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PROBLEM SHEET 3: NUMERICAL INTEGRATION. GAUSSIAN QUADRATURE

Gaussian quadrature methods can be used for the evaluation of integrals of the type

$$\int_a^b W(x)f(x)\,dx \tag{1}$$

where $W(x)$ is an "ill-behaved" function, for example a function with singularities at the end-points a and b. Examples include

$$\int_1^1 \frac{f(x)}{\sqrt{1-x^2}}, \qquad \int_0^1 f(x)\log(\frac{1}{x})\,dx; \qquad \int_0^1 \frac{f(x)}{\sqrt{x}}\,dx$$

To approximate (1), we write

$$\int_a^b W(x)f(x)\,dx \approx w_1 f(x_1) + w_2 f(x_2) + \cdots + w_n f(x_n) \tag{2}$$

and determine the nodes x_i and weights w_i so that the polynomials 1, x, $x^2, \ldots, x^{2n-1}$ are integrated exactly.

Examples:

1. Obtain a one-point Gaussian quadrature formula for the general integral

$$\int_0^1 \frac{f(x)}{\sqrt{x}}\,dx$$

and use it to approximate

$$\int_0^1 \frac{\cos(\pi x)}{\sqrt{x}}\,dx.$$

Repeat the problem for a two-point quadrature formula.

2. Obtain a one-point Gaussian quadrature formula for the general integral

$$\int_{-1}^1 \frac{f(x)}{\sqrt{1-x^2}}\,dx$$

and use it to approximate

$$\int_{-1}^1 \frac{\cos(x)}{\sqrt{1-x^2}}\,dx.$$

Repeat the problem for a two-point quadrature formula.