UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

Faculty of Science and Engineering

Department of Mathematics & Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4702	SEMESTER: Spring 2012/13
MODULE TITLE: Technological Mathematics 2	DURATION OF EXAMINATION: 2.5 hours
LECTURER: Dr. Páraic Treacy	PERCENTAGE OF TOTAL MARKS: 80 %
EXTERNAL EXAMINER: Prof. B. Murphy	

INSTRUCTIONS TO CANDIDATES:

Questions One is **compulsory** and carries 40 marks.

Answer any other **four** questions worth 15 marks each.

Log tables and graph paper are available from the invigilators.

N.B. There are some useful formulae at the end of the paper.

Question 1

(a) Find $f^{-1}(x)$ the inverse of the function

$$f(x) = \frac{1}{2x - 5}$$ **(4)**

(b) Find the domain and the range of the function:

$$f(x) = 7 + 2\sin x$$ **(4)**

(c) Find the equations of the vertical and horizontal asymptotes of the following function:

$$f(x) = \frac{5x^2 - 7x + 2}{2x^2 - 1}$$ **(8)**

(d) Find:

$$\lim_{x \to \infty} \frac{2x^2 - 8x}{4x^2 - 7}$$ **(4)**

(e) Evaluate the following indefinite integral:

$$\int 3x^2 + 2e^x - 1 \, dx$$ **(4)**

(f) Evaluate the following definite integral:

$$\int_4^9 \frac{1}{\sqrt{x}} \, dx$$ **(4)**

(g) Evaluate all the first partial derivatives $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ of the following function:

$$z = 2x^3 y + x \sin y$$ **(4)**

(h) Check whether the following function is an odd or an even function:

$$f(x) = \frac{e^x - e^{-x}}{2}$$ **(4)**

(i) Find the sum of the following geometric series:

$$3 + 6 + 12 + 24 + \ldots + 3072$$ **(4)**

Question 2

Consider the function $f(x) = x^4 - 8x^2 + 7$

(a) Find the y intercept of the function $f(x)$. **(2)**

(b) Find the three turning points of the function $f(x)$ and classify them as local maxima or local minima. **(3)**

(c) Find the two points of inflection of the function $f(x)$. **(3)**

(d) Determine the behaviour of the function $f(x)$ as $x \to +\infty$ and as $x \to -\infty$. **(3)**

(e) Sketch the graph of the function $f(x)$ illustrating the features of the curve obtained in parts (a – d). **(4)**

Question 3

(a) Evaluate the following definite and indefinite integrals.

(i)

$$\int_0^2 \frac{2x+3}{x^2+3x+1}\,dx$$ **(3)**

(ii)

$$\int \cosh x \sinh^3 x \; dx$$ **(3)**

(iii) Use integration by parts to find:

$$\int xe^x dx$$ **(3)**

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(b) An object moves in a straight line with acceleration $a(t) = \cos 4t$ where t represents time in seconds and acceleration is measured in m/s^2.
At time $t = 0$ it has a velocity $v = 2m/s$. Find its velocity at all times t. **(6)**

Question 4

(a) Find the area enclosed by the curve $y = 4 - x^2$ and the line $y = x + 2$ **(5)**

(b) Find the area enclosed by the curve $f(x) = 3x^2 - 4x + 1$ and the x axis. **(5)**

(c) A current $i(t) = 7 + 4\sin 2t$ passes through a capacitor at time t.
The capacitor is uncharged at time $t = 0$. Find the charge $q(t)$ at all times t. **(5)**

Question 5

(a) Find the Maclaurin series of $f(x) = \cos x$ up to and including the term containing x^4.

Use your answer to estimate the value of $\cos(1)$ **(9)**

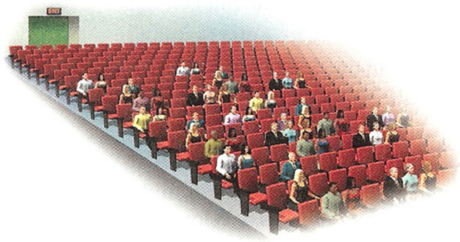
(b) **(i)** Find $\frac{\partial z}{\partial x}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y \partial x}$ of the function $z = 2x^2y + 4x^2y^3 - 7x^2$. **(3)**

(ii) Prove that the function $z = \cos(x + 3y)$ satisfies the partial differential equation: **(3)**

$$\frac{\partial^2 z}{\partial y^2} - 9\frac{\partial^2 z}{\partial x^2} = 0.$$

Question 6

(a) Cinema theatres are often built with more seats per row as the rows move towards the back. In screen 3 of the Odeon cinema complex in Castletroy there are 22 seats in the first row, 24 in the second, 26 in the third, and so on, for 18 rows. How many seats are there in screen 3?

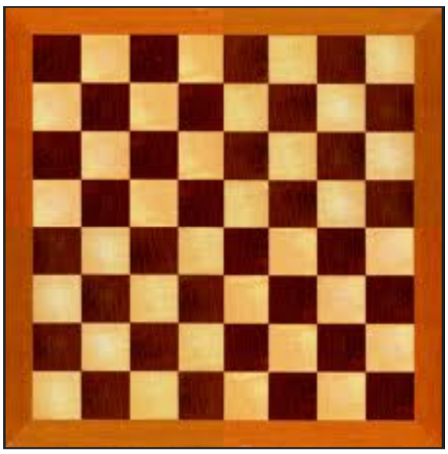


(6)

(b) Legend tells us that the game of chess was invented hundreds of years ago by the Grand Vizier Sissa Ben Dahir for King Shirham of India. The King loved the game so much that he offered his Grand Vizier any reward he wanted for having created the game.

"Majesty, give me one grain of wheat to go on the first square of the chess board, two grains to place on the second square, four grains to place on the third square, eight on the fourth square and so on until all the squares on the board are covered". Note: there are 64 squares on a chess board.

How many grains of wheat did the King owe Sissa Ben Dahir through this deal? Hint: use a geometric series to help determine the total number of grains of wheat. **(9)**



Useful Formulae

Logs

If $a^b = c$

Then $\log_a c = b$

Differentiation

Product Rule:

$$y = uv$$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient Rule:

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Further formulae and special cases on pages 41 & 42 of the log tables provided.

Hyperbolic functions

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

Integration

Integration by parts:

$$\int u\,dv = uv - \int v\,du$$

Further formulae and special cases on pages 41 & 42 of the log tables provided.

Sequences & Series

Arithmetic Series:

$$u_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Geometric Series:

$$u_n = ar^{n-1}$$

$$S_n = a\left(\frac{1-r^n}{1-r}\right)$$

$$S_\infty = a\left(\frac{1}{1-r}\right)$$

Maclaurin Series:

$$f(x) = f(0) + xf'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \cdots$$

Ratio Test:

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = r$$