Numerical Methods 22 February 2012
Dr Dana Mackey (Dana.Mackey@dit.ie)

Problem Sheet 2: Numerical integration. Composite Newton-Cotes quadrature

1. Determine the number of intervals needed, in both the composite trapezoidal rule and the composite Simpson's rule, such that when approximating the value of the integral
$$I = \int_0^1 \mathrm{e}^{-x^4}\, dx$$
the error is less than 10^{-5}. Calculate such an approximation to I, using both methods.
Hint: You may assume that, if $f(x) = \mathrm{e}^{-x^4}$ *then*
$$\max_{0\le x\le 1} |f''(\xi)| \le 3.5 \qquad \text{and} \qquad \max_{0\le x\le 1} |f^{(4)}(\xi)| \le 95$$

2. Repeat Question 1 for the integral
$$I = \int_1^3 \log(x)\, dx$$

3. Derive the composite midpoint rule and check that it has order of convergence $O(h^2)$ by approximating the value of the integral
$$I = \int_0^1 \sqrt{1+x^3}\, dx$$
Hint: The explicit form of the error term for the midpoint rule is given below:
$$I(f) = \int_a^b f(x)\, dx = (b-a) f\left(\frac{a+b}{2}\right) + \frac{(b-a)^3}{24} f''(\xi), \qquad a \le \xi \le b.$$

4. Approximate the value of the following integrals using the composite trapezoidal rule, the composite midpoint rule and the composite Simpson's rule. For each method, use the smallest value of n that will guarantee an absolute error less than 5×10^{-5}.
$$\text{(i)} \int_1^2 \frac{1}{x}\, dx; \qquad \text{(ii)} \int_0^1 \mathrm{e}^{-x}\, dx$$

5. Approximate the value of the integral
$$I = \int_0^1 2x f(x)\, dx$$
where f is given by

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
f(x)	0.667	0.671	0.689	0.711	0.742	0.790	0.841	0.910	0.975	1.052	1.130