Q1

(a) $F(x) = \dfrac{1}{2x-5}$

$$y = \frac{1}{2x-5}$$

$$\frac{1}{y} = 2x - 5$$

$$\frac{1}{2y} + \frac{5}{2} = x$$

$$F^{-1}(x) = \frac{1}{2x} + \frac{5}{2}$$

(b) Domain: $(-\infty, \infty)$

Range: $[5, 9]$

(c) Vertical: $2x^2 - 1 = 0$

$$x^2 = \frac{1}{2}$$

$$x = \pm \frac{1}{\sqrt{2}}$$

Horizontal: $\lim\limits_{x \to \infty} \dfrac{5x^2 - 7x + 2}{2x^2 - 1}$

$$= \lim_{x \to \infty} \frac{5 - \frac{7}{x} + \frac{2}{x^2}}{2 - \frac{1}{x^2}} = \frac{5 - \frac{7}{\infty} + \frac{2}{\infty^2}}{2 - \frac{1}{\infty^2}} = \frac{5}{2}$$

(d) $\lim_{x\to\infty} \dfrac{2x^2 - 8x}{4x^2 - 7} = \lim_{x\to\infty} \dfrac{2 - \frac{8}{x}}{4 - \frac{7}{x^2}}$

$= \dfrac{2 - \frac{8}{\infty}}{4 - \frac{7}{\infty^2}} = \dfrac{2 - 0}{4 - 0} = \dfrac{1}{2}$

(e) $\int 3x^2 + 2e^x - 1 \, dx$

$= x^3 + 2e^x - x + C$

(F) $\int_4^9 x^{-\frac{1}{2}} \, dx = \left. \dfrac{x^{\frac{1}{2}}}{\frac{1}{2}} \right|_4^9$

$= \left. 2x^{\frac{1}{2}} \right|_4^9 = \left. 2\sqrt{x} \right|_4^9$

$= 2\sqrt{9} - 2\sqrt{4}$

$= 2$

(g) $$Z = 2x^3y + x\sin y$$

$$\frac{dz}{dx} = 6x^2y + \sin y$$

$$\frac{dz}{dy} = 2x^3 + x\cos y$$

(h) $$F(-x) = \frac{e^{-x} - e^{x}}{2}$$

$$-F(x) = -\left(\frac{e^{x} - e^{-x}}{2}\right) = \frac{e^{-x} - e^{x}}{2}$$

$$F(-x) = -F(x)$$

$\Rightarrow$ Odd Function.

(i) $a = 3$ $\cancel{R} = 2$ $U_n = aR^{n-1}$

$$aR^{n-1} = 3072$$

$$3(2^{n-1}) = 3072$$

$$2^{n-1} = 1024$$

$$\log_2 1024 = n - 1$$

$$n = 11$$

$$S_n = a\left(\frac{1-R^n}{1-R}\right)$$

$$S_{11} = 3\left(\frac{1-2^{11}}{1-2}\right)$$

$$S_{11} = 6141$$

Q2. $F(x) = x^4 - 8x^2 + 7$

a) y-int:

$y = (0)^4 - 8(0)^2 + 7$

$y = 7$

y-int: $(0, 7)$

(b) $F'(x) = 4x^3 - 16x$

$4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$x = 0 \quad x = \pm 2$

$x = 0 \qquad y = 7 \qquad (0, 7)$

$x = 2 \qquad y = (2)^4 - 8(2)^2 + 7 = -9 \qquad (2, -9)$

$x = -2 \qquad y = -9 \qquad (-2, -9)$

$F''(x) = 12x^2 - 16$

$x = 0 \qquad F''(0) = 12(0)^2 - 16 = -16$

$\Rightarrow$ Max at $(0, 7)$

$$F''(x) = 12x^2 - 16$$

$x = -2$ $\quad F''(-2) = 12(-2)^2 - 16 = 32$

Min. point at $(-2, -9)$

$x = 2$ $\quad F''(2) = 12(2)^2 - 16 = 32$

Min point at $(2, -9)$

(c) $\quad F''(x) = 12x^2 - 16$

$$12x^2 - 16 = 0$$

$$x^2 = \frac{4}{3}$$

$$x = \pm\sqrt{\frac{4}{3}}$$

$x = \sqrt{\frac{4}{3}}$ $\quad y = \left(\sqrt{\frac{4}{3}}\right)^4 - 8\left(\sqrt{\frac{4}{3}}\right)^2 + 7$

$$= \frac{16}{9} - \frac{32}{3} + 7 = -\frac{17}{9}$$

$x = -\sqrt{\frac{4}{3}}$ $\quad y = -\frac{17}{9}$

Points of Inflection: $\left(\sqrt{\frac{4}{3}}, \frac{-17}{9}\right), \left(-\sqrt{\frac{4}{3}}, \frac{-17}{9}\right)$

