Section A

1a $f(x) = \frac{1}{\sqrt{1-x^2}}$, $g(x) = 2x-5$

(i) $f(\cos x) = \frac{1}{\sqrt{1-\cos^2 x}}$

$= \frac{1}{\sqrt{\sin^2 x}}$

$= \frac{1}{\sin x}$

(ii) $f(x) = \frac{1}{\sqrt{1-x^2}}$

$f(-x) = \frac{1}{\sqrt{1-(-x)^2}}$

$= \frac{1}{\sqrt{1-x^2}}$

$= f(x)$

$\Rightarrow$ even function

$g: y = 2x-5$

$g^{-1}: x = 2y-5$

$x+5 = 2y$

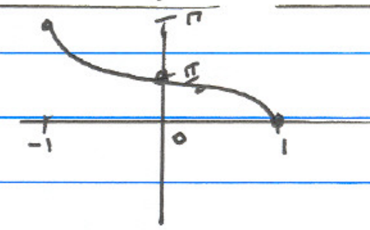
$\frac{x+5}{2} = y. = g^{-1}(x)$

$\Rightarrow g^{-1} \circ g(x) = \frac{(2x-5)+5}{2} = x.$

(b) $\tan^{-1}(-2)$

$\Rightarrow -63.43^0$

OR -1.1072 Rad.



(c) $\sinh^2 x = \frac{1}{2}(\cosh 2x - 1)$

$\left(\frac{e^x - e^{-x}}{2}\right)^2$

$\Rightarrow \frac{(e^x-e^{-x})(e^x-e^{-x})}{4}$

$\Rightarrow \frac{e^{2x} - e^0 - e^0 + e^{-2x}}{4}$

$\Rightarrow \frac{e^{2x} - 2 + e^{-2x}}{4}$

$\frac{1}{2}\left(\frac{e^{2x}+e^{-2x}}{2} - 1\right)$

$\Rightarrow \frac{1}{2}\left(\frac{e^{2x}+e^{-2x}-2}{2}\right)$

$\Rightarrow \frac{e^{2x} - 2 + e^{-2x}}{4}.$

2. $y = f(x) = \frac{x}{x+2}$

(i) $f(0) = \frac{0}{2} = 0$

$\Rightarrow (0,0)$

(ii) $f'(x) = \frac{(x+2).1 - x(1)}{(x+2)^2}$

$= \frac{x+2-x}{(x+2)^2}$

$= \frac{2}{(x+2)^2}$

$\Rightarrow \frac{2}{(x+2)^2} \neq 0$

$\Rightarrow$ no solution

$\Rightarrow$ no turning point

(iii) $f'(x) = \frac{2}{(x+2)^2} > 0$

$\Rightarrow$ increasing function

(iv) $y = \frac{x}{x+2}$

Vertical asymptote

$x+2=0$

$\Rightarrow$ line $x=-2$

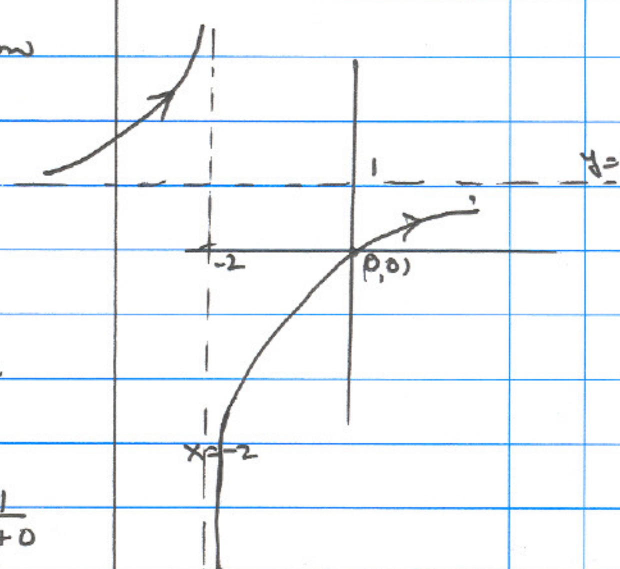
(v) Horizontal asymptote

$\lim_{x\to\infty} \frac{x}{x+2}$

$\Rightarrow \lim_{x\to\infty} \frac{1}{1+\frac{2}{x}} = \frac{1}{1+0}$

$\Rightarrow 1$

$\Rightarrow$ line $y=1$ is asymptote



Section B

3 (a)

