Asymptotes

Vertical asymptotes are vertical lines which correspond to the zeroes of the denominator of a rational function. (They can also arise in other contexts, such as logarithms, but you'll almost certainly first encounter asymptotes in the context of rationals.)

Asymptotes

Let's consider the following equation:

$$y = [x^2 + 2x - 3]/[x^2 - 5x - 6]$$

This is a rational function. More to the point, this is a fraction. Can you have a zero in the denominator of a fraction?

Asymptotes

No. So if I set the denominator of the above fraction equal to zero and solve, this will tell me the values that x cannot be:

$$x^2 5x 6 = 0$$

$$(x 6)(x + 1) = 0$$

$$x = 6 or 1$$

So x cannot be 6 or 1, because then I'd be dividing by zero.

Asymptotes

Now look at the graph:

Asymptotes

- You can see how the graph avoided the vertical lines $x = 6$ and $x = 1$.
- This avoidance occurred because x cannot be 1 or 6.
- In other words, the fact that the function's domain is restricted is reflected in the function's graph.
- More usefully, you can use the domain to help you graph, because whichever values are not allowed in the domain will be vertical asymptotes on the graph.

Asymptotes

You can draw the vertical asymptote as a dashed line to remind you not to graph there, like this:
(It's alright that the graph appears to climb right up the sides of the asymptote on the left. This is common. As long as you don't draw the graph crossing the vertical asymptote, you'll be fine.)

Asymptotes

Let's review this relationship between the domain and the vertical asymptotes.
Find the domain and vertical asymptotes(s), if any, of the following function:

$$y = [x + 2]/[x^2 + 2x - 8]$$

Asymptotes

The domain is the set of all x-values that I'm allowed to use. The only values that could be disallowed are those that give me a zero in the denominator. So I'll set the denominator equal to zero and solve.

$$x2 + 2x8 = 0$$

$$(x + 4)(x2) = 0$$

$$x = 4orx = 2$$

Asymptotes

Since I can't have a zero in the denominator, then I can't have $x = 4$ or $x = 2$ in the domain. This tells me that the vertical asymptotes (which tell me where the graph can not go) will be at the values $x = 4$ or $x = 2$.

- domain: all x not equal to -4 or 2
- vertical asymptotes: $x = 4, 2$

Note that the domain and vertical asymptotes are "opposites". The vertical asymptotes are at 4 and 2, and the domain is everywhere but 4 and 2. This is always true.

Asymptotes

Find the domain and vertical asymptote(s), if any, of the following function:

$$y = [x^3 - 8]/[x^2 + 9]$$

To find the domain and vertical asymptotes, I'll set the denominator equal to zero and solve. The solutions will be the values that are not allowed in the domain, and will also be the vertical asymptotes.

$$x^2 + 9 = 0$$

$$x^2 = 9$$

Asymptotes

Oops! That doesn't solve! So there are no zeroes in the denominator. Since there are no zeroes in the denominator, then there are no forbidden x-values, and the domain is "all x". Also, since there are no values forbidden to the domain, there are no vertical asymptotes.

domain: all x vertical asymptotes: none

Asymptotes

Note again how the domain and vertical asymptotes were "opposites" of each other.
Find the domain and vertical asymptote(s), if any, of the following function:

$$y = [x^3 - 8]/[x^2 + 5x + 6]$$

Asymptotes

I'll check the zeroes of the denominator:

$x2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = 3$ or $x = 2$

Asymptotes

Since I can't divide by zero, then I have vertical asymptotes at $x = 3$ and $x = 2$, and the domain is all other x-values.
domain: for x not equal to -3 or -2 vertical asymptotes: $x = 3$ and $x = 2$

Asymptotes

When graphing, remember that vertical asymptotes stand for x-values that are not allowed. Vertical asymptotes are sacred ground. Never, on pain of death, can you cross a vertical asymptote. Don't even try!