(d) $\lim\limits_{x \to +\infty} x^4 - 8x^2 + 7$

$$= \lim_{x \to +\infty} x^4 \left(1 - \frac{8}{x^2} + \frac{7}{x^4}\right)$$

$$= \infty \left(1 - \frac{8}{\infty^2} + \frac{7}{\infty^4}\right)$$

$$= \infty (1 - 0 + 0) = +\infty$$

$$\lim_{x \to -\infty} x^4 - 8x^2 + 7 = \lim_{x \to -\infty} x^4 \left(1 - \frac{8}{x^2} + \frac{7}{x^4}\right)$$

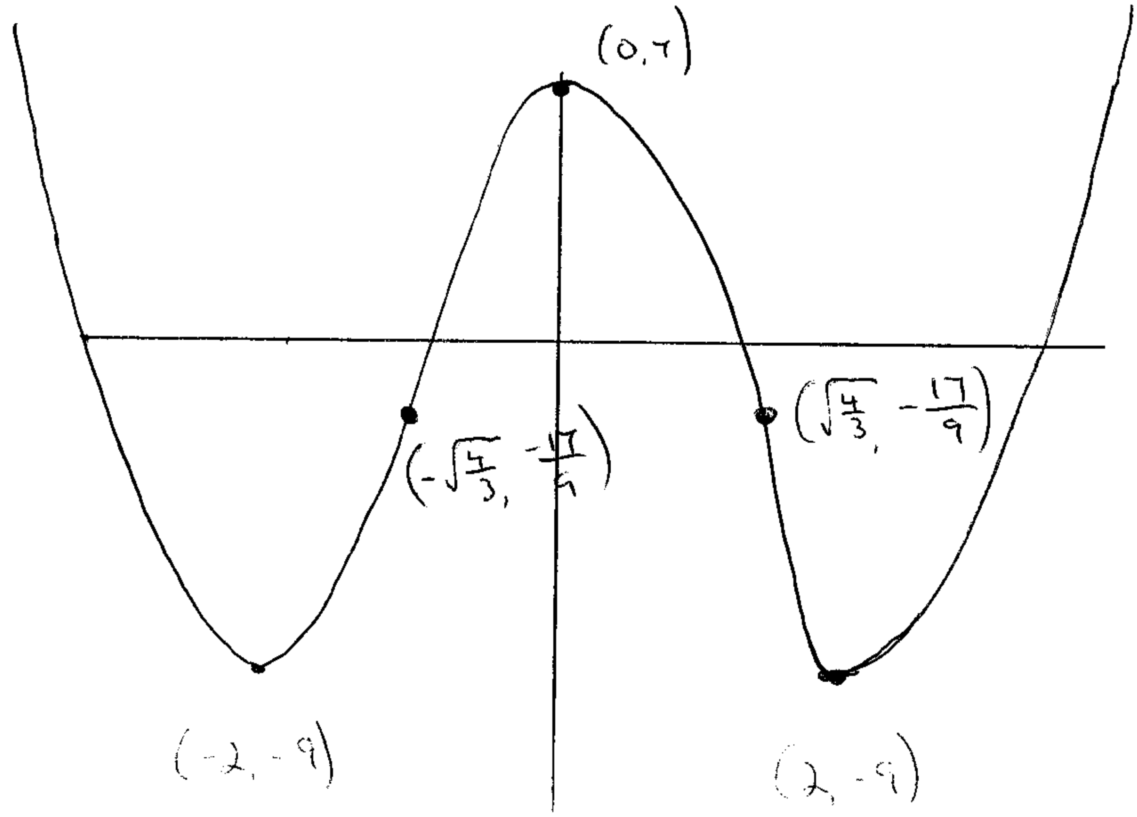
$$= (-\infty)^4 \left(1 - \frac{8}{(-\infty)^2} + \frac{7}{(-\infty)^4}\right)$$

$$= +\infty$$

Conclusion: As $x \to +\infty$ $y \to +\infty$

As $x \to -\infty$ $y \to +\infty$.

