

26 April 2012

## PROBLEM SHEET 6: INTRODUCTION TO RANDOM VARIABLES

1. A poker hand is dealt fairly. Let  $X$  be the number of clubs in the hand. What is the probability distribution of the random variable  $X$ ? What is the expectation of  $X$ ?
2. There are 5 bad apples in a crate of 40. A child checks 10 apples selected at random, without replacing them. Find the probability of finding at least one bad apple.
3. Determine  $P(X \leq 12)$  when  $X$  is a binomial random variable with parameters 20 and 0.4.
4. Suppose that every microchip produced is independently defective with probability 0.01. Find the expected value and variance of the number of defective chips in a shipment of 1000.
5. Suppose  $X$  is the random variable with distribution  $N(0, 1)$ . Find: (i)  $P(X < 3)$ ; (ii)  $P(X < -3)$ ; (iii)  $P(-3 < X < 3)$ ; (iv)  $P(X > 1.26)$ ; (v)  $P(X < 0.195)$ .
6. If  $X$  is  $N(1, 4)$ , find: (i)  $P(X > 2)$ ;  $P(-2 < X < 1.4)$
7. Suppose  $X$  is normally distributed with mean  $\mu$  and standard deviation  $\sigma$ . Then
  - (a) The probability of  $X$  being within  $\sigma$  of the mean is 68%.
  - (b) The probability of  $X$  being within  $2\sigma$  of the mean is 95%.
  - (c) The probability of  $X$  being within  $3\sigma$  of the mean is 99.7%.
8. The height of male students at a particular university is normally distributed with mean  $\mu = 70$ in and standard deviation  $\sigma = 4$ in. Calculate: (i) the probability that a randomly selected male is between 67 and 76 inches; (ii) the probability that a randomly selected male is taller than 80 inches.