MA4F02 (2010)

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Section A

(a) (i) $f(x) = \sqrt{2x-6}$

Domain: $2x - 6 \geq 0$

$2x \geq 6$

$x \geq 3$

$\Rightarrow [3, \infty)$

Range: $y \geq 0$

$[0, \infty)$

(ii) $y = \log_e(2x+3)$

$\Rightarrow g^{-1}: x = \log_e(2y+3)$

$\Rightarrow e^x = 2y + 3$

$\Rightarrow e^x - 3 = 2y$

$= \dfrac{e^x - 3}{2} = y$

$\Rightarrow g^{-1}(x) = \dfrac{e^x - 3}{2}$

(iii) $f(x) = \dfrac{e^x - e^{-x}}{2}$

$f(-x) = \dfrac{e^{-x} - e^{x}}{2}$

$= -\dfrac{(e^x - e^{-x})}{2}$

$= -f(x)$

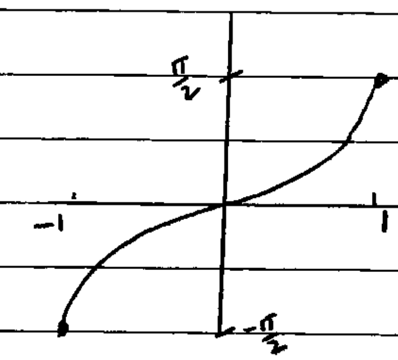
$\Rightarrow f(-x) = -f(x)$

$\Rightarrow$ odd function

(b) $\sin^{-1}\left(\frac{1}{4}\right) = 14.47^0$ or 0.2526 Radians

(ii) $y = \sin^{-1} x$

		radians
$\sin^{-1}(-1)$	$= -90^0$	$= -\frac{\pi}{2}$
$\sin^{-1}(0)$	$= 0^0$	$= 0$
$\sin^{-1}(1)$	$= 90^0$	$= \frac{\pi}{2}$



(c)

$\cosh 2x$

$\Rightarrow \dfrac{e^{2x} + e^{-2x}}{2}$

$\cosh^2 x + \sinh^2 x$

$\Rightarrow$

$\left(\dfrac{e^x + e^{-x}}{2}\right)^2 + \left(\dfrac{e^x - e^{-x}}{2}\right)^2$

$\Rightarrow \dfrac{(e^x + e^{-x})(e^x + e^{-x})}{4} + \dfrac{(e^x - e^{-x})(e^x - e^{-x})}{4}$

$\Rightarrow \dfrac{e^{2x} + e^0 + e^0 + e^{-2x}}{4} + \dfrac{e^{2x} - e^0 - e^0 + e^{-2x}}{4}$

$\Rightarrow \dfrac{e^{2x} + \cancel{1} + \cancel{1} + e^{-2x} + e^{2x} - \cancel{1} - \cancel{1} + e^{-2x}}{4}$

$\Rightarrow \dfrac{2e^{2x} + 2e^{-2x}}{4}$

$\Rightarrow \dfrac{e^{2x} + e^{-2x}}{2}$

$\Rightarrow \cosh^2 x = \cosh^2 x + \sinh^2 x$

(2) $y = f(x) = x^4 - 2x^2$

(i) $x=0$ $\quad f(0) = 0 \Rightarrow (0,0)$

$y=0$
$\Rightarrow x^4 - 2x^2 = 0$
$\Rightarrow x^2(x^2-2) = 0$
$\Rightarrow x^2 = 0 \;|\; x^2 - 2 = 0$
$\Rightarrow x = 0 \;|\; x^2 = 2$
$x = \pm\sqrt{2}$

$\Rightarrow (0,0)\ (\sqrt{2},0)\ (-\sqrt{2},0)$

(ii) $f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x$

Stationary points $\Rightarrow 4x^3 - 4x = 0$

$\Rightarrow x^3 - x = 0$

$\Rightarrow x(x^2-1) = 0$

$\Rightarrow x(x-1)(x+1) = 0$

$\Rightarrow x = 0 \;|\; x - 1 = 0 \;|\; x + 1 = 0$
$\Rightarrow x = 1 \;|\; x = -1$
$\Rightarrow f(0) = 0 \;|\; f(1) = -1 \;|\; f(-1) = -1$

