Question 1 (25 Marks)

Part A

Given the matrices

$$A = \begin{pmatrix} 2 & 3 & 0 & -1 & 4 \end{pmatrix}; B = \begin{pmatrix} 1 & 4 \\ 0 & 5 \\ -1 & 0 \\ 4 & 1 \\ 1 & 0 \end{pmatrix}; C = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

calculate the products AB and CA.

Part B

For the matrices below, evaluate the following expressions where it is possible.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & 0 \\ 1 & -7 \end{bmatrix}, C = \begin{bmatrix} 3 & 2 & -2 \\ 4 & 8 & 2 \end{bmatrix}, D = \begin{bmatrix} 3 & 2 & -2 \\ 4 & 8 & 2 \end{bmatrix},$$

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, F = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \\ 3 & 1 & 0 \end{bmatrix},$$

1. $2A + 3B$
2. $3C - D$
3. $8A + 4C$
4. $2000A + 3000B$
5. $E - F$
6. $\det(A) + \det(B)$
7. $\det(A + B)$
8. $\det(C)$

Part A. Addition and Subtraction of Matrice

(a)

(b) Suppose A is a lower triangular matrix of the form

$$\begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix}$$

(i) State the transpose of A.

(ii) Compute B where B = $A \times A^T$

(iii) B is a symmetric matrix. What is meant by this?

(c) Let A and B be $m \times n$ matrices. Then:

(i) $(kA)^T = kA^T$

(ii) $(A+B)^T = A^T + B^T$

(iii) $(AB)^T = B^T A^T$

(d) For a square matrix A show that:

(i) AA^T and $A + A^T$ are symmetric

(ii) $A - A^T$ is skew symmetric

(iii) A can be expressed as the sum of a symmetric matrix, $\frac{1}{2}(A + A^T)$ and a skew symmetric matrix $\frac{1}{2}(A - A^T)$

Invertible Matrices

Show that if A is an $n \times n$ invertible matrix that satisfies

$$9A^3 + A^2 - 3A = 0$$

where $A^n = \underbrace{A \ldots A}_{n \text{ times}}$, I is the $n \times n$ identity matrix and 0 is the $n \times n$ zero matrix, then the inverse of A is given by

$$A^{-1} = \frac{1}{3}I + 3A.$$