Q9 2010 Yellow

- The function f(x) is defined below.

$$f(x) = \begin{cases} -1 \quad , & \text{if } -1 \leq x < 0 \\ 1 \quad , & \text{if } 0 \leq x < 1 \end{cases}$$

  is periodic with period 2.

- Range = -1 to 1. $L = 1 \therefore 1/L = 1$ Also remark : this is an odd function.

- We use the following definition:

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x) sin(\frac{n\pi x}{L})dx$$

- $$b_n = \int_{-1}^{0} (-1) sin(n\pi x)dx + \int_{0}^{1} sin(n\pi x)dx$$

- $$b_n = -1 \times \left[\frac{-cos(n\pi x)}{n\pi}\right]_{-1}^{0} + \left[\frac{-cos(n\pi x)}{n\pi}\right]_{0}^{1}$$

- $$b_n = \left[\frac{cos(n\pi x)}{n\pi}\right]_{-1}^{0} + \left[\frac{-cos(n\pi x)}{n\pi}\right]_{0}^{1}$$

- Remember $cos(-x) = cos(x)$

$$b_n = \left[\left(\frac{cos(0)}{n\pi}\right) - \left(\frac{cos(\boldsymbol{n\pi})}{n\pi}\right)\right] + \left[\left(\frac{-cos(\boldsymbol{n\pi})}{n\pi}\right) - \left(\frac{-cos(0)}{n\pi}\right)\right]$$

- $cos(0) = 1$

$$b_n = \left[\left(\frac{1}{n\pi}\right) - \left(\frac{cos(\boldsymbol{n\pi})}{n\pi}\right)\right] + \left[\left(\frac{-cos(\boldsymbol{n\pi})}{n\pi}\right) - \left(\frac{-1}{n\pi}\right)\right]$$

- Simplifying

$$b_n = \left(\frac{2}{n\pi}\right) - \left(\frac{2cos(\boldsymbol{n\pi})}{n\pi}\right) = \frac{2}{n\pi}\left(1 - cos(n\pi)\right)$$

Q9 2011 Yellow

- The function f(x) is defined below.

$$f(x) = \begin{cases} 1 & , \quad \text{if} - 1 \leq x < 0 \\ -1 & , \quad \text{if} 0 \leq x < 1 \end{cases}$$

  is periodic with period 2.

- Range = -1 to 1. $L = 1 \therefore 1/L = 1$ Also remark : this is an odd function.

- We use the following definition:

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x) sin(\frac{n\pi x}{L})dx$$

- $$b_n = \int_{-1}^{0} (1) sin(n\pi x)dx + \int_{0}^{1} (-1) sin(n\pi x)dx$$

- (We move the (-1) in the second term outside - changing the plus sign to minus)

- $$b_n = \left[\frac{-cos(n\pi x)}{n\pi}\right]_{-1}^{0} - \left[\frac{-cos(n\pi x)}{n\pi}\right]_{0}^{1}$$

- $$b_n = \left[\left(\frac{-cos(0)}{n\pi}\right) - \left(\frac{-cos(\boldsymbol{n\pi})}{n\pi}\right)\right] - \left[\left(\frac{-cos(\boldsymbol{n\pi})}{n\pi}\right) - \left(\frac{-cos(0)}{n\pi}\right)\right]$$

- $cos(0) = 1$

$$b_n = \frac{2cos(n\pi) - 2}{n\pi}$$

- $cos(n\pi) = (-1)^n$

$$b_3 = \frac{2cos(3\pi) - 2}{3\pi} = \frac{2(-1)^3 - 2}{3\pi} = \frac{-4}{3\pi}$$

Q9 2011 Green

- The function $f(x) = -x$ is defined below.
- function is periodic with period 2π.
- Range $= -\pi$ to π. $L = \pi \therefore 1/l = 1/\pi$
- Also remark : this is an odd function.
- We use the following definition:

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)sin(nx)dx$$

- Consider problem in following form

$$b_n = \frac{I}{-\pi}$$

- Require integration by parts of integral (divide it later by $-\pi$).

$$I = \int udv = uv - \int vdu$$

- Let $u = x$ then $du = dx$
- Let $dv = sin(nx)$ then

$$v = \int vdv = \frac{-cos(nx)}{n}$$

- $$I = \frac{-xcos(nx)}{n} + \frac{1}{n}\int cos(nx)dx$$
- (Comment upon the sign change , and the term (1/n) being removed)
- $$I = \frac{-xcos(nx)}{n} + \frac{1}{n} \times \left(\frac{sin(nx)}{n}\right)$$

- Remark: second term cancels to zero because $sin(n\pi) = sin(-n\pi) = 0$
- Even functions $cos(n\pi) = cos(-n\pi)$
- $$I = \frac{-2\pi cos(n\pi)}{n}$$
- $$b_n = \frac{I}{-\pi} = \frac{2cos(n\pi)}{n}$$

Q9 2010 Green

- The function $f(x) = -x$ is defined below.
- function is periodic with period 2.
- Range $= -1$ to 1. $L = 1 \therefore 1/L = 1$
- Also remark : this is an odd function.
- We use the following definition:

$$b_n = -\int_{-1}^{1} (x) sin(n\pi x) dx$$

- Find the integral I then negate it to find b_n.
- Require integration by parts of integral (divide it later by -1).

