MA4004 Tutorial 5 (Week 8) Single Sample Tests

Hypothesis Tests

Four Step process

1) formal statement of null and alternative hypothesese
2) Determination of the critical values
3) Calculation of the test statistic
4) Decision based on a comparison of the test statistic and critical value

Test statistic

$$\frac{\text{Sample Value} - \text{Null value}}{\text{Standard Error}}$$

Standard Errors

$$\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \qquad \sqrt{\frac{p(1-p)}{n}} \qquad \sqrt{\frac{s_1}{n_1}+\frac{s_2}{n_2}}$$

Question 1

Given $\bar{x} = 121 \quad s = 49$
Significance $\alpha = 0.05$

hypotheses

$$\mathrm{H_0}: \mu = 100$$
$$\mathrm{H_A}: \mu \neq 100$$

Test statistic

Standard Error $\quad S.E.(\overline{X}) = \frac{14}{\sqrt{49}} = 14/7 = 2$

Test Statistic $\quad \frac{\bar{x} - \mu_0}{S.E.(\overline{X})} = \frac{121-100}{2} = 10.5$

Critical values

Population standard deviation σ is unknown.

The sample size (n=49) is large $(n>30)$. Use t distribution with ∞ degrees of Freedom.

The test is two tailed. k= 2 ("$\neq$" symbol in the alternative hypothesis).

Column = $\frac{\alpha}{k} = \frac{0.05}{2} = 0.025$

Murdoch Barnes table 7

Row: ∞
Column $\sigma = 0.025$

Critical value = **1.96**

Decision rule

Is the test statistic value greater than the critical value

If Yes: we reject the null hypothesis

If No: We fail to reject the null hypothesis. (not enough evidence)

Here TS = 10.5 is greater than CV = 1.96.

We reject the null hypothesis. We accept the null hypothesis that this is an unusual group.

95% confidence

Point estimate 121
Quantile 1.96
Std. Error = 2

Confidence Interval is $121 \pm (1.96 \times 2)$ = $121 \pm (3.92)$ = $(117.08, 124.92)$

Question 2

Given population parameters

$\mu = 4700 \qquad \sigma = 1460$

sample size $n = 100$

$\bar{x} = 5000 \text{ hrs}$

Significance $\alpha = 0.01$

hypotheses

The alternative hypothesis states that the process significantly increases the life span of the component.

$$\mathrm{H}_0: \mu \leq 4700$$
$$\mathrm{H}_\mathrm{A}: \mu > 4700$$

Test statistic

$$S.E.(\overline{X}) = \frac{1460}{\sqrt{100}} = 146$$
$$\frac{\overline{x} - \mu_0}{S.E.(\overline{X})} = \frac{5000 - 4700}{146} = 2.054$$

Critical values

Population standard deviation σ is known.

Use Normal distribution (equivalently ***t*** distribution with ∞ degrees of Freedom).

The test is one tailed. k= 1 .

Column = $\frac{\alpha}{k} = \frac{0.01}{1} = 0.01$

Murdoch Barnes table 7

Row: df = ∞
Column $\alpha = 0.01$

Critical value = **2.326**

Decision rule

Is the test statistic value greater than the critical value

No Here TS = 2.054 is less than CV = 2.326.

We fail to reject the null hypothesis. Not enough evidence to suggest that the new process increases lifespan of TV tubes.

Question 3

Given

Sample size $n = 106$

sample estimate $\overline{X} = 98.2$

sample standard deviation $s = 0.62$

significance level $\alpha = 0.05$

Hypotheses

$H_o: \mu = 98.6$ Mean body Temp is 98.6 degrees
$H_a: \mu \neq 98.6$ Mean body Temp is not 98.6 degrees

Test Statistic

$$S.E.(\overline{X}) = \frac{0.62}{\sqrt{106}} = 0.06$$

$$\frac{\overline{x} - \mu_0}{S.E.(\overline{X})} = \frac{98.2 - 98.6}{0.06} = -6.6$$

Critical values

Population standard deviation σ is unknown. But large Sample size $(n > 30)$

Use ***t*** distribution with ∞ degrees of Freedom.

The test is two tailed. k= 2 .

Column = $\frac{\alpha}{k} = \frac{0.05}{2} = 0.025$

Murdoch Barnes table 7

Row: df = ∞
Column $\alpha = 0.025$

Critical value = **1.96**

Decision rule

Is the test statistic (here - the absolute value) value greater than the critical value

No Here TS = 6.6 is greater than CV = 1.96.

We reject the null hypothesis. The mean body termperature is significantly different from 98.2.

Question 4

Given

Sample size $n = 50$

sample mean $\overline{X} = 2177$ Euros

sample standard deviation $s = 1257$ Euros

Population standard deviation σ is unknown.

Hypotheses

$$H_o : \mu \leq 2000$$
$$H_a : \mu > 2000$$

Test Statistic

$$S.E.(\overline{X}) = \frac{1257}{\sqrt{50}} = 177.77$$
$$\frac{\overline{x} - \mu_0}{S.E.(\overline{X})} = \frac{2177 - 2000}{177.7} = 0.99$$

Critical values

Population standard deviation σ is unknown.
But large Sample size $(n > 30)$

Use Normal distribution (equivalently ***t*** distribution with ∞ degrees of Freedom).

The test is one tailed. k= 1 .

Column = $\frac{\alpha}{k} = \frac{0.025}{1} = 0.025$

Murdoch Barnes table 7

Row: df = ∞
Column $\alpha = 0.025$

Critical value = **1.96**

Decision rule

Is the test statistic value greater than the critical value

No Here TS = 0.99 is less than CV = 1.96.

We fail to reject the null hypothesis. Not need to implement the monitoring system.