Examinable Material Checklist

Important: Past Paper short questions and lecture note examples are indicative of what is going to be asked.

- Evaluation of Functions

- Cross Multiplication

- Floor and Ceiliing Functions

- Exponentials and Logarithms

  – Solving Equations with Exponentials and Logarithms terms (See Tutorial Sheet Question 8)

- Difference of Two Squares and Two Cubes

  – Formula for Difference of Two Cubes in Formula

- Evaluating Limits (Conventional Evaluation)

- Evaluating Limits (Indeterminate Answers - Factorize and Simplify)

- Evaluating Limits (Indeterminate Answers - Divide by Highest power)

- Piece Wise Functions

  – Evaluating Functions from the Left
  – Evaluating Function from the Right
  – Are Functions Continuous?

- *(Trigonometric Functions are not examinable for the mid-term)*

- Arithmetic Sequences - Determining the $n - th$ term

- Arithmetic Series - Summation of n terms

- Geometric Sequences - Determining the common ratio r

- Geometric Sequences - Determining the $n-th$ term

- Geometric Series - Summation

- Summation of Integers (See Tutorial Sheet Question 7)

- Partition of Summations (See Tutorial Sheet Question 7)

- Sum to Infinity of a Geometric Series

- Repeating Decimals - Expressing them as a fraction

- Telescoping Series

Examination Procedures

- This exam will start at 11:05, and will last 45 minutes.

- Each Question will be worth Either 1 or 2 Marks. There are 15 Marks worth of questions.

- All Questions must be attempted (LENS students please see below)

- Write all of your answers in the exam script. Write the script number on any other documents you submit.

- It is your responsibility to return the script to collection box. An audit of scripts will take place immediately after the exam. If your script is account for in that audit, you are deemed to be absent, and will receive no marks.

- **LENS Student** Specifically approved LENS students have to answer any selection of questions that have an aggregate mark of 12 Marks.

  - They may skip any three 1-Mark Questions
  - OR - They may skip a 1-Mark Question and a 2-Mark Question
  - The mark will be rescaled by 125 %.

Formula Sheet

Sum and Difference of Two Cubes

$$a^3 + b^3 = (a - b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Sequences and Series

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_n = a\left(\frac{1 - r^n}{1 - r}\right)$$

$$S_\infty = \frac{a}{1 - r}$$

Sample Paper 1 - Finished

Part A Fundamentals of Mathematics (5 Marks)

(i) (2 Marks) Determine the values of A and B from the following expression

$$\frac{2x+1}{x^2-4x+3} = \frac{A}{x-3} + \frac{B}{x-1}$$

(ii) (2 Marks) Evaluate the function for the values of $x = \{-1, 0.5, 1\}$

$$f(x) = \frac{e^x + e^{-x}}{2}$$

(iii) (1 Mark) Compute the floor and ceiling functions for $x = -1.25$.

Part B Exponentials and Logarithms (2 Marks)

(i) (1 Mark) Find the value of x

$$e^{2x-5} = 3.$$

(ii) (1 Mark) Find the value of x

$$log_2(x+1) + log_2(5) = 3$$

Part C Limits of Functions (3 Marks)

(i) Compute the limit of the following function

$$\lim_{x \to 3} \frac{x^2 - 12}{x + 3}$$

(ii) Compute the limit of the following function

$$\lim_{x \to 3} \frac{x^2 + x - 12}{x - 3}$$

(iii) Compute the limit of the following function

$$\lim_{x \to \infty} \frac{2x^2 - 8x}{4x^2 - 7}$$

Part D Sequences and Series (5 Marks)

(i) (1 Mark) Compute the following summation

$$\sum_{i=1}^{75} i$$

(ii) (1 Mark) Compute the following summation

$$\sum_{i=33}^{75} i$$

(iii) (1 Mark) Determine the common ratio r of the following geometric sequence.

$$6, 2, 0.666, 0.222\ldots$$

(iv) (1 Mark) Compute the sum to infinity of the following geometric series

$$6 + 2 + 0.666 + 0.22 + \ldots$$

(v) (1 Mark) Express the following repeating decimal number as fractions.

$$0.2424242424....$$

Sample Paper 2

Part A Fundamentals of Mathematics (5 Marks)

(i) (2 Marks) Determine the values of A and B from the following expression

$$\frac{3}{(3n+1)(3n+4)} = \frac{A}{n+1} + \frac{B}{3n+4}$$

(ii) (2 Marks) Evaluate the function for the values of x $= \{-1, 0.5, 1\}$

$$f(x) = \frac{e^x - e^{-x}}{2}$$

(iii) (1 Mark) Compute the floor and ceiling functions for $x = -1.25$.

