Question 3

Part A. Equation of a Line (4 Marks)

Find the equation of the line ℓ in $\mathbb{R}^2$, which passes through the points $(2, 1)$ and $(1, 3)$. Let $Q = (1, -3)$ be a point in $\mathbb{R}^2$.

(i) Verify that Q does not lie on the line ℓ.

(ii) Find the distance between the point Q and the line ℓ.

Part B. Row-Echelon Form of a Matrix (4 Marks)

Consider the matrices U, V, W and X presented below. For each matrix state one reason why that matrix in not in row-echelon form. Provide distinct answers for each of the four matrices.

$$U = \begin{pmatrix} 1 & 2 & 6 & 3 & 5 \\ 0 & 1 & 4 & 0 & 6 \\ 0 & 1 & 2 & 1 & 7 \\ 0 & 0 & 1 & 1 & 7 \end{pmatrix} \qquad V = \begin{pmatrix} 1 & 1 & 6 & 8 & 5 \\ 0 & 2 & 6 & 0 & 6 \\ 0 & 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 1 & 7 \end{pmatrix}$$

$$W = \begin{pmatrix} 1 & 1 & 6 & 8 & 5 \\ 0 & 1 & 6 & 0 & 6 \\ 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 1 & 1 & 7 \end{pmatrix} \qquad X = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 6 & 8 & 5 \\ 0 & 1 & 6 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(**Marking Scheme**: 4×1 *Mark where 1 Mark is awarded for each valid and distinct reason*)

Part C. Distance from Planes

Ggive the general form of the equation of the plant π in $\mathbb{R}^3$ passing throughthe point $P_0 = (1, 0, 2)$ with the vector $n = (-5, 3, 2)$ as the normal.

Part D. Equations of Planes

Show that the point $Q = (1, -1, 1)$ does not lie in the plane π and find its distance from π.

- Are the points

$$P_1 = (1, 2, 0), \quad P_2 = (3, 5, 0), \quad P_3 = (7, 3, 0), \quad P_4 = (-5, 3, 0)$$

coplanar? If yes, what is the equation of the plane containing them?

1. Find the equation of the line ℓ in $\mathbb{R}^2$, which passes through the points $(2, 1)$ and $(1, 3)$.
2. Let $Q = (1, -3)$ be a point in $\mathbb{R}^2$.
   (a) Verify that Q does not lie on the line ℓ.
   (b) Find the distance between the point Q and the line ℓ.