Question 1

1) Basic Exercises Matrix Multiplication Arranging Formulaes Identity Matrix Equations Proofs

Given the following matrices

$$A = \begin{pmatrix} 1 & 4 & -2 \\ 2 & 1 & 0 \end{pmatrix}; \qquad B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}; \qquad C = \begin{pmatrix} -1 & -3 & 4 \\ 2 & 4 & -3 \end{pmatrix}; \qquad D = \begin{pmatrix} 3 & 1 \\ 4 & 0 \\ -2 & 5 \end{pmatrix},$$

calculate (if possible) the following operations (justify your answer for any operation you think may not be performed)

(a) AB; (b) BA; (c) $A + C$; (d) BC; (e) $(A + 2C)D$.

$$\begin{array}{|ccc|} \hline a & & \\ \hline b & c & \\ \hline d & e & f \\ \hline \end{array}$$

. Fundamental Theorem of Invertible Matrices Rank Trace

Quesiton 2

2) Inverting a Matrix using Co-Factors Matrix of Minors Determinant of a 3 by 3 matrix

- Evaluate the minors and cofactors of A, for A given by

$$A = \begin{pmatrix} 2 & 3 & -1 \\ 0 & 1 & 3 \\ -3 & 0 & 0 \end{pmatrix}; \qquad A = \begin{pmatrix} 1 & 5 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 7 \end{pmatrix};$$

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 8 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \qquad A = \begin{pmatrix} 1 & 3 & 4 \\ 0 & 1 & 1 \\ -6 & 0 & -1 \end{pmatrix}.$$

and hence, in each case, construct the cofactor matrix Cof(A) of A.

Quesiton 3

3) Planes Distance

Quesiton 4

4) Vectors / Systems of Linear Equations Cross Product Scalar Triple Product

Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & -2 & 4 \\ 1 & -4 & 1 \\ -3 & 0 & -1 \end{pmatrix}.$$

using elementary row operations.

Given the matrix

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 1 & 1 & 0 \\ 2 & -1 & 2 \end{pmatrix}.$$

calculate

- the determinant of A;
- the cofactor matrix of A;
- and **hence** the inverse matrix A^{-1} .

Quesiton 5

5) Eigenvalues / Diagonalization Characteristic Polynomial Power Formula

- Given $u,\ u',\ v,\ v', w\ w'$, with

$$\begin{aligned} u &= (1,3,0); \qquad & u' &= (-3,1,5) \\ v &= (5,0,4); \qquad & v' &= (-4,3,5) \\ w &= (3,2,7); \qquad & w' &= (1,0,1), \end{aligned}$$

  calculate $u \cdot u'$, $v \cdot v'$, $w \cdot w'$. Which of the pairs are orthogonal vectors?

- Calculate the (Euclidean) norm of the following vectors

$$\begin{aligned} u &= (1,2) \\ v &= (3,0) \\ w &= (4,0,3) \\ 0 &= (0,0,0). \end{aligned}$$

- Calculate the scalar triple product

$$u \cdot (v \times w)$$

  for

  1. $u = (1,3,5)$; $v = (0,5,3)$; $w = (3,0,7)$;
  2. $u = (0,1,2)$; $v = (5,0,1)$; $w = (2,2,2)$.

- Are the points

$$P_1 = (1, 2, 0), \quad P_2 = (3, 5, 0), \quad P_3 = (7, 3, 0), \quad P_4 = (-5, 3, 0)$$

coplanar? If yes, what is the equation of the plane containing them?

- 1. Find the equation of the line ℓ in $\mathbb{R}^2$, which passes through the points $(2, 1)$ and $(1, 3)$.
  2. Let $Q = (1, -3)$ be a point in $\mathbb{R}^2$.
     (a) Verify that Q does not lie on the line ℓ.
     (b) Find the distance between the point Q and the line ℓ.