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Problem Sheet 2: Partial Differentiation

1. Calculate the first order partial derivatives

$$\text{(i) } f(x,y) = (x^2-1)(y+2); \qquad \text{(ii) } f(x,y) = \sqrt{x^2+y^2}; \qquad \text{(iii) } \frac{1}{x+y};$$
$$\text{(iv) } \frac{x}{x^2+y^2}; \qquad \text{(v) } f(x,y) = \sin(x+2y); \qquad \text{(vi) } f(x,y) = y^2x^4\mathrm{e}^x + 2x.$$

2. Which order of differentiation will calculate $\frac{\partial^2 f}{\partial x \partial y}$ faster, x first or y first?

$$\text{(i) } f(x,y) = x\sin(y) + \mathrm{e}^y; \qquad \text{(ii) } f(x,y) = \frac{1}{x}; \qquad \text{(iii) } f(x,y) = x\ln(xy);$$
$$\text{(iv) } f(x,y) = y + \frac{x}{y}; \qquad \text{(v) } f(x,y) = y + x^2y + 4y^3 - \ln(y^2+1)$$

3. Find all the second order partial derivatives for each of the following functions

$$\text{(i) } f(x,y) = 3x^2y - 5xy^4 \qquad \text{(ii) } f(x,y) = x^2\sin(y) - y^3\cos(x);$$
$$\text{(iii) } f(x,y) = \frac{x}{x^2-y^3}; \qquad \text{(iv) } f(x,y) = (x-1)\mathrm{e}^{xy}$$

4. Show that if $f(x,y) = (x^2+y^2)\ \ln(xy)$ then

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

5. A *harmonic function* (in 3 variables) is a function $f(x,y,z)$ which satisfies Laplace's equation, that is

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0.$$

(For example, the potential function for an electrostatic field is harmonic in any region of space which is free of electrostatic charge. Similarly, the potential function for a gravitational field is harmonic in any region where there is no mass.)

Show that the following functions are harmonic:

(i) $f(x,y,z) = 2z^3 - 3(x^2+y^2)z$; (ii) $f(x,y) = \mathrm{e}^{-2y}\cos(2x)$; (iii) $f(x,y,z) = x^2+y^2-2z^2$