MA4505 Week 9 Questions from Tutorial Sheets 3 and 4.

Question 3.1

A driver passes through 3 traffic lights. The chance he/she will stop at the first is 1/2 , at the second 1/3 and at the third ¼ independently of what happens at any of the other lights. What is the probability that

i) the driver makes the whole journey without being stopped at any of the lights
ii) the driver is only stopped at the first and third lights
iii) the driver is stopped at just one set of lights.

$$\begin{aligned} P[F] &= 0.5 & P[F^c] &= 0.5 \\ P[S] &= 0.333 & P[S^c] &= 0.666 \\ P[T] &= 0.25 & P[T^c] &= 0.75 \end{aligned}$$

Probability of not getting stopped at all three lights

$$P[0] = P[F^c] \times P[S^c] \times P[T^c] = 0.5 \times 0.666 \times 0.75 = 0.25$$

Probability of only getting stopped at first lights

$$P[F\ only] = P[F] \times P[S^c] \times P[T^c] = 0.5 \times 0.666 \times 0.75 = 0.25$$

Probability of only getting stopped at second lights

$$P[S\ only] = P[F^c] \times P[S] \times P[T^c] = 0.5 \times 0.333 \times 0.75 = 0.125$$

Probability of only getting stopped at third lights
$P[T\ only] = P[F^c] \times P[S^c] \times P[T] = 0.5 \times 0.666 \times 0.25 = 0.083$

Probability of getting stopped at one lights only

$P[1\ only] = P[F\ only] + P[S\ only] + P[T\ only]$

$P[1\ only] = 0.125 + 0.25 + 0.083 = 0.458$

Question 3.2

What is the probability of getting a number divisible by 3 in each of 3 throws of a dice?

Solution

Numbers divisible by 3 : 3 and 6 probability of throwing 3 or 6:

Probability of throwing 3 or 6 three times in a row (Each throw of a dice is an independent event.)

$$P[3T] = P[T] \times P[T] \times P[T] = \left(\tfrac{1}{3}\right)^3 = \tfrac{1}{27}$$

Question 3.3

Which are the following pairs of events are mutually exclusive?

i) Two dice are thrown: A is the event the sum is 10, B is the event the sum is 11

ii) A hand of two cards is dealt: A is the event that the hand includes at least one red card, B is the event that the hand includes at least one black card.

iii) student is chosen from the class at random: A is the event that the student is female, B is the event that a student is left-handed.

Solution

(i) is mutually exclusive. cant throw 10 and 11 in same throw of two dice.

(ii) not mutually exlusive: can have one red card and one black card.

(iii) not mutually exclusine: can have a lefthanded female

Question 3.4

The following contingency table shows the age and sex of derby winners

	age =3	**age =4**	**age =5**	**Total**
Stallion	10	30	20	**60**
Filly	20	20	10	**50**
Total	**30**	**50**	**30**	**110**

A winner is chosen at random. Calculate the probability that
i) the horse is a filly
ii) the horse won as a 5-year old.
iii) the horse was a stallion, given it won as a 3-year old
iv) the horse was a 4-year old, given it was a filly.

Solutions

110 derby winners. 50 winners were fillies.

answer (i) = 50/110 = **45.45 %**

30 winners were 5 years old

answer (ii) = 30/110 = **27.27%**

30 winners were three year olds. Of that 30, 10 were stallions.

answer (iii) = 10/30 = **33.33%**

50 winners were fillies. Of that 50, 20 were 4 year olds

answer (iv) = 20/50 = **40%**

Question 3.5

A card is drawn at random from a standard pack of playing cards. It is an ace. What is the probability that it is the ace of diamonds?

Solution

- What is the probability the ace picked is the ace of diamonds (given that we **know** that it is an ace).
- Wording of this question is very important.
- There are four card suits (hearts, diamonds, clubs, spades)
- The card has a '**one in four**' chance of being an ace of diamonds.

