FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS AND STATISTICS

END OF SEMESTER EXAMINATION

MODULE CODE: MA4702	SEMESTER: Spring 2016
MODULE TITLE: Technology Mathematics 2	DURATION OF EXAM: 2.5 hours
LECTURER: Kevin O'Brien	GRADING SCHEME: 100 marks 60% of total module marks
EXTERNAL EXAMINER: Prof. John King	

INSTRUCTIONS TO CANDIDATES

This paper comprises six questions. Question 1 is compulsory and is worth 40 Marks.
You must also attempt any four of the other five questions, each of which are worth 15 marks.
Scientific calculators approved by the University of Limerick can be used.
Formula sheet and statistical tables are provided.

Question 1 - Short Questions (40 Marks)

(i) (4 Marks) Find $f^{-1}(x)$ the inverse of the function

$$f(x) = e^{3x}.$$

(ii) (4 Marks) Solve the following limit:

$$\lim_{x\to 5} \frac{x^2 - 7x + 10}{x - 5}.$$

(iii) (4 Marks) The domain and range of the function $f(x) = \cos(x)$ are $(-\infty, \infty)$ and $[-1, 1]$ respectively. State the domain and the range of the following functions:

(a) $f(x) = \cos^2(x)$,

(b) $f(x) = \cos(2x)$,

(c) $f(x) = 2\cos(x)$,

(d) $f(x) = \cos(x) + 2$.

(iv) (4 Marks) Find the x-intercepts and the y-intercept of the following function:

$$f(x) = x^2 + x - 20.$$

(v) (4 Marks) Determine the vertical asymptote(s) of the following function:

$$y = f(x) = \frac{4x^2}{x^2 - 25}.$$

(vi) (4 Marks) Determine the horizontal asymptote(s) of the following function:

$$y = f(x) = \frac{4x^2}{x^2 - 25}.$$

(vii) (4 Marks) Evaluate the following indefinite integrals:

(a) $\displaystyle\int e^{3x} + x^2 \, dx,$ (b) $\displaystyle\int \sin(2x) + \cosh(x) \, dx$

(viii) (4 Marks)Evaluate the following definite integral:

$$\int_2^5 2x + 5 \, dx.$$

(ix) (4 Marks) Find $\dfrac{\partial^2 z}{\partial x^2}$ and $\dfrac{\partial^2 z}{\partial y^2}$ for both of the following functions:

(a) $z = 9y^2 + 10x^3.$ (b) $z = 3x^2y^3 + y\cosh(x).$

(x) (4 Marks) Compute the following summation:

$$\sum_{i=10}^{50} i.$$

Question 2 - Limits and Functions (15 Marks)

Part A - Limits (5 Marks)

(i) (2 Marks) Compute the limit of the following function.

$$\lim_{x \to 6} \frac{x^2 + 2x - 10}{x - 4}.$$

(iii) (3 Marks) Compute the limit of the following function.

$$\lim_{x \to \infty} \frac{3 + 2x^2 - 6x^3}{4x^3 - 5x + 7}.$$

Part B - Functions (6 Marks)

(i) (3 Marks) Determine if the function $f(x) = x^3 \sin(x)$ is an even function, an odd function or neither.

(ii) (3 Marks) Determine if the function $f(x) = x^2 + x$ is an even function, an odd function or neither.

Part C - Hyperbolic Functions (4 Marks)

(i) (4 Marks) Using their definition in terms of exponentials, prove the following hyperbolic identity

$$\sinh^2(x) = \frac{1}{2}\left[\cosh(2x) - 1\right]$$

Question 3 - Curve Sketching (15 Marks)

Consider the function $f(x) = x^4 - 8x^2 + 9$

(i) (2 Marks) Find the y intercept of the function $f(x)$.

(ii) (3 Marks) Find the three turning points of the function $f(x)$ and classify them as local maxima or local minima.

(iii) (3 Marks) Find the two points of inflection of the function $f(x)$.

(iv) (3 Marks) Determine the behaviour of the function $f(x)$ as $x \to +\infty$ and as $x \to -\infty$.

(v) (4 Marks) Sketch the graph of the function $f(x)$ illustrating the features of the curve obtained in parts (i iv).

