Quesiton 5

5) Eigenvalues / Diagonalization Characteristic Polynomial Power Formula

Part A. Eigenvalues and Eigenvectors

1. Let $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

   (a) Determine the eigenvalues and corresponding eigenvectors of A.

   (b) Diagonalise A; i.e, give a matrix P and a diagonal matrix D, such that $A = PDP^{-1}$.

   (c) Hence, evaluate A^5.

Part D. Euclidean Norms

- Given $u,\ u',\ v,\ v', w\ w'$, with

$$\begin{aligned} u &= (1,3,0); \qquad & u' &= (-3,1,5) \\ v &= (5,0,4); \qquad & v' &= (-4,3,5) \\ w &= (3,2,7); \qquad & w' &= (1,0,1), \end{aligned}$$

  calculate $u \cdot u'$, $v \cdot v'$, $w \cdot w'$. Which of the pairs are orthogonal vectors?

- Calculate the (Euclidean) norm of the following vectors

$$\begin{aligned} u &= (1,2) \\ v &= (3,0) \\ w &= (4,0,3) \\ 0 &= (0,0,0). \end{aligned}$$