The Shape of a Graph

Example 3 Find and classify all the critical points of the following function. Give the intervals where the function is increasing and decreasing.

The Shape of a Graph

Solution

First well need the derivative so we can get our hands on the critical points. Note as well that well do some simplification on the derivative to help us find the critical points.

The Shape of a Graph

So, it looks like well have four critical points here. They are, Finding the intervals of increasing and decreasing will also give the classification of the critical points so lets get those first. Here is a number line with the critical points graphed and test points.

The Shape of a Graph

So, it looks like weve got the following intervals of increasing and decreasing.
From this it looks like and are neither relative minimum or relative maximums since the function is increasing on both side of them.
On the other hand, is a relative maximum and is a relative minimum.

The Shape of a Graph

For completeness sake here is the graph of the function.

The Shape of a Graph

In the previous example the two critical points where the derivative didnt exist ended up not being relative extrema. Do not read anything into this. They often will be relative extrema. Check out this example in the Absolute Extrema to see an example of one such critical point.

The Shape of a Graph

Lets work a couple more examples.
Example 4 Suppose that the elevation above sea level of a road is given by the following function.

$$E(x) = 500 + cos(\frac{x}{4}) + \sqrt{3}sin(\frac{x}{4})$$

where x is in miles.

- Assume that if x is positive we are to the east of the initial point of measurement and if x is negative we are to the west of the initial point of measurement.
- If we start 25 miles to the west of the initial point of measurement and drive until we are 25 miles east of the initial point how many miles of our drive were we driving up an incline?

The Shape of a Graph

Solution

- Okay, this is just a really fancy way of asking what the intervals of increasing and decreasing are for the function on the interval [-25,25].
- So, we first need the derivative of the function.

$$E'(x) = \frac{-1}{4} sin(\frac{x}{4}) + \frac{\sqrt{3}}{4} cos(\frac{x}{4})$$

The Shape of a Graph

Setting this equal to zero gives,

$$\frac{-1}{4} sin(\frac{x}{4}) + \frac{\sqrt{3}}{4} cos(\frac{x}{4}) = 0$$

$$tan(\frac{x}{4}) = \sqrt{3}$$

The solutions to this and hence the critical points are,
Ill leave it to you to check that the critical points that fall in the interval that were after are,

The Shape of a Graph

Here is the number line with the critical points and test points.
So, it looks like the intervals of increasing and decreasing are,

The Shape of a Graph

Notice that we had to end our intervals at -25 and 25 since weve done no work outside of these points and so we cant really say anything about the function outside of the interval [-25,25].

The Shape of a Graph

From the intervals we can actually answer the question. We were driving on an incline during the intervals of increasing and so the total number of miles is,

The Shape of a Graph

Even though the problem didnt ask for it we can also classify the critical points that are in the interval [-25,25].

The Shape of a Graph

Example 5 The population of rabbits (in hundreds) after t years in a certain area is given by the following function,

Determine if the population ever decreases in the first two years.

The Shape of a Graph

Solution So, again we are really after the intervals and increasing and decreasing in the interval [0,2].
We found the only critical point to this function back in the Critical Points section to be,

The Shape of a Graph

Here is a number line for the intervals of increasing and decreasing. So, it looks like the population will decrease for a short period and then continue to increase forever.
Also, while the problem didnt ask for it we can see that the single critical point is a relative minimum.

The Shape of a Graph

In this section weve seen how we can use the first derivative of a function to give us some information about the shape of a graph and how we can use this information in some applications.
Using the first derivative to give us information about a whether a function is increasing or decreasing is a very important application of derivatives and arises on a fairly regular basis in many areas.