Function Concavity

Example 2 Use the second derivative test to classify the critical points of the function,

Solution Note that all were doing here is verifying the results from the first example. The second derivative is,

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The three critical points (, , and) of this function are all critical points where the first derivative is zero so we know that we at least have a chance that the Second Derivative Test will work. The value of the second derivative for each of these are,

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The second derivative at is negative so by the Second Derivative Test this critical point this is a relative maximum as we saw in the first example. The second derivative at is positive and so we have a relative minimum here by the Second Derivative Test as we also saw in the first example.

In the case of the second derivative is zero and so we cant use the Second Derivative Test to classify this critical point. Note however, that we do know from the First Derivative Test we used in the first example that in this case the critical point is not a relative extrema.

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Lets work one more example.

Example 3 For the following function find the inflection points and use the second derivative test, if possible, to classify the critical points. Also, determine the intervals of increase/decrease and the intervals of concave up/concave down and sketch the graph of the function.

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Solution Well need the first and second derivatives to get us started.
The critical points are,

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Notice as well that we wont be able to use the second derivative test on to classify this critical point since the derivative doesnt exist at this point. To classify this well need the increasing/decreasing information that well get to sketch the graph.

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We can however, use the Second Derivative Test to classify the other critical point so lets do that before we proceed with the sketching work. Here is the value of the second derivative at . So, according to the second derivative test is a relative maximum.

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Now lets proceed with the work to get the sketch of the graph and notice that once we have the increasing/decreasing information well be able to classify .
Here is the number line for the first derivative.

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So, according to the first derivative test we can verify that is in fact a relative maximum. We can also see that is a relative minimum.

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Be careful not to assume that a critical point that cant be used in the second derivative test wont be a relative extrema. Weve clearly seen now both with this example and in the discussion after we have the test that just because we cant use the Second Derivative Test or the Test doesnt tell us anything about a critical point doesnt mean that the critical point will not be a relative extrema. This is a common mistake that many students make so be careful when using the Second Derivative Test.

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Okay, lets finish the problem out. We will need the list of possible inflection points. These are,

Here is the number line for the second derivative. Note that we will need this to see if the two points above are in fact inflection points.

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So, the concavity only changes at and so this is the only inflection point for this function.

Here is the sketch of the graph.

The change of concavity at is hard to see, but it is there its just a very subtle change in concavity.