Section A

(a)

(i) $f(x) = \sqrt{2x-8}$

$f(2x^2+4) = \sqrt{2(2x^2+4)-8}$

$= \sqrt{4x^2+8-8}$

$= \sqrt{4x^2}$

$= 2x$

(ii) $f(x) = x\sin x$

$f(-x) = -x\sin(-x)$

$= -x(-\sin x)$

$= x\sin x$

$\Rightarrow f(x) = f(-x)$

$\Rightarrow$ even function

(iii) $g(x) = y = e^{3x}$

$f^{-1}: \Rightarrow x = e^{3y}$

$\Rightarrow \log_e x = 3y$

$\Rightarrow \frac{\ln x}{3} = y$

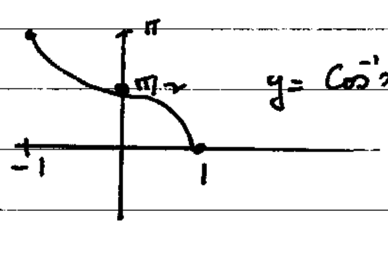
$\Rightarrow \cancel{f}\ g^{-1}(x) = \frac{\ln x}{3}.$

(b)(i) $\cos^{-1}(-\frac{1}{5}) = 101.53^\circ$

$\Rightarrow 101^\circ 32'$

$\Rightarrow 1.772$ Radians

(ii)



$y = \cos^{-1}x.$

(c) $\sinh^2 x$

$\Rightarrow \left(\frac{e^x - e^{-x}}{2}\right)^2$

$\Rightarrow \frac{(e^x - e^{-x})(e^x - e^{-x})}{4}$

$\Rightarrow \frac{e^{2x} - e^0 - e^0 + e^{-2x}}{4}$

$\Rightarrow \frac{e^{2x} - 1 - 1 + e^{-2x}}{4}$

$\Rightarrow \frac{e^{2x} - 2 + e^{-2x}}{4}$

$\frac{1}{2}(\cosh 2x - 1)$

$\Rightarrow \frac{1}{2}\left(\frac{e^{2x}+e^{-2x}}{2} - 1\right)$

$\Rightarrow \frac{1}{2}\left(\frac{e^{2x}+e^{-2x}-2}{2}\right)$

$\Rightarrow \frac{e^{2x} - 2 + e^{-2x}}{4}$

$\Rightarrow \sinh^2 x = \frac{1}{2}(\cosh 2x - 1).$

2 $\quad f(x) = \frac{1}{x-3}$

(i) $f(0) = -\frac{1}{3} \Rightarrow (0, -\frac{1}{3})$.

