Tutorial Sheet 3

Let the function f be given by the rule $f(x) = x - \log(x) - \sqrt{2}$.

1. Using intervals of 0.5, evaluate f from $x = 0.5$ to $x = 3$ and hence sketch the graph of $f(x)$ over this interval.

2. Use the bisection method to estimate the root of $f(x)$ in the interval $[0.5, 3]$. Start with an interval of length 1 and iterate until the size of the interval is less than 0.01.

3. Given that $f'(x) = 1 - \frac{1}{x}$, use the Newton-Raphson method to estimate the root of $f(x)$ in the interval $[0.5, 3]$. Use an integer as an initial estimate for the root. Ans $\cong 2.20489216557483$

n		x_n		$f(x_n)$		$f'(x_n)$		$f(x_n)/f'(x_n)$
0								
1								
2								
3								
4								

4. Use Newton's Method to find a root of $f(x)$ starting with an initial guess of 0.5. Ans $\cong 0.342380252644745$

n		x_n		$f(x_n)$		$f'(x_n)$		$f(x_n)/f'(x_n)$
0								
1								
2								
3								
4								
5								

Tutorial Sheet 4

Derivatives of polynomials $f(x) = x^n, \quad f'(x) = nx^{n-1} \qquad f(x) = C, f'(x) = 0$
$f(x) = Ag(x) + Bh(x), f'(x) = Ag'(x) + Bh'(x)$

1. Find the derivatives of the following polynomials

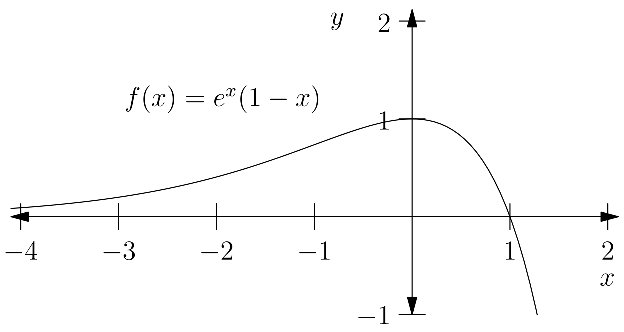
   i) $x^2 + x + 1$ Ans. $2x + 1$
   ii) $3x^2 - 9x + 4$ Ans. $6x - 9$
   iii) $\frac{1}{x}$ Ans. $-\frac{1}{x^2}$
   iv) $\frac{4}{x^3}$ Ans. $-\frac{12}{x^4}$
   v) $\frac{10}{x^4} - \frac{3}{x^5}$ Ans. $\frac{15}{x^6} - \frac{40}{x^5}$
   vi) $\frac{x^2}{3} + \frac{4x}{5}$ Ans. $\frac{2x}{3} + \frac{4}{5}$
   vii) $\sqrt{x}$ Ans. $\frac{1}{2\sqrt{x}}$
   viii) $11x^7 - 7x^{11} + 12$ Ans. $77(x^6 - x^{10})$
   ix) $7x^4 - x^3 + x(x-1)$ Ans. $28x^3 - 3x^2 + 2x - 1$

2. The Newton-Heron method to compute $\sqrt[k]{D}$ starts from an initial guess x_0 and updates according to the rule

$$x_{n+1} = \left(\frac{k-1}{k}\right) x_n + \frac{D}{kx_n^{k-1}}$$

   Use Newton's method to derive this formula by considering the problem of finding the k^{th} root as an inverse problem.

3.



Consider the function

$$f(x) = e^x(1-x)$$

Show that this function has a root at $x = 1$. Comment on using Newton's method to estimate this root using the initial guesses $-1, 0, 1, 2$.

4. By considering the inverse problem, use Newton's method to estimate $\cosh^{-1}(2)$ using an initial guess of $\cosh^{-1}(2) \cong 1$. Note that the derivative of $\cosh(x)$ is $\sinh(x)$. Stop when the cosh of your estimate is within 0.001 of 2.

5. Given $f(x) = \tan^{-1}(x)$ and $f'(x) = \frac{1}{1+x^2}$, use Newton's method to estimate the root of f using the following initial estimates and comment on the results.

   i) $x_0 = 0$ ii) $x_0 = 1$ iii) $x_0 = 2$