Section A

1 (a)

(i) $f.g(x)$

$\Rightarrow f(2x^2+4)$

$\Rightarrow \sqrt{2(2x^2+4)-8}$

$\Rightarrow \sqrt{4x^2+8-8}$

$\Rightarrow \sqrt{4x^2}$

$\Rightarrow 2x$

(ii) $y = f(x) = \sqrt{2x-8}$

$f^{-1}: \quad x = \sqrt{2y-8}$

$\Rightarrow x^2 = 2y-8$

$\Rightarrow x^2+8 = 2y$

$\Rightarrow \frac{x^2+8}{2} = y$

$\Rightarrow f^{-1}(x) = \frac{x^2+8}{2}$

(iii) $f(x) = \frac{x}{x^2-4}$

$f(-x) = \frac{-x}{(-x)^2-4}$

$= \frac{-x}{x^2-4}$

$-f(x) = -\frac{x}{x^2-4}$

$\Rightarrow f(-x) = -f(x)$

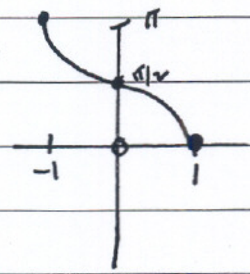
$\Rightarrow$ odd function

(b)

(i) $\cos^{-1}\left(-\frac{1}{3}\right)$

$\Rightarrow 109.47^0$

(ii)



(c)

$\sinh 2x = \frac{e^{2x}-e^{-2x}}{2}$

$2\sinh x \cosh x$

$\Rightarrow \cancel{2}\frac{(e^x-e^{-x})}{\cancel{2}} \cdot \frac{(e^x+e^{-x})}{2}$

$\Rightarrow \frac{e^{2x}-e^0+e^0-e^{-2x}}{2}$

$\Rightarrow \frac{e^{2x}\cancel{-1+1}-e^{-2x}}{2}$

$\Rightarrow \frac{e^{2x}-e^{-2x}}{2}$

$\Rightarrow \sinh 2x = 2\sinh x \cosh x$

2 (i)

(i) $y = f(x) = x^4-6x^2+2$

$f(0) = 2$

$\Rightarrow \boxed{(0,2)}$

(ii) $f'(x) = 4x^3-12x$

$\Rightarrow x(4x^2-12) = 0$

$\Rightarrow x=0 \quad | \quad 4x^2-12=0$

$4x^2 = 12$

$x^2 = 3$

$x = \pm\sqrt{3}$

$x=0, y=2 \quad | \quad x=\sqrt{3}, y=-7 \quad | \quad x=-\sqrt{3}, y=-7$

$\Rightarrow (0,2) \; (\sqrt{3},-7) \; (-\sqrt{3},-7)$

(iii) $f''(x) = 12x^2-12$

$f''(0) = -12 < 0 \Rightarrow$ max. turning point

$f''(\sqrt{3}) = 24 > 0 \Rightarrow$ min

$f''(-\sqrt{3}) = 24 > 0 \Rightarrow$ min

$\Rightarrow (0,2)$ is max turning point

$(\sqrt{3},-7)$ is min turning point

$(-\sqrt{3},-7)$ is min turning point

(iii) $f''(x) = 12x^2-12 = 0$

$\Rightarrow x^2-1 = 0$

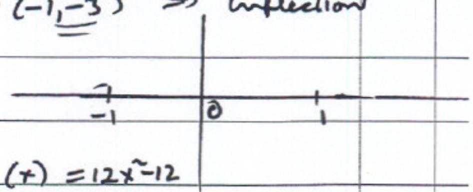
$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

$\Rightarrow x=1, y=-3 \quad | \quad x=-1, y=-3$

$\Rightarrow (1,-3) \; (-1,-3) \Rightarrow$ inflection

(iv)



$f''(x) = 12x^2-12$

$f''(-2) = 36 > 0 \quad +$

$f''(0) = -12 < 0 \quad -$

$f''(2) = 36 > 0 \quad +$

$\Rightarrow$ Concave up $x<-1$, $x>1$.

Concave down $-1 < x < 1$.