(i) $\int x^2 (x^3+1)^4 dx$

let $u = x^3+1$

$\frac{du}{dx} = 3x^2$

$du = 3x^2 dx$

$\frac{du}{3x^2} = dx$

$\Rightarrow \int \cancel{x^2} . u^4 . \frac{du}{3\cancel{x^2}}$

$\Rightarrow \frac{1}{3}\int u^4 du$

$\Rightarrow \frac{1}{3}\left(\frac{u^5}{5}\right) + c$

$\Rightarrow \frac{u^5}{15} + c$

$\Rightarrow \frac{(x^3+1)^5}{15} + c$

(ii) $\int_0^{\pi/2} e^{\sin x} \cos x \, dx$

let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x dx$

$\frac{du}{\cos x} = dx$

$\Rightarrow \int e^u \cancel{\cos x} . \frac{du}{\cancel{\cos x}}$

$\Rightarrow \int e^u du$

$\Rightarrow \cancel{e^{\sin x}}$

$\Rightarrow e^u$

$\Rightarrow e^{\sin x} \Big|_{x=0}^{x=\pi/2}$

$\Rightarrow e^{\sin \pi/2} - e^{\sin 0}$

$\Rightarrow e - e^0$

$\Rightarrow e - 1$

(iii) $\int x \sinh x \, dx$

$u = x$

$\frac{du}{dx} = 1$

$du = dx$

$dv = \sinh x \, dx$

$v = \int \sinh x \, dx$

$v = \cosh x.$

$\int u dv = uv - \int v du.$

$\Rightarrow \int x \sinh x = x \cosh x - \int \cosh x \, dx$

$= x \cosh x - \sinh x + c$

(b) $a = \cos 2t$

$v = \int \cos 2t \, dt$

$v = \frac{\sin 2t}{2} + c$ when $t = 0$ $v = 0$ ~~when t=0~~

$0 = \frac{\sin 0}{2} + c$

$\Rightarrow 0 = \frac{0}{2} + c$

$\Rightarrow \boxed{0 = c}$

~~$v = \frac{1}{2}\int \frac{\sin 2t}{2} dt$~~ $v = \frac{1}{2}\sin 2t$

$\Rightarrow s = \frac{1}{2}\int \sin 2t \, dt$

$= \frac{1}{2}\left(-\frac{\cos 2t}{2}\right) + c$

$= -\frac{\cos 2t}{4} + c$

$s = 0$ when $t = 0$

$\Rightarrow 0 = -\frac{\cos 0}{4} + c$

$0 = -\frac{1}{4} + c$

$\boxed{\frac{1}{4} = c}$

$\Rightarrow s = -\frac{\cos 2t}{4} + \frac{1}{4}.$

4 (a) $y = x^2 - 4$

$\underline{y = 4 - x^2}$

$x^2 - 4 = 4 - x^2$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

$\Rightarrow A = \int_{-2}^{2} f(x) - g(x)\, dx.$

$= \int_{-2}^{2} 4 - x^2 - (x^2 - 4)\, dx$

$= \int_{-2}^{2} 4 - x^2 - x^2 + 4\, dx$

$= \int_{-2}^{2} 8 - 2x^2\, dx$

$= 8x - \frac{2x^3}{3} \Big|_{x=-2}^{x=2}$

$= \left(16 - \frac{16}{3}\right) - \left(-16 + \frac{16}{3}\right)$

$\Rightarrow \frac{32}{3} + \frac{32}{3}$

$\Rightarrow \boxed{\frac{64}{3}}$

(b) $\int_0^2 \sqrt{1 + e^{-x}}\, dx$

$h = \frac{2-0}{4} = \frac{1}{2}$

x =	0	.5	1	1.5	2
y =	1.414	1.267	1.169	1.105	1.065

$\text{Area} = \frac{0.5}{3}\left[(1.414 + 1.065) + 4(1.267 + 1.105) + 2(1.169)\right]$

$\Rightarrow \frac{1}{6}(14.305)$

$\Rightarrow \boxed{2.384}$

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Section C

5 (a) $\sum_{n=1}^{\infty} \frac{4}{(4n-1)(4n+3)}$

$$\frac{4}{(4n-1)(4n+3)} = \frac{1}{4n-1} - \frac{1}{4n+3}$$

$$\sum_{n=1}^{\infty} \frac{4}{(4n-1)(4n+3)} = \sum_{n=1}^{\infty}\left(\frac{1}{4n-1} - \frac{1}{4n+3}\right)$$

$$= \frac{1}{3} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{7}} - \cancel{\frac{1}{11}} + \cancel{\frac{1}{11}} - \cancel{\frac{1}{15}} + \ldots$$

$$= \frac{1}{3}$$

or

$$a_n = \frac{1}{4n-1} - \frac{1}{4n+3}$$

$$a_1 = \frac{1}{3} - \cancel{\frac{1}{7}}$$

$$a_2 = \cancel{\frac{1}{7}} - \cancel{\frac{1}{11}}$$

$$a_3 = \cancel{\frac{1}{11}} - \cancel{\frac{1}{15}}$$

$$\vdots$$

$$a_n = \cancel{\frac{1}{4n-1}} - \frac{1}{4n+3}$$

$$S_n = \frac{1}{3} - \frac{1}{4n+3}$$

$$\lim_{n\to\infty} S_n = \frac{1}{3} - \frac{1}{\infty}$$

$$= \frac{1}{3} - 0$$

$$= \frac{1}{3}$$

(b) (i) $\sum_{n=1}^{\infty} \frac{n-1}{4n+1}$

$$\lim_{n\to\infty} \frac{n-1}{4n+1} = \frac{\infty}{\infty}$$

$$\Rightarrow \lim_{n\to\infty} \frac{1-\frac{1}{n}}{4+\frac{1}{n}}$$

$$\Rightarrow \frac{1-0}{4+0}$$

$$\Rightarrow \frac{1}{4} \neq 0$$

Series is divergent

(ii) $\sum_{n=1}^{\infty} \frac{2n+1}{2n^3+3n+4}$

Compare with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$$\Rightarrow \frac{\left(\frac{2n+1}{2n^3+3n+4}\right)}{\frac{1}{n^2}} = \frac{(2n+1)}{2n^3+3n+4} \cdot \frac{n^2}{1}$$

