Laplace Transform

- It turns out that many problems are greatly simplified when converted to the complex frequency domain.
- For example, integration and differentiation in the time domain become simple algebraic expressions in the complex frequency domain.
- The Laplace transform converts a problem between these two domains.
- This type of calculation can be done entirely in the time domain, but it requires solving differential equations, which is challenging and time-consuming.
- The Laplace transform technique is a huge improvement over working directly with differential equations.

Laplace Transform

- The *Laplace* transform of $f(t)$ is denoted $F(s)$
- $f(t)$ is a function in the time domain
- $F(S)$ is the function in the complex frequency domain that corresponds to $F(s)$
- The *Laplace* transform operator is denoted $\mathscr{L}$

Laplace Transforms

- We will use a set of tables of *Laplace* transforms.

Laplace Transforms : Example 1

Compute the Laplace transforms of the time domain function $f(t)$

$$f(t) = t^4$$

Laplace Transforms : Example 1

Table: Laplace Transforms Tables

f(t)	F(s)
$\ldots$	$\ldots$
t^n	$\frac{n!}{s^{n+1}}$
$\ldots$	$\ldots$

Laplace Transforms : Example 2

Compute the Laplace transforms of the time domain function $f(t)$

$$f(t) = 4\sin(2t)$$

Laplace Transforms : Example 2

Table: Laplace Transforms Tables

f(t)	F(s)
$\dots$	$\dots$
$\sin(at)$	$\frac{a^2}{s^2+a^2}$
$\dots$	$\dots$

Laplace Transforms : Example 3

Compute the Laplace transforms of the time domain function $f(t)$

$$f(t) = \frac{e^{-t}}{2}$$

Laplace Transforms : Example 3

Table: Laplace Transforms Tables

f(t)	F(s)
$\ldots$	$\ldots$
e^{at}	$\frac{1}{s-a}$
$\ldots$	$\ldots$