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DUBLIN INSTITUTE OF TECHNOLOGY
KEVIN STREET, DUBLIN 8

B.Sc. in Physics Technology

B.Sc. in Science with Nanotechnology

B.Sc. in Clinical Measurement

B.Sc. in Physics with Medical Physics and Bioengineering

YEAR II

SUPPLEMENTAL EXAMINATIONS 2012

MATHEMATICS FOR THE PHYSICAL SCIENCES II

DR. D. MACKEY
DR. C. HILLS

Full marks for complete answers to **FOUR** questions.

Graph Paper, Mathematics Tables.

1 (a) Verify that the function $f(x) = \dfrac{\cos x}{x}$ is a solution of the following initial value problem

$$x\,\frac{df}{dx} + f(x) = -\sin x, \qquad f(\frac{\pi}{2}) = 0.$$

[10 marks]

(b) Using the method of the integrating factor, solve the linear first order differential equation

$$\frac{df}{dx} + \frac{f(x)}{x+5} = 4$$

with initial condition $f(0) = 0$.

[15 marks]

2 (a) Let $f(x, y) = xy\,\mathrm{e}^x + x^2 + y^2$. Calculate **all** second order partial derivatives of f and verify that

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

[15 marks]

(b) Let

$$f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4.$$

Find the critical points for this function and decide whether each of them is a maximum, minimum or saddle point.

[10 marks]

3 Find the eigenvalues and the associated eigenvectors of the matrix

$$\begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}.$$

[25 marks]

4 (a) Evaluate the following double integral

$$\int_1^4 \int_{-2}^3 (x^2 - 2xy^2 + y^3)\, dxdy.$$

[10 marks]

(b) Evaluate the double integral

$$\iint_R x^2 y\, dxdy$$

where R is the triangular area bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$. Show that the same result is obtained when the order of integration is reversed.

[15 marks]

5 (a) Find the inverse of the matrix A below using elementary row operations

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 4 & 0 & -1 \\ 4 & -2 & 0 \end{pmatrix}.$$

[15 marks]

(b) Use the result in (a) to solve the following system of equations

$$x - y + z = -6$$
$$4x - z = 3$$
$$4x - 2y = -3$$

[10 marks]

6 A spring-mass system which is being driven by a motor on a surface with friction satisfies the differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} = 6x(t) = 4\cos(2t),$$

where $x(t)$ is the displacment of the mass. Assuming that the mass is located at $x = 1$ at time $t = 0$ and is released from rest, find the subsequent displacement $x(t)$ and hence determine the steady state of the system.

[25 marks]