (a) Evaluate the following definite and indefinite integrals:

   (i) $\int_0^{\pi/2} \cos x(1+\sin x)^3 dx$ (ii) $\int x(x^2-1)^5 dx.$

   (iii) Use integration by parts to find $\int x \ln x dx.$ 15

   (b) A car has acceleration $a(t) = 2 + 4\sin 4t$ at time t. It starts from rest at time $t = 0$. Find its velocity at all time t. 5

4. (a) Find the area enclosed by the curve $y = 9 - x^2$ and the line $y = x + 3$. 10

   (b) Use Simpson's Rule with 4 equal subintervals to find an approximation for $\int_0^2 \sinh x^2 dx$. 10

SECTION C

5. (a) Find the sum of the telescoping series

$$\sum_{n=1}^{\infty} \frac{2}{(2n-1)(2n+1)}$$ 5

(b) Test the following series for convergence

(i) $\sum\limits_{n=1}^{\infty} \frac{n+4}{5n+3}$

(ii) $\sum\limits_{n=1}^{\infty} \frac{2n+3}{n^2+4n+1}$

(iii) $\sum\limits_{n=1}^{\infty} \frac{n+1}{3^n}$ 15

6. (a) Find the Maclaurin series of $\sin x$ up to and including the term containing x^5.

Use your answer to find the Maclaurin series of

(i) $\cos x$

(ii) $\cos 3x$. 10

(b) (i) Find $\frac{\partial z}{\partial x}$, $\frac{\partial^2 z}{\partial x^2}$ and $\frac{\partial^2 z}{\partial y \partial x}$ of the function $z = 3x^2 y + 2x \sin y$.

(ii) Prove that the function $z = \sin(2x + t)$ satisfies the partial differential equation $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial t^2} = 0$. 10

SECTION D

7. Write down the Maple commands which implement the following;

   (Do not attempt to find the answers of the Maple output.)

   (a) Evaluate $\left(\frac{6^7}{\sqrt{2^4-5}}\right)^3$ to 20 significant figures. 3

   (b) Substitute $x = 1$ into $(\cos x + 1)e^{2x+3}$. 2

   (c) Find the factors of the quartic polynomial: $2x^4 - x^3 - 45x^2 + 58x + 40$. 3

   (d) Plot $y = \ln x$ for $-1 \leq x \leq 7$. 3

   (e) Find the first derivative of $\frac{4\sin 3x}{x^2-2}$ with respect to x and simplify the answer. 3

   (f) Find the second derivative of $\frac{(8+x)^2}{\sin^2 x+1}$ with respect to x. 3

   (g) Evaluate the definite integral $\int_1^2 \sin x \cosh x dx$. 3

8. The output of a Maple session, investigating the properties of some function $y = f(x)$ is represented on the next page.

   (a) Based on this output:

   (i) Find the x and y intercepts of $f(x)$ (if any).

   (ii) Find the x and y co-ordinates of all maxima and minima turning points of $f(x)$.

   (iii) Find the x and y co-ordinates of all points of inflection of $f(x)$.

   (iv) Discuss the behaviour of $f(x)$ as $x \to +\infty$ and $x \to -\infty$. 15

   (b) Based on the information given in the output, plot $y = f(x)$ in the domain $-5 \leq x \leq 5$ labelling the parts found in (a). 5

```
> solve(y=0);
```

$$2,\ \frac{1}{2},-1$$

```
> subs(x=0,y);
```

$$2$$

```
> y1:=diff(y,x):
> solve(y1=0);
```

$$\frac{1}{2}+\frac{1}{2}\sqrt{3},\frac{1}{2}-\frac{1}{2}\sqrt{3}$$

```
> evalf(%,5);
```