(ii) $f(x) = \frac{1}{x-3}$

$$f'(x) = \frac{\cancel{(x-3).0} - 1(1)}{(x-3)^2}$$

$$= \frac{-1}{(x-3)^2}$$

$\Rightarrow -\frac{1}{(x-3)^2} \neq 0$

$\Rightarrow$ no turning point

(iii) $\frac{dy}{dx} = f'(x) = -\frac{1}{(x-3)^2} < 0$

$\Rightarrow$ decreasing function for all x $(x \neq 3)$

(iv) $f(x) = \frac{1}{x-3}$

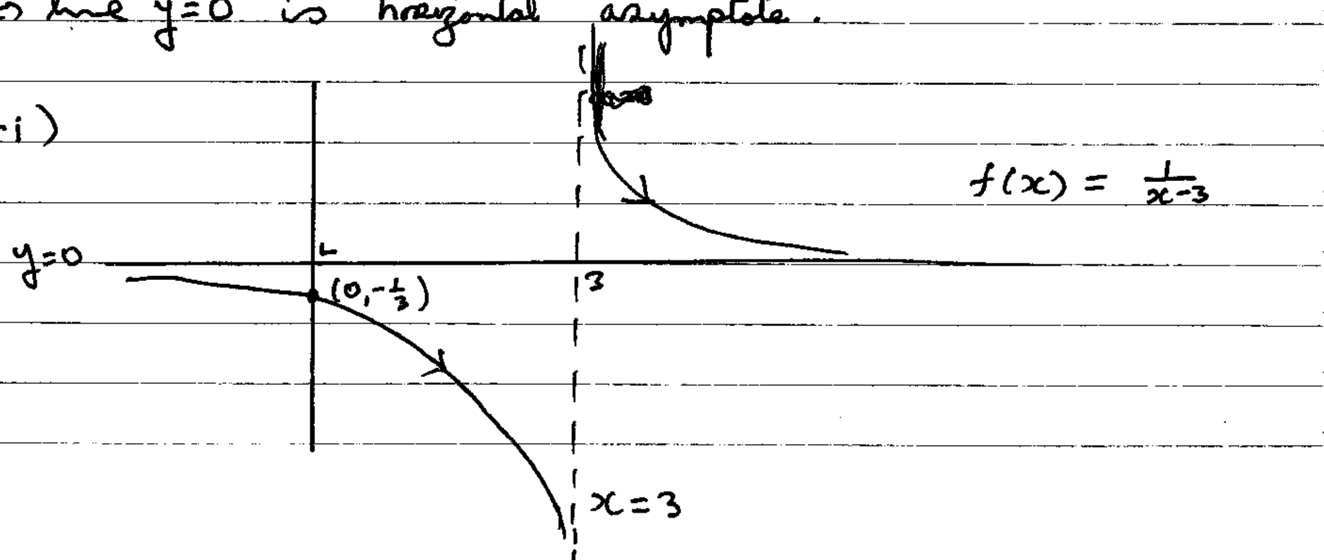
$f(3) = \frac{1}{0}$

$\Rightarrow$ line $x = 3$ is vertical asymptote.

(v) $\lim\limits_{x \to \infty} \left(\frac{1}{x-3}\right) = \frac{1}{\infty} = 0$

$\Rightarrow$ line $y = 0$ is horizontal asymptote.

(vi)



ection B

3 (a) (i) $\int_1^2 2x \sinh(x^2-1)\,dx$

let $u = x^2-1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$\Rightarrow \int \cancel{2x} \sinh u \frac{du}{\cancel{2x}}$

$\Rightarrow \int \sinh u\,du$

$\Rightarrow \cosh u$

$\Rightarrow \cosh(x^2-1)\Big|_{x=1}^{x=2}$

$\Rightarrow \cosh 3 - \cosh 0$

$\Rightarrow 9.06$

(ii) $\int \sin x\, e^{\cos x}\,dx$

let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$\Rightarrow du = -\sin x\,dx$

$\Rightarrow \frac{du}{-\sin x} = dx$

$\Rightarrow \int \cancel{\sin x}\, e^u \frac{du}{-\cancel{\sin x}}$

$\Rightarrow -\int e^u\,du$

$= -e^u + c$

$\Rightarrow -e^{\cos x} + c$

(iii) $\int x \sin x\,dx$

let $u = x$ | $dv = \sin x\,dx$

$\frac{du}{dx} = 1$ | $v = \int \sin x\,dx$

$\Rightarrow du = dx$ | $\Rightarrow v = -\cos x$

$\Rightarrow \int x \sin x\,dx = x(-\cos x) - \int -\cos x\,dx$

$= -x\cos x + \int \cos x\,dx$

$= -x\cos x + \sin x + c$

(b) $i(t) = 4 + 6\cos 2t$

$q(t) = \int 4 + 6\cos 2t\,dt$

$= 4t + 6\frac{\sin 2t}{2} + c$

$\Rightarrow q(t) = 4t + 3\sin 2t + c$

$q(0) = 0$ given

$\Rightarrow 0 = 4(0) + 3\sin 2(0) + c$

$0 = 0 + 3\sin 0 + c$

$0 = 0 + 3(0) + c$

$\Rightarrow 0 = c.$

$\Rightarrow q(t) = 4t + 3\sin 2t + 0$

$= 4t + 3\sin 2t.$

4 (a) 

$y = x^2 + 1$

$y = 9 - x^2$

$\Rightarrow 9 - x^2 = x^2 + 1$

$\Rightarrow -2x^2 = -8$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

$\Rightarrow A = \int_{-2}^{2} (9 - x^2) - (x^2 + 1)\, dx$

$= \int_{-2}^{2} 9 - x^2 - x^2 - 1\, dx$

$= \int_{2}^{2} 8 - 2x^2\, dx$

$= 8x - \frac{2x^3}{3} \Big|_{x=-2}^{x=2}$

$= \left(16 - \frac{16}{3}\right) - \left(-16 + \frac{16}{3}\right)$

$= \frac{32}{3} - \left(-\frac{32}{3}\right)$

$= \frac{32}{3} + \frac{32}{3}$

$\Rightarrow \frac{64}{3}.$

(b) $y = \sqrt{1 + e^x}$

$h = \frac{1}{4} = 0.25$

1 1.25 1.5 1.75 2.

$y = \sqrt{1 + e^x}$

$x =$	1	1.25	1.5	1.75	2
$y =$	1.928	2.119	2.341	2.598	2.896

$A \approx \frac{.25}{3}\left[1.928 + 2.896 + 4(2.119 + 2.598) + 2(2.341)\right]$

$= \frac{.25}{3}(4.824 + 18.868 + 4.682)$

$= 2.3645$

ction C

5 (a) $\sum_{n=1}^{\infty} \frac{6}{(6n+1)(6n+7)}$

$\Rightarrow \sum_{n=1}^{\infty} \left(\frac{1}{6n+1} - \frac{1}{6n+7} \right)$

$\Rightarrow \left(\frac{1}{7} - \cancel{\frac{1}{13}} \right) + \left(\cancel{\frac{1}{13}} - \cancel{\frac{1}{19}} \right) + \left(\cancel{\frac{1}{19}} - \cancel{\frac{1}{25}} \right) + \cdots$

