FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS AND STATISTICS

END OF SEMESTER EXAMINATION

MODULE CODE: MA4702 SEMESTER: Spring 2015

MODULE TITLE: Technology Mathematics 2 DURATION OF EXAM: 2.5 hours

LECTURER: Kevin O'Brien GRADING SCHEME: 100 marks
70% of total module marks

EXTERNAL EXAMINER: Prof. John King

INSTRUCTIONS TO CANDIDATES

This paper is comprised of six questions. Question 1 is compulsory and is worth 40 Marks. You must also attempt any four of the other five questions, each of which are worth 15 marks.
Scientific calculators approved by the University of Limerick can be used.
Formula sheet and statistical tables are provided.

Question 1

(i) Find $f^{-1}(x)$ the inverse of the function

$$f(x) = \sqrt{2x+3}$$

(ii) Solve the following limit:

$$\lim_{x \to 3} \frac{x^2 - 4x + 3}{x - 3}$$

(iii) Find the domain and the range of the function:

$$f(x) = \frac{6x}{3x+2}$$

(iv) Determine the vertical asymptotes of the following function

$$f(x) = \frac{4x+3}{x-7}$$

(v) Determine the horizontal asymptotes of the following function

$$f(x) = \frac{4x+3}{x-7}$$

(vi) Evaluate the following indefinite integral:

$$\int (\sin(2x) + e^{4x}) dx$$

(vii) Evaluate the following definite integral:

$$\int_1^3 (12x^3 - 48x^2 + 36x) dx$$

(viii) Determine both of the first order partial derivatives with respect to z of the following expression.

$$z = \frac{x^2 y^3}{3} + y\cos(x)$$

(ix) Find $\dfrac{\partial^2 z}{\partial^2 x}$ and $\dfrac{\partial^2 z}{\partial^2 y}$ for the following function

$$z = x^2 y + 3x^2 y^3 - 9x^2.$$

(x) Compute the following summation

$$\sum_{i=16}^{50} i$$

Question 2 - Limits and Functions

Part A - Limits

(i) (2 Marks) Compute the limit of the following function.

$$\lim_{x\to 5} \frac{x^2 - 15}{x - 4}.$$

(iii) (3 Marks) Compute the limit of the following function.

$$\lim_{x\to\infty} \frac{3 + 2x^2 - 9x^3}{3x^3 - 7x + 5}.$$

Part B - Functions

(i) (3 Marks) Determine if the function $f(x) = x^2 \cos(x)$ is an even function, an odd function or neither.

(ii) (2 Marks) Given the functions $g(x) = x^2 - 1$ and $f(x) = 2x + 1$ determine expressions for $f \circ g(x)$ and $g \circ f(x)$.

Part C - Hyperbolic Functions

(i) (5 Marks) Using their definition in terms of exponentials, prove the following hyperbolic identity

$$\cosh^2(x) = \frac{1 + \cosh(2x)}{2}.$$

Question 3 - Curve Sketching

The concentration of a drug in a patient's bloodstream 7 hours after it was injected is given by

$$A(h) = \frac{0.17h}{h^2 + 2}$$

(i) (4 Marks) Find the axis intercepts of $A(h)$.

(ii) (5 Marks) Find and classify the critical points of $A(h)$ as local maxima or local minima.

(iii) (3 Marks) Determine the behaviour of $A(h)$ as $h \to +\infty$.

(iv) (3 Marks) Sketch the graph of $y = A(h)$ for $h \geq 0$ illustrating clearly the features of the curve obtained in parts (i - iii).

Question 4 - Sequences and Series

(i) (4 Marks) The second term u_2 of a geometric sequence is 24. The third term u_3 is -96. Answer the following questions. Both questions are worth 2 Marks each.

(a) Find the common ratio r.

(b) Find the first and fourth term u_1 and u_4.

(ii) (3 Marks) Three consecutive terms of an arithmetic series are

$$4x - 1, 2x + 11, 3x + 41.$$

Find the value of x.

(iii) (3 Marks) Suppose that the following term is the general term for a series. Use the Ratio Test to test this series for convergence

$$u_n = \frac{n!n!}{(2n)!}$$

(iv) (3 Marks) Find the sum of the telescoping series

$$\sum_{n=1}^{\infty} \frac{2}{(2n-1)(2n+1)}$$

(v) (2 Marks) Express the following repeating decimal number as a simple fraction. Show your workings.

$$0.162162162162....$$

Question 5 - Integration

(i) (2 Marks) Evaluate the definite integral

$$\int_1^2 x^2 + \sqrt{x} + \cos(5x) dx.$$

(ii) (3 Marks) By finding a good substitution, evaluate the indefinite integral

$$\int \frac{10x+4}{5x^2+4x+3} dx.$$

(iii) (3 Marks) By finding a good substitution, evaluate the indefinite integral

$$\int 2x(x^2+2)^9 dx.$$

(iv) (3 Marks) Use integration by parts to evaluate the indefinite integral

$$\int xe^{4x} dx.$$

(v) (4 Marks) By first performing a partial fraction expansion (that is, by writing the integrand as follows)

$$\frac{A}{x+1} + \frac{b}{x+3},$$

evaluate the definite integral

$$\int \frac{3x+5}{(x+1)(x+3)} dx.$$

Question 6 - Applications of Integration and Partial Derivatives

Part A - Applications of Integration

(i) (5 Marks) Find the area enclosed by the curves $y = x^2 - 6x + 5$ and $y = x - 5$.

(ii) (5 Marks) A current $i(t) = 4+6\cos(2t)$ passes through a capacitor at time t. The capacitor is uncharged at time $t = 0$. Find the charge $q(t)$ at all times t.

Part B - Partial Derivatives

(i) (5 Marks) Show that the function $z = e^{2t}\sin(4x)$ satisfied the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial t^2} = 0.$$

Formula Sheet

Logarithms

If $a^b = c$ then $\log_a c = b$.

Sum and Difference of Two Cubes

$$a^3 + b^3 = (a - b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Sequences and Series

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Arithmetic Series Summation:

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

Geometric Series Summation:

$$S_n = a\left(\frac{1 - r^n}{1 - r}\right)$$

$$S_\infty = \frac{a}{1 - r}$$

Ratio Test

For a series with general term u_n, if

$$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = r$$

then the series converges (absolutely) if $r < 1$

Curve Sketching

Horizontal Asymptote: The horizontal asymptote is computed as

$$\lim_{x\to\infty} f(x)$$

Maclaurin Series

$$f(x) = f(0) + f'(0) + \frac{f''(0)}{2!} + \frac{f''(0)}{2!} + \frac{f'''(0)}{3!} + \dots$$

Hyperbolic Functions

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

Rules of Differentiation

Product Rule: with $y = uv$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient Rule:

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Chain Rule:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Integration

Integration by parts:

$$\int udv = uv - \int vdu$$

Further formulae and special cases on pages 41 & 42 of the log tables provided.

Dynamics

Where $s(t)$ denotes displacement at time t, $v(t)$ denotes the velocity at time t and $a(t)$ denotes the acceleration at time t,

$$\frac{ds(t)}{dt} = v(t),$$

$$\frac{dv(t)}{dt} = a(t).$$

Electrical Circuits

Where $q(t)$ denotes the charge at time t and $i(t)$ denotes the current at time t,

$$\frac{dq(t)}{dt} = i(t).$$