

1. Simplify the following:

(i) i^2

(iii) $3i$

(ii) i^5

(iv) i^3

2. Evaluate the following:

(i) $(2 + 3i) + (5 + 7i)$,

(iv) $(35i)^3$,

(ii) $(2 + 3i)(5 + 7i)$,

(v) $\frac{35i}{12i}$

(iii) $(2 + 3i)(23i)$,

3. For each part of question 2, draw the complex numbers on an Argand diagram, and express in the form $re^{i\theta}$

4. Evaluate $(-2 - 5i)(3 - 2i)$.

5. Evaluate $(6 - 2i)(1 - i)(2 - 2i)$.

6. Evaluate $\frac{1}{i} \times \frac{6 - 2i}{1 - i}$

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7. Express $z = e^{2+i(\pi/4)}$ in the form $a + bi$.

8. Express $3 - 2i$ in polar form.

9. Find the square roots of

$$z = 4(\cos 3 + i \sin 3)$$

, and draw these on an Argand diagram. Identify the principal root.

10. For $z = 3(\cos \pi/6 + i \sin \pi/6)$, calculate z^4 in polar form.

11. For $z = 3 - 2i$, find the five roots $z^{1/5}$, and plot these on an Argand diagram indicating the principal root.

12. Find the values of x, y (both real) which satisfy the equation

$$x(x + y) + xyi = 13i$$

13. Two competing *probability amplitudes* A_1 and A_2 for a quantum mechanical transition from some initial state —ii to some final state —fi are given by $A_1 = ae^{i\psi_1}, A_2 = be^{i\psi_2}$. and the total probability amplitude (A) for the process is given by $A_1 + A_2$. Given that the probability for a process is given by AA^* , calculate the probability for the transition from —ii to —fi. Using this, calculate the probability if $\psi_1 = \psi_2$.

1. Perform the indicated operation and write the answers in standard form.

(i)

(ii)

(iii)

2. Multiply each of the following and write the answers in standard form.

(i)

(ii)

(iii)

(iv)

3. Write each of the following in standard form.

(i)

(ii)

(iii)

(iv)

$$\text{Exercises[edit]} \quad (7 + 2i) + (11 - 6i) = (8 - 3i) - (6i) = (9 + 4i)(3 - 16i) = 3i \times \times 9i =$$

$$\frac{i}{2+i} = \frac{i}{2+i} = \frac{2+i}{11+3i} = \frac{2+i}{11+3i} = \frac{\sqrt{3}-4i}{\sqrt{3}-4i} =$$

$$(x + yi)^{-1} = (x + yi)^{-1} = \text{Answers[edit]} \quad 18 - 4i \quad 8 - 3i - 6i = 8 - 9i \quad 27 - 144i + 12i -$$

$$64i^2 = 91 - 132i - 27. \quad (i \times i = i^2 = -1 \quad i \times i = i^2 = -1)$$

$$\frac{i}{2+i} \times \frac{2-i}{2-i} = \frac{1+2i}{5} = \frac{1}{5} + \frac{2}{5}i \quad \frac{i}{2+i} \times \frac{2-i}{2-i} = \frac{1+2i}{5} = \frac{1}{5} + \frac{2}{5}i$$

$$\frac{11+3i}{\sqrt{3}-4i} \times \frac{\sqrt{3}+4i}{\sqrt{3}+4i} = \frac{11+3i}{\sqrt{3}-4i} \times \frac{\sqrt{3}+4i}{\sqrt{3}+4i} =$$

$$\frac{11\sqrt{3}+44i+3i\sqrt{3}+12i^2}{11\sqrt{3}+44i+3i\sqrt{3}+12i^2} = \frac{11\sqrt{3}+44i+3i\sqrt{3}+12i^2}{11\sqrt{3}+44i+3i\sqrt{3}+12i^2} =$$

