Numerical Methods 15 October 2012
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Problem Sheet 2: The secant methd. The fixed-point iteration method.

1. Use the secant method to calculate a root for the equation $x^3 + 2x^2 - 3x - 1 = 0$, for two different sets of starting points: (i) $x_0 = -3$, $x_1 = -2$ and (ii) $x_0 = -4$, $x_1 = -2$. Iterate until $|x_n - x_{n-1}| < 5 \times 10^{-7}$. Compare the number of iterations needed in each case.

2. Use the secant method to evaluate a root for each of the equations on Sheet 1, subject to the required accuracy restrictions. Compare the secant method with the previous methods in each case.

3. Convert the equation $x^2 - 5 = 0$ to the fixed-point problem

$$x = g(x) \equiv x + c(x^2 - 5)$$

where c is a constant. Find the possible values of c that ensure convergence of the algorithm $x_{n+1} = g(x_n)$ to $\alpha = \sqrt{5}$.

4. Use the method in the previous problem to approximate the root of $x^3 - 3 = 0$.

5. Use fixed-point iteration algorithms to find both roots of the equation $3x^2 - 6x - 2 = 0$.

6. Consider the equation $x^3 + x^2 - 3x - 3 = 0$ (which has a unique root between 1 and 2) and the following four fixed point iteration schemes for solving it:

$$x_{n+1} = g_k(x_n) \qquad \text{where} \qquad g_1(x) = \frac{1}{3}(x^3 + x^2 - 3),$$

$$g_2(x) = -1 + \frac{3x+3}{x^2}, \quad g_3(x) = \sqrt[3]{3 + 3x - x^2}, \quad g_4(x) = \sqrt{\frac{3 + 3x - x^2}{x}}$$

Calculate 10 iterations (with 6 exact digits) for each of the four algorithms and decide which of these converge to the root. Explain your observations using the convergence criterion.

7. The function $f(x) = e^x + x^2 - x - 4$ has a unique zero on the interval (1,2). Create three different iteration functions corresponding to this function and compare their convergence properties for approximating the root. Use the same starting approximation for all iteration schemes.