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1 Euler's Homogeneous Function Theorem

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Let $f(x,y)$ be a homogeneous function of order n so that

$$f(tx,ty) = t^n f(x,y).$$

Then define $x' = xt$ and $y' = yt$. Then

$$\begin{aligned} n t^{n-1} f(x,y) &= (\partial f)/(\partial x')(\partial x')/(\partial t) + (\partial f)/(\partial y')(\partial y')/(\partial t) \\ &= x(\partial f)/(\partial x') + y(\partial f)/(\partial y') \\ &= x(\partial f)/(\partial(xt)) + y(\partial f)/(\partial(yt)). \end{aligned}$$

Let $t=1$, then

$$x(\partial f)/(\partial x) + y(\partial f)/(\partial y) = n f(x,y).$$