Question 3.6

A dice is thrown 5 times. Calculate the probability of

i) Obtaining exactly one six

ii) Obtaining at least one six

iii) Calculate the (theoretical) mean and variance of the number of sixes obtained?

$$P(x = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Part 1

$$P(x = 1) = \binom{5}{1} \times \left(\tfrac{1}{6}\right)^1 \times \left(\tfrac{5}{6}\right)^4$$

$$\binom{5}{1} = \tfrac{5!}{1! \times 4!} = 5$$

$$P(x = 1) = 5 \times \left(\tfrac{1}{6}\right)^1 \times \left(\tfrac{5}{6}\right)^4 = 0.401$$

Part 2

obtaining at least one head is complement of obtaining zero heads

$$P(x \geq 1) = 1 - p(x = 0)$$

$$P(x = 0) = \binom{5}{0} \times \left(\frac{1}{6}\right)^0 \times \left(\frac{5}{6}\right)^5$$

$$\binom{5}{0} = \frac{5!}{0! \times 5!} = 1$$

$$P(x \geq 1) = 1 - 0.401 = 0.599$$

Question 3.7

A doctor treating a patient issues a prescription for antibiotics and provides for two repeat prescriptions. The probability that the infection will be cleared by the first prescription is $p_1 = 0.6$.

The probability that successive treatments are successful, given that previous prescriptions were not successful are $p_2 = 0.5$, $p_3 = 0.4$.

Calculate the probability that

i) the patient is still infected after the third prescription

ii) the patient is cured by the second prescription.

Solution

$P(need\ 2nd) = P(F^c) = 1 - P(F) = 0.4$

$P(need\ 3rd) = P(S^c|F^c) = P(S^c) \times P(F^c) = 0.5 \times 0.4 = 0.2$

$P(not\ cured) = P(T^c|\ need\ 2nd\ ) = P(T^c) \times P(S^c) \times P(F^c)$
$= 0.6 \times 0.5 \times 0.4 = 0.12$

$P(2nd\ cured) = P(S) \times P(F^c) = 0.5 \times 0.4 = 0.2$

Alternative solution
[lets use cohort of 1000 patients]

probability that a person is cured after first prescription $P[F]$ = 0.6 [600 patients]
probability that a person is still infected after first prescription $P[F^c]$ = 1-0.6 = 0.4 [400 patients]

[400 patients will need second prescription]

probability that a person is cured after second prescription $P[S]$ = 0.5 [200 patients]
probability that a person is still infected after second prescription $P[S^c]$ = 1-0.5 = 0.5 [200 patients]

[200 patients will need third prescription. 800 patients now cured]

probability that a person is cured after third prescription $P[T]$ = 0.4 [80 patients]
probability that a person is still infected after third prescription $P[T^c]$ = 1-0.4 = 0.6 [120 patients]

[120 patients will need treatment. 880 patients now cured]

Question 4.1

The gestation period of horses is approximately normally distributed with a mean of 337 days and a standard deviation of 4.5 days.

Estimate the probability that the gestation period is

i) greater than 340 days
ii) less than 330 days
iii) between 335 and 345 days.
iv) What gestation period is surpassed by 2.5% of the population?

X : Gestation period of horses

$P(X \geq 340) = 0.2524$ [Z=0.66]

$P(X \leq 330) = 0.0599$

$P(335 \leq X \leq 345) = 0.6339$

Find X_o such that $P(X \geq X_0) = 0.025$

X_o **= 345.82** **[ANS]**

Question 4.2

The length of the jump of an athlete has a normal distribution with mean 7m and standard deviation 0.1m.

1. Calculate the probability that he jumps at least 7.15m
2. Calculate the probability that he jumps between 6.9 and 7.05m
3. Find the probability that if he jumps 3 times all the jumps will be less than 7.15m (assume the lengths of the jumps are independent and use the answer to first part.

$\mu = 7.00\ m \quad \sigma = 0.1\ m$

Part 1. Determine $P(X \geq 7.15)$ $\qquad Z_o = \frac{X_o - \mu}{\sigma} = \frac{7.15 - 7}{0.1} = \frac{0.15}{0.10} = 1.5$

From tables $P(Z \geq 1.5) = 0.0668$ $\qquad$ Therefore $P(X \geq 7.15) = 0.0668$

[ANS]

Part 2. Determine $P(6.9 \leq X \leq 7.15)$

$P(6.9 \leq X \leq 7.15) = 1 - P(X \leq 6.9) - P(X \geq 7.05)$

$P(X \geq 7.05) = P(Z \geq 0.5) = 0.3085$

$P(X \leq 6.90) = P(Z \leq -1) = P(Z \geq 1) = 0.1586$

$P(6.9 \leq X \leq 7.15) = 0.5328$ $\qquad$ **[ANS]**

Part 3. : $\qquad P(X \leq 7.15) = 0.9332$ $\qquad$ **Probability =** $(0.9332)^2 = 0.8126$