Question 4 - Sequences and Series (15 Marks)

(i) (3 Marks) The third term u_3 of a geometric sequence is 24. The fourth term u_4 is -48.
Answer the following questions. Both questions are worth 2 Marks each.

(a) Find the common ratio r.

(b) Find the first and fifth term u_1 and u_5.

(ii) (3 Marks) Three consecutive terms of an arithmetic series are

$$4x + 4, \quad 6x - 6, \quad 7x - 5.$$

Find the values for x and constant difference d.

(iii) (2 Marks) Express the following repeating decimal number as a simple fraction. Show your workings.

$$0.2162162162162....$$

(iv) (4 Marks) Suppose that the following term is the general term for a series. Use the Ratio Test to test this series for convergence

$$u_n = \frac{2^n}{(2n)!}$$

(v) (3 Marks) Find the sum of the telescoping series

$$\sum_{n=1}^{\infty} \frac{4}{(2n-1)(2n+1)}$$

Question 5 - Integration (15 Marks)

(i) (3 Marks) Evaluate the definite integral

$$\int_1^3 3x^2 + \sqrt{x} + \frac{e^x}{4} dx.$$

(ii) (3 Marks) By finding a good substitution, evaluate the indefinite integral

$$\int \frac{12x + 12}{3x^2 + 6x + 8} dx.$$

(iii) (3 Marks) By finding a good substitution, evaluate the indefinite integral

$$\int 2x\sqrt{x^2 + 2} dx.$$

(iv) (3 Marks) Use integration by parts to evaluate the indefinite integral

$$\int 2xe^{2x}\, dx.$$

(v) (3 Marks) By first performing a partial fraction expansion (that is, by writing the integrand as follows)

$$\frac{A}{x-2} + \frac{B}{x+3},$$

evaluate the definite integral

$$\int \frac{7x + 6}{(x-2)(x+3)} dx.$$

Question 6 - Applications of Calculus (15 Marks)

Part A - Applications of Integration

(i) (5 Marks) Find the area enclosed by the curve $y = 1 + 2x - x^2$ and the line $y = x - 1$. Clearly state the points of intersection in your answer.

(ii) (5 Marks) A current $i(t) = 4+5\cos(5t)$ passes through a capacitor at time t. The capacitor is uncharged at time $t = 0$. Find the charge $q(t)$ at all times t.

Part B - Partial Derivatives

(i) (5 Marks) Show that the function $z = e^{-3y}\cos(x)$ satisfies the partial differential equation

$$9\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$

Formula Sheet

Logarithms

If $a^b = c$ then $\log_a c = b$.

Change of Base Formula

$$\log_A(B) = \frac{\log_e(B)}{\log_e(A)}$$

Sum and Difference of Two Cubes

$$a^3 + b^3 = (a-b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Sequences and Series

Arithmetic Series Summation:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

Geometric Series Summation:

$$S_n = a\left(\frac{1-r^n}{1-r}\right) \qquad S_\infty = \frac{a}{1-r}$$

Ratio Test

For a series with general term u_n, if

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = r$$

then the series converges (absolutely) if $r < 1$

Curve Sketching

Horizontal Asymptote: The horizontal asymptote is computed as

$$\lim_{x\to\infty} f(x)$$

Maclaurin Series

$$f(x) = f(0) + f'(0) + \frac{f''(0)}{2!} + \frac{f'''(0)}{3!} + \dots$$

Hyperbolic Functions

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \sinh(x) = \frac{e^x - e^{-x}}{2}$$

Integration

Integration by parts:

$$\int u dv = uv - \int v du$$

Further formulae and special cases on pages 25 & 26 of the log tables provided.

Dynamics

Where $s(t)$ denotes displacement at time t, $v(t)$ denotes the velocity at time t and $a(t)$ denotes the acceleration at time t,

$$\frac{ds(t)}{dt} = v(t), \qquad \frac{dv(t)}{dt} = a(t).$$

Electrical Circuits

Where $q(t)$ denotes the charge at time t and $i(t)$ denotes the current at time t,

$$\frac{dq(t)}{dt} = i(t).$$