25 October 2012

# Mathematics for Physical Sciences II
## Continuous Assessment Test -Sample

**Full marks for complete answers to any FOUR questions.**
**All questions carry equal marks.**
**Time allowed: 50 minutes**

1. Find all the second order derivatives for the function

$$f(x, y) = x^2 y + \cos(y) + y\,\mathrm{e}^x$$

and verify that $\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial^2 f}{\partial y \partial x}$.

2. Find the critical points for the following function and then decide whether they are maximum, minimum or saddle points.

$$f(x, y) = x^2 + 2y^2 - 4xy + 4y$$

3. Let $Z_1 = 4 - i$ and $Z_2 = 1 + 2i$. Evaluate the real and imaginary part of the complex expression

$$\frac{Z_1 + Z_2}{Z_1 - Z_2}.$$

4. Sketch the position of the complex number $Z = 2 + 2i$ in the complex plane and then express this number in its polar form. Use de Moivre's formula to express $Z^8$ in the form $a + ib$.

5. Evaluate the following double integrals

   (a) $\displaystyle\int_0^1 \int_1^2 (xy + x + 2y + 5)\, dxdy$

   (b) The integral of $f(x, y) = xy$ over the square $1 \leq x \leq 2$, $1 \leq y \leq 2$.

6. Calculate the integral

$$\int_0^1 \int_2^{4-2x} dydx$$

Sketch the region of integration and then write the integral with the order of integration reversed. Show that the same result is obtained when integrating with respect to $x$ first.