Tutorial Sheet 5

Sets:

$\mathbb{R}$ - All real numbers positive and negative
$\mathbb{R}^+$ - All positive real numbers including 0
$\mathbb{R}^-$ - All negative real numbers including 0

$[a, b]$ - All real numbers x such that $a \leq x \leq b$
(a, b) - All real numbers x such that $a < x < b$
$[a, \infty)$ - All real numbers x such that $a \leq x$
(a, ∞) - All real numbers x such that $a < x$

1. Which of the following functions are well defined functions? If the function is not well defined, give a counterexample showing that it is not.

   i) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = x^2 + 1$
   ii) $f : \mathbb{R}^+ \to \mathbb{R}^+, \quad f(x) = x^2 + 1$
   iii) $f : \mathbb{R}^+ \to [1, 10], \quad f(x) = x^2 + 1$
   iv) $f : \mathbb{R}^+ \to [1, \infty), \quad f(x) = x^2 + 1$
   v) $f : \mathbb{R}^+ \to \mathbb{R}^+, \quad f(x) = \sqrt[+]{x}$
   vi) $f : \mathbb{R}^- \to \mathbb{R}^-, \quad f(x) = \sqrt[+]{x}$
   vii) $f : \mathbb{R}^+ \to \mathbb{R}^-, \quad f(x) = \sqrt[+]{x}$
   viii) $f : \mathbb{R}^+ \to \mathbb{R}, \quad f(x) = \sqrt[+]{x}$
   ix) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = \frac{1}{x}$
   x) $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}, \quad f(x) = \frac{1}{x}$
   xi) $f : \mathbb{R}^+ \setminus \{0\} \to \mathbb{R}, \quad f(x) = \frac{1}{x}$
   xii) $f : \mathbb{R}^+ \setminus \{0\} \to \mathbb{R}, \quad f(x) = \frac{1}{x-1}$
   xiii) $f : \mathbb{R}^+ \setminus \{1\} \to \mathbb{R}^+, \quad f(x) = \frac{1}{x-1}$
   xiv) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = e^x$
   xv) $f : \mathbb{R} \to \mathbb{R}^+, \quad f(x) = e^x - 1$
   xvi) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = \ln(x)$
   xvii) $f : \mathbb{R}^+ \setminus \{0\} \to \mathbb{R}^+, \quad f(x) = \ln(x)$
   xviii) $f : \mathbb{R}^+ \setminus \{0\} \to \mathbb{R}, \quad f(x) = \ln(x)$
   xix) $f : (1, \infty) \to \mathbb{R}, \quad f(x) = \ln(x + 1)$

2. For each of the following well defined functions, say whether the function is one-to-one, onto, or invertible. In the case of invertible functions, give the inverse function. In the case of non-invertible functions, modify the domain and codomain of the functions to make them invertible and give the corressponding inverse function.

   i) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = 2x + 4$
   ii) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = x$
   iii) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = x^2$
   iv) $f : \mathbb{R} \to \mathbb{R}^+, \quad f(x) = x^2 + 4$
   v) $f : \mathbb{R}^+ \to \mathbb{R}, \quad f(x) = \sqrt[+]{x}$
   vi) $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}, \quad f(x) = \frac{1}{x}$
   vii) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = e^x$
   viii) $f : \mathbb{R}^+ \to [1, \infty), \quad f(x) = e^x$
   ix) $f : \mathbb{R}^+ \to \mathbb{R}^+, \quad f(x) = e^x + 1$
   x) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = \sin(x)$
   xi) $f : (\text{-}\pi, \pi) \to [\text{-}1, 1], \; f(x) = \sin(x)$
   xii) $f : (\text{-}\frac{\pi}{2}, \frac{\pi}{2}) \to [\text{-}1, 1], \; f(x) = \sin(x)$

3. Graph the well defined function $f : \mathbb{R} \to \mathbb{R}, \; f(x) = \cosh(x)$ on the interval $[-2, 2]$. Based on the graph, give a suitable domain and codomain of the function to make it invertible.

4. For each of the following graphs,

   i) Use the vertical line test to determine whether it is a graph of a well defined function mapping subsets of the reals to the reals.

   ii) Use the horizontal line test to determine over which domains and codomains(on the graph) the function is one-to-one, onto, or invertible.