MA4702 Integration

Integration

Constants of integration omitted.

$f(x)$	$\int f(x)dx$
$x^n, (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln\lvert x\rvert$
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax}$
$a^x \quad (a>0)$	$\frac{a^x}{\ln a}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\tan x$	$\ln\lvert\sec x\rvert$
$\frac{1}{\sqrt{a^2-x^2}} \quad (a>0)$	$\sin^{-1}\frac{x}{a}$
$\frac{1}{x^2+a^2} \quad (a>0)$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$

Addition and Subtraction Rules of Integration

$$\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx.$$

$$\int_a^b (f(x)-g(x))dx = \int_a^b f(x)dx - \int_a^b g(x)dx.$$

The Power Rule for Integration

The power rule for derivatives can be reversed to give us a way to handle integrals of powers of x. Since

$$\frac{d}{dx}x^n = nx^{n-1},$$

we can conclude that

$$\int nx^{n-1}\,dx = x^n + C,$$

or, a little more usefully,

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

Question 33 : Introduction to Integration

Part A

Using appropriate substitutions, evaluate the indefinite integrals:

(i)

$$\int (s-4)^5 ds$$

(ii)

$$\int \frac{3}{(x+1)^4} dx$$

(iii)

$$\int (2y+3)(y^2+3y+2)^2 dy$$

Part B

Using appropriate substitutions, evaluate the indefinite integrals:

(i)

$$\int 3x^2(x^3+1)^5\,dx$$

(ii)

$$\int x^4 \sin(x^5)\,dx$$

Question 34 : Integration

Evaluate the following indefinite integrals using partial fractions:

(i)

$$\int \frac{x}{x^2-9} dx$$

(ii)

$$\int \frac{x-2}{x^2-4x+3} dx$$

(iii)

$$\int \frac{2x-4}{x^2-4x+8} dx$$

Question 35 : Integration by Parts

Evaluate the following using integration by parts.

(i)
$$\int -4\ln(x)\,dx$$

(ii)
$$\int (-7x+38)\cos(x)\,dx$$

(iii)
$$\int_0^{\frac{\pi}{2}} (-6x+45)\cos(x)\,dx$$

(iv)
$$\int (5x+1)(x-6)^4\,dx$$

(v)
$$\int_{-1}^{1} (2x+8)^3(-x+2)\,dx$$

(vi)
$$\int \sin(x)\,e^x\,dx$$

Question 36 : Integration by Parts

Formula:
If u and v are functions of x that have continuous derivatives, then

$$\int udv = uv - \int vdu$$

The LIPET rule

It is considered a rule of thumb to remember the acronym **LIPET** when performing integration by parts. This acronym will help you to determine what to use as u.

L -logarithms,

I -inverse trigonometric functions,

P -polynomials (i.e. x, x^2) ,

E -exponentials (i.e. e^x, e^{3x}),

T -trigonometric functions.

- $\cosh(x)$ is both the derivative and integral of $\sinh(x)$
- $\sinh(x)$ is both the derivative and integral of $\cosh(x)$

Question 37 : Integration by Parts

Evaluate the following indefinite integrals by integration by parts:

(a) $\int x^2 e^x \, dx$

(b) $\int x \ln x \, dx$

(c) $\int x^2 \cos x \, dx$

(d) $\int x \sin x \, dx$

(e) $\int e^x \sin x \, dx$

(f) $\int \ln x \, dx$

Question 38 : Integration (Video)

Evaluate the following:

(i) $\int x^2 - (2x)^2 \, dx$

(ii) $\int 8x^3 \, dx$

(iii) $\int (4x^2 + 11x^3) \, dx$

(iv) $\int (31x^{32} + 4x^3 - 9x^4) \, dx$

(v)

$$\int 5x^{-2}\,dx$$

Question 39 : Definite Integrals (Video)

Evaluate the following definite integrals

(i)

$$\int_1^2 (x^2-1)dx$$

(ii)

$$\int_0^{\frac{\pi}{2}} \cos x dx$$

(iii)

$$\int_0^{\pi} \cos x dx$$

(iv)

$$\int_1^2 (y^2-y^{-2})dy$$

(v)

$$\int_{-3}^1 (6x^2-5x+2)dx$$

(vi)

$$\int_4^0 \sqrt{t}(t-2)dt$$

Hint:

$$\int \sqrt{t}(t-2)dt$$

$$\sqrt{t}(t-2) = t^{1/2} \times (t-2) = t^{3/2} - 2t^{1/2}$$

Question 40 : Definite Integrals (Video)

Evaluate the following definite integrals:

(a) $\int_{-2}^{2} \frac{1}{x+3}\,dx$

(b) $\int_{0}^{2} (x^4 + 3x^2 + 2)\,dx$

(c) $\int_{-\pi}^{\pi} (5\sin x - 7\cos x)\,dx$

(d) $\int_{-3}^{2} 2x\, e^{(x^2+1)}\,dx$

(e) $\int_{0}^{\sqrt{\pi}} x\cos\left(x^2 - \frac{\pi}{2}\right)\,dx$

(f) $\int_{0}^{\pi} x\sin x\,dx$

(g) $\int_{0}^{1} \frac{1}{x^2-4}\,dx$

(h) $\int_{0}^{2} \frac{1}{x^2+4}\,dx$

Question 41 : Definite Integrals

Exercise: Evaluate the following definite integral

$$\int_1^3 \frac{x}{3}dx$$

Solution

$$\int_1^3 \frac{x}{3}dx = \left[\frac{x^4}{4}\right]_1^3 = \frac{81}{4} - \frac{1}{4} = 20$$

Exercise: Evaluate the following definite integral

$$\int_1^3 \frac{x^2-4x+3}{x-3}dx$$

Factorize the numerator $x^2-4x+3=(x-1)(x-3)$

Treat it as an indefinite integral for time being.

$$\int \frac{x^2-4x+3}{x-3}dx = \int \frac{(x-1)(x-3)}{x-3}dx = \int (x-1)dx = \frac{x^2}{2}-x+c$$

$$\left[\frac{x^2}{2}-x\right]_1^3 = (4.5-3)-(0.5-1)=2$$

Question 42 : Integration by Parts (Exam Standard)

the following questions are from previous past papers. Please be advised of the notes below.

(i) (2005) Use integration by parts to find $\int xe^x dx$

(ii) (2006) Use integration by parts to find $\int xln(x)dx$

(iii) (2007) Use integration by parts to find $\int xsinh(x)dx$

(iv) (2008) Use integration by parts to find $\int xcos(x)dx$

(v) (2009) Use integration by parts to find $\int xcosh(x)dx$

(vi) (2010) Use integration by parts to find $\int xe^x dx$

Important:

- You should expect to see hyperbolic functions (i.e. $\cosh(x)$ and $\sinh(x)$) in the end of semester exam.
- However you should expect to see terms like x^2, e^{2x} and $\ln(x)$, as well as what was in previous exams.
- **VERY Important:** Make sure you know how to integrate and differentiate expressions of the form e^{ax}, $\cos(ax)$, $\cosh(ax)$, $\sin(ax)$ and $\sinh(ax)$.

Question 43 : Definite Integrals

Evaluate the following definite integrals

(i) Find the area between $f(x) = x^2 + 4x$ and the $x-$axis between $x = -4$ and $x = 3$.

(ii) Calculate the following:

$$\int_0^1 \frac{4x^3}{x^4+1} dx$$

(iii) Evaluate

$$\int_0^{\frac{\pi}{2}} \cos^4(x)\sin(x) dx$$