

Question 2 (25 Marks)

Reduce the following matrix to row echelon form

Part B. Inverting a Matrix using Co-Factors

Given the matrix

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 1 & 1 & 0 \\ 2 & -1 & 2 \end{pmatrix}.$$

calculate

- the determinant of A ;
- the cofactor matrix of A ;
- and **hence** the inverse matrix A^{-1} .
- Evaluate the minors and cofactors of A , for A given by and hence, in each case, construct the cofactor matrix $\text{Cof}(A)$ of A .

Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & -2 & 4 \\ 1 & -4 & 1 \\ -3 & 0 & -1 \end{pmatrix}.$$

using elementary row operations.

2. Let a triangular matrix be a square matrix with either all (i, j) entries zero for either $i < j$ (in which case it is called a lower triangular matrix) or for $j < i$ (in which case it is called an upper triangular matrix). Show that any triangular matrix satisfying $AA^T = A^T A$ is a diagonal matrix.

This is also expressed by saying that Ax is a linear combination of the columns of A .

Part C. System of Linear Equations

1. Consider the linear system

$$\begin{aligned}x_1 + x_3 &= 4 \\2x_1 + 4x_2 + x_3 &= -3 \\x_2 + 3x_3 &= 7.\end{aligned}$$

- Write down the coefficient matrix and the augmented matrix of this system.
- What can you say about the solution set of the system? Justify your answer.
- Solve the system of equations, using any appropriate method.

2. Consider the homogeneous system:

$$\begin{aligned}x_1 + x_3 &= 0 \\2x_1 + 4x_2 + x_3 &= 0 \\x_2 + 3x_3 &= 0.\end{aligned}$$

What can you say about its solution set?

1. Prove that, for any $u, v \in \mathbb{R}^3$,

$$(u \times v) \times w = (u \cdot w)v - (v \cdot w)u.$$

(It is sufficient to verify this property for one component.)

2. Consider the three vectors in \mathbb{R}^3 :

$$u = (1, 2, 0), \quad v = (0, 1, 3), \quad w = (1, 2, 3).$$

(a) Evaluate $\|u\|$, $\|v\|$, $u \cdot v$, $u \times v$ and the angle between u and v .

(b) Calculate the scalar triple product $u \cdot (v \times w)$.

Fundamental Theorem of Invertible Matrices Rank Trace