

## PROBLEM SHEET 4: EIGENVALUES AND EIGENVECTORS

1. Perform five iterations of the power method to approximate the dominant eigenvalue for each of the following matrices (with starting vector  $\mathbf{x}^{(0)}$ )

$$(i) \quad A = \begin{pmatrix} 3 & 2 & -2 \\ -3 & -1 & 3 \\ 1 & 2 & 0 \end{pmatrix} \quad \text{and } \mathbf{x}^{(0)} = [1, 0, 0]^T$$

$$(ii) \quad A = \begin{pmatrix} 19 & -9 & -6 \\ 25 & -11 & -9 \\ 17 & -9 & -4 \end{pmatrix} \quad \text{and } \mathbf{x}^{(0)} = [0, 0, 1]^T$$

$$(iii) \quad A = \begin{pmatrix} 1 & 4 & 5 \\ 4 & -3 & 0 \\ 5 & 0 & 7 \end{pmatrix} \quad \text{and } \mathbf{x}^{(0)} = [1, 0, 1]^T$$

2. For each of the matrices listed below, estimate the eigenvalue which is closest to the given number and determine its associated eigenvector.

$$(i) \quad A = \begin{pmatrix} 1 & 4 & 5 \\ 4 & -3 & 0 \\ 5 & 0 & 7 \end{pmatrix} \quad q = 1$$

$$(ii) \quad A = \begin{pmatrix} 4 & 1 & -2 & 2 \\ 1 & -1 & 1 & -1 \\ 1 & 0 & 2 & 0 \\ 1 & 1 & 3 & 4 \end{pmatrix} \quad q = 2$$

$$(iii) \quad A = \begin{pmatrix} 21 & 7 & -1 \\ 5 & 7 & 7 \\ 4 & -4 & 20 \end{pmatrix} \quad q = 15$$

3. Evaluate the smallest eigenvalue and its associated eigenvector for each of the following matrices.

$$(i) \quad A = \begin{pmatrix} 4 & 2 & -2 & 2 \\ 1 & 3 & 1 & -1 \\ 1 & 0 & 2 & 0 \\ 1 & 1 & -3 & 4 \end{pmatrix} \quad (ii) \quad A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -4 & -1 \\ 3 & -1 & 0 \end{pmatrix}$$

$$(iii) \quad A = \begin{pmatrix} -10 & -4 & 0 & -4 \\ -4 & 5 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ -4 & 1 & 0 & 5 \end{pmatrix}$$

4. For each of the following matrices, sketch the Gerschgorin circles and obtain an approximation for each eigenvalue contained in an isolated circle.

$$(i) \quad A = \begin{pmatrix} 16 & -8 & 2 & 1 \\ 2 & -12 & 1 & 0 \\ -1 & 1 & -4 & 1 \\ 0 & -1 & 2 & 3 \end{pmatrix} \quad (ii) \quad A = \begin{pmatrix} 8 & 2 & 0 & 1 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & -8 \end{pmatrix}$$

$$(iii) \quad A = \begin{pmatrix} 2 & -1/2 & 0 \\ -1/2 & 3 & 1/2 \\ 0 & 1/2 & 6 \end{pmatrix}$$