Example 2: Evaluating a Function

Evaluate the following function for x $=$1,2 and 5 respectively.

$$f(x) = \frac{e^x + e^{-x}}{2}$$

Remark: Example 1 was done in previous class.

Example 3: Evaluating a Function

Evaluate the function for each of the following values : 0.5,1,1.25,2.

$$f(x) = \sqrt{1 + e^x}$$

Four decimal places will suffice.

x	e^x	$1+e^x$	$\sqrt{1+e^x}$
0.5			
1			
1.25			
2			

Recall : Sets of Numbers

- $\mathbb{N}$ Set of all natural numbers
- $\mathbb{Z}$ Set of all integers
- $\mathbb{Q}$ Set of all rational numbers
- $\mathbb{R}$ Set of all real numbers

There are, of course, other numbers sets, but we will not be encountering them on the course.

Recall : Sets of Numbers

- $\mathbb{Z}^+$ Set of all positive integers
- $\mathbb{Z}^-$ Set of all negative integers
- $\mathbb{R}^+$ Set of all positive real numbers
- $\mathbb{R}^-$ Set of all negative real numbers

Special Functions

- Absolute Value Function
- The Sign Function
- Floor and Ceiling Functions
- Hyperbolic Functions

Absolute Value Function

- The absolute value (or modulus) $|x|$ of a real number x is the non-negative value of x without regard to its sign.

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0. \end{cases}$$

Topic 1 : Absolute Value Function

- For a positive x, $|x| = x$
- For a negative x (in which case x is positive) $|x| = -x$
- The absolute value of 0 is 0: $|0| = 0$.
- For example, the absolute value of 4 is 4, and the absolute value of -4 is also 4.
- IMPORTANT: The input to this function is any real number. The output of this function will always be a positive real numbers.

Sign Function

- The sign function $sng(x)$ of a real number x is a signed value of absolute value of 1, dependent on the sign of x.
- IMPORTANT: The input to this function is any real number. The output of this function will always be either 1 or -1.

$$sgn(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ -1, & \text{if } x < 0. \end{cases}$$

Topic 1 : Floor and Ceiling Functions

- The floor and ceiling functions map a real number to the largest previous or the smallest following integer, respectively.
- More precisely,

$$floor(x) = \lfloor x \rfloor$$

is the largest integer not greater than x and

$$ceiling(x) = \lceil x \rceil$$

is the smallest integer not less than x.

Topic 1 : Floor and Ceiling Functions

Examples

$$\lfloor 3.14 \rfloor = 3 \quad (1)$$
$$\lceil -4.5 \rceil = -5 \quad (2)$$
$$|-4| = 4 \quad (3)$$

Remark: Input to the floor and ceiling function can be any really number, but outputs are always integers.

Even and Odd functions

Even Functions

Then f is even if the following equation holds for all x and -x in the domain of f:

$$f(x) = f(-x),$$

Geometrically speaking, the graph face of an even function is symmetric with respect to the y-axis, meaning that its graph remains unchanged after reflection about the y-axis.

Odd Functions

Let f(x) be a real-valued function of a real variable. Then f is odd if the following equation holds for all x and -x in the domain of f:

$$-f(x) = f(-x),$$

or

$$f(x) + f(-x) = 0.$$

Even and Odd functions

- **Important:** A function may be neither even nor odd.
- Discussion with examples on Blackboard
- Examples of Questions from Past Papers done on board

Cross Multiplication

- Can simplify an expression by multiplying both the numerator and denominator by same term.
- This does not change the value of the expression.
- Remark

$$\frac{A}{B} + \frac{X}{Y} = \frac{AY}{BY} + \frac{BX}{BY} = \frac{AY + BX}{BY}$$

Cross Mutliplication

$$\frac{p}{x+a}+\frac{q}{x+b} = \frac{p(x+b)+q(x+a)}{(x+a)(x+b)} = \frac{(p+q)x+(pb}{(x+a)(x+}$$

- $\{p, q, a, b\} \in R$

Cross Mutlplication

$$
\begin{aligned}
\frac{4}{x+2} + \frac{2}{x-1} &= \frac{4(x-1) + 2(x+2)}{(x+2)(x-1)} \\
&= \frac{(4+2)x + (4(-1) + (2 \times 2)}{(x+2)(x-1)} \\
&= \frac{2x}{x^2 + x - 2}
\end{aligned}
$$

Cross-Multiplication: Example 1

- Solve the following Equation for A and B
- $A, B \in \mathbb{R}$

$$\frac{2x+5}{x^2-4x-12} = \frac{A}{x-6} + \frac{B}{x+2}$$

Cross-Multiplication: Example 2

- Solve the following Equation for A and B
- $A, B \in \mathbb{R}$

$$\frac{5}{x^2 - 4x - 12} = \frac{A}{x-6} + \frac{B}{x+2}$$

Topic 2 : Laws of Logarithms

- Law 1 : Multiplcation of Logarithms

$$Log(a) \times Log(b) = Log(a + b)$$

- Law 2 : Division of Logarithms

$$\frac{Log(a)}{Log(b)} = Log(a - b)$$

- Law 3 : Powers of Logarithms

$$Log(a^b) = b \times Log(a)$$

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