# Partial Differentiation

Compute the partial derivatives of the expression $w$ with respect to $x$, $y$ and $z$.

$$w = x^3y - 12y^2z^3 + 42\sqrt{z}$$

# Partial Differentiation

Part 1. Differentiate with respect to $x$

$$w = x^3 y - 12y^2 z^3 + 42\sqrt{z}$$

# Partial Differentiation

Part 1. Differentiate with respect to $x$

$$w = x^3 y - 12y^2 z^3 + 42\sqrt{z}$$

$$\frac{\partial w}{\partial x} = 3x^2.y$$

# Partial Differentiation

Part 2. Differentiate with respect to $y$

$$w = x^3y - 12y^2z^3 + 42\sqrt{z}$$

# Partial Differentiation

Part 2. Differentiate with respect to $y$

$$w = x^3 y - 12y^2 z^3 + 42\sqrt{z}$$

$$\frac{\partial w}{\partial y} = 3x^3 + 24yz^3$$

# Partial Differentiation

$$w = x^3 y - 12y^2 z^3 + 42\sqrt{z}$$

$$\frac{\partial w}{\partial z} = 3x^3 + 24yz^3$$

## Partial Derivatives

$$f(x,y) = \frac{x^2}{x+y} + x\, sin\left(\frac{x}{y}\right)$$

Verify that

$$f(x,y) = \frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$$

## Partial Derivatives

- We will split $f(x,y)$ into two expressions such that $f(x,y) = f_1(x,y) + f_2(x,y)$
- The first part is

$$f_1(x,y) = \frac{x^2}{x+y}$$

- The second component is

$$f_2(x,y) = x\ sin\left(\frac{x}{y}\right)$$

## Partial Derivatives

- Differentiate the expression $f_1(x, y)$ with respect to both $x$ and $y$.

$$f_1(x, y) = \frac{x^2}{x + y}$$

- To differentiate with respect to $x$, we use the **Chain Rule**.

# Partial Derivatives