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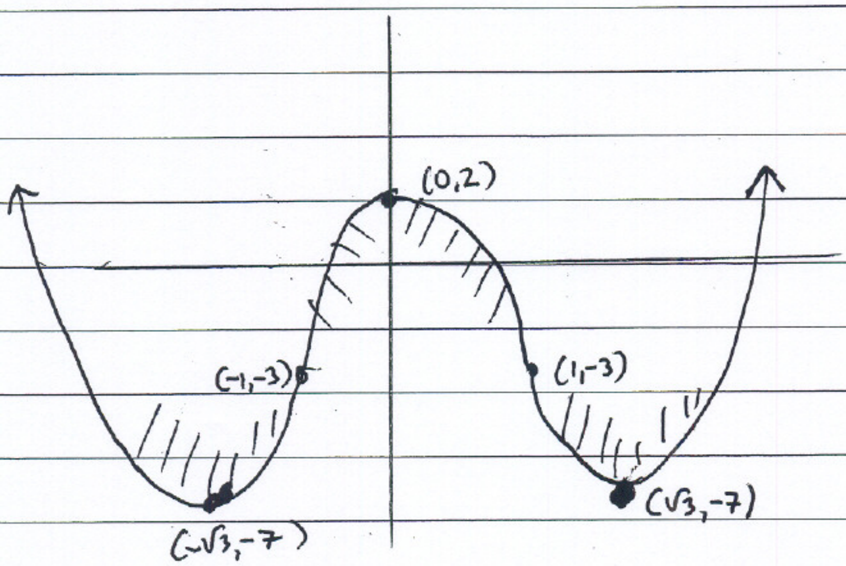
(v) as $x \to +\infty$, $y \to +\infty$

as $x \to -\infty$, $y \to +\infty$



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Section B

3 (a)

(i) $\int (2x-1)(x^2-x+3)^4\,dx$

let $u = x^2-x+3$

$\frac{du}{dx} = 2x-1$

$\Rightarrow du = (2x-1)dx$

$\Rightarrow \frac{du}{2x-1} = dx$

$\Rightarrow \int (2x-1).u^4.\frac{du}{(2x-1)}$

$\Rightarrow \int u^4 du$

$\Rightarrow \frac{u^5}{5} + c$

$\Rightarrow \frac{(x^2-x+3)^5}{5} + c$

(ii) $\int \sin^6 x \cos x\, dx$

let $u = \sin x$

$\Rightarrow \frac{du}{dx} = \cos x$

$\Rightarrow du = \cos x . dx$

$\Rightarrow \frac{du}{\cos x} = dx$

$\Rightarrow \int u^6 . \cos x . \frac{du}{\cos x}$

$\Rightarrow \int u^6 du$

$\Rightarrow \frac{u^7}{7}$

$\Rightarrow \frac{\sin^7 x}{7}\Big|_{x=0}^{x=\pi/2}$

$\Rightarrow \frac{\sin^7 \frac{\pi}{2}}{7} - \frac{\sin^7 0}{7}$

$\Rightarrow \frac{1}{7} - 0$

$\Rightarrow \frac{1}{7}$

(iii) $\int xe^x\,dx$

$u = x$	$dv = e^x dx$
$\frac{du}{dx} = 1$	$v = \int e^x dx$
$\Rightarrow du = dx$	$\Rightarrow v = e^x$

$\Rightarrow \int xe^x dx = xe^x - \int e^x dx$

$= xe^x - e^x + c$

(b) $a = 4e^{-2t}$

$v = \int 4e^{-2t} dt$

$v = \frac{4e^{-2t}}{-2} + c$

$\Rightarrow v = -2e^{-2t} + c$

$t = 0,\ v = 0$

$\Rightarrow 0 = -2e^0 + c$

$0 = -2 + c$

$\Rightarrow 2 = c$

$\Rightarrow v = -2e^{-2t} + 2$

$\Rightarrow s = \int -2e^{-2t} + 2\, dt$

$s = \frac{-2e^{-2t}}{-2} + 2t + c$

$s = e^{-2t} + 2t + c$

$t=0,\ s=0 \Rightarrow 0 = e^0 + 0 + c$

$0 = 1 + c$

$-1 = c$

$\Rightarrow s = e^{-2t} + 2t - 1$

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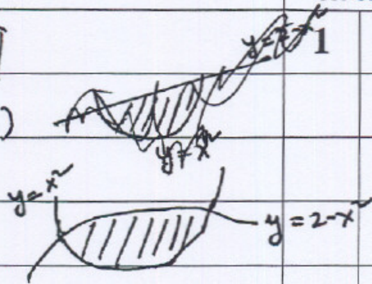
(a)



