Definite Integrals Examples

Definite Integral – Ex. 3

Find the area between $f(x) = \sin x$ and the x axis between $x = \pi$ and $x = 2\pi$

Solution:

$$\int_{\pi}^{2\pi} \sin x \, dx$$

$$= [-\cos x]_{\pi}^{2\pi}$$

$$= (-\cos 2\pi) - (-\cos \pi)$$

$$= (-1) - (1)$$

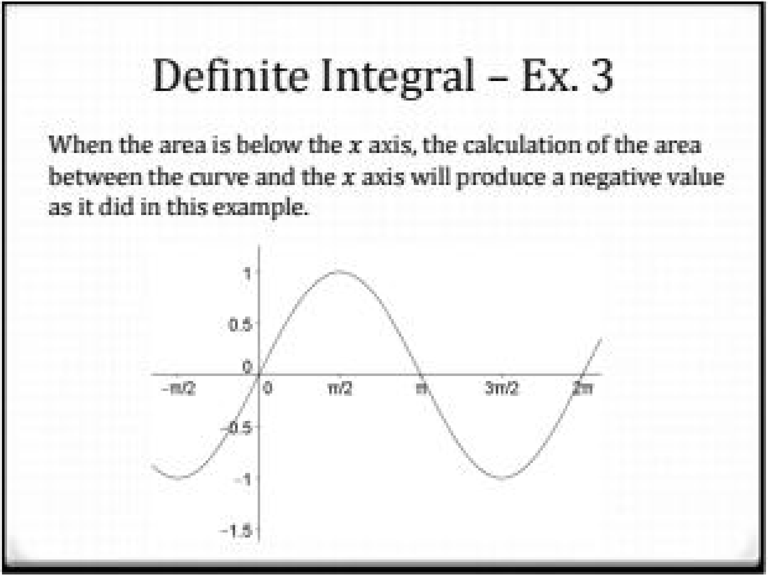
$$= -2$$

Because it is an area that we are calculating, we ignore the minus.

The area between $f(x) = \sin x$ and the x axis between $x = \pi$ and $x = 2\pi$ is 2 units^2

Definite Integrals Examples

Definite Integrals Examples



Definite Integrals Examples

Definite Integrals Examples

Definite Integral – Ex. 4

Find the area between $f(x) = \sin x$ and the x axis between $x = 0$ and $x = 2\pi$

Solution:

$$\int_0^{2\pi} \sin x \, dx$$

$$= [-\cos x]_0^{2\pi}$$

$$= (-\cos 2\pi) - (-\cos 0)$$

$$= (-1) - (-1)$$

$$= 0$$

This suggest that there is no area between $f(x) = \sin x$ and the x axis between $x = 0$ and $x = 2\pi$

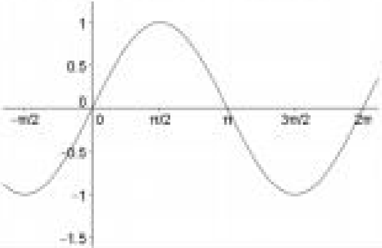
But if we look at the graph of $f(x) = \sin x$ we can see that this is obviously not true…

Definite Integrals Examples

Definite Integrals Examples

Definite Integral – Ex. 4

The positive area between $x = 0$ and $x = \pi$ was cancelled out by the negative area between $x = \pi$ and $x = 2\pi$. As such, the area between the curve and the x axis above the x axis needs to be calculated separately to the area between the curve and the x axis below the x axis, then combined to find the full area.



Definite Integrals Examples

Definite Integrals Examples

Definite Integral – Ex. 4

In Example 2, we found:

$$\int_0^{\pi} \sin x \, dx = 2 \text{ units}^2$$

In Example 3, we found:

$$\int_{\pi}^{2\pi} \sin x \, dx = 2 \text{ units}^2$$

This means that the area between $f(x) = \sin x$ and the x axis between $x = 0$ and $x = 2\pi$ is:

$$2 + 2 = 4 \text{ units}^2$$

Definite Integrals Examples

Definite Integrals Examples

Definite Integral – Ex. 5

Find the area between $f(x) = x^2 + 4x$ and the x axis between $x = -4$ and $x = 3$

Solution:

We must check if the curve crosses the x axis and how that might affect our calculations.

$$f(x) = x^2 + 4x$$

Crosses x axis when $y = 0$

$$0 = x^2 + 4x$$

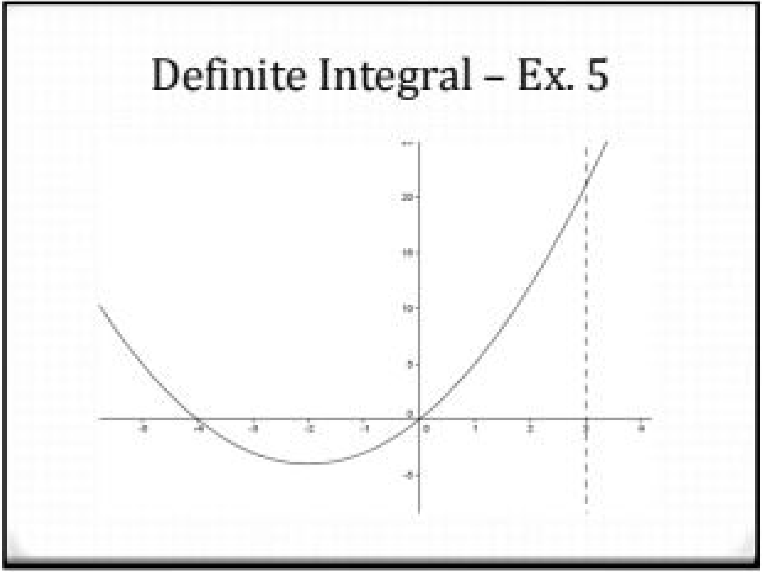
$$x(x+4) = 0$$

$$x = 0 \quad or \quad x = -4$$

Crosses x axis at 0 and -4

Definite Integrals Examples

Definite Integrals Examples



Definite Integrals Examples

Definite Integrals Examples

Definite Integral – Ex. 5

As such, we need to find the area below the x axis and the area above the x axis separately.

We'll start with the area below the x axis:

$$\int_{-4}^{0} (x^2 + 4x)dx$$

$$= \left[\frac{x^3}{3} + 2x^2\right]_{-4}^{0}$$

$$= \left(\frac{(0)^3}{3} + 2(0)^2\right) - \left(\frac{(-4)^3}{3} + 2(-4)^2\right)$$

$$= 0 - \left(-\frac{64}{3} + 32\right)$$

$$= -\frac{32}{3}$$

The area is $\frac{32}{3}$ units^2

Definite Integrals Examples

Definite Integrals Examples

Definite Integral – Ex. 5

Next we need to find the area above the x axis:

$$\int_0^3 (x^2+4x)\,dx$$

$$= \left[\frac{x^3}{3} + 2x^2\right]_0^3$$

$$= \left(\frac{(3)^3}{3} + 2(3)^2\right) - \left(\frac{(0)^3}{3} + 2(0)^2\right)$$

$$= (9 + 18) - (0)$$

$$= 27 \text{ units}^2$$

The area above the x axis is 27 units^2

Definite Integrals Examples

Definite Integrals Examples

Definite Integral – Ex. 5

To find the area between $f(x) = x^2 + 4x$ and the x axis between $x = -4$ and $x = 3$

We add the two areas we found:

$$Total\ Area = \frac{32}{3} + 27 = 37\frac{2}{3}$$

Answer: Area is $37\frac{2}{3}$ units^2