UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

Faculty of Science and Engineering
Department of Mathematics & Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4702

SEMESTER: Spring 2011

MODULE TITLE: Technological Mathematics 2

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Mr. J. O'Shea

PERCENTAGE OF TOTAL MARKS: 85 %

EXTERNAL EXAMINER: Prof. T. Myers

INSTRUCTIONS TO CANDIDATES: **Answer 5 questions, one each from sections A, B, C, D, and any other question.**

N.B. There are some useful formulae at the end of the paper.

University of Limerick approved calculators may be used.

SECTION A

1 (a) (i) $f(x) = \sqrt{2x-8}$, find $f(2x^2+4)$ and simplify answer.

(ii) Prove that the function $f(x) = x \sin x$ is even.

(iii) Find $g^{-1}(x)$ the inverse of the function $g(x) = e^{3x}$. 10

(b) (i) Evaluate $\cos^{-1}\left(-\frac{1}{5}\right)$.

(ii) Sketch the graph of $\cos^{-1} x$ (the principal value of the inverse cosine curve) indicating clearly the domain and range of the function. 5

(c) Using their definition in terms of exponentials, prove the following hyperbolic identity:

$$\sinh^2 x = \frac{1}{2}(\cosh 2x - 1)$$ 5

2 Consider the function $y = f(x) = \frac{1}{x-3}$ $(x \neq 3)$.

(i) Find the y intercept of $f(x)$. 2

(ii) Show that the function has no local maximum or local minimum turning point. 5

(iii) Explain why the function is decreasing for all values of x. 2

(iv) Find the equation of the vertical asymptote. 3

(v) Find the equation of the horizontal asymptote. 3

(vi) Sketch the graph of $y = f(x)$ indicating clearly the features of the curve obtained in parts (i) - (v). 5

SECTION B

3 (a) Evaluate the following definite and indefinite integrals:

(i) $\int_1^2 2x \sinh(x^2 - 1)dx$ (ii) $\int \sin x \, e^{\cos x} dx$.

(iii) Use integration by parts to find $\int x \sin x dx$. 15

(b) A current $i(t) = 4 + 6\cos 2t$ passes through a capacitor at time t. The capacitor is uncharged initially. Find the charge $q(t)$ on the capacitor at all times t. 5

4 (a) Find the area enclosed by the curves $y = x^2 + 1$ and $y = 9 - x^2$. 10

(b) Use Simpson's Rule with 4 equal subintervals to find an approximation for $\int_1^2 \sqrt{1 + e^x} dx$. 10

SECTION C

5 (a) Find the sum of the telescoping series

$$\sum_{n=1}^{\infty} \frac{6}{(6n+1)(6n+7)}$$ 5

(b) Test the following series for convergence

(i) $\sum_{n=1}^{\infty} \frac{n+5}{3n+4}$

(ii) $\sum_{n=1}^{\infty} \frac{n^2+5n}{n^4+2n-1}$

(iii) Use the ratio test to find the values of x for which the series $\sum_{n=1}^{\infty} \frac{x^{n+1}}{4^n}$ is convergent. 15

6 (a) Find the Maclaurin series of $\cosh x$ up to and including the term containing x^6.

Use your answer to find

(i) the Maclaurin series of $\sinh x$.

(ii) the approximate value of $\cosh 0.3$. 10

(b) (i) Find $\frac{\partial^2 z}{\partial y \partial x}$ for the function $z = 5x^2y^4 + 2xy^3$.

(ii) Prove that the function $z = e^{2t} \sin 4x$ satisfies the partial differential equation $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial t^2} = 0$. 10

SECTION D

7 Write down the Maple commands which implement the following;

(Do not attempt to find the answers of the Maple output.)

(a) Evaluate $\left(\frac{9^3-\sqrt{15}+20}{\sqrt{8}}\right)^5$ to 20 significant figures. 3

(b) Substitute $x = 1 + \sqrt{2}$ into $x^2 e^{3x}$. 2

(c) Find the factors of the cubic polynomial: $3x^3 - 12x^2 + 44x - 100$. 3

(d) Plot $y = \sin^{-1} x$ for $-1 \leq x \leq 1$. 3

(e) Find the first derivative of $\frac{\cos x}{x+\sin^2 x}$ with respect to x and simplify the answer. 3

(f) Find the second derivative of $\frac{\cos x}{x+\sin^2 x}$ with respect to x and simplify the answer. 3

(g) Evaluate the definite integral $\int_1^2 5x^2 \sinh 4x dx$. 3

8 The output of a Maple session, investigating the properties of some function $y = f(x)$ is represented on the next page.