$$\frac{3+16}{(11\sqrt{3}-12)+(44i+3i\sqrt{3})} = \frac{3+16}{(11\sqrt{3}-12)+(44i+3i\sqrt{3})} =$$

$$\frac{11\sqrt{3}-12}{19} + \frac{44+3\sqrt{3}}{19}i \quad \frac{11\sqrt{3}-12}{19} + \frac{44+3\sqrt{3}}{19}i$$

$$\text{Recall that } x^{-1} = \frac{1}{x}x^{-1} = \frac{1}{x} \text{ can be seen as the division of two complex numbers:}$$

$$(x + yi)^{-1} = \frac{1}{x + yi} \times \frac{x - yi}{x - yi} = (x + yi)^{-1} = \frac{1}{x + yi} \times \frac{x - yi}{x - yi} =$$

$$\frac{x - yi}{x^2 - y^2i^2} = \frac{x}{x^2 + y^2} - \frac{y}{x^2 + y^2}i \quad \frac{x - yi}{x^2 - y^2i^2} = \frac{x}{x^2 + y^2} - \frac{y}{x^2 + y^2}i$$

MA4604: Science Maths 4, Homework Week 1

1. Write each of the complex numbers below in the form $a + ib$, that is simplify each expression to find the real numbers a and b .

(a) $(3 + 8i) + (2i - 6) - (-5 + i)$	(d) $\frac{34i}{2i}$
(b) $(3 - i)(5 + 6i)$	(e) $\frac{5i}{23i}$
(c) $(1 + i)(2 - 5i)(7 + 3i)$	(f) $\frac{2 + 4i}{i(1i)}$

2. Find the real number(s) t that makes each expression below real:

(a) $(4 + 6i)(3i)(t + 6i)$

(b) $i(t + 4i)^2$

(c) $5 - 10i - 4i + t$.

3. Solve the complex equation $2z + i\bar{z} = 5 + 4i$ (hint: write $z = x + iy$).

4. Find all the roots of, and hence factorise fully, each given polynomial:

(a) $x^2 - 8x + 25$

(c) $x^4 - 16$

(b) $z^2 + (4 + 3i)z + 14 + 6i$

(d) $x^3 + 8$.

5. Given that $x = 1 + 4i$ is a root of the quartic polynomial $x^4 + 13x^2 + 34x$, find the other three roots and write it as a product of linear factors.

6. Plot each of the given complex numbers in an Argand diagram:

(a) $2 + i$

(d) $2 - 2i$

(g) 36

(b) $1 + 3i$

(e) 26

(c) $2i$

(f) 46

7. Write each of the complex numbers in Q.6

(e) through (l) in the form $x + iy$, that is convert them from polar or exponential form to standard form.

MA4604: Science Maths 4, Homework Week 2

1. Write each of the complex numbers below in polar form $r(\cos \theta + i \sin \theta)$ and in exponential form $re^{i\theta}$ (hint: first find $r = |z|$ and $\theta = \arg(z)$):

(i) $z = -2$

(iii) $z = 3 - 3i$

(ii) $z = -5i$

(iv) $z = -3 - 3i$.

2. For each complex number given below in exponential form rei , find its absolute value $|z|$ and its argument $\arg(z)$; express z in the form $x + iy$:

3. Use de Moivre's theorem to simplify the given powers:

(a) $(1 + i)^{20}$

(b) $(1 + i)^{12}$.

4. Use de Moivre's theorem with $n = 3$ to write $\cos(3)$ in terms of \cos and to write $\sin(3)$ in terms of \sin (hint: you'll also need to use the fact that $\cos^2 + \sin^2 = 1$).

5. Find the twelve twelfth roots of unity in exponential and in Cartesian form.

6. By first writing the given number in complex exponential form, evaluate each of the following in Cartesian form:

(a) all cube roots of 27;

(d) all fourth roots of 81;

(b) all cube roots of -64;

(e) all fourth roots of -4;

(c) all cube roots of -64i;

(f) all square roots of -9i.