$\Rightarrow (0,0)\ (1,-1)\ (-1,-1)$

Classification: $f'(x) = 4x^3 - 4x$

$f''(x) = 12x^2 - 4$

$f''(0) = -4 < 0$

$\Rightarrow (0,0)$ is max. turning point

$f''(1) = 8 > 0$

$\Rightarrow (1,-1)$ is min. turning point

$f''(-1) = 8 > 0$

$\Rightarrow (-1,-1)$ is min. turning point

(iii) $f''(x) = 12x^2 - 4 = 0$

$\Rightarrow 12x^2 = 4$

$\Rightarrow 3x^2 = 1$

$\Rightarrow x^2 = \frac{1}{3}$

$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$

$f(\frac{1}{\sqrt{3}}) = \frac{1}{9} - \frac{2}{3} = -\frac{5}{9}$

$f(-\frac{1}{\sqrt{3}}) = \frac{1}{9} - \frac{2}{3} = -\frac{5}{9}$

$\Rightarrow (\frac{1}{\sqrt{3}}, -\frac{5}{9})\ (-\frac{1}{\sqrt{3}}, -\frac{5}{9}) \rightarrow$ inflection points if change in concavity

$\frac{1}{\sqrt{3}} \approx .6$

(iv) Concavity

$+ \quad -\frac{1}{\sqrt{3}} \quad - \quad 0 \quad \frac{1}{\sqrt{3}} \quad +$

$f''(x) = 12x^2 - 4$

$f''(-1) = 8 > 0$

$f''(0) = -4 < 0$

$f''(1) = 8 > 0$

$\Rightarrow$ function is concave up for $x < -\frac{1}{\sqrt{3}}$, $x > \frac{1}{\sqrt{3}}$

Concave down for $-\frac{1}{3} < x < \frac{1}{\sqrt{3}}$

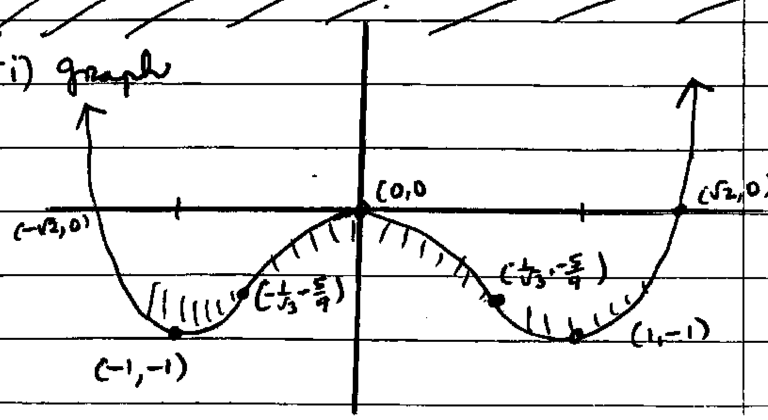
$\Rightarrow$ ~~infl~~

(v) $y = f(x) = x^4 - 2x^2$

$x \to +\infty \Rightarrow y \to +\infty$

$x \to -\infty \Rightarrow y \to +\infty$.