Part B Limits of Functions (3 Marks)

(i) (1 Mark) Compute the limit of the following function

$$\lim_{x \to 8} \frac{x^2 - 9}{11}$$

(ii) (1 Mark) Compute the limit of the following function

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

(iii) (1 Mark) Compute the limit of the following function

$$\lim_{x \to \infty} \frac{6x^2 - 9x}{3x^2 - 7x^3}$$

Part C Sequences and Series (7 Marks)

(i) (1 Mark) Compute the following summation

$$\sum_{i=1}^{45} i$$

(ii) (1 Mark) Find the sum of the following geometric series:

$$3 + 6 + 12 + 24 + \ldots + 1536$$

(iii) (1 Mark) Determine the common ratio r of the following geometric sequence.

$$5, 1, 0.2, 0.04 \ldots$$

(iv) (1 Mark) Compute the sum to infinity of the following geometric series

$$5 + 1 + 0.2 + 0.04 \ldots$$

(v) (2 Marks) Find the sum of the following telescoping series

$$\sum_{n=1}^{\infty} \frac{3}{(3n+1)(3n+4)}$$

(vi) (1 Mark) Express the following repeating decimal as a fraction

$$0.4545454545.....$$

Sample Paper 3

Part A Fundamentals of Mathematics, Logarithms and Exponentials (5 Marks)

(i) (2 Marks) Determine the values of A and B from the following expression

$$\frac{5}{x^2 - 1} = \frac{A}{x - 1} + \frac{B}{x + 1}$$

(ii) (1 Mark) Compute the floor and ceiling functions for $x = 1.25$.

(iii) (1 Mark) Find the value of x

$$ln(e^x + 2) = 4$$

(iv) (1 Mark) Find the value of x

$$log_3(2x - 1) + log_3(5) = 3$$

Part B Limits of Functions (3 Marks)

(i) (1 Mark) Compute the limit of the following function

$$\lim_{x \to 7} \frac{x^2 + 3}{11}$$

(ii) (1 Mark) Compute the limit of the following function

$$\lim_{x \to 4} \frac{x^2 - 9x - 20}{x - 4}$$

(iii) (1 Mark) Compute the limit of the following function

$$\lim_{x \to \infty} \frac{6x^2 - 9x^3}{3x^3 - 7x}$$

Part C Piece Wise Functions (4 Marks)

Consider the following piecewise function:

$$f(x) = \begin{cases} (x - 3)^2 & x < 3 \\ x - 2 & x \geq 3 \end{cases}$$

(i) (1 Mark) Evaluate the following limit

$$\lim_{x \to 3^-} f(x)$$

(ii) (1 Mark) Evaluate the following limit

$$\lim_{x \to 3^+} f(x)$$

(iii) (2 Marks) Is $f(x)$ continuous at $x = 3$? Justify your answer.

Part D Sequences and Series (3 Marks)

(i) (1 Mark) Compute the following summation

$$\sum_{i=1}^{65} i$$

(ii) (2 Marks) In an arithmetic sequence, three consecutive terms have a sum of 15 and a product of 80. Find the second of the three terms, and common difference d for these terms.

Hint : Write terms as $x - d, x, x + d$ - the second term is x

(i) (2 Marks) Determine if the function $f(x) = x^3 sin(x)$ is an even function, an odd function or neither.

(ii) (1 Mark) Given $f(x) = sqrt2x - 8$, find $f(2x^2+4)$ and simplify the answer.

(iii) (2 Marks) Find $g^{-1}(x)$ the inverse of the function $g(x) = e^{3x}$

(iv) (2 Marks) Given the functions $g(x) = x^2 + 1$ and $f(x) = (x - 1)/2$ determine the values of $f \circ g(1)$ and $g \circ f(x1)$

(v) (2 Marks) Find the Maclaurin Series of e^x up to and including the term containing x^6 Use this answer to evaluate $e^{-0.4}$ and e^3

(vi) (2 Marks) Find the domain and the range of the function:

$$f(x) = 7 + 2sin(x)$$

(vii) (1 Mark) Consider the function $f(x) = x^2 - 8x + 7$. Find the y intercept of the function f(x).