27 February 2012

Mathematics for Physical Sciences III
Continuous Assessment Test - SAMPLE

Full marks for complete answers to any FOUR questions.
All questions carry equal marks.
Time allowed: 50 minutes

1. If $\Phi = x^2yz^3$ and $\mathbf{A} = xz\mathbf{i} - y^2\mathbf{j} + 2x^2y\mathbf{k}$ find $\text{div}(\mathbf{A})$ and $\nabla\Phi$.

2. If $\mathbf{A} = x^2\mathbf{i} - y\mathbf{j} + xz\mathbf{k}$ and $\mathbf{B} = y\mathbf{i} + x\mathbf{j} - xyz\mathbf{k}$, calculate
$$\frac{\partial}{\partial x}(\mathbf{A} \times \mathbf{B})$$
either by finding the cross product and then differentiating, or by using the product rule.

3. Find a unit normal to the surface given by the equation
$$\Phi(x, y, z) = x^2 + 2xy - y^2 + z^2 = 7$$
at the point $(1, -1, 3)$.

4. Evaluate the line integral
$$\int_{(0,0,0)}^{(1,1,1)} xy\,dx + yz\,dy + xz\,dz$$
along the path given by $x = t$, $y = t^2$, $z = t^4$.

5. Show that the vector field $\mathbf{A} = (y + z)\mathbf{i} + (x + z)\mathbf{j} + (y + x)\mathbf{k}$ is conservative and construct a potential function Φ.

6. Using Green's Theorem, evaluate the integral $\int_C y^2dx + x^2dy$, where C is the triangle bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$, by transforming it into a double integral.