Convolution

- Convolution is a mathematical operation on two functions $f(t)$ and $g(t)$, creating a third function that can be considered a "blending" of the two component functions.
- The convolution of functions is denoted $(f*g)(t)$, and can be evaluated using this formula:

$$(f*g)(t) = \int_{-\infty}^{\infty} f(u)\, g(t-u)\, du$$

- Convolution is quite useful in a lot of software and engineering applications, such as image processing.

Using Laplace Transforms

We can compute $(f*g)(t)$, the convolution of two functions $f(t)$ and $g(t)$, by following these steps:

- Get the Laplace transforms of the two component functions : $\mathscr{L}[f(t)] = F(s)$ and $\mathscr{L}[g(t)] = G(s)$
- Multiply these two Laplace transforms: $F(s) \times G(s)$
- Find the inverse Laplace transform of the product: $\mathscr{L}^{-1}[F(s) \times G(s)]$

Example 1

Use Laplace transforms to compute $t * t^2$, the convolution of t and t^2

First compute the Laplace transforms of the two component functions:

- $\mathscr{L}[t]$
- $\mathscr{L}[t^2]$

Example 1

The Laplace transform of the convolution is the product of the Laplace transforms of two component functions.

Example 1

Compute the inverse Laplace transform to find the convolution of the functions.

Example 1

Using the table of formulae:

$$\mathscr{L}^{-1}\left[k \times \frac{n!}{s^{n+1}}\right] = k \times t^n$$

Example 2

Use Laplace transforms to compute $e^t * e^{-t}$, the convolution of e^t and e^{-t}

First compute the Laplace transforms of the two component functions:

- $\mathscr{L}[e^t]$
- $\mathscr{L}[e^{-t}]$

Example 2

The Laplace transform of the convolution is the product of the Laplace transforms of two component functions.

Example 2

Compute the inverse Laplace transform to find the convolution of the functions.