UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

Faculty of Science and Engineering
Department of Mathematics & Statistics

End OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4702 **SEMESTER:** Spring 2012

MODULE TITLE: Technological Mathematics 2 **DURATION OF EXAMINATION:** $2\frac{1}{2}$ hours

LECTURER: J. O'Shea **PERCENTAGE OF TOTAL MARKS:** 85 %

EXTERNAL EXAMINER: Prof. T. Myers

INSTRUCTIONS TO CANDIDATES: Answer 5 questions, one each from sections A, B, C, and any other two questions.

N.B. There are some useful formulae at the end of the paper.

University of Limerick approved calculators may be used.

SECTION A

1 (a) (i) $f(x) = \sqrt{1 - x^2}$, find $f(\cos x)$ and simplify answer.

(ii) Prove that the function $f(x) = \dfrac{x - x^3}{2}$ is odd.

(iii) Find $g^{-1}(x)$, the inverse of the function $g(x) = \log_e 4x$. 10

(b) (i) Evaluate $\sin^{-1}\left(-\frac{1}{4}\right)$.

(ii) Sketch the graph of $\sin^{-1} x$ (the principal value of the inverse sine curve) indicating clearly the domain and range of the function. 5

(c) Using their definition in terms of exponentials, prove the following hyperbolic identity:

$$\cosh^2 x - \sinh^2 x = 1.$$ 5

2 Consider the function $y = f(x) = x^4 - 6x^2 + 10$.

(i) Find the y intercept of the graph of the function $y = f(x)$. 1

(ii) Show that $(\sqrt{3}, 1)$ is a stationary point of the function. Find the other two stationary points and classify the three points as local maxima or local minima. 6

(iii) Find the two inflection points of $f(x)$. 3

(iv) Find the x values for which $y = f(x)$ is concave up/down. 3

(v) Determine the behaviour of y as $x \to +\infty$ and as $x \to -\infty$. 2

(vi) Sketch the graph of $y = f(x)$ indicating clearly the features of the curve obtained in parts (i) - (v). 5

3 (a) The equation of a curve is $y = e^{x^2}$. Find $\dfrac{dy}{dx}$ and hence the turning point of the curve. 6

(b) Consider the function $f(x) = \dfrac{x}{x-2} \quad (x \neq 2)$.

(i) Prove that the graph of the function has no turning points.

(ii) Find the equations of the vertical and horizontal asymptotes. 8

(c) Find

(i) $\int(4x + 3 + \frac{1}{x^2})\,dx$.

(ii) The area enclosed between the curve $y = \cos x$ and the x axis between $x = 0$ and $x = \frac{\pi}{2}$. 6

SECTION B

4 (a) Evaluate the following definite and indefinite integrals:

(i) $\int \dfrac{4x+1}{2x^2+x+3}dx$ (ii) $\int_0^{\pi/2} \cos x \sin^4 x dx.$

(iii) Use integration by parts to find $\int x \cos x dx.$ 15

(b) An object moves in a straight line with acceleration $a(t) = 3t - 1$. If the object has a velocity $v = 3m/s$ at time $t = 2$ seconds, find its velocity at all times t. 5

5 (a) Find the area enclosed by the curves $y = 4 - x^2$ and $y = 1 - 2x$. 10

(b) Use Simpson's Rule with 4 equal subintervals to find an approximation for

$$\int_0^1 \sinh(x^2+1)dx.$$

10

SECTION C

6 (a) Find the sum of the telescoping series

$$\sum_{n=1}^{\infty} \frac{2}{(2n-1)(2n+1)}$$ 5

(b) Test the following series for convergence

(i) $\sum_{n=1}^{\infty} \frac{n+2}{3n+1}$

(ii) $\sum_{n=1}^{\infty} \frac{n^2+3}{4n^3+2n+1}$

(iii) $\sum_{n=1}^{\infty} \frac{2^n}{(n+1)!}$ 15

7 (a) Find the Maclaurin series of e^x up to and including the term containing x^4.

Use your answer

(i) to find the Maclaurin series of e^{-x}.

(ii) to approximate $e^{0.4}$. 10

(b) (i) Find the first partial derivatives of the function

$$z = 4x^2y + 3xy^2 + y^3.$$

(ii) Show that the function $z = e^{-2t}\cos x$ satisfies the partial differential equation $4\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = 0$. 10

Formulae

1. Hyperbolic functions :

$$\sinh x = \frac{1}{2}(e^x - e^{-x})\ ; \qquad \cosh x = \frac{1}{2}(e^x + e^{-x})$$

2. Differentiation

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
e^{ax}	ae^{ax}
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\cosh x$	$\sinh x$
$\sinh x$	$\cosh x$

Product Rule:

$$\begin{aligned} y &= uv \\ \frac{dy}{dx} &= u\frac{dv}{dx} + v\frac{du}{dx} \end{aligned}$$

Quotient Rule:

$$\begin{aligned} y &= \frac{u}{v} \\ \frac{dy}{dx} &= \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \end{aligned}$$

3. Integration (constants of integration omitted)

$f(x)$	$\int f(x)dx$
$x^n, (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln \|x\|$
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$

4. $a^x = y \Longleftrightarrow \log_a y = x$.

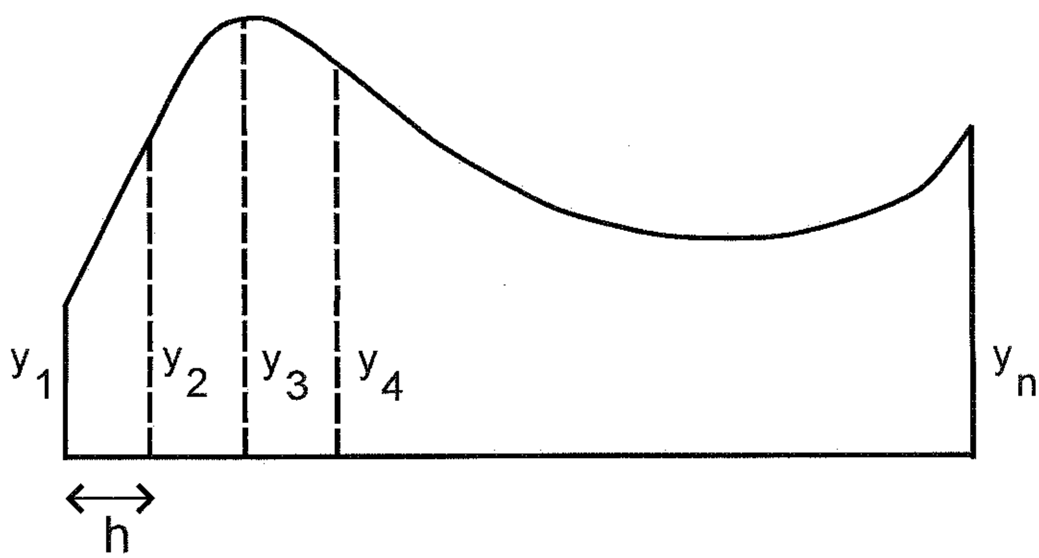
5. Integration by parts

$$\int u dv = uv - \int v du$$

6. Maclaurin Series

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \ldots + \frac{f^{(R)}(0)x^R}{R!} + \ldots$$

7. Simpson's Rule for odd n



A represents the area of the shape.

$$A \approx \frac{h}{3}\left[y_1 + y_n + 2(y_3 + y_5 + \ldots + y_{n-2}) + 4(y_2 + y_4 + \ldots y_{n-1})\right]$$

8. Trigonometry: Tables (old) Page 9.