PROBLEM SHEET 7: TRIPLE INTEGRALS

1. Calculate the triple integrals

   (a) $\displaystyle\int_0^1 \int_1^2 \int_2^3 4xyz\,dxdydz$

   (b) $\displaystyle\int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dzdydx$

   (c) $\displaystyle\int_0^1 \int_0^{\pi} \int_0^2 (x\sin(y) - xz)\,dzdydx$

   (d) $\displaystyle\int_0^{\pi/2} \int_0^y \int_0^x \cos(x+y+z)\,dzdxdy$

   (e) $\displaystyle\int_{-1}^1 \int_0^2 \int_0^1 (x^2 y + xe^z)\,dzdydx$

2. Calculate the triple integral
$$\iiint_D x^2 y\,dxdydz$$
   over the 3-dimensional region D bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x+y+z = 1$.

3. A solid region in the first octant is bounded by the coordinate planes and the plane $x+y+z = 2$. Find the mass of the solid if the density function is given by $\delta(x, y, z) = 2x$.

4. Suppose D is the cube with side lengths 2 centered at the origin. If the density of the cube is $\delta(x, y, z) = x^2 + y^2 + z^2$, calculate the mass of the cube.