Dixon Q-Test for Outliers

Use the Dixon Q-Test to determine whether or not there is an outlier value present in this data set.
Use a 5% significance level.

$19, 36, 33, 25, 30, 28, 31, 36, 29, 37$

(i) Arrange the data set into ascending order.

$$19, 25, 28, 29, 30, 31, 33, 36, 36, 37$$

(ii) Here the potential outlier is the lowest value, i.e. 19

(iii) We can formally state the null and alternative hypothesis as follows

$\mathbf{H}_0$ There are no outliers present in the data.

$\mathbf{H}_1$ There is one outlier present
(i.e. the lowest value 19)

(iv) The test statistic for this procedure is as follows:

$$Q_{TS} = \frac{\text{Gap}}{\text{Range}}$$

(v) The gap is the difference of the outlier from the next value. In this case , the next value is 25, so the gap is

$$\text{Gap} = 25 - 19 = 6$$

(vi) The range is simply the difference between the maximum and minimum value.

$$\text{Range} = 37 - 19 = 18$$

(vii)

$$Q_{TS} = \frac{\text{Gap}}{\text{Range}} = \frac{6}{18} = 0.333$$

(viii) Before we look at the critical value, we confirm the size of the data set is $n = 10$.

(ix) The critical value can be determined from the following table.

- The column to chose is the significance level (here 5% or 0.05).
- The row to use is n, the number of items in the data set.

n	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
3	0.941	0.970	0.994
4	0.765	0.829	0.926
5	0.642	0.710	0.821
6	0.560	0.625	0.740
7	0.507	0.568	0.680
8	0.468	0.526	0.634
9	0.437	0.493	0.598
10	0.412	0.466	0.568
11	0.392	0.444	0.542
12	0.376	0.426	0.522
13	0.361	0.410	0.503
14	0.349	0.396	0.488
15	0.338	0.384	0.475

(x) **Rule of Thumb**

- If the Test Statistic is greater than the Critical value, reject the null hypothesis

$$Q_{TS} > Q_{CV}$$

- Otherwise we fail to reject the null hypothesis

$$\text{Expected Value} = \frac{\text{Column Total} \times \text{Row Total}}{\text{Overall Total}}$$

- If the Test Statistic is greater than the Critical value, reject the null hypothesis

$$\chi^2_{TS} > \chi^2_{CV}$$

- Otherwise we fail to reject the null hypothesis