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MA4702 Tech Maths 2 Syllabus

On successful completion of this module, students should be able to:

1. Define the domain and range of a function and define and plot simple inverse trigonometric and hyperbolic functions.
2. Sketch curves using properties such as symmetry, intercepts, discontinuities, turning points and asymptotic behaviour.
3. Sum arithmetic, geometric and telescoping series; test series for convergence; find the Maclaurin series of a function; manipulate power series; use l'Hopital's rule.

MA4702 Tech Maths 2 Syllabus

4. Integrate standard functions using substitution and parts;
   Apply to calculation of areas and volumes.
   Integrate numerically using Simpson's rule.
5. Find partial derivatives of functions of two variables as well as higher partial derivatives;
   apply to analysis of small errors.

Tutorials

- Tutorials start in Week 2. T
- I will use the "Teaching Week" naming convention, not the "Timetable Week" convention as in the SAA calendar.
- There is no lectures in Reading Week (Teaching Week 13), but we will probably schedule a revision session and tutorials.

Mid Terms

Details about Mid-Term Exams

- Three Mid-Terms worth 10%, 15 % and 15 % respectively.
- Mid Term 1 will take place on Week 5.
- Mid Term 2 will take place on Week 8.
- Mid Term 3 will take place on Week 11.
- Precise Dates to be confirmed (I want to see which of our rooms is most suitable holding an exam).
- End of Year Exam is worth 60%.

MA4701 Tech Maths 1 Syllabus

1. Define elementary functions including polynomials, exponential logarithms and graph simple examples.
2. Define trigonometric functions and use formulas and identities including sine and cosine rules.
3. Differentiate elementary functions using the laws of differentiation and apply to curve-sketching.

- Other sections of Tech Maths 1 are not relevant for this module

Quick Check

Quick Check on MA4701

- Trigonometric Functions
- Differentiation - Please Revise

Revision and Fundamental Concepts

Revision and Fundamental Concepts

- In this first section we will review various fundamental theorems and concepts that will feature in the course.
- Please be mindful of these, and regularly refer back to this section throughout the semester.
- Expect some short questions from this section in all of the exams

Lecture 1A

Revision and Fundamentals

- Numbers and Number Sets (notation)
- (Quick) Revision of Functions
- Exponents and powers
- Logarithms
- Special functions and operators
- Cross multiplication (fractions expansion)

Topic 1 : Sets of Numbers

- $\mathbb{N}$ Set of all natural numbers
- $\mathbb{Z}$ Set of all integers
- $\mathbb{Q}$ Set of all rational numbers
- $\mathbb{R}$ Set of all real numbers

There are, of course, other numbers sets, but we will not be encountering them on the course.

Topic 1 : Sets of Numbers

- $\mathbb{Z}^+$ Set of all positive integers
- $\mathbb{Z}^-$ Set of all negative integers
- $\mathbb{R}^+$ Set of all positive real numbers
- $\mathbb{R}^-$ Set of all negative real numbers

Topic 1 : The e constant

- The number e is an important mathematical constant (another is π) that is the base of the natural logarithm.
- It is approximately equal to 2.71828.

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \cdots$$

Topic 2: The Factorial Operator

Topic 2 : The Factorial Function
This is the product of the positive integers from 1 to n inclusive is denoted by $n!$, read as **"n factorial".**

Namely:

$$n! = 1 \times 2 \times 3 \times \ldots \times (n-2) \times (n-1) \times n$$

- Accordingly, $1! = 1$ and $n! = n(n-1)!$.
- **Important** It is also convenient to define $0! = 1$.

Topic 2 : The Factorial Operator

$$\frac{5!}{3!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 5 \times 4 = 20$$

- Remark : We will use the factorial operator frequently in this module.
  You are also now expected to be familiar with it for future modules.

Topic 3 : Mathematical Operations

- Power (also known as exponents)
- Root Functions
- Exponentials
- Logarithns

Topic 3 : Revision of Power Rules

$$(a^b)^c = a^{b \times c}$$

$$64^{2/3} = (4^3)^{2/3} = 4^{3 \times 2/3} = 4^2 = 16$$

$$(a^b) \times (a^c) = a^{b+c}$$

$$(3^2) \times (3^3) = 3^{2+3} = 3^5 = 243$$

Topic 3 : Revision of Power Rules

$$(e^y)^2 = e^{y \times 2} = e^{2y}$$

$$(e^y) \times (e^{-y}) = e^{y+(-y)} = e^0 = 1$$

Cube Roots, Fourth Roots etc

$$\sqrt[a]{b} = c$$

Also

$$b^{1/a} = c$$

necessarily

$$c \times c \times c = b$$

$$\sqrt[3]{27} = 3$$

Topic 3 : Roots

Sign of Roots

Remark: In this course, we will assume the positive square root for a function, in the first instance. (*We will only consider the negative root of a function in some special cases.*)

$$\sqrt[2]{4x^2} \equiv 2x$$

The $\equiv$ symbols is the symbol for equivalence. You would use it to say that two expressions are equivalent, although the *equals* sign is conventionally used also.

Cube Roots

- For this course, only positive numbers have square roots.
- The square roots are also positive numbers.
- *(This statement is not strictly true. The square root of a negative number is called a* **complex number***. However this is not part of the course).*

Negative numbers can have cube roots

$$-27 = -3 \times -3 \times -3$$

$$\sqrt[3]{-27} = -3$$

Topic 4 : Laws of Logarithms

- Law 1 : Multiplcation of Logarithms

$$Log(a) \times Log(b) = Log(a + b)$$

- Law 2 : Division of Logarithms

$$\frac{Log(a)}{Log(b)} = Log(a - b)$$

- Law 3 : Powers of Logarithms

$$Log(a^b) = b \times Log(a)$$

Example 1: Evaluating a Function

Evaluate the following function for x =1,2 and 3 respectively.

$$f(x) = \frac{e^x}{x!}$$

Example 2: Evaluating a Function Evaluate the following function for x =1,2 and 5 respectively.

$$f(x) = \frac{e^x + e^{e^{-x}}}{2}$$

Worked Example : Evaluating a Function
Evaluate the function for each of the following values : 0.5,1,1.25,2.

$$f(x) = \sqrt{1 + e^x} dx$$

Four decimal places will suffice.

Topic 5 : Functions

x	e^x	$1 + e^x$	$\sqrt{1 + e^x}$
0.5			
1			
1.25			
2			

Topic 6 : Special Functions

- Absolute Value Function
- Floor and Ceiling Functions
- Hyperbolic Functions

Absolute Value Function

- The absolute value (or modulus) $|x|$ of a real number x is the non-negative value of x without regard to its sign.

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0. \end{cases}$$

Topic 6 : Absolute Value Function

- For a positive x$|x| = x$, for a negative x (in which case x is positive) $|x| = -x$, and $|0| = 0$.

- For example, the absolute value of 4 is 4, and the absolute value of -4 is also 4.
- **IMPORTANT:** The input to this function is an real number. The output of this function will always be a positive real numbers.

Topic 6 : Floor and Ceiling Functions

- The floor and ceiling functions map a real number to the largest previous or the smallest following integer, respectively.
- More precisely,

$$floor(x) = \lfloor x \rfloor$$

is the largest integer not greater than x and

$$ceiling(x) = \lceil x \rceil$$

is the smallest integer not less than x.

Topic 6 : Floor and Ceiling Functions

$$\lfloor 3.14 \rfloor = 3 \tag{1}$$

$$\lceil -4.5 \rceil = -5 \tag{2}$$

$$|-4| = 4 \tag{3}$$

Topic 7 : Cross Multiplication

- Can simplify an expression by multiplying both the numerator and denominator by same term.
- This does not change the value of the expression.
- Remark

$$\frac{A}{B} + \frac{X}{Y} = \frac{AY}{BY} + \frac{BX}{BY} = \frac{AY + BX}{BY}$$

Cross Mutliplication

$$\frac{p}{x+a} + \frac{q}{x+b} = \frac{p(x+b)+q(x+a)}{(x+a)(x+b)}$$

$$= \frac{(p+q)x+(pb+aq)}{(x+a)(x+b)}$$

- $\{p, q, a, b\} \in R$

Cross Mutliplication

$$\frac{4}{x+2} + \frac{2}{x-1} = \frac{4(x-1)+2(x+2)}{(x+2)(x-1)}$$

$$= \frac{(4+2)x + (4(-1)+(2\times 2)}{(x+2)(x-1)}$$

$$= \frac{2x}{x^2+x-2}$$

Cross-Multiplication: Example 1

- Solve the following Equation for A and B
- $A, B \in \mathbb{R}$

$$\frac{2x+5}{x^2-4x-12} = \frac{A}{x-6} + \frac{B}{x+2}$$

Cross-Multiplication: Example 2

- Solve the following Equation for A and B
- $A, B \in \mathbb{R}$

$$\frac{11}{x^2 - 4x - 12} = \frac{A}{x - 6} + \frac{B}{x + 2}$$

Revision Topics

- Exponents
- Laws of Logarithms
- The Natural Logarithm