$$1.3660,-0.36605$$

```
> evalf(subs(x=(1+sqrt(3))/2,y),5);
```

$$-2.5984$$

```
> evalf(subs(x=(1-sqrt(3))/2,y),5);
```

$$2.5981$$

```
> y2:=diff(y1,x):
> subs(x=(1+sqrt(3))/2,y2);
```

$$6\sqrt{3}$$

```
> subs(x=(1-sqrt(3))/2,y2);
```

$$-6\sqrt{3}$$

```
> solve(y2=0);
```

$$\frac{1}{2}$$

```
> subs(x=1/2,y);
```

$$0$$

```
> evalf(subs(x=1000,y));
```

$$1.996997002\ 10^{9}$$

```
> evalf(subs(x=-1000,y));
```

$$-2.002996998\ 10^{9}$$

Formulae

1. Trigonometry : Tables (Old) Page 9.
2. Logarithms :

$$a^x = y \iff \log_a y = x$$

3. Hyperbolic functions :

$$\sinh x = \frac{1}{2}(e^x - e^{-x}) \; ; \qquad \cosh x = \frac{1}{2}(e^x + e^{-x})$$

4. Calculus

Derivatives

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
e^{ax}	ae^{ax}
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\cosh x$	$\sinh x$
$\sinh x$	$\cosh x$

Product Rule:

$$\begin{aligned} y &= uv \\ \frac{dy}{dx} &= u\frac{dv}{dx} + v\frac{du}{dx} \end{aligned}$$

Quotient Rule:

$$\begin{aligned} y &= \frac{u}{v} \\ \frac{dy}{dx} &= \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \end{aligned}$$

Integrals (constants of integration omitted)

$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln \lvert x \rvert$
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$

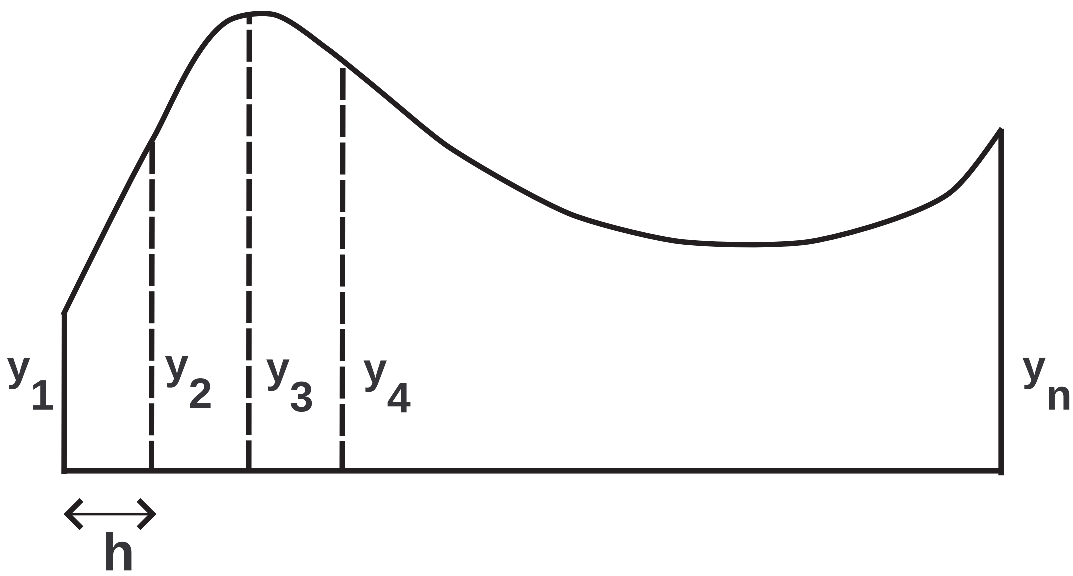
Integration by parts

$$\int udv = uv - \int vdu$$

5. Maclaurin Series

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \ldots + \frac{f^{(R)}(0)x^R}{R!} + \ldots$$

6. Simpson's Rule for odd n



A represents the area of the shape.

$$A \approx \frac{h}{3}\left[y_1 + y_n + 2(y_3 + y_5 + \ldots + y_{n-2}) + 4(y_2 + y_4 + \ldots y_{2n-1})\right]$$