UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

MODULE CODE: MA4702 **SEMESTER: Annual Repeats 08/09**

MODULE TITLE: **Technological Mathematics 2** **DURATION OF EXAM: 2.5 hours**

LECTURER: Mr J. O'Shea **GRADING SCHEME**:
Examination: 100%

EXTERNAL EXAMINER:

INSTRUCTIONS TO CANDIDATES

Answer 5 questions, one each from sections A, B, C, D, and any one other question.

Mathematics tables may be used.

University of Limerick approved calculators may be used

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SECTION A

1 (a)

(i) $f(x) = 3 - x$, find the composite function $fof(x)$.

(ii) Prove that the function $g(x) = x\sin(x)$ is even.

(iii) Find $f^{-1}(x)$ the inverse of the function $f(x) = \log_e(x+3)$. **10**

1 (b)

(i) Evaluate $\tan^{-1}(4)$.

(ii) Sketch the graph of $\sin^{-1}(x)$ (The principal value of the inverse sine curve) indicating clearly the domain and range of the function. **5**

1 (c) Using their definition in terms of exponentials, prove the following hyperbolic identity:

$$\cosh^2 x = \frac{1}{2}(1 + \cosh 2x).$$ **5**

2 Consider the function $y = f(x) = x^4 - 4x^3$

(i) Find the x and y intercepts of $f(x)$. **2**

(ii) Find the critical points of $f(x)$ and classify them. **6**

(iii) Find the two points of inflection. **3**

(iv) Find the x values for which $y = f(x)$ is concave up/down. **2**

(v) Determine the behaviour of y as $x \to +\infty$ and as $x \to -\infty$. **2**

(vi) Sketch the graph of $y = f(x)$ illustrating the features of the curve obtained in parts (i-v). **5**

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SECTION B

3 (a) Evaluate the following definite and indefinite integrals

(i) $\int 3x^2 \cos(x^3 - 2)dx$.

(ii) $\int_3^4 \frac{2x-6}{x^2-6x+10}\ dx.$

(iii) Use integration by parts to find $\int x\cos x dx.$ **15**

(b) An object moves in a straight line with velocity $v(t) = \sin 4t$.
It starts from position $s = 0$ at time $t = 0$.
Find the distance travelled at all times $t \geq o$.
5

4 (a) Find the area bounded by the curve $y = x^2 - 3x + 2$ and the line $y = x + 2$. **10**

(b) Use Simpson's rule with 4 equal subintervals to find an approximation for $\int_1^2 \sqrt{1+x^3}\ dx.$ **10**

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SECTION C

5 (a) Find the sum of the telescoping series

$$\sum_{n=1}^{\infty} \frac{2}{(2n-1)(2n+1)} \, .$$ **5**

(b) (i) Prove that the series $\sum_{n=1}^{\infty} \frac{n+4}{3n+5}$ is divergent.

Test the following series for convergence

(ii) $\sum_{n=1}^{\infty} \frac{3n^2-1}{n^3+4n}$

(iii) $\sum_{n=1}^{\infty} \frac{4^n}{(n+1)!}$.

15

6 (a) Find the Maclaurin series of e^{2x} up to and including the term containing x^3.

Use your answer to approximate $e^{0.4.}$ **10**

(b) Find all the first and second partial derivatives of the function

$z = \sinh(4x + 3y - 5)$. **10**

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SECTION D

7. Write down Maple 12 commands which implement the following:
(*Do not attempt to find the answers or Maple 12 output*)

(a) Evaluate $\left(\frac{7^3-\sqrt{21}+312}{\sqrt{7}}\right)^4$ to 20 significant digits. **3**

(b) Substitute $x=\frac{4}{5}$ into $x^3 \ln x$. **2**

(c) Find the factors of the cubic polynomial

$3x^3 - 12x^2 - 100 + 44x$. **3**

(d) Plot $y = \text{Cos}^{-1} x$ for $-3 \le x \le 4$. **3**

(e) Find the first derivative of $\frac{\mathbf{6}\sin^2 x}{8+e^{2x}}$ with respect to x and simplify the answer. **3**

(f) Find the second derivative of $\frac{\mathbf{6}\sin^2 x}{8+e^{2x}}$ with respect to x and simplify the answer. **3**

(g) Evaluate the definite integral $\int_{-1}^{4} \cosh x \tan x dx$. **3**

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SECTION D

8. The output of a Maple 12 session, investigating the properties of some function $y = f(x)$ is represented on the **next page**.

(a) Based on this output

(i) Find the x and y intercepts of $f(x)$ (if any).

(ii) Find the x and y coordinates of all maxima and minima of $f(x)$ (if any).

(iii) Find the x and y coordinates of all points of inflection of $f(x)$.

(iv) Discuss the behaviour of $f(x)$ as $x \to +\infty$ and as $x \to -\infty$. **15**

(b) Based on the information given in the output, plot $y = f(x)$ in the domain $-3 \le x \le 3$, labelling the parts found in (a). **5**

Maple 12 Output:

```
solve(y = 0);
```

$$1, 2, 3$$

```
subs(x = 0, y);
```

-6

```
y1 := diff(y, x) :
solve(y1 = 0);
```

$$2 + \frac{1}{3}\sqrt{3}, 2 - \frac{1}{3}\sqrt{3}$$

```
evalf(%, 5);
```

2.5774, 1.4226

```
subs(x = 2.5774, y);
```

-0.38490018

```
subs(x = 1.4226, y);
```

0.384900175

```
y2 := diff(y1, x) :
evalf(subs(x = 2.5774, y2));
```

3.4644

```
evalf(subs(x = 1.4226, y2));
```

-3.4644

```
evalf(solve(y2 = 0));
```

2

```
subs(x = 2, y);
```

0

```
evalf(subs(x = 10000, y));
```

$$9.99400110010^{11}$$

```
evalf(subs(x = -10000, y));
```

$$-1.00060011010^{12}$$