

# Calculus for Engineers

## Inverse Laplace Transforms

[kobriendublin.wordpress.com](http://kobriendublin.wordpress.com)

Youtube: StatsLabDublin

# Convolution

The convolution of  $e^t$  with  $e^{-t}$  (also denoted  $e^t * e^{-t}$ )

- 1 Determine the Laplace transforms of both terms (find  $G_1(s)$  and  $G_2(s)$ ).
- 2 Multiply these terms ( find  $G_1(s) \times G_2(s)$ ).
- 3 Determine the inverse Laplace transform of that product (  $\mathcal{L}^{-1} (G_1(s) \times G_2(s))$ ).

## Convolution : Step 1

Determine the Laplace transform of  $e^t$  (using tables)

$$G_1(s) = \mathcal{L}[e^t] = \frac{1}{s+1}$$

Determine the Laplace transform of  $e^{-t}$  (using tables)

$$G_1(s) = \mathcal{L}[e^{-t}] = \frac{1}{s-1}$$

(Note: see Formula sheet entry 6)

## Convolution : Step 2

Multiply the terms  $G_1(s)$  and  $G_2(s)$

$$G_1(s) \cdot G_2(s) = \frac{1}{s+1} \times \frac{1}{s-1}$$

$$G_1(s) \cdot G_2(s) = \frac{1}{s^2 - 1}$$

(Note: see next slide for workings)

## Convolution : Step 2 (Workings)

Using the cross-multiplication technique

$$\frac{1}{s+1} \times \frac{1}{s-1} = \frac{1}{s+1} \times \frac{1}{s-1}$$

## Convolution : Step 3

Find the inverse laplace transform of  $G_1(s) \times G_2(s)$

## Convolution : Step 3

$$e^t * e^{-t} = \sinh(t)$$