R222/227/229/235/702

DUBLIN INSTITUTE OF TECHNOLOGY
KEVIN STREET, DUBLIN 8

B.Sc. in Physics Technology

B.Sc. in Science with Nanotechnology

B.Sc. in Clinical Measurement

B.Sc. in Physics with Medical Physics and Bioengineering

YEAR II

SUPPLEMENTAL EXAMINATIONS 2011

Mathematics for the Physical Sciences II

Dr. D. Mackey
Dr. C. Hills

Full marks for complete answers to **FOUR** questions.

Graph Paper, Mathematics Tables.

1 (a) Verify that the function $f(x) = \dfrac{\cos x}{x}$ is a solution of the following initial value problem

$$x\,\frac{df}{dx} + f(x) = -\sin x, \qquad f(\frac{\pi}{2}) = 0.$$

[8 marks]

(b) The differential equation for Newton's law of cooling is written in the form

$$\frac{dT}{dt} = -k\,\left(T(t) - T_S\right), \qquad T(0) = T_0.$$

where k is a proportionality constant.

i. Solve this separable equation and show the solution is given by

$$T(t) = T_S + (T_0 - T_S)\,\mathrm{e}^{-kt}$$

[9 marks]

ii. A cup of tea is at 70°C but after 10 minutes has cooled to 50°. If the ambient temperature is 25°C find the total time required for the temperature to drop to 30°C.

[8 marks]

2 (a) Let $f(x, y) = \mathrm{e}^x\,(x\cos(y) - y\sin(x))$. Calculate **all** second order partial derivatives of f and verify that

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

[15 marks]

(b) Let

$$f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4.$$

Find the critical points for this function and decide whether each of them is a maximum, minimum or saddle point.

[10 marks]

3 (a) Let $z = \sqrt{3} - 2i$. Evaluate the real and imaginary part of the complex expression

$$\frac{1}{2}\left(\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right).$$

[7 marks]

(b) Let $z = a + ib$ be a complex number, $z \neq 1$, and let

$$W = \frac{z+1}{z-1}.$$

Show that, if $|z| = 1$ then the real part of W is zero.

[8 marks]

(c) Use DeMoivre's formula to express the complex number

$$(4 + 4i)^7$$

in the form $a + ib$. [10 marks]

4 (a) Evaluate the following double integral

$$\int_{\pi}^{2\pi} \int_{0}^{\pi} (\sin(x) + \cos(y))\, dxdy.$$

[10 marks]

(b) Evaluate the double integral

$$\iint_R x^2 y\, dxdy$$

where R is the triangular area bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$. Show that the same result is obtained when the order of integration is reversed.

[15 marks]

5 (a) Plot the cardioid $r = 1 - \cos(\theta)$ from $\theta = 0$ to $\theta = 2\pi$ using intervals of length $\frac{\pi}{6}$.

[10 marks]

(b) Use integration in polar coordinates to determine the area of the region that lies outside the circle $r = 1$ and inside the cardioid $r = 1 - \cos(\theta)$.

[15 marks]

6 Find the eigenvalues and the associated eigenvectors of the matrix

$$\begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}.$$

[25 marks]