Problem Sheet 3: Applications of partial differentiation

1. Find the critical points for each of the following functions and then decide whether they are maximum, minimum or saddle points.

$$f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4, \quad f(x,y) = 5xy - 7x^2 + 3x - 6y + 2$$
$$f(x,y) = x^3 - y^3 - 2xy + 6, \quad f(x,y) = \frac{1}{x} + \frac{1}{y} + xy$$

2. Find three real numbers whose sum is 9 and whose sum of squares is as small as possible.

3. A delivery company accepts only rectangular boxes whose length and girth (perimeter of a cross section) do not sum over 108in. Find the dimensions of the acceptable box of largest volume.

4. Find the minimum value of $f(x,y) = xy$ on the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

5. Find the maximum volume of a box such that the sum of the lengths of the edges of the box is equal to 6.

6. Find the point on the plane $x + 2y + 3z = 13$ closest to the point $(1,1,1)$.
Hint: The square of the distance from $(1,1,1)$ to a point (x,y,z) is given by

$$(x-1)^2 + (y-1)^2 + (z-1)^2.$$

Minimize this function subject to the constraint given by the plane equation.

7. Find the points on the curve $xy^2 = 54$ nearest the origin.