Numerical Methods 12 November 2012
Dr Dana Mackey (Dana.Mackey@dit.ie)

Problem Sheet 3: Gaussian elimination

1. For what value of k does the following system of equations have infinitely many solutions?

$$\begin{aligned} x + ky + z &= 1 \\ -y + z &= 2 \\ x + y + 2z &= 3 \end{aligned}$$

2. Find the value of k for which the following system is *consistent* (that is, it has at least one solution) and then find the solution when k has this particular value.

$$\begin{aligned} x - y + 2z &= -2 \\ 2x + 3y + 4z &= 7 \\ 4x - 7y + 5z &= k \\ 8x - 4y + 6z &= 2 \end{aligned}$$

3. Using Gaussian elimination, solve the following system

$$\begin{aligned} 3x + 5y + 2z + 4w &= 0 \\ x + 2y - z + w &= -2 \\ -2x - y + z - w &= 2 \\ 4x + 3y + 3z + 3w &= 4 \end{aligned}$$

4. Consider the following system

$$\begin{aligned} 0.002x_1 + 1.231x_2 + 2.471x_3 &= 3.704 \\ 1.196x_1 + 3.165x_2 + 2.543x_3 &= 6.904 \\ 1.475x_1 + 4.271x_2 + 2.142x_3 &= 7.888. \end{aligned}$$

   (a) By direct substitution, show that the exact solution is $x_1 = 1$, $x_2 = 1$ and $x_3 = 1$.

   (b) Use Gaussian elimination with and without pivoting to solve this system. Use 4-digit rounding arithmetic.

   (c) Calculate the relative errors for each of the methods used in part (b). Can you explain why the results are so different?

5. For each of the following systems, use Gaussian elimination without pivoting, with partial pivoting and scaled partial pivoting and compare the results obtained from each technique with the exact solution. Use 3-digit rounding arithmetic.

   (a)

$$\begin{aligned} 3.03x_1 - 12.1x_2 + 14x_3 &= -119 \\ -3.03x_1 + 12.1x_2 - 7x_3 &= 120 \\ 6.11x_1 - 14.2x_2 + 21x_3 &= -139 \end{aligned}$$

   (Exact solution: $(0, 10, \frac{1}{7})$.)

   (b)

$$\begin{aligned} x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 &= 2 \\ \frac{1}{2}x_1 + \frac{1}{3}x_2 + \frac{1}{4}x_3 &= -1 \\ \frac{1}{3}x_1 = \frac{1}{4}x_2 + \frac{1}{5}x_3 &= 0 \end{aligned}$$

   (Exact solution: $(54, -264, 240)$.)

   (c)

$$\begin{aligned} -149x_1 - 50x_2 - 154x_3 &= 353 \\ 537x_1 + 180x_2 + 546x_3 &= -1263 \\ -27x_1 - 9x_2 - 25x_3 &= 61 \end{aligned}$$

   (Exact solution: $(-1, -1, -1)$.)