(a) Based on this output:

(i) Find the x and y intercepts of $f(x)$ (if any).

(ii) Find the x and y co-ordinates of all maxima and minima turning points of $f(x)$.

(iii) Find the x and y co-ordinates of all points of inflection of $f(x)$.

(iv) Discuss the behaviour of $f(x)$ as $x \to +\infty$ and $x \to -\infty$. 15

(b) Based on the information given in the output, plot $y = f(x)$ in the domain $-5 \leq x \leq 5$ labelling the parts found in (a). 5

```
> solve(y=0);
```

$$\sqrt{2},\ -\sqrt{2}$$

```
> subs(x=0,y);
```

$$-2$$

```
> y1:=diff(y,x):
> solve(y1=0);
```

$$0$$

```
> y2:=diff(y1,x):
> subs(x=0,y2);
```

$$6$$

```
> solve(y2=0);
```

$$-1/3\,\sqrt{3},\ 1/3\,\sqrt{3}$$

```
> evalf(%,5);
```

$$-0.57736,\ 0.57736$$

```
> evalf(subs(x=-sqrt(3)/3,y));
```

$$-1.250000000$$

```
> evalf(subs(x=sqrt(3)/3,y));
```

$$-1.250000000$$

```
> evalf(subs(x=1000,y));
```

$$0.9999970000$$

```
> evalf(subs(x=-1000,y));
```

$$0.9999970000$$

Formulae

1. Trigonometry : Tables (Old) Page 9.
2. Logarithms :

$$a^x = y \iff \log_a y = x$$

3. Hyperbolic functions :

$$\sinh x = \frac{1}{2}(e^x - e^{-x}) \ ; \qquad \cosh x = \frac{1}{2}(e^x + e^{-x})$$

4. Calculus

Derivatives

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
e^{ax}	ae^{ax}
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\cosh x$	$\sinh x$
$\sinh x$	$\cosh x$

Product Rule:

$$y = uv$$
$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient Rule:

$$y = \frac{u}{v}$$
$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Integrals (constants of integration omitted)

$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln\|x\|$
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$

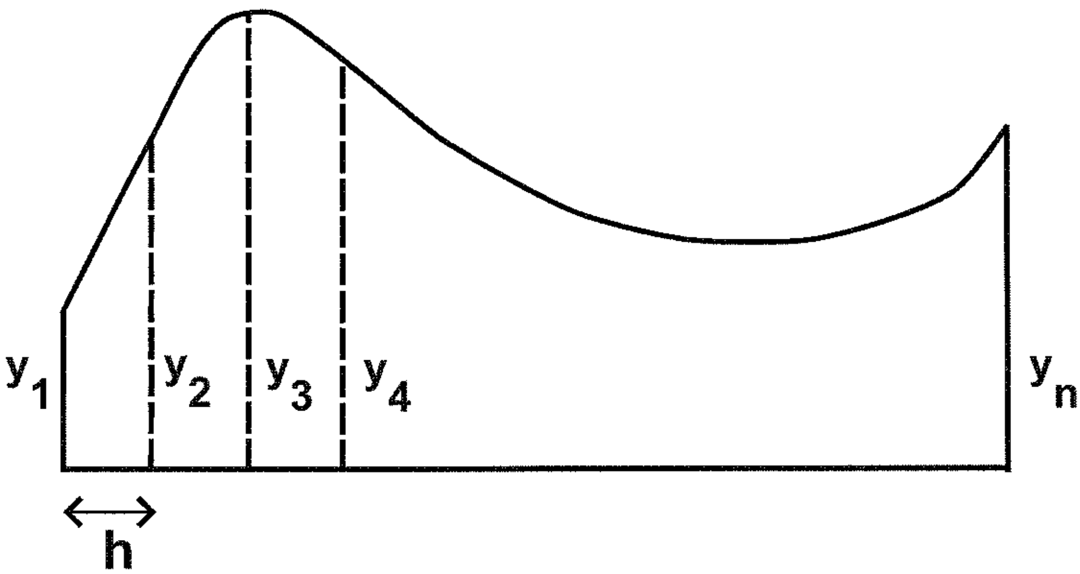
Integration by parts

$$\int u dv = uv - \int v du$$

5. Maclaurin Series

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \ldots + \frac{f^{(R)}(0)x^R}{R!} + \ldots$$

6. Simpson's Rule for odd n



A represents the area of the shape.

$$A \approx \frac{h}{3}\left[y_1 + y_n + 2(y_3 + y_5 + \ldots + y_{n-2}) + 4(y_2 + y_4 + \ldots y_{2n-1})\right]$$