Quesiton 2

2) Inverting a Matrix using Co-Factors Matrix of Minors Determinant of a 3 by 3 matrix

- Evaluate the minors and cofactors of A, for A given by and hence, in each case, construct the cofactor matrix Cof(A) of A.

1. Let A and B be $m \times n$ matrices. Then:

(i) $(kA)^T = kA^T$

(ii) $(A+B)^T = A^T + B^T$

(iii) $(AB)^T = B^T A^T$

2. Let a triangular matrix be a square matrix with either all (i,j) entries zero for either $i < j$ (in which case it is called an lower triangular matrix) or for $j < i$ (in which case it is called an upper triangular matrix). Show that any triangular matrix satisfying $AA^T = A^T A$ is a diagonal matrix.

3. For a square matrix A show that:

(i) AA^T and $A + A^T$ are symmetric

(ii) $A - A^T$ is skew symmetric

(iii) A can be expressed as the sum of a symmetric matrix, $\frac{1}{2}(A + A^T)$ and a skew symmetric matrix $\frac{1}{2}(A - A^T)$

4. Suppose A is a mn matrix and x is a $n \times 1$ column vector. Show that if

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

and

$$A = \begin{pmatrix} c_1 & c_2 & \cdots c_n \end{pmatrix} where c_j = \begin{pmatrix} A_{1j} \\ A_{2j} \\ \vdots \\ A_{mj} \end{pmatrix}$$

then

$$Ax = x_1 c_1 + x_2 c_2 + \cdots x_n c_n.$$

This is also expressed by saying that Ax is a linear combination of the columns of A.

Question 3

3) Planes Distance

give the general form of the equation of the plant π in $\mathbb{R}^3$ passing throughthe point $P_0 = (1, 0, 2)$ with the vector $n = (-5, 3, 2)$ as the normal.

1. Consider the linear system

$$\begin{aligned} x_1 + x_3 &= 4 \\ 2x_1 + 4x_2 + x_3 &= -3 \\ x_2 + 3x_3 &= 7. \end{aligned}$$

   (a) Write down the coefficient matrix and the augmented matrix of this system.

   (b) What can you say about the solution set of the system? Justify your answer.

   (c) Solve the system of equations, using any appropriate method.

2. Consider the homogeneous system:

$$\begin{aligned} x_1 + x_3 &= 0 \\ 2x_1 + 4x_2 + x_3 &= 0 \\ x_2 + 3x_3 &= 0. \end{aligned}$$

   What can you say about its solution set?

Quesiton 4

4) Vectors / Systems of Linear Equations Cross Product Scalar Triple Product

Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & -2 & 4 \\ 1 & -4 & 1 \\ -3 & 0 & -1 \end{pmatrix}.$$

using elementary row operations.

Given the matrix

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 1 & 1 & 0 \\ 2 & -1 & 2 \end{pmatrix}.$$

calculate

- the determinant of A;
- the cofactor matrix of A;
- and **hence** the inverse matrix A^{-1} .

Quesiton 5

5) Eigenvalues / Diagonalization Characteristic Polynomial Power Formula

Show that the point $Q = (1, -1, 1)$ does not lie in the plane π and find its distance from π.

1. Let $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

   (a) Determine the eigenvalues and corresponding eigenvectors of A.

   (b) Diagonalise A; i.e, give a matrix P and a diagonal matrix D, such that $A = PDP^{-1}$.

   (c) Hence, evaluate A^5.

- Given $u,\ u',\ v,\ v', w\, w'$, with

$$\begin{aligned} u &= (1,3,0); & u' &= (-3,1,5) \\ v &= (5,0,4); & v' &= (-4,3,5) \\ w &= (3,2,7); & w' &= (1,0,1), \end{aligned}$$

  calculate $u \cdot u'$, $v \cdot v'$, $w \cdot w'$. Which of the pairs are orthogonal vectors?

- Calculate the (Euclidean) norm of the following vectors

$$\begin{aligned} u &= (1,2) \\ v &= (3,0) \\ w &= (4,0,3) \\ 0 &= (0,0,0). \end{aligned}$$

- Calculate the scalar triple product

$$u \cdot (v \times w)$$

  for

  1. $u = (1,3,5)$; $v = (0,5,3)$; $w = (3,0,7)$;
  2. $u = (0,1,2)$; $v = (5,0,1)$; $w = (2,2,2)$.

- Are the points

$$P_1 = (1, 2, 0), \quad P_2 = (3, 5, 0), \quad P_3 = (7, 3, 0), \quad P_4 = (-5, 3, 0)$$

coplanar? If yes, what is the equation of the plane containing them?

- 1. Find the equation of the line ℓ in $\mathbb{R}^2$, which passes through the points $(2, 1)$ and $(1, 3)$.
  2. Let $Q = (1, -3)$ be a point in $\mathbb{R}^2$.
     (a) Verify that Q does not lie on the line ℓ.
     (b) Find the distance between the point Q and the line ℓ.