DUBLIN INSTITUTE OF TECHNOLOGY
KEVIN STREET, DUBLIN 8

B.Sc. in Physics Technology

B.Sc. in Science with Nanotechnology

B.Sc. in Clinical Measurement

B.Sc. in Physics with Medical Physics and Bioengineering

YEAR II

WINTER EXAMINATIONS 2011

MATHEMATICS FOR THE PHYSICAL SCIENCES II

DR. D. MACKEY
DR. C. HILLS

Tuesday, 11 January 2011   9.30 a.m. to 11.30 a.m.

Full marks for complete answers to **FOUR** questions.

Graph Paper, Mathematics Tables.

1 (a) Show that the function $f(x) = \frac{1}{x} + \frac{x}{2}$, where $x \neq 0$, is a solution of the initial value problem
$$\frac{df}{dx} = 1 - \frac{1}{x}f(x), \qquad f(2) = \frac{3}{2}.$$
[8 marks]

(b) Using the method of the integrating factor, solve the linear first order differential equation
$$\frac{df}{dx} + \frac{f(x)}{x+5} = 4$$
with initial condition $f(0) = 0$.
[9 marks]

(c) If 16 grams of a radioactive substance were present at time $t = 1$ year and 2 grams were present at time $t = 4$ years, how much was present initially $(t = 0)$ and what is the half-life of the substance?

(You may assume that the amount $N(t)$ present at time t decays exponentially, according to a law of the form $N(t) = N_0 e^{-kt}$, where $N_0 = N(0)$.)
[8 marks]

2 (a) Find all the second order partial derivatives for the following function
$$f(x, y) = x^2 \ln(2x + 3y)$$
and verify that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
[10 marks]

(b) Let $f(x, y) = 2x^3 - 6xy + 3y^2$. Show that f has two critical points and decide whether each of them is a maximum, minimum or saddle point.
[15 marks]

3 (a) Let $Z_1 = 4 - 3i$ and $Z_2 = 2 + i$. Evaluate the real and imaginary part of the complex expression

$$\frac{1}{Z_1 - Z_2} + \frac{1}{Z_1 Z_2}.$$

[7 marks]

(b) Show that i is a root of the following equation and determine the other 3 roots

$$z^4 - 2z^3 + 6z^2 - 2z + 5 = 0.$$

[8 marks]

(c) Use DeMoivre's formula to express the complex number

$$\left(\frac{-1 + i\sqrt{3}}{2}\right)^5 + \left(\frac{-1 - i\sqrt{3}}{2}\right)^5$$

in the form $a + ib$. [10 marks]

4 (a) Evaluate the following double integral

$$\int_1^4 \int_{-2}^3 (x^2 - 2xy^2 + y^3)\, dxdy$$

[10 marks]

(b) Sketch the region of integration, reverse the order of integration and evaluate the following integral

$$\int_0^1 \int_y^1 x^2 e^{xy}\, dxdy.$$

[15 marks]

5 (a) Plot the cardioid $r = 2(1 + \cos(\theta))$ from $\theta = 0$ to $\theta = 2\pi$ using $\frac{\pi}{6}$ intervals.

[10 marks]

   (b) Use integration in polar coordinates to determine the area enclosed by the cardioid in part (a).

[15 marks]

6 (a) One of the eigenvalues of the matrix A, where

$$A = \begin{pmatrix} 4 & -6 & 2 \\ -1 & 9 & -2 \\ -4 & 12 & -2 \end{pmatrix}$$

is 6. Find the other eigenvalues of A.

[10 marks]

   (b) Given that

$$\begin{pmatrix} -2 \\ 1 \\ \alpha \end{pmatrix}$$

is an eigenvector of the matrix

$$B = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix}$$

find α and the corresponding eigenvalue.

[15 marks]