$\Rightarrow \frac{1}{7}.$

(b) (i) $\sum_{n=1}^{\infty} \frac{n+5}{3n+4} \Rightarrow \lim_{n\to\infty} \frac{n+5}{3n+4}$

$\Rightarrow \lim_{n\to\infty} \frac{1+\frac{5}{n}}{3+\frac{4}{n}} = \frac{1+0}{3+0}$

$\Rightarrow \frac{1}{3}$

$\frac{1}{3} \neq 0 \Rightarrow$ Series is divergent

(ii) $\sum_{n=1}^{\infty} \frac{n^2+5n}{n^4+2n-1}$ Compare with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$\Rightarrow \lim_{n\to\infty} \frac{\left(\frac{n^2+5n}{n^4+2n-1} \right)}{\left(\frac{1}{n^2} \right)}$

$\Rightarrow \lim_{n\to\infty} \left(\frac{n^2+5n}{n^4+2n-1} \right) \cdot \frac{n^2}{1}$

$\Rightarrow \lim_{n\to\infty} \frac{n^4+5n^3}{n^4+2n-1}$ Divide by n^4

$\Rightarrow \lim_{n\to\infty} \left(\frac{1+5/n}{1+\frac{2}{n^3}-\frac{1}{n^4}} \right) = \frac{1+0}{1+0-0} \Rightarrow 1$

by Limit Comparison Test, as $\sum \frac{1}{n^2}$ is cgt.

$\Rightarrow \sum^{\infty} \frac{n^2+5n}{n^4+2n-1}$ is Convergent.

(iii) $\sum_{n=1}^{\infty} \frac{x^{n+1}}{4^n}$

$a_n = \frac{x^{n+1}}{4^n}$

$a_{n+1} = \frac{x^{n+2}}{4^{n+1}}$

$\Rightarrow \frac{a_{n+1}}{a_n} = \frac{\left(\frac{x^{n+2}}{4^{n+1}} \right)}{\left(\frac{x^{n+1}}{4^n} \right)}$

$= \frac{x^{n+2}}{4^{n+1}} \cdot \frac{4^n}{x^{n+1}}$

$= \frac{x}{4}$

$\Rightarrow \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{x}{4} \right| = \left| \frac{x}{4} \right|$

If series is Convergent

$\left| \frac{x}{4} \right| < 1$

$\Rightarrow |x| < 4.$

6 (a) $f(x) = \cosh x$

$f(0) = \cosh 0 = 1$

$f'(x) = \sinh x$

$f'(0) = \sinh 0 = 0$

$f''(x) = \cosh x$

$f''(0) = 1$

$f'''(x) = \sinh x$

$f'''(0) = 0$

$f^{iv}(x) = \cosh x$

$f^{iv}(0) = 1.$

$f^{v}(x) = \sinh x$

$f^{v}(0) = 0$

$f^{vi}(x) = \cosh x$

$f^{vi}(0) = 1$

$\Rightarrow f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \cdots$

$\Rightarrow \cosh x = 1 + \cancel{0(x)} + \frac{(1)x^2}{2!} + \cancel{\frac{0(x^3)}{3!}} + \frac{1(x)^4}{4!} + \cancel{\frac{0(x^5)}{5!}} + \frac{1(x^6)}{6!}$

$\Rightarrow \cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!}$

(i) Differentiate

$\sinh x = \frac{2x}{2!} + \frac{4x^3}{4!} + \frac{6x^5}{6!} + \cdots$

$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$

(ii) $\cosh 0.3$

$\Rightarrow 0.3 + \frac{(0.3)^2}{2!} + \frac{(0.3)^4}{4!} + \frac{(0.3)^6}{6!}$

$\Rightarrow 1 + .045 + .000375 + .000001$

$\Rightarrow 1.045$

b (i) $z = 5x^2y^4 + 2xy^3$

$\frac{\partial z}{\partial x} = 10xy^4 + 2y^3$

$\frac{\partial^2 z}{\partial y \partial x} = 40xy^3 + 6y^2$

(ii) $z = e^{2t} \sin 4x$

$\frac{\partial z}{\partial x} = e^{2t} \cos 4x \, (4)$

$= 4e^{2t} \cos 4x$

$\frac{\partial^2 z}{\partial x^2} = 4e^{2t}(-\sin 4x \cdot 4)$

$\frac{\partial^2 z}{\partial x^2} = \underline{-16e^{2t} \sin 4x}$

$\frac{\partial z}{\partial t} = 2e^{2t} \sin 4x$

$\frac{\partial^2 z}{\partial t^2} = 2(2)e^{2t} \sin 4x$

$\underline{\frac{\partial^2 z}{\partial t^2} = 4e^{2t} \sin 4x}$

$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial t^2}$

$\Rightarrow -16e^{2t} \sin 4x + 4(4)e^{2t} \sin 4x$

$= -16e^{2t} \cancel{\sin 4x} + 16e^{2t} \cancel{\sin 4x}$

$\Rightarrow 0.$