Problem Sheet 5: Double integrals

1. Evaluate the following double integrals

$$(i) \int_0^4 \int_0^2 (x^2+y^2)\,dydx \quad (ii) \int_1^4 \int_{-2}^3 (x^2-2xy^2+y^3)\,dxdy, \quad (iii) \int_{-1}^0 \int_{-1}^1 (x+y+1)\,dxdy$$

$$(iv) \int_0^3 \int_{-2}^0 (x^2y-2xy)\,dydx, \quad (v) \int_0^3 \int_0^2 (4-y^2)\,dydx, \quad (vi) \int_1^{\ln(8)} \int_0^{\ln(y)} \mathrm{e}^{x+y}\,dxdy$$

2. Sketch the region of integration, reverse the order of integration and evaluate the following integrals

$$(i) \int_0^2 \int_0^{4-y^2} y\,dxdy \quad (ii) \int_0^1 \int_2^{4-2x} dydx \quad (iii) \int_1^2 \int_y^{y^2} dxdy \quad (iv) \int_0^{\pi} \int_0^x x\sin(y)\,dydx$$

3. Evaluate the double integral

$$\iint_R x^2y\,dxdy$$

where R is the triangular area bounded by the lines $x=0$, $y=0$ and $x+y=1$. Show that the same result is obtained when the order of integration is reversed.

4. Evaluate the double integral

$$\iint_R xy\,dxdy$$

where R is the triangular area bounded by the lines $y=x$, $y=2x$ and $x+y=2$. Show that the same result is obtained when the order of integration is reversed.

5. Sketch the regions described below then calculate their areas.

   (a) The region bounded by the parabola $x=-y^2$ and the line $y=x+2$;

   (b) The region bounded by the curve $y=\mathrm{e}^x$ and the lines $y=0$, $x=0$ and $x=\ln(2)$;

   (c) The region bounded by the parabola $y=x^2-x$ and the line $y=x$.