UNIVERSITY of LIMERICK

OLLSCOIL LUIMNIGH

Faculty of Science and Engineering
Department of Mathematics & Statistics

Special Mathematics Entrance Examination
Higher Level

DATE: Thursday 23 August 2018

TIME: 14.30-17.30 (3 HOURS)

INSTRUCTIONS TO CANDIDATES:

There are two sections in this examination paper.

Section A: 6 questions, 25 marks each.
Section B: 3 questions, 50 marks each.

ANSWER ALL QUESTIONS

The invigilator will provide answer books, graph paper and a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination.
You are not allowed to bring your own copy into the examination.

You **will** lose marks if all necessary work is not clearly shown.

Answers should include appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) on the cover of your answer books.

Section A (6 Questions, 25 Marks Each)

Question 1

(a) Solve the following simultaneous equations for x, y and z

$$\begin{aligned} 3x - y - 2z &= 1 \\ 2x + 4y - 3z &= -9 \\ -y + 5z &= 16 \end{aligned}$$

(b) (i) For what range of values of $x \in \mathbb{R}$ is $|x-5| \leq |x+1|$?

(ii) For what values of $a \in \mathbb{R}$ does the quadratic equation

$$x^2 + 2ax + 5a - 6 = 0$$

have equal roots ?

Question 2

(a) Let $z_1 = 2 - i$, where $i^2 = -1$.

(i) Write z_1 in polar form giving the modulus correct to 2 decimal places and the argument in radians correct to 2 decimal places.

(ii) Given that z_1 is a root of $f(z) = z^3 - 2z^2 - kz + 10 = 0$, find the value of k for $k \in \mathbb{R}$.

(iii) Find the other two roots of $f(z) = 0$.

(b) Write *de Moivre* 's theorem for $n = 3$. Use this result and the binomial theorem to show that $\cos(3A) = 4\cos^3(A) - 3\cos(A)$.

Question 3

(a) Evaluate the follow expressions:

(i)

$$\lim_{x\to\infty} \frac{2x^2+5}{3x^2+5x}$$

(ii)

$$\lim_{x\to 0} \left(x+\frac{1}{5x}\right)^2 - \left(\frac{1}{5x}-4x\right)^2$$

(b) A curve has equation $y = \dfrac{x^3}{24} - \ln(2x), \quad x > 0.$

(i) Find the slope of the tangent to the curve at $x = 1$.

(ii) Find the coordinates of the turning point of the curve.

Question 4

The circle with equation $x^2 + y^2 + 2x - 6y - 6 = 0$ cuts the x-axis at the points A and B.

(a) Write down the coordinates of the centre of the circle and the length of its radius.

(b) Find the coordinates of the points A and B.

(c) Find the equations of the tangents to the circle at the points A and B.

(d) Show that the tangents found in part(c) are not perpendicular to each other.

Question 5

(a) The first two terms of a geometric series are $T_1 = x+2$ and $T_2 = x^2 - x - 6$, where $x \neq -2$.

(i) What is the common ratio of the series in terms of x ?

(ii) If the sum to infinity of the series is 3, find the value of x.

(iii) For what range of values of x is the sum to infinity well defined?

(b) Sketch the curve $y = \frac{2}{4+x^2}$.

The area between the curve $y = \dfrac{2}{4+x^2}$ and the lines $x = 0$ and $x = 1$ is equal to the area between the curve and the lines $x = 1$ and $x = s$, where $1 < s$. Find the value of s.

Question 6

A game consists of tossing two fair six-sided dice.

(a) If x is the sum of the scores on the two dice, find

(i) $P(x = 5)$,

(ii) $P(x > 5)$,

(iii) $P(x = 7 | x > 5)$.

(b) Jo plays this game where she tosses two dice. If the sum is 5, she wins €3. If the sum is greater than 5, she wins €1. If the sum is less than 5, she loses q euros. Find the value of q for which Jo neither wins nor loses.

Section B (3 Questions, 50 Marks Each)

Question 7

A researcher wants to investigate whether the length of time people spend in education affects the income they earn. In a survey of twelve adults, they are asked to state their annual income and the number of years they spent in full-time education. The resultant data are given in the table below.

Length of Education (Years)	Income (€1000)
11	28
12	30
13	35
13	45
14	55
15	38
16	45
16	38
17	45
17	60
17	30
19	58

(i) Identify the dependent and independent variables.

(ii) Represent the data in a scatter plot, showing the relevant axes.

(iii) Calculate the coefficient of linear correlation.

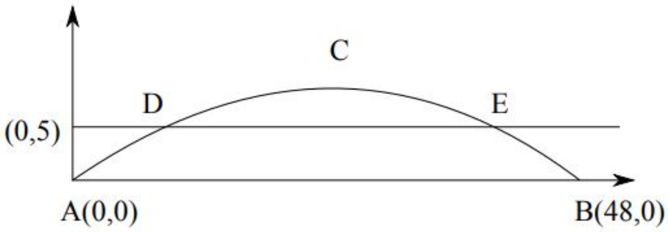
(iv) What can you conclude from your results in parts (ii) and parts (iii)?

(v) Add the line of best fit to the completed scatterplot in (ii).

(vi) Use the line of best fit to estimate the annual income of a person with 14 years full-time education.

(vii) By taking suitable readings from your digram, or otherwise, calculate the line of best fit.

(viii) Explain how to interpret the slope of the line in this context.



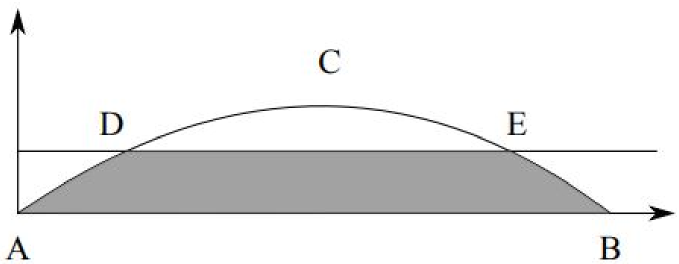
Question 8

The arch of a bridge is in the shape of a parabola, as shown. The length of the span of the arch, [AB], is 48 metres.

(i) Using the coordinate plane, with A(0, 0) and B(48, 0), the equation of the parabola is $y = 0.624x - 0.013x^2$.
Find the coordinates of C, the highest point of the arch.

(ii) The perpendicular distance between the walking deck, $[DE]$, and $[AB]$ is 5 metres. Find the coordinates of D and of E.
Give your answers correct to the nearest whole number.

(iii) Using integration, find the area of the shaded region, $ABED$, shown in the diagram below.
Give your answer correct to the nearest whole number.



(iv) Write the equation of the parabola in part (i) in the form $y - k = p(x - h)^2$, where k, p and h are constants.

(v) Using what you learned in part (iv) above, or otherwise, write down the equation of a parabola for which the coefficient of x^2 is -2 and the coordinates of the maximum point are $(3, -4)$.

Question 9

(a) A farmer buys 1000m of fencing with the intention of fencing off a rectangular field that borders a straight river. There is no need to fence along the river.

(i) What are the dimensions of the field that will give the largest area?

(ii) Calculate the maximum area.

(b) A man launches his boat from point A on a bank of a straight river, 4 km wide, and wants to reach point B, 10 km downstream on the opposite bank, as quickly as possible. He could row his boat directly across the river to point C and then run to B, or he could row directly to B, or he could row to some point D between C and B and then run to B. If he can row at 5 km/h and run at 13 km/h, where should he land to reach B as soon as possible? (Assume that the speed of the water is negligible compared with the speed at which the man rows.)