Integer Programming

Recommended Text Operations Research- An Introduction. Hamdy A. Taha (003/TAH) The 7th edition is the most up to date in the library. The library has 4 copies of this edition. In addition, there are around 25 copies of older editions. Also, Introduction to Operations Research. Hillier and Lieberman. Course notes and tutorial sheets will be available on the Internet. 2

Integer Programming

Module Outline

1. Integer Programming. Applications to the travelling salespersons problem and the knapsack (packing) problem.
2. Dynamic Programming. Deterministic. Stochastic. Finite horizon. Infinite horizon problems. Discounting and average-cost problems.

CHAPTER 1 - INTEGER PROGRAMMING AND THE BRANCH AND BOUND ALGORITHM

- 1.1 The definition of an integer programming problem An integer programming problem can be defined by the description of a linear programming problem together with the constraint that the variables must be integers. It is assumed that the variables are constrained to be non-negative.

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Example of an integer programming problem

```
max z = 4x1 + 5x2
subject to
x1 + x2  5
6x1 + 10x2  45,
where x1, x2 are non-negative integers
```

- Note that we can describe any integer programming problem as a maximisation problem (the minimisation of 4x1 + 5x2 is equivalent to the maximisation of 4x1 5x2).
- The function z = 4x1 + 5x2 is called the **objective function**.

1.2 The Branch and Bound Method - The initial bound

- **Important** We may find an initial upper bound on the optimal value in the integer programming problem by solving the corresponding linear programming problem.
- Consider the problem given previously. If the values of x1 and x2 at the optimal solution to this problem are integers, then this must be the optimal solution to the integer programming problem.
  (Remark: That is a big "If".)

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The Branch and Bound Method - The initial branching step

- Suppose x1 = c at this optimal solution, where c is not an integer.
- We can split the initial linear programming problem into two linear programming problems.
- The first problem is obtained by adding the constraint x1 bcc *(bcc is the largest integer not greater than c)*.
- The second is obtained by adding the constraint x1 dce *(dce is the smallest integer not less than c)*.

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The initial branching step

- The union of the feasible sets for these two LP problems is the feasible set for the initial LP problem with the set of (x1, x2) satisfying $bcc < x1 < dce$ removed.
- Since no feasible solutions in which both x1 and x2 are integers have been removed (see diagram below, the dots show the feasible points in the integer programming problem), the optimal solution to the integer programming problem must lie in one of the feasible regions of these two new problems.