(vi) graph



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Section B

3 (a) (i) $\int_0^1 \frac{2x+1}{x^2+x+5}\,dx$

let $u = x^2+x+5$

$\frac{du}{dx} = 2x+1$

$\Rightarrow du = (2x+1)\,dx$

$\Rightarrow \frac{du}{(2x+1)} = dx$

$\Rightarrow \int \frac{(2x+1)}{u} \cdot \frac{du}{(2x+1)}$

$\Rightarrow \int \frac{1}{u}\,du$

$\Rightarrow \log_e|u|$

$\Rightarrow \log_e|x^2+x+5| \Big|_0^1$

$\Rightarrow \log 7 - \log_e 5$

$\Rightarrow \cdot 3364$

(ii) $\int \cosh x \sinh^4 x\,dx$

let $u = \sinh x$

$\frac{du}{dx} = \cosh x$

$\Rightarrow du = \cosh x\,dx$

$\Rightarrow \frac{du}{\cosh x} = dx$

$\Rightarrow \int \cosh x \cdot u^4 \cdot \frac{du}{\cosh x}$

$\Rightarrow \int u^4\,du$

$\Rightarrow \frac{u^5}{5}$

$\Rightarrow \frac{\sinh^5 x}{5} + c.$

(iii) $\int x e^x\,dx$

let $u = x$ | $dv = e^x\,dx$

$\frac{du}{dx} = 1 \Rightarrow du = dx$ | $v = \int e^x\,dx = e^x$

$\Rightarrow \int u\,dv = uv - \int v\,du$

$\Rightarrow \int x e^x\,dx = x e^x - \int e^x\,dx$

$= x e^x - e^x + c$

(b) $a(t) = \cos 4t$

$v = \int \cos 4t\,dt$

$v = \frac{\sin 4t}{4} + c$

$t = 0,\ v = 2$

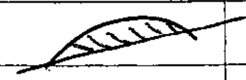
$\Rightarrow 2 = \frac{\sin 0}{4} + c$

$\Rightarrow 2 = \frac{0}{4} + c$

$\Rightarrow 2 = c$

$\Rightarrow v = \frac{\sin 4t}{4} + 2$

4 (a) $y = 4 - x^2$, $y = x+2$



limits: $y = 4 - x^2$, $y = x+2$

$\Rightarrow 4 - x^2 = x+2$

$\Rightarrow 0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$\Rightarrow x+2 = 0,\ x = -2$ | $x - 1 = 0,\ x = 1$

$\Rightarrow A = \int_{-2}^{1} (4-x^2) - (x+2)\,dx$

$= \int_{-2}^{1} 4 - x^2 - x - 2\,dx$

$= \int_{-2}^{1} 2 - x - x^2\,dx$

$= 2x - \frac{x^2}{2} - \frac{x^3}{3} \Big|_{x=-2}^{x=1}$

$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right)$

$\Rightarrow 9/2$

(b) $\int_0^2 \sqrt{1+3x^2}$

0 .5 1 1.5 2

$h = \frac{2-0}{4} = \frac{1}{2} = 0.5$

$y = \sqrt{1+3x^2}$

$x =$	0	.5	1	1.5	2
$y =$	1 (y_1)	1.322 (y_2)	2 (y_3)	2.783 (y_4)	3.605 (y_5)

$A = \frac{.5}{3}\left[(1+3.605) + 4(1.322+2.783) + 2(2)\right]$

$= \frac{1}{6}(4.605 + 16.42 + 4)$

$= \frac{1}{6}(25.025)$

$= 4.17$

Section C

5(a) $\sum_{n=1}^{\infty} \frac{4}{(4n+1)(4n+5)}$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(4n+1)} - \frac{1}{(4n+5)}$

$\Rightarrow \left(\frac{1}{5} - \cancel{\frac{1}{9}}\right) + \left(\cancel{\frac{1}{9}} - \cancel{\frac{1}{13}}\right) + \left(\cancel{\frac{1}{13}} - \frac{1}{17}\right) + \cdots$

$\Rightarrow \frac{1}{5}$

(b)(i) $\sum_{n=1}^{\infty} \frac{2n-1}{4n+3}$

$\Rightarrow \lim_{n\to\infty} \frac{2n-1}{4n+3}$

$\Rightarrow \lim_{n\to\infty} \left(\frac{2-\frac{1}{n}}{4+\frac{3}{n}}\right)$

$\Rightarrow \frac{2-0}{4-0}$

$\Rightarrow \frac{1}{2} \neq 0$

$\Rightarrow$ dgt. series

(ii) $\sum_{n=1}^{\infty} \frac{n+4}{2n^3+n+3}$ Compare with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$\lim_{n\to\infty} \left(\frac{\frac{n+4}{2n^3+n+3}}{\frac{1}{n^2}}\right) = \lim_{n\to\infty} \frac{n^3+4n}{2n^3+n+3}$ divide by n^3.

$\Rightarrow \lim_{n\to\infty} \left(\frac{1+\frac{4}{n^2}}{2+\frac{1}{n^2}+\frac{3}{n^3}}\right)$

$\Rightarrow \frac{1+0}{2+0+0} = \frac{1}{2}$

as $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is cgt.

$\Rightarrow \sum_{n=1}^{\infty} \frac{n+4}{2n^3+n+3}$ is cgt.

