Quesiton 2

2) Inverting a Matrix using Co-Factors Matrix of Minors Determinant of a 3 by 3 matrix

- Evaluate the minors and cofactors of A, for A given by and hence, in each case, construct the cofactor matrix Cof(A) of A.

1. Let A and B be $m \times n$ matrices. Then:

(i) $(kA)^T = kA^T$

(ii) $(A + B)^T = A^T + B^T$

(iii) $(AB)^T = B^T A^T$

Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & -2 & 4 \\ 1 & -4 & 1 \\ -3 & 0 & -1 \end{pmatrix}.$$

using elementary row operations.

2. Let a triangular matrix be a square matrix with either all (i, j) entries zero for either $i < j$ (in which case it is called an lower triangular matrix) or for $j < i$ (in which case it is called an upper triangular matrix). Show that any triangular matrix satisfying $AA^T = A^T A$ is a diagonal matrix.

This is also expressed by saying that Ax is a linear combination of the columns of A.

Fundamental Theorem of Invertible Matrices Rank Trace

1. Consider the linear system

$$\begin{aligned} x_1 + x_3 &= 4 \\ 2x_1 + 4x_2 + x_3 &= -3 \\ x_2 + 3x_3 &= 7. \end{aligned}$$

   (a) Write down the coefficient matrix and the augmented matrix of this system.

   (b) What can you say about the solution set of the system? Justify your answer.

   (c) Solve the system of equations, using any appropriate method.