(e)


(0,7)
$\left(-\sqrt{\frac{4}{3}}, -\frac{17}{9}\right)$
$\left(\sqrt{\frac{4}{3}}, -\frac{17}{9}\right)$
(-2, -9)
(2, -9)

Q3.

(a) (i) $\int_0^2$ Let $u = x^2 + 3x + 1$

$x = 2 \quad u = (2)^2 + 3(2) + 1$

$= 11$

$x = 0 \quad u = (0)^2 + 3(0) + 1$

$= 1$

$\frac{du}{dx} = 2x + 3$

$\frac{du}{2x+3} = dx$

$$\int_1^{11} \frac{\cancel{2x+3}}{u} \frac{du}{\cancel{2x+3}} = \int_1^{11} \frac{1}{u}\, du$$

$$= \ln|u| \Big|_1^{11}$$

$$= \ln|11| - \ln|1|$$

$$= 2.398$$

(ii) $\int \cosh x \sinh^3 x \, dx$

let $u = \sinh x$

$\frac{du}{dx} = \cosh x$

$\frac{du}{\cosh x} = dx$

$= \int \cancel{\cosh x}\, u^3 \frac{du}{\cancel{\cosh x}}$

$= \int u^3 \, du$

$= \frac{u^4}{4} + C$

$= \frac{\sinh^4 x}{4} + C$

(iii) $\int x e^x \, dx$

$\int u \, dv = uv - \int v \, du$

$u = x \quad dv = e^x \, dx$

$\frac{du}{dx} = 1 \quad v = e^x$

$\int x e^x dx = x e^x - \int e^x \, dx$

$= x e^x - e^x + C$

(b) $a(t) = \cos 4t$

$$v(t) = \int \cos 4t \, dt$$

$$v(t) = \frac{\sin 4t}{4} + C$$

$$v(0) = \frac{\sin 4(0)}{4} + C$$

$$v(0) = C$$

$$v(0) = 2$$

$$\Rightarrow C = 2$$

$$\Rightarrow v(t) = \frac{\sin 4t}{4} + 2.$$

Q4 $\quad y = 4 - x^2$

$y = x + 2$

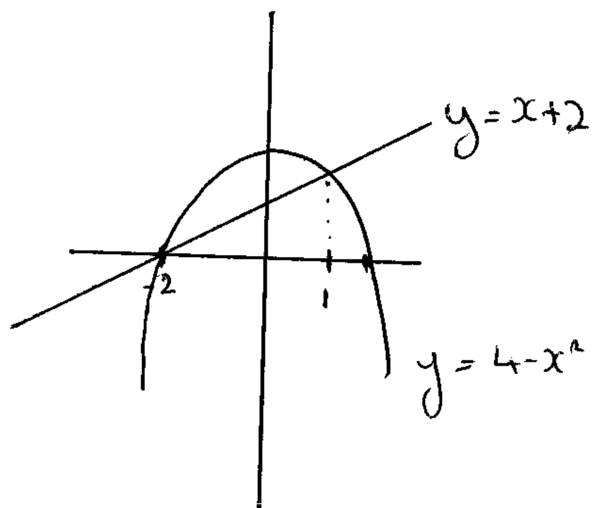
Point of intersection : $\quad 4 - x^2 = x + 2$

$$x^2 + x - 2 = 0$$

$$(x + 2)(x - 1) = 0$$

$$x = -2 \qquad x = 1$$

$$\int_{-2}^{1} (4 - x^2) - (x + 2)\, dx$$



$$= \int_{-2}^{1} \cancel{2} - x^2 - x \, dx$$

$$= \int_{-2}^{1} 2 - x^2 - x \, dx$$

$$= 2x - \frac{x^3}{3} - \frac{x^2}{2} \Bigg|_{-2}^{1}$$

$$2x - \frac{x^3}{3} - \frac{x^2}{2}\Big|_{-2}$$

$$= \left[2(1) - \frac{(1)^3}{3} - \frac{(1)^2}{2}\right] - \left[2(-2) - \frac{(-2)^3}{3} - \frac{(-}{2}\right.$$