$$= \frac{2n^3+n^2}{2n^3+3n+4}$$

$$\Rightarrow \lim_{n\to\infty} \frac{2n^3+n^2}{2n^3+3n+4} = \frac{\infty}{\infty}$$

$$\Rightarrow \lim_{n\to\infty} \left(\frac{2+\frac{1}{n}}{2+\frac{3}{n^2}+\frac{4}{n^3}}\right)$$

$$\Rightarrow \frac{2+0}{2+0+0} = \frac{2}{2}$$

$$\Rightarrow 1.$$

as $\sum_{n=1}^{\infty} \frac{1}{n^2}$ $(P>1)$ is Convergent

$\Rightarrow \sum_{n=1}^{\infty} \frac{2n+1}{2n^3+3n+4}$ is Convergent.

(iii) $\sum_{n=1}^{\infty} \frac{4^n}{(n+1)!}$

$$a_n = \frac{4^n}{(n+1)!}$$

$$a_{n+1} = \frac{4^{n+1}}{(n+2)!}$$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{4^{n+1}}{(n+2)!}}{\frac{4^n}{(n+1)!}}$$

$$= \frac{\cancel{4^{n+1}}\,4}{\cancel{(n+2)!}\,n+2} \cdot \frac{\cancel{(n+1)!}}{\cancel{4^n}}$$

$$= \frac{4}{n+2}$$

$$\lim_{n\to\infty} \left|\frac{4}{n+2}\right| = \left|\frac{4}{\infty}\right| = |0| = 0$$

$$0 < 1$$

$\Rightarrow$ by Ratio Test

$\sum \frac{4^n}{(n+1)!}$ is Convergent

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6 (a)

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \cdots$$

$f(x) = \sin x$

$f(0) = \sin 0 = 0$

$f'(x) = \cos x$

$\boxed{f'(0) = \cos 0 = \boxed{1}}$

$f''(x) = -\sin x$

$f''(0) = -\sin 0 = 0$

$f'''(x) = -\cos x$

$\boxed{f'''(0) = -\cos 0 = -1}$

$f^{iv}(x) = -(-\sin x) = \sin x$

$f^{iv}(0) = \sin 0 = 0$

$f^{v}(x) = \cos x$

$\boxed{f^{v}(0) = \cos 0 = 1}$

$\Rightarrow \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

Differentiate w.r.t x

$\Rightarrow \cos x = 1 - \frac{3x^2}{3!} + \frac{5x^4}{5!}$

$\Rightarrow \boxed{\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}}$

(b) (i) $z = x^2 \sin y + y^2 \cos x$

$\frac{\partial z}{\partial x} = 2x \sin y + y^2(-\sin x)$

$= 2x\sin y - y^2 \sin x$

$\frac{\partial z}{\partial y} = x^2 \cos y + 2y \cos x$

(ii) $z = e^{3x} \sin 3y$

$\frac{\partial z}{\partial x} = 3e^{3x} \sin 3y$

$\frac{\partial^2 z}{\partial x^2} = 3e^{3x} . 3 \sin 3y$

$= \boxed{9e^{3x} \sin 3y}$

$\frac{\partial z}{\partial y} = e^{3x} \cos 3y . 3$

$= 3e^{3x} \cos 3y$

$\frac{\partial^2 z}{\partial y^2} = 3e^{3x}(-\sin 3y . 3)$

$= \boxed{-9e^{3x} \sin 3y}$

$\Rightarrow \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$

$= 9e^{3x} \sin 3y - 9e^{3x} \sin 3y$

$\Rightarrow 0.$

Section D

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7 (a) Evalf((7 - 2^3 + Sqrt(17))/3^2, 17);

  (b) Subs(x=8, Sqrt((cos(x) - 2)* x^3));

  (c) Factor(-5*x^3 + 8*x^2 + 19*x + 6);

  (d) plot(arccos(x), x = -1..1);

  (e) Diff(exp(3*x)/(x^2+1), x);
          Simplify(%);

  (f) Diff(exp(3*x)/(x^2+1), x $2);
          Simplify(%);

  (g) Int(2*x*(1+exp(x^2), x = 0..1);
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8 (i) (−1,0) (2,0) (−3,0)
(0,−6)

(ii) (0.7862996483, −8.208820736) min turning point
(−2.119632982, 4.06067259) max turning point

(iii) (−0.6666666667, −2.074074073) inflection point

(iv) $x \to +\infty$, $y \to +\infty$
$x \to -\infty$, $y \to -\infty$

graphs