$\left.\begin{array}{l} y = x^2 \\ y = 2-x^2 \end{array}\right\}$ solve

$\Rightarrow x^2 = 2-x^2$

$\Rightarrow 2x^2 = 2$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$ limits

$\Rightarrow A = \int_{-1}^{1} 2 - x^2 - x^2\, dx$

$= \int_{-1}^{1} 2 - 2x^2\, dx$

$\Rightarrow 2x - \frac{2x^3}{3}\Big|_{x=-1}^{x=1}$

$\Rightarrow (2 - \frac{2}{3}) - (-2 + \frac{2}{3})$

$\Rightarrow \boxed{\frac{8}{3}}$

(b) $h = \frac{1}{4} = 0.25$

$y = \sinh(x^2)$

$x =$	1	1.25	1.5	1.75	2
$y =$	1.175	2.280	4.691	10.667	27.289

$\Rightarrow$ evaluation

$\frac{.25}{3}\left[(1.175 + 27.289) + 4(2.28 + 10.667) + 2(4.691)\right]$

$\Rightarrow \frac{.25}{3}(28.464 + 51.788 + 9.382)$

$\Rightarrow \frac{1}{12}(89.634)$

$\Rightarrow 7.4695$

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Section C

5(a)

$$\frac{2}{(2n+1)(2n+3)} = \frac{1}{(2n+1)} - \frac{1}{(2n+3)}$$

$$\Rightarrow a_n = \frac{1}{2n+1} - \frac{1}{2n+3}$$

$$\Rightarrow a_1 = \frac{1}{3} - \cancel{\frac{1}{5}}$$
$$a_2 = \cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}$$
$$a_3 = \cancel{\frac{1}{7}} - \frac{1}{9}$$
$$\vdots$$
$$a_n = \frac{1}{2n+1} - \frac{1}{2n+3}$$

$$S_n = \frac{1}{3} - \frac{1}{2n+3}$$

$\Rightarrow$ as $n \to \infty$

$$\Rightarrow S_\infty = \frac{1}{3} - 0$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{2}{(2n+1)(2n+3)} = \frac{1}{3}$$

(b) $\sum \frac{n-5}{2n+3}$

(i)

$$\lim_{n\to\infty} \frac{n-5}{2n+3}$$

$$\Rightarrow \lim_{n\to\infty} \frac{1-\frac{5}{n}}{2+\frac{3}{n}}$$

$$\Rightarrow \frac{1-0}{2+0}$$

$$\Rightarrow \frac{1}{2} \neq 0$$

$\Rightarrow$ divergent series

(ii) $\sum_{n=1}^{\infty} \frac{n^2+2n-1}{n^4-n+3}$ Compare with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$$\Rightarrow \lim_{n\to\infty} \frac{\left(\frac{n^2+2n-1}{n^4-n+3}\right)}{\frac{1}{n^2}} \rightarrow \frac{(n^2+2n-1)}{n^4-n+3} \cdot \frac{n^2}{1}$$

$$\Longrightarrow \cancel{n^4+2n^3-n} \quad \frac{n^4+2n^3-n^2}{n^4-n+3}$$

$$\Rightarrow \lim_{n\to\infty} \frac{n^4+2n^3-n^2}{n^4-n+3}$$

$$\Rightarrow \lim_{n\to\infty} \frac{1+\frac{2}{n}-\frac{1}{n^2}}{1-\frac{1}{n^3}+\frac{3}{n^4}}$$

$$\Rightarrow \frac{1+0-0}{1-0+0}$$

$$\Rightarrow 1$$

as $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is cgt $\Rightarrow \sum_{n=1}^{\infty} \frac{n^2+2n-1}{n^4-n+3}$ is convergent

