Section A (6 Questions, 25 Marks Each)

Question 1

(a) Solve the following simultaneous equations for x and y :

$$\frac{x+1}{2} - \frac{y+3}{3} = 4 \qquad ; \; x + \frac{y-3}{2} = \frac{1}{2}$$

(b) Let $f(x) = ax^3 - 5x^2 - bx + 18$

(i) If -2 and $+3$ are roots of the equation $f(x) = 0$, find the values of a and b.

(ii) If $f(k) = 0$, with $k \neq -2, 3$, find the value of k

Question 2

(a) Let $f(x) = 2x^3 - 9x^2 + 20x - 8$

(i) Verify that $f(1/2) = 0$.

(ii) Plot on an argand diagram the three roots of $f(x) = 0$.

(iii) Find the area of the triangle formed by the three points found in part (ii) above.

(b) Let

$$\frac{\sqrt{2}\,(\cos\theta + i\sin\theta)}{2+i} = \frac{1-3i}{5}$$

where $i = \sqrt{-1}$

(i) Show that $\cos\theta + i\sin\theta = \dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}i$.

(ii) Hence find the value of θ, where $0 \leq \theta < 2\pi$.

Question 3

(a) Evaluate the follow expressions:

(i) $$\lim_{x\to\infty} \frac{2x^2+5}{x^2+3x}$$

(ii) $$\lim_{x\to 0}\left(2x+\frac{1}{x}\right)^2-\left(\frac{1}{x}-3x\right)^2$$

(b) Differentiate the function $(3x-5)^2$ from first principles, with respect to x.

(c) Find the slope of the tangent to the curve, $y = x\sin(\frac{1}{x})$ when $x = \frac{4}{\pi}$, correct to one decimal place.

Question 4

The straight line $2x + 2y = a$ intersects the circle $x^2 + y^2 = a^2$ at the points P and Q.

(i) Find the coordinates of P and Q in terms of a, where $a \in \mathbb{R}$.

(ii) Find the equation of the straight lines joining P and Q to the origin O.

(iii) Find the angle $\angle POQ$.

(iv) Hence, or otherwise, find the smaller area enclosed by the chord PQ and the circle.

Question 5

(a) Evaluate the follow expressions:

(i) $\int_0^{\ln 4} e^x \, dx$

(ii) $\int_0^4 \frac{x^3 dx}{\sqrt{3x^2+1}} \, dx$

(b) Sketch the curve $y = 4/x, \ x > 0$.

The area between the curve $y = 4/x$ and the lines $x = 1$ and $x = 2$ is equal to the area between the curve and the lines $x = t$ and $x = 1$, where $0 < t < 1$. Find the value of t.

Question 6

(a) Three students A,B and C are given a puzzle to solve. If on past performance, their probabilities of success are $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{3}$, find the probability that

(i) all three are successful.

(ii) only one is successful.

(iii) at least one is successful.

(b) $\{1, a, b\}$ is a set of numbers, whose mean is 5.

(i) Write b in terms of a.

(ii) Write the standard deviation, σ, in terms of a.

(iii) If $\sigma = \sqrt{14}$, find the values of a and b.

Section B (3 Questions, 50 Marks Each)

Question 7

In the planning phase, Irish Water needed to estimate water usage in households around the country. Water meters were installed in a sample of houses, with the following results:

Household Size	Weekly Water Consumption (Litres)
2	650
7	1200
9	1300
4	450
10	1400
6	900
8	1800
3	650
3	800
2	900

(i) Identify the dependent and independent variables.

(ii) Represent the data in a scatterplot, showing the relevant axes.

(iii) Calculate the coefficient of linear correlation.

(iv) What can you conclude from your results in parts (ii) and parts (iii)?

(v) Add the line of best fit to the completed scatterplot in (ii).

(vi) Use the line of best fit to estimate the weekly consumption of a household with 5 occupants.

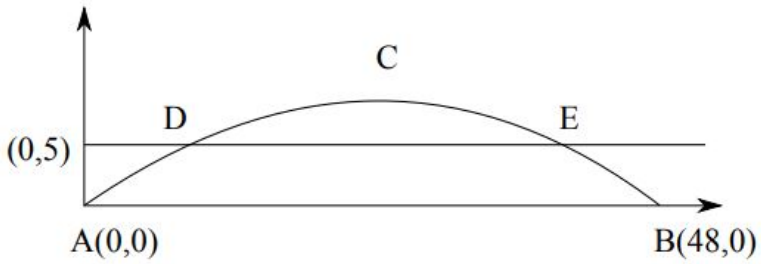
(vii) By taking suitable readings from your digram, or otherwise, calculate the line of best fit.

(viii) Explain how to interpret the slope of the line in this context.

Suppose that instead of measuring the water consumption in litres, the engineers had measured the water consumption in gallons. If you were to convert the values in the table above from litres to gallons, what effect would this have on the correlation correlation? (**Remark:** *one gallon = 4.55 litres).*

Question 8

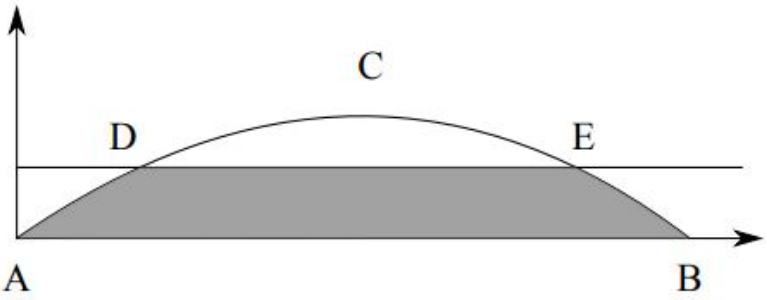
The arch of a bridge is in the shape of a parabola, as shown. The length of the span of the arch, [AB], is 48 metres.



(i) Using the coordinate plane, with A(0, 0) and B(48, 0), the equation of the parabola is $y = 0.013x^2 + 0.624x$.
Find the coordinates of C, the highest point of the arch.

(ii) The perpendicular distance between the walking deck, $[DE]$, and $[AB]$ is 5 metres. Find the coordinates of D and of E.
Give your answers correct to the nearest whole number.

(iii) Using integration, find the area of the shaded region, $ABED$, shown in the diagram below.
Give your answer correct to the nearest whole number.



(iv) Write the equation of the parabola in part (i) in the form $y - k = p(x - h)^2$, where k, p and h are constants.

(v) Using what you learned in part (iv) above, or otherwise, write down the equation of a parabola for which the coefficient of x^2 is -2 and the coordinates of the maximum point are $(3, -4)$.

Question 9

(a) The cost €C of running a ship per hour is given by

$$C = 4 + \frac{V^3}{2500},$$

where V is the speed in knots through the water.

(i) What is the most economical speed through the water at which a voyage can be made against a current of 5 knots?

(ii) Calculate the minimum cost to the nearest 10 euro?

(iii) Sketch the graph of the cost (C) versus the speed (V) in the interval $0 < V < 20$.

(b) Oil is poured into a cylindrical tank of radius 1.2m, length 2.4 m with its axis horizontal. When the depth of the oil is 0.6 m, the level of the oil is rising at the rate of 0.25cm per second. How fast is the oil pouring into the tank? Give your answer in cubic meters per minute.