Mathematics for Physical Sciences II 17 September 2012
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Problem Sheet 1: Review of Differentiation

1. Differentiate the following functions using the rules.

$$(i)\ 4x^7+9x^5-x^3+2x^2+11; \quad (ii)\ \frac{x^2+3x+2}{x^4+x^2+1}; \quad (iii)\ (x^2+7x+1)(x^3-x^2-x);$$
$$(iv)\ \frac{1}{x^2+1}; \quad (v)\ \frac{1}{x+1}; \quad (vi)\ x^4+\sin(x); \quad (vii)\ \frac{1}{x^2+1}+x^5\cos(x);$$
$$(viii)\ \frac{1}{2+\cos(x)}; \quad (ix)\ \frac{x\sin(x)}{1+x^2}; \quad (x)\ \frac{x+\sqrt{x}}{\sin(x)}.$$

2. Differentiate the following functions using the power rule:

$$\text{(i)}\quad x^{10}+3x^7+\frac{1}{2}x^5+14x^2; \quad \text{(ii)}\ x^{1/3}+2x^{1/4}; \quad \text{(iii)}\ \frac{1}{x^2}+\frac{1}{x^6}; \quad \text{(iv)}\ 2\sqrt{x}+\sqrt[3]{x}$$
$$\text{(v)}\quad \frac{1}{\sqrt[4]{x^3}}; \quad \text{(vi)}\ \frac{\sqrt{x}}{x^{1/6}\cdot\sqrt[3]{x}}; \quad \text{(vii)}\ x^{\pi}.$$

3. Use the chain rule to differentiate the functions

$$(i)\ \sin(x^3-5x^2); \quad (ii)\ \tan(\frac{1}{x^4+2}); \quad (iii)\ 5\cos^6(2x+1);$$
$$(iv)\ \exp(\tan(x)); \quad (v)\ \sqrt{1+2x^2}; \quad (vi)\ (\frac{x^5}{4}+\frac{1}{x}+2x)^3; \quad (vii)\ \cot(\frac{1}{x^3}).$$

4. Use the chain rule to differentiate the functions

$$(i)\ (x^2+1)^{10}; \quad (ii)\ \frac{\sin^2(x)}{\sin(2x)}; \quad (iii)\ \sqrt{1+\sqrt{x}};$$
$$(iv)\ \exp(x^3-1); \quad (v)\ \ln(\sin(x)); \quad (vi)\ \tan(x^3);$$

5. Find the extreme values of $f(x) = x^3 - 12x - 5$. Determine whether they are maximum or minimum points.

6. The height of an object moving vertically is given by

$$H(t) = -16t^2 + 96t + 112$$

where $H(t)$ is the height at time t. (H is measured in metres and t is measured in seconds.) Find: (i) the object's velocity when $t = 0$; (ii) its maximum height and when it occurs; (iii) its velocity when $H = 0$.

Differentiation rules

Addition rule

$$\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x) \qquad \text{or} \quad (f+g)' = f' + g'$$

Product Rule:

$$\frac{d}{dx}\left(f(x)g(x)\right) = \left(\frac{d}{dx}f(x)\right)g(x) + f(x)\left(\frac{d}{dx}g(x)\right) \qquad \text{or} \quad (fg)' = f'g + fg'$$

Quotient rule

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{g(x)^2} \qquad \text{or} \quad (\frac{f}{g})' = \frac{gf' - fg'}{g^2}$$

Chain rule:

$$\frac{d}{dx}\left(f(g(x)\right) = \frac{df}{dx}(g(x)).\frac{d}{dx}g(x) \qquad \text{or} \quad (f(g))' = f'(g).f'$$

Derivatives of elementary functions

$f(x)$	$f'(x)$	
x^n	nx^{n-1}	(**Power rule**)
$\sin x$	$\cos x$	
$\cos x$	$-\sin x$	
$\tan x$	$1/\cos^2 x = 1 + \tan^2 x$	
$\cot x$	$-1/\sin^2 x = -(1+\cot^2 x)$	
e^x	e^x	
a^x	$a^x(\ln a)$	
$\ln(x)$	$\frac{1}{x}$	
$\log_a(x)$	$\frac{1}{x \ln a}$	
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	

Maximum and minimum points

A point x_0 such that $f'(x_0) = 0$ is called a **critical point** or **extreme point** of the function f.
Recall the **Second Derivative Test for Extreme Points:**
If $f'(x_0) = 0$ and $f''(x_0) < 0$ then x_0 is a maximum point.
If $f'(x_0) = 0$ and $f''(x_0) > 0$ then x_0 is a minimum point.