$$= \frac{7}{6} - \left(-\frac{10}{3}\right)$$

$$= \frac{7}{6} + \frac{10}{3} = \frac{27}{6} = \frac{9}{2}.$$

$$\text{Area} = \underline{4.5} \text{ units}^2.$$

4 (b) $F(x) = 3x^2 - 4x + 1$

x-int : $0 = 3x^2 - 4x + 1$

$$(3x-1)(x-1) = 0$$

$$x = \frac{1}{3} \quad \text{or} \quad x = 1$$

$$\int_{\frac{1}{3}}^{1} 3x^2 - 4x + 1 \, dx$$

$$= x^3 - 2x^2 + x \Big|_{\frac{1}{3}}^{1}$$

$$= \left[(1)^3 - 2(1)^2 + 1\right] - \left[\left(\frac{1}{3}\right)^3 - 2\left(\frac{1}{3}\right)^2 + \frac{1}{3}\right]$$

$$= 0 - \left(\frac{4}{27}\right) = -\frac{4}{27}$$

Area $= \frac{4}{27}$ units^2

$= 0.148$ units^2.

4(c) $i(t) = 7 + 4\sin 2t$

$$q(t) = \int 7 + 4\sin 2t \, dt.$$

$$= 7t \frac{-4\cos 2t}{2} + C$$

$$q(t) = 7t - 2\cos 2t + C.$$

$$q(0) = 7(0) - 2\cos 2(0) + C$$

$$q(0) = -2 + C$$

$$q(0) = 0$$

$$\Rightarrow -2 + C = 0$$

$$C = 2.$$

$$q(t) = 7t - 2\cos 2t + 2.$$

Q5(a)

$F(x) = \cos x$	$F(0) = 1$
$F'(x) = -\sin x$	$F'(0) = 0$
$F''(x) = -\cos x$	$F''(0) = -1$
$F'''(x) = \sin x$	$F'''(0) = 0$
$F^{iv}(x) = \cos x$	$F^{iv}(0) = 1$

$$\cos x = 1 + x(0) + \frac{x^2(-1)}{2!} + \frac{x^3(0)}{3!} + \frac{x^4(1)}{4!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$\cos(1) = 1 - \frac{(1)^2}{2!} + \frac{(1)^4}{4!}$$

$$\cos(1) = 0.5417$$

Q5(b)(i) $Z = 2x^2y + 4x^2y^3 - 7x^2$

$$\frac{dz}{dx} = 4xy + 8xy - 14x$$

$$\frac{d^2Z}{dx^2} = 4y + 8y - 14$$

$$\frac{d^2Z}{dydx} = 4x + 8x$$

(ii) $Z = \cos(x+3y)$

$$\frac{dz}{dx} = -\sin(x+3y)$$

$$\frac{d^2z}{dx^2} = -\cos(x+3y)$$

$$\frac{dz}{dy} = -3\sin(x+3y)$$

$$\frac{d^2z}{dy^2} = -9\cos(x+3y)$$

$$\frac{d^2z}{dy^2} - 9\frac{d^2z}{dx^2} = 0.$$

$$-9\cos(x+3y) - 9(-\cos(x+3y)) = 0$$

$$-9\cos(x+3y) + 9\cos(x+3y) = 0.$$

true

(QED).

$22 + 24 + 26 \ldots\ldots$ $a = 22$ $d = 2$.

$U_n = a + (n-1)d$

$U_{18} = 2 + (18-1)2$

$= 2 + 34 = 36$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{18} = \frac{18}{2}[2(22) + (18-1)2]$

$= 9(44 + 34)$

$= 702$ seats.

$$1 + 2 + 4 + 8 + 16 \ldots$$

$a = 1 \qquad R = 2$

$$S_n = a\left(\frac{1-R^n}{1-R}\right)$$

$$S_{64} = 1\left(\frac{1-2^{64}}{1-2}\right)$$

$= 1.84467 \times 10^{19}$ grains of wheat.