Numerical Methods 15 February 2012
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Problem Sheet 1: Numerical integration. Newton-Cotes formulas.

1. Use the midpoint, trapezoidal and Simpson's rule to approximate the integral

$$I = \int_0^1 \sqrt{x}\, dx$$

Evaluate the error in each case.

2. Show that the closed Newton-Cotes formula for $n = 3$ (known as the "three-eights rule") is

$$I_3(f) = \frac{b-a}{8}\,[f(a) + 3f(a+\Delta x) + 3f(a+2\Delta x) + f(b)]$$

where $\Delta x = (b-a)/3$. Verify directly that this rule has degree of precision equal to 3.

3. Show that the open Newton-Cotes formula for $n = 2$ is

$$I_2(f) = \frac{b-a}{3}\,[2f(a+\Delta x) - f(a+2\Delta x) + 2f(a+3\Delta x)]$$

where $\Delta x = (b-a)/4$. Verify directly that this rule has degree of precision equal to 3.

4. Determine the degree of precision for each of the following quadrature rules

$$\text{(i)} \int_{-1}^1 f(x)\, dx \approx f\left(-\frac{\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right)$$

$$\text{(ii)} \int_{-1}^1 f(x)\, dx \approx \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right)$$