(iii) $a_n = \frac{x^n}{n+1}$

$$a_{n+1} = \frac{x^{n+1}}{n+2}$$

$$\frac{a_{n+1}}{a_n} = \frac{x^{n+1}}{n+2} \cdot \frac{n+1}{x^n}$$

$$= \frac{x(n+1)}{n+2}$$

$$\Rightarrow \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$$

$$\Rightarrow \lim_{n\to\infty} \left|\frac{x(n+1)}{n+2}\right|$$ Divide by n

$$\Rightarrow \lim_{n\to\infty} \left|\frac{x(1+\frac{1}{n})}{(1+\frac{2}{n})}\right|$$

$$\Rightarrow \left|\frac{x(1+0)}{(1+0)}\right| \Rightarrow \left|\frac{x}{1}\right| \Rightarrow |x|$$

$\Rightarrow$ Series is convergent for $|x| < 1$

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[6] (a)

$f(x) = \cosh x$

$f(0) = \cosh 0 = 1$

$f'(x) = \sinh x$

$f'(0) = \sinh 0 = 0$

$f''(x) = \cosh x$

$f''(0) = 1$

$f''(x) = \sinh x$

$f''(0) = 0$

$f'''(x) = \cosh x$

$f'''(0) = 1$

$f^{iv}(x) = \sinh x$

$f^{iv}(0) = 0$

$f^{v}(x) = \cosh x$

$f^{v}(0) = 1.$

$f^{vi}(x) = \sinh x$

$f^{iv}(0) = 0$

$$f(x) = f(0) + x f'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \frac{x^4 f^{iv}(0)}{4!} + \frac{x^5 f^{v}(0)}{5!} + \frac{x^6 f^{vi}(0)}{6!}$$

$\Rightarrow \cancel{\cosh x = 1 + \frac{x^2}{2!} +}$

$$\Rightarrow \cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!}$$

Differentiate both sides W.R.T. x

$$\Rightarrow \sinh x = \frac{2x}{2!} + \frac{4x^3}{4!} + \frac{6x^5}{6!}$$

$$\Rightarrow \sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!}$$

(b) ~~z = e^{2t} sin x~~

(i) $z = 2x^2y - x^3y^2 + 4y$

$$\frac{\partial z}{\partial x} = 4xy - 3x^2y^2$$

$$\frac{\partial z}{\partial y} = 2x^2 - 2x^3y + 4.$$

(ii) $z = e^{2t}\sin x$

$$\frac{\partial z}{\partial t} = 2e^{2t}\sin x$$

$$\frac{\partial^2 z}{\partial t^2} = 4e^{2t}\sin x$$

$$\frac{\partial z}{\partial x} = e^{2t}\cos x$$

$$\frac{\partial^2 z}{\partial x^2} = -e^{2t}\sin x$$

$$\frac{\partial^2 z}{\partial t^2} + 4\frac{\partial^2 z}{\partial x^2}$$

$$\Rightarrow 4e^{2t}\sin x - 4e^{2t}\sin x = 0$$

Proved.

Section D

7

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(a) Evalf(((2+Sqrt(13)+3^4)/Sqrt(7))^21, 15);
(b) ~~[illegible]~~ Subs(x=7, (Sin(x)+7)*exp(21*x));
(c) factor(3*x^2 + 6*x - x^3 - 8);
(d) plot(arccos(x), x=-1..1);
(e) diff(x^5*ln(x)/(1+(cos(x))^2)^7, x);
    Simplify(%);
(f) diff(x^5*ln(x)/(1+(cos(x))^2)^7, x$2);
(g) Int(4*x^3/(x^4 + cosh(x)), x = 0..1);
```

8 (a)

(i) (1,0) (−2,0) (4,0)
(0,−8)

(ii) (−0·732------, −10·392-----) min turning point

(2·732------, 10·392------) max. turning point

(iii) (1,0)

(iv) $x \to +\infty$, $y \to -\infty$
$x \to -\infty$, $y \to +\infty$