FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS AND STATISTICS

END OF SEMESTER EXAMINATION PAPER 2015

MODULE CODE: MS4131 SEMESTER: Spring 2016

MODULE TITLE: Linear Algebra 1 DURATION OF EXAM: 2.5 hours

LECTURER: Mr. Kevin O'Brien GRADING SCHEME: 80 marks
100% of module grade

EXTERNAL EXAMINER: Prof. J. King

INSTRUCTIONS TO CANDIDATES

Scientific calculators approved by the University of Limerick can be used.
Students must attempt any 4 questions from 5.

Question 1

Part A. Matrix Multiplication (4 Marks)

Given the matrices

$$A = \begin{pmatrix} 2 & 2 & 1 & -1 & 4 \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 4 \\ 0 & 5 \\ -1 & 0 \\ 7 & 1 \\ 1 & 3 \end{pmatrix}; \quad C = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

calculate the products AB and CA.

Part B. Diagonal Matrices (3 Marks)

Consider the following diagonal matrix D. In terms of the values a, b and c ;

$$D = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

(i) (1 Mark) Write an expression for the trace of the matrix D.

(ii) (1 Mark) State the inverse of D, i.e. D^{-1}.

(iii) (1 Mark) State the matrix D^3.

Part C. Matrix Multiplication (4 Marks)

Suppose A is a lower triangular matrix of the form;

$$A = \begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix}$$

(i) (1 Mark) State the transpose of A.

(ii) (2 Marks) Compute B where B = $A \times A^T$.

(iii) (1 Mark) B is a symmetric matrix. What is meant by this?

Question 2

Part A. Fundamental Theorem of Invertible Matrices (5 Marks)

The Fundamental Theorem of Invertible Matrices states that a set of mathematical expressions concerning a $n \times n$ matrix A are each equivalent to one another.

(i) (4×1 Mark) State any four of these expressions.

(ii) (1 Mark) What is the rank of a matrix.

Part B. Invertible Matrices (5 Marks)

Show that if A is an $n \times n$ invertible matrix that satisfies

$$9A^3 + A^2 - 3A = 0$$

where $A^n = \underbrace{A \dots A}_{n \text{ times}}$, I is the $n \times n$ identity matrix and 0 is the $n \times n$ zero matrix, then the inverse of A is given by

$$A^{-1} = \frac{1}{3}I + 3A.$$

Part C. Inverting a Matrix with E.R.O.s (5 Marks)

Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & -2 & 4 \\ 1 & -4 & 1 \\ -3 & 0 & -1 \end{pmatrix}.$$

using elementary row operations.

Question 3

Part A. System of Linear Equations

Consider the linear system

$$\begin{aligned} x_1 + x_3 &= 4 \\ 2x_1 + 4x_2 + x_3 &= -3 \\ x_2 + 3x_3 &= 7. \end{aligned}$$

(i) (1 Mark) Write down the coefficient matrix and the augmented matrix of this system.

(ii) (1 Mark) What can you say about the solution set of the system? Justify your answer.

(iii) (4 Marks) Solve the system of equations, using any appropriate method.

Question 4

Consider the three vectors in $\mathbb{R}^3$:

$$u = (1, 2, 0), \quad v = (0, 1, 3), \quad w = (1, 2, 3).$$

(i) Evaluate $\|u\|$, $\|v\|$, $u \cdot v$, $u \times v$ and the angle between u and v.

(ii) Calculate the scalar triple product $u \cdot (v \times w)$.

Question 5