# Problem Sheet 4: Complex numbers

1. Solve the quadratic equation
$$z^2 - 4iz + 5 = 0.$$

2. Let $z = \sqrt{3} - 2i$. Evaluate the real and imaginary part of the complex expression
$$\frac{1}{2}\left(\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right).$$

3. Let $z = a + ib$ be a complex number, $z \neq 1$, and let
$$W = \frac{z+1}{z-1}.$$
Show that, if $|z| = 1$ then the real part of $W$ is zero.

4. Use DeMoivre's formula to express each of the complex numbers
$$(4+4i)^7; \qquad (\sqrt{3}+i)^{20}; \qquad (2-2i)^5$$
in the form $a + ib$.

5. Use DeMoivre's formula to express the complex number
$$\left(\frac{-1+i\sqrt{3}}{2}\right)^5 + \left(\frac{-1-i\sqrt{3}}{2}\right)^5$$
in the form $a + ib$.

6. Use DeMoivre's formula to express the complex number
$$(2+i)^5 + (2-i)^5$$
in the form $a + ib$.

7. Verify that $z = i$ is a root of the equation
$$z^4 - 2z^3 + 6z^2 - 2z + 5 = 0$$
and find the other three roots.