UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

Faculty of Science and Engineering

Department of Mathematics & Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4702

SEMESTER: Spring 2013/14

MODULE TITLE: Technological Mathematics 2

DURATION OF EXAMINATION: 2.5 hours

LECTURER: Dr Páraic Treacy

PERCENTAGE OF TOTAL MARKS: 80 %

EXTERNAL EXAMINER: Prof. B. Murphy

INSTRUCTIONS TO CANDIDATES:

Question One is **compulsory** and carries 40 marks.

Answer any other **four** questions worth 15 marks each.

Log tables and graph paper are available from the invigilators.

N.B. There are some useful formulae at the end of the paper.

Question 1

(a) Find $f^{-1}(x)$ the inverse of the function

$$f(x) = \sqrt{2x+3}$$

(4)

(b) Find the domain and the range of the function:

$$f(x) = \ln x$$

(4)

(c) Find the equations of the vertical and horizontal asymptotes of the following function:

$$f(x) = \frac{x^2 + x + 9}{2x^2 - 18}$$

(8)

(d) Find:

$$\lim_{x \to 3} \frac{x-3}{x^2 - 9}$$

(4)

(e) Evaluate the following indefinite integral:

$$\int x^2 - 4e^x + 2\, dx$$

(4)

(f) Evaluate the following definite integral:

$$\int_1^3 \frac{1}{x}\, dx$$

(4)

(g) Evaluate all the first partial derivatives $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ of the following function:

$$z = -x^2 y + x^4 e^y$$

(4)

(h) Check whether the symmetry of the following function is odd, even, or neither:

$$f(x) = \frac{e^x + e^{-x}}{2}$$

(4)

(i) Find the sum of the following geometric series:

$$4 + 12 + 36 + 108 + \ \dots + 8748$$

(4)

Question 2

The population of bacteria (in millions) in a certain culture x hours after an experimental nutrient is introduced to the culture is represented by the function

$$f(x) = \frac{25x}{x^2 + 4}$$

(a) Find the y intercept of the function $f(x)$. **(2)**

(b) Find the turning points of the function $f(x)$ and classify them as local maxima or local minima. **(3)**

(c) Check whether the function is even, odd, or neither. **(3)**

(d) Determine the behaviour of the function $f(x)$ as $x \to +\infty$ and as $x \to -\infty$. **(3)**

(e) Sketch the graph of the function $f(x)$ illustrating the features of the curve obtained in parts (a – d). **(4)**

Question 3

(a) An object moves in a straight line with acceleration $a(t) = 3t^2 + 2$ where t represents time in seconds and acceleration is measured in m/s^2.
At time $t = 0$ it has a velocity $v = 2m/s$. Find its velocity at all times t. **(6)**

(b) Based on data from the New York City Police department, the number of homicides per year in New York City between 1998 and 2009 can be approximated by

$$f(t) = 5.45t^3 - 105t^2 + 391t + 1798$$

where t is the number of years since January 1st 1998. Find the total number of homicides during the 12 year period from the beginning of 1998 to the end of 2009.

(9)

Question 4

(a) Evaluate the following integral

$$\int \frac{4x-5}{2x^2-5x+3}\,dx$$ **(5)**

(b) Find the area enclosed by the curve $f(x) = x^2 - 7x + 10$ and the x axis. **(5)**

(c) A population of E. coli bacteria will grow at a rate given by

$$w'(t) = (3t+4)^{\frac{1}{3}}$$

where *w(t)* is the weight in milligrams after *t* hours. Find the total change in weight of the population in the first 5 hours i.e. from $t = 0$ to $t = 5$. **(5)**

Question 5

(a) Find the Maclaurin series of $f(x) = e^x$ up to and including the term containing x^6.

Use your answer to estimate the value of e^3 **(9)**

(b) (i) Find $\frac{\partial z}{\partial x}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y \partial x}$ of the function $z = 2x^2y + 4x^2y^3 - 7x^2$. **(3)**

(ii) Prove that the function $z = \cos(x+3y)$ satisfies the partial differential equation:

$$\frac{\partial^2 z}{\partial y^2} - 9\frac{\partial^2 z}{\partial x^2} = 0.$$ **(3)**

Question 6

West Ham Utd. midfielder Joe Cole's contract has just expired. His agent informs him that the club want him to sign a new contract and have given him two options:

- Option 1: A 1-year contract with no bonuses through which he is guaranteed to earn €1,400,000 over the course of the contract.

- Option 2: A 1-year contract which will pay him based <u>only</u> on the number of competitive games he appears in (as either a sub or starter) – he will get €35,000 for the first game, €38,000 for the second, €41,000 for the third game, €44,000 for the fourth game and so on. In other words, he will have €3,000 added to his payment each time he plays a game.

(a) Joe Cole appeared in 24 games in the most recent season and thinks he will appear in about the same number of games in the coming season. If he plays in 24 games in the coming season, how much would he earn through option 2? **(5)**

(b) West Ham typically play 44 competitive games per season (this includes Premier League and domestic cup competitions). If West Ham play 44 games in the coming season, what is the maximum amount of money Joe Cole can earn through option 2? **(5)**

(c) How many games would Joe Cole need to play in to earn more than €1,400,000 over the course of the 1-year contract if he were to pick option 2? **(5)**

Useful Formulae

Logs

If $a^b = c$

Then $\log_a c = b$

Differentiation

Product Rule:

$$y = uv$$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient Rule:

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Further formulae and special cases on pages 41 & 42 of the log tables provided.

Hyperbolic functions

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

Integration

Integration by parts:

$$\int u\,dv = uv - \int v\,du$$

Further formulae and special cases on pages 41 & 42 of the log tables provided.

Sequences & Series

Arithmetic Series:

$$u_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Geometric Series:

$$u_n = ar^{n-1}$$

$$S_n = a\left(\frac{1-r^n}{1-r}\right)$$

$$S_\infty = a\left(\frac{1}{1-r}\right)$$

Maclaurin Series:

$$f(x) = f(0) + xf'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \cdots$$

Ratio Test:

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = r$$