Mathematics for Physical Sciences III 6 February 2012
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Problem Sheet 1: Introduction to Vector Calculus

1. The position vector of a particle moving in the plane is given by

   (i) $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j}$
   (ii) $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j}$
   (iii) $\mathbf{r}(t) = t\mathbf{i} + \mathrm{e}^t\mathbf{j}$

   Calculate the velocity, speed, direction of motion and acceleration at each moment of time t for all paths above.

2. For each of the following pairs of three-dimensional vectors, $\mathbf{A}$ and $\mathbf{B}$, calculate the dot product $\mathbf{A} \cdot \mathbf{B}$ and the cross product, $\mathbf{A} \times \mathbf{B}$.

   $$\mathbf{A} = -\mathbf{i} + \mathbf{k}; \qquad \mathbf{B} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$$
   $$\mathbf{A} = 4\mathbf{i} - 2\mathbf{j}; \qquad \mathbf{B} = -2\mathbf{i} - \mathbf{k}$$
   $$\mathbf{A} = -3\mathbf{i} + \mathbf{j} - 2\mathbf{k}; \qquad \mathbf{B} = -3\mathbf{j}$$

3. If $\mathbf{A} = x^2\mathbf{i} - y\mathbf{j} + xz\mathbf{k}$, $\mathbf{B} = y\mathbf{i} + x\mathbf{j} - xyz\mathbf{k}$ and $\mathbf{C} = \mathbf{i} - y\mathbf{j} + x^3z\mathbf{k}$ find

   $$\frac{\partial^2}{\partial x \partial y}(\mathbf{A} \times \mathbf{B}) \qquad \text{and} \qquad \frac{\partial}{\partial x}(\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}))$$

4. If $\Phi = xy + yz + zx$ and $\mathbf{A} = x^2y\mathbf{i} + y^2z\mathbf{j} + z^2x\mathbf{k}$, find (a) $\mathbf{A} \cdot \nabla\Phi$; (b) $\Phi\nabla \cdot \mathbf{A}$ and (c) $(\nabla\Phi) \times \mathbf{A}$ at the point (3,-1,2).

5. If $\mathbf{A} = 3xz^2\mathbf{i} - xz\mathbf{j} + (x + 2z)\mathbf{k}$, find $\mathrm{curl}(\mathrm{curl}(\mathbf{A})) = \nabla \times (\nabla \times \mathbf{A})$.