Numerical Methods 1 October 2012
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Problem Sheet 1: The bisection method. The Newton-Raphson method

1. Let $f(x) = 3x^4 - 8x^2 + 1$.

   (a) Using sign changes, show that $f(x) = 0$ has four roots between -2 and 2.

   (b) Use the bisection method to evaluate one root of your choice.

   (c) Use Newton's method to evaluate the same root as in (b).

   (d) How do these two methods compare?

   Use an error tolerance of $\epsilon = 0.01$.
   (*Answer:* -1.592226039, -0.3626057200, 0.3626057200, 1.592226039)

2. Approximate $\sqrt[3]{13}$ to three decimal places by applying the bisection method to the equation $x^3 - 13 = 0$.
   (*Answer: 2.351334688...*)

3. It can be shown that the equation $\frac{3}{2}x - 6 - \frac{1}{2}\sin(2x) = 0$ has a unique real root.
   (*The value is 4.261483697...*)

   (a) Find an interval on which the root is guaranteed to exist.

   (b) Using the bisection method, approximate this root to within a tolerance of 10^{-4}.

4. Let $f(x) = x^3 - x^2 + 3x - 1$.

   (a) Show that $f(x) = 0$ has at least one root between -1 and 1.

   (b) Use five iterations of Newton's method to find an approximation for this root.

   (c) How many iterations of the bisection method would be needed in order to produce the same accuracy as in part (ii)?

   Answer: 0.3611030805...

5. Consider the function $f(x) = \tan(\pi x) - x - 6$.

   (a) Show that $f(x) = 0$ has a root between 0 and 1.

   (b) Use Newton-Raphson method to evaluate this root. Try the following initial approximations: 0.48, 0.4 and 0. (Use 6 exact digits and do not exceed 10 iterations each time.) Comment briefly on the results.

   (*The root is 0.4510472588.*)

6. Consider the nonlinear equation

$$x^4 - 18x^2 + 45 = 0.$$

   (a) Show that the equation has a root in the interval $(1, 2)$.

   (b) Use five iterations of Newton's method to find an approximation for this root.

   (c) How many iterations of the bisection method would be needed in order to produce the same accuracy as in part (ii)?

   (*Hint: The exact root is* $\sqrt{3} = 1.73205080$.)