(iii) $\sum_{n=1}^{\infty} \frac{5^n}{n!}$ $\quad a_n = \frac{5^n}{n!}$

$a_{n+1} = \frac{5^{n+1}}{(n+1)!}$

$\frac{a_{n+1}}{a_n} = \frac{\frac{5^{n+1}}{(n+1)!}}{\frac{5^n}{n!}} = \frac{5^{n+1}}{(n+1)!} \cdot \frac{n!}{5^n}$

$= \frac{5}{n}$

$\Rightarrow \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{5}{n}\right| = |0| = 0$

$0 < 1$

$\Rightarrow$ Series is cgt.

6(a)

$f(x) = \cos x$

$f(0) = \cos 0 = 1$

$f'(x) = -\sin x$

$f'(0) = -\sin 0 = 0$

$f''(x) = -\cos x$

$f''(0) = -\cos 0 = -1$

$f'''(x) = -(-\sin x) = \sin x$

$f'''(0) = \sin 0 = 0$

$f^{iv}(x) = \cos x$

$f^{iv}(0) = \cos 0 = 1$

$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!}$

$\Rightarrow \cos x = 1 + (0)x + (-1)x^2 + \frac{(0)x^3}{3!} + \frac{(1)x^4}{4!}$

$\Rightarrow \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

(i) $\cos x^2 = 1 - \frac{(x^2)^2}{2!} + \frac{(x^2)^4}{4!}$

$\Rightarrow \cos x^2 = 1 - \frac{x^4}{2!} + \frac{x^8}{4!}$

(ii) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

Differentiate

$-\sin x = -\frac{2x}{2!} + \frac{4x^3}{4!} - \frac{6x^5}{6!}$

$\Rightarrow -\sin x = -x + \frac{x^3}{3!} - \frac{x^5}{5!}$

$\Rightarrow \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

(b) $z = 2x^2y + 4xy^3 + 5x^2$

(i) $\boxed{\frac{\partial z}{\partial x} = 4xy + 4y^3 + 10x}$

$\frac{\partial^2 z}{\partial x^2} = 4y + 0 + 10$

$= \boxed{4y + 10}$

$\frac{\partial^2 z}{\partial y \partial x} = 4x + 12y^2 + 0$

$= \boxed{4x + 12y^2}$

(ii) $z = \cos(x+2y)$

$\frac{\partial z}{\partial x} = -\sin(x+2y).$

$= -\sin(x+2y)$

$\frac{\partial^2 z}{\partial x^2} = -\cos(x+2y).1$

$\boxed{\frac{\partial^2 z}{\partial x^2} = -\cos(x+2y)}$

$\frac{\partial z}{\partial y} = -\sin(x+2y).2$

$\frac{\partial^2 z}{\partial y^2} = -\cos(x+2y).4$

$\boxed{\frac{\partial^2 z}{\partial y^2} = -4\cos(x+2y)}$

$\frac{\partial^2 z}{\partial y^2} - 4\frac{\partial^2 z}{\partial x^2}$

$= -4\cos(x+2y) - 4(-\cos(x+2y))$

$\Rightarrow -4\cos(x+2y) + 4\cos(x+2y)$

$\Rightarrow 0$

Section D

7(a)

(a) Evalf((10^2 +2)/Sqrt(3))^5 , 20);

(b) Subs(x= Sqrt(20), x^2 + 5*x);

(c) Factor (2*x^3 +10*x^2 -5);

(d) plot (cos(x), x = -5..5);

(e) Diff (2*ln(x)/((Cos(x))^2+1), x);

(f) Diff (2*ln(x)/((Cos(x))^2+1) x$2);

g Int (1/(Tan(x))^2, x=-1..2);

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(i) x intercepts 1, 3, 4 $\Rightarrow$ (1,0) (3,0) (4,0)
y intercepts −12 $\Rightarrow$ (0,−12)

(ii) (3.54...., −0.63....) ~~max~~ min turning pt
(1.78...., 2.11,....) max turning pt

(iii) (2.66.., 0.741..) inflection pt.

(iv) $x \to +\infty$, $y \to +\infty$
$x \to -\infty$, $y \to -\infty$.

(v)