1. Simplify the following:

   (i) i^2

   (ii) i^5

   (iii) $3i$

   (iv) i^13

2. Evaluate the following:

   (i) $(2 + 3i) + (5 + 7i)$,

   (ii) $(2 + 3i)(5 + 7i)$,

   (iii) $(2 + 3i)(23i)$,

   (iv) $(35i)^3$,

   (v) $\dfrac{35i}{12i}$

3. For each part of question 2, draw the complex numbers on an Argand diagram, and express in the form $re^{i\theta}$

4. Evaluate $(-2 - 5i)(3 - 2i)$.

5. Evaluate $(6 - 2i)(1 - i)(2 - 2i)$.

6. Evaluate $\dfrac{1}{i} \times \dfrac{6 - 2i}{1 - i}$

   .

7. Express $z = e^{2+i(\pi/4)}$ in the form a + bi.

8. Express $3 - 2i$ in polar form.

9. Find the square roots of

$$z = 4(cos3 + isin3)$$

   , and draw these on an Argand diagram. Identify the principal root.

10. For $z = 3(cos\pi/6 + isin\pi/6)$, calculate z 4 in polar form.

11. For $z = 3 - 2i$, find the five roots $z^{1/5}$, and plot these on an Argand diagram indicating the principal root.

12. Find the values of x, y (both real) which satisfy the equation

$$x(x+y)+xyi=13i$$

.

13. Two competing *probability amplitudes* A1 and A2 for a quantum mechanical transition from some initial state —ii to some final state —fi are given by $A1 = aei\psi 1, A2 = bei\psi 2$. and the total probability amplitude (A) for the process is given by A1 + A2. Given that the probability for a process is given by AA , calculate the probability for the transition from —ii to —fi. Using this, calculate the probability if $\psi 1 = \psi 2$.

1. Perform the indicated operation and write the answers in standard form.

   (i)

   (ii)

   (iii)

2. Multiply each of the following and write the answers in standard form.

   (i)

   (ii)

   (iii)

   (iv)

3. Write each of the following in standard form.

(i)

(ii)

(iii)

(iv)

Exercises[edit] $(7+2i)+(11-6i) = (8-3i)-(6i) = (9+4i)(3-16i) = 3i\times \times 9i =$
$\frac{i}{2+i} = \frac{i}{2+i} =$
$\frac{11+3i}{\sqrt{3}-4i} = \frac{11+3i}{\sqrt{3}-4i} =$
$(x+yi)^{-1} = (x+yi)^{-1} =$ Answers[edit] 18 - 4i 8 - 3i - 6i = 8 - 9i 27 - 144i + 12i - 64i2 = 91 - 132i -27. $(i \times i = i^2 = -1 i \times i = i^2 = -1)$
$\frac{i}{2+i} \times \frac{2-i}{2-i} = \frac{1+2i}{5} = \frac{1}{5} + \frac{2}{5}i \frac{i}{2+i} \times \frac{2-i}{2-i} = \frac{1+2i}{5} = \frac{1}{5} + \frac{2}{5}i$
$\frac{11+3i}{\sqrt{3}-4i} \times \frac{\sqrt{3}+4i}{\sqrt{3}+4i} = \frac{11+3i}{\sqrt{3}-4i} \times \frac{\sqrt{3}+4i}{\sqrt{3}+4i} =$
$\frac{11\sqrt{3}+44i+3i\sqrt{3}+12i^2}{3+16} = \frac{11\sqrt{3}+44i+3i\sqrt{3}+12i^2}{3+16} =$
$\frac{(11\sqrt{3}-12)+(44i+3i\sqrt{3})}{19} = \frac{(11\sqrt{3}-12)+(44i+3i\sqrt{3})}{19} =$
$\frac{11\sqrt{3}-12}{19} + \frac{44+3\sqrt{3}}{19}i \frac{11\sqrt{3}-12}{19} + \frac{44+3\sqrt{3}}{19}i$
Recall that $x^{-1} = \frac{1}{x} x^{-1} = \frac{1}{x}$ can be seen as the division of two complex numbers:
$(x+yi)^{-1} = \frac{1}{x+yi} \times \frac{x-yi}{x-yi} = (x+yi)^{-1} = \frac{1}{x+yi} \times \frac{x-yi}{x-yi} =$
$\frac{x-yi}{x^2-y^2i^2} = \frac{x}{x^2+y^2} - \frac{y}{x^2+y^2}i \frac{x-yi}{x^2-y^2i^2} = \frac{x}{x^2+y^2} - \frac{y}{x^2+y^2}i$

MA4604: Science Maths 4, Homework Week 1

1. Write each of the complex numbers below in the form a + ib, that is simplify each expression to find the real numbers a and b.

   (a) (3 + 8i) + (2i - 6) - (-5 + i)

   (b) (3 - i)(5 + 6i)

   (c) (1 + i)(2 5i)(7 + 3i)

   (d) $\frac{34i}{2i}$

   (e) $\frac{5i}{23i}$

   (f) $\frac{2+4i}{i(1i)}$

2. Find the real number(s) t that makes each expression below real:

(a) (4 + 6i)(3i)(t + 6i)

(b) i(t + 4i) 2

(c) 5 10i 4i + t .

3. Solve the complex equation 2z + i$\bar{z}$ = 5 + 4i (hint: write z = x + iy).

4. Find all the roots of, and hence factorise fully, each given polynomial:

   (a) x 2 8x + 25

   (b) z 2 + (4 + 3i)z + 14 + 6i

   (c) x 4 16

   (d) x 3 + 8.

5. Given that x = 1 + 4i is a root of the quartic polynomial x 4 + 13x 2 + 34x, find the other three roots and write it as a product of linear factors.

6. Plot each of the given complex numbers in an Argand diagram:

(a) 2 + i

(b) 1 + 3i

(c) 2i

(d) 2 2i

(e) 26

(f) 46

(g) 36

7. Write each of the complex numbers in Q.6

(e) through (l) in the form x + iy, that is convert them from polar or exponential form to standard form.

MA4604: Science Maths 4, Homework Week 2

1. Write each of the complex numbers below in polar form r6 and in exponential form rei (hint: first find r = —z— and = arg(z)):

   (i) z = 2

   (ii) z = 5i

   (iii) z = 3 3i

   (iv) z = 3 3i.

2. For each complex number given below in exponential form rei, find its absolute value —z— and its argument Arg(z); express z in the form x + iy: .

3. Use de Moivres theorem to simplify the given powers:

(a) (1 + i) 20

(b) (1 + i 3)12 .

4. Use de Moivres theorem with n = 3 to write cos(3) in terms of cos and to write sin(3) in terms of sin (hint: youll also need to use the fact that cos2 + sin2 = 1).

5. Find the twelve twelfth roots of unity in exponential and in Cartesian form.

6. By first writing the given number in complex exponential form, evaluate each of the following in Cartesian form:

   (a) all cube roots of 27;

   (b) all cube roots of -64;

   (c) all cube roots of -64i;

   (d) all fourth roots of 81;

   (e) all fourth roots of -4;

   (f) all square roots of -9i.