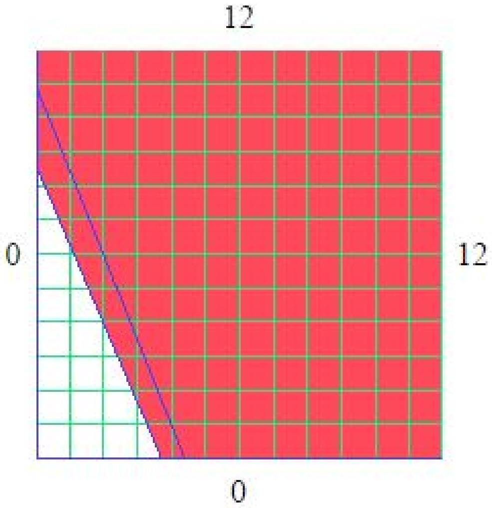
Consider the IP:

$$\begin{aligned} \max\ 16x_1 + 7x_2 & \\ 7x_1 + 3x_2 &\le 26 \\ 5x_1 + 2x_2 &\le 22 \\ x_1, x_2 \ge 0 \text{ and integer.} & \end{aligned}$$




The feasible region is shown in white.

**MS4315 2014 Exam Question**

- You will be asked to partly solve the IP, using the **Branch and Bound Method**.
- You should draw an enumeration tree/diagram to keep track of your progress draw the enumeