

Engineering Calculus (MA4003)

Laplace Transforms (Qs 1 and 2)

- (a1) Number each entry of the Laplace Transforms - you should have 22 entries. (e.g. Heaviside function is entry 16).
- (a2) Questions 1 and 2 typically involve using one of the table entries to find the Laplace transform.

Inverse Laplace Transforms (Q3)

- (b1) Consider function in form $f(t-a) \times u_a(t)$. The value of a should be evident. Determine $f(t-a)$ and hence $f(t)$. From $f(t)$ compute $F(s)$.

Inverse Laplace Transforms (Qs 4 and 5)

- (c1) Always (attempt to) factorize the quadratic component.
- (c2) Use table entries 8 and 9 for the following form:

$$\frac{s+k}{(s+a)(s+b)} = \frac{s}{(s+a)(s+b)} + \frac{k}{(s+a)(s+b)}$$

- (c3) Factorize because sometimes terms cancel each other out.

$$\frac{s+a}{(s^2+(a+b)s+ab)} = \frac{s+a}{(s+a)(s+b)} = \frac{1}{(s+b)}$$

- (c4) Sometimes there is no obvious way to factorize the denominator. Try a different approach:

$$\frac{s+a}{(s+a)^2+m} \text{ remark: We can use now shifting theorem}$$

- (c5) Numerator can be re-expressed as sum of two terms. $s = (s-a) + a$

$$\frac{s}{(s+a)^2} = \frac{s-a}{(s+a)^2} + \frac{a}{(s+a)^2} = \frac{1}{s+a} + \frac{a}{(s+a)^2}$$

Convolution (Q6)

- (d1) Find the Laplace transform of both terms individually : $F(s)$ and $G(s)$.
- (d2) The Laplace transform of the convolution result $f(t) * g(t)$ is the product of $F(s)$ and $G(s)$.

$$\mathcal{L}[f(t) * g(t)] = F(S)G(S)$$

- (d3) To find $f(t) * g(t)$, Compute the inverse Laplace Transform of $F(S) \times G(S)$

$$\mathcal{L}^{-1}[F(S)G(S)] = f(t) * g(t)$$

Period of a function (old questions)

1. Period of a function $Trig(kx)$:

$$p = \frac{2\pi}{k}$$

Even and Odd Functions (Q8)

(f1) Even Functions: $f(-a) = f(a)$

(f2) Odd Functions: $f(-a) = -f(a)$

Fourier Coefficients

$$(g1) \quad a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$

(g2) Important - Revise “Integration by Parts”.