Question 3

- Fourier Series
- Differential Equation

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)dx \tag{0.1}$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)cos(\frac{n\pi x}{L})dx \tag{0.2}$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)sin(\frac{n\pi x}{L})dx \tag{0.3}$$

- **"a for even"**: If function is odd : $a_0 = 0$ and $a_n = 0$
- **"b for odd"**: If function is even : $b_n = 0$

$$I_2 = \int_{-\pi}^{\pi} f(x) cos(nx) dx$$

This integral can be simplifies as follows (on acccount of the fact that it is even)

$$I_2 = 2 \times \int_{0}^{\pi} f(x) cos(nx) dx$$

$$I_2 = 2 \times \left(\int_{0}^{\pi/2} 0 \times cos(nx) dx + \int_{pi/2}^{\pi} 1 \times cos(nx) dx \right)$$

$$I_2 = 2 \times \int_{\pi/2}^{\pi} cos(nx) dx$$

$$I_2 = 2 \left[\frac{-sin(nx)}{n} \right]_{\pi/2}^{\pi}$$

$$I_2 = 2 \left[\frac{-sin(n\pi)}{n} - \frac{-sin(n\pi/2)}{n} \right]$$

$sin(n\pi) = 0$

$$I_2 = \frac{2sin(n\pi/2)}{n}$$

$$I_1 = \int_{-\pi}^{\pi} f(x) dx$$

$$I_2 = 2 \times \left(\int_{0}^{\pi/2} 0 \times cos(nx) dx + \int_{pi/2}^{\pi} 1 \times cos(nx) dx \right)$$

$$I_2 = \frac{2sin(n\pi/2)}{n\pi}$$

$$a_n = \frac{I_2}{L} = \frac{2sin(n\pi/2)}{n\pi}$$

Computing a_0

$$I_1 = \int_{-\pi}^{\pi} f(x)dx$$

$$I_1 = 2 \times \left(+ \int_{pi/2}^{\pi} 1 \times dx \right)$$

$$I_1 = 2 \times (x)_{\pi/2}^{\pi} = \pi$$

$$a_0 = \frac{I_2}{L} = 1$$

Evaluation of a series

Hence evaluate the series

$$1 - \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots$$

Differential equation

Use the Fourier series of part (a) to find a particular solution of the differential equation