$$I = \int udv = uv - \int vdu$$

- Let $u = x$ then $du = dx$
- Let $dv = sin(n\pi x)$ then

$$v = \int vdv = \frac{-cos(n\pi x)}{n\pi}$$

- $$I = \frac{-xcos(n\pi x)}{n\pi} + \frac{1}{n\pi} \int cos(n\pi x) dx$$
- (Comment upon the sign change , and the divisor being moved outside)
- $$I = \frac{-xcos(n\pi x)}{n\pi} + \frac{1}{n\pi} \times \left(\frac{sin(n\pi x)}{n\pi} \right)$$
- Remark: once limits are applied second term cancels to zero because $sin(n\pi) = sin(-n\pi) = 0$

- $$I = \left[\frac{-xcos(n\pi x)}{n\pi}\right]_{-1}^{1}$$
- Even functions $cos(n\pi) = cos(-n\pi)$
$$I = \left[\frac{-cos(n\pi)}{n\pi}\right] - \left[\frac{-(-1)\boldsymbol{cos(n\pi)}}{n\pi}\right]$$
-
- $$I = \frac{-2cos(n\pi)}{n\pi}$$
- $$b_n = \frac{2(-1)^n}{n\pi}$$

Revision

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)dx \tag{0.1}$$
$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)cos(\frac{n\pi x}{L})dx \tag{0.2}$$
$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)sin(\frac{n\pi x}{L})dx \tag{0.3}$$

- **"a for even"**: If function is odd : $a_0 = 0$ and $a_n = 0$
- **"b for odd"**: If function is even : $b_n = 0$

Q10 2011 Yellow

- Re-express function
$$f(x) = cosh(x) = \frac{e^x + e^{-x}}{2}$$
- Range = -1 to 1. $L = 1 \therefore 1/L = 1$ Also remark : this is an even function
-
$$\int_{-1}^{1} f(x)dx = 2\int_{0}^{1} f(x)dx = 2\int_{0}^{1} \frac{e^x}{2} + \frac{e^{-x}}{2}dx$$
-
$$2\int_{0}^{1} \frac{e^x}{2} + \frac{e^{-x}}{2}dx = 2 \times \left[\frac{e^x}{2} - \frac{e^{-x}}{2}\right]_0^1$$
- remark :
$$\frac{e^x}{2} - \frac{e^{-x}}{2} = \frac{e^x - e^{-x}}{2} = sinh(x)$$
- Simplifying
$$2 \times \left[\frac{e^x}{2} - \frac{e^{-x}}{2}\right]_0^1 = 2 \times [sinh(1) - sinh(0)] = 2sinh(1)$$

Q10 2011 Green

- Re-express function
$$f(x) = |x|$$
- Range = -1 to 1. $L = 1$ ($\therefore 1/L = 1$) Also remark : this is an even function.
- $$\int\limits_{-1}^{1} f(x)dx = \int\limits_{-1}^{0} (-x)dx + \int\limits_{0}^{1} (x)dx$$
- $$\int\limits_{-1}^{1} f(x)dx = 2\int\limits_{0}^{1} f(x)dx = 2 \times \int\limits_{0}^{1} (x)dx$$
- $$2 \times \int\limits_{0}^{1} xdx = 2 \times \left[\frac{x^2}{2}\right]_0^1 = 2 \times \left[\frac{1^2}{2} - \frac{0}{2}\right] = 1$$

Q10 2010 Green

- Re-express function
$$f(x) = xcos(x)$$
- This is an **ODD** function. a_0 is necessarily 0.
- You can check this by trying out some trial values
  - * $f(-\pi) = -\pi \times (-1) = \pi$
  - * $f(\pi) = \pi \times (-1) = -\pi$

Q10 2010 Green

- Re-express function
$$f(x) = -x^3$$

- This is an **ODD** function. a_0 is necessarily 0.
- Lets do it out just to make sure.
- Range = -1 to 1. $L = 1$ ($\therefore 1/L = 1$)
- $$\int_{-1}^{1} f(x)dx = \int_{-1}^{1} -x^3 dx = \left[\frac{-x^4}{4}\right]_{-1}^{1} = \left[\left(-\frac{1}{4}\right) - \left(-\frac{1}{4}\right)\right] = 0$$
- Remark: Be VERY Careful with signs

Question 7

- The period of a function p may be identified by the following term : $f(t) = f(t + p)$
- Period of a function is inversely proportional to the coefficient.
- If $f(x)$ has period p , then $f(2x)$ has period $p/2$
- For older questions a periodic trigonometric function of form $trig(kx)$ has period $2\pi/k$

Q7 2011 Yellow

- If $f(x)$ has period 2 , then $f(2x)$ has period 1

Q7 2011 Green

- If $f(x)$ has period 2π , then $f(2x)$ has period π

Q7 2010 Yellow

- $trig(kx)$ k= $\frac{pi}{2}$
- $p = \frac{2\pi}{pi/2}$ =4

Q7 2010 Green

- $trig(kx)$ $k = \frac{1}{2}$
- $p = \frac{2\pi}{1/2} = 4\pi$

0.1 Q8 2010 Green

- $f(x) = x - x^5$
- $f(-1) = (-1) - (-1)^5 = 0$
- $f(1) = (1) - (1)^5 = 0$
- use a different number instead
- $f(-1/2) = (-1/2) - (-1/2)^5 = 0$
- $f(1/2) = (1/2) - (1/2)^5 = 0$
- $f(x)$ is odd
- $g(x) = x^2 sinx$
- $g(-1) = (-1^2) \times sin(-1)$
- $g(1) = (1^2) \times sin(-1)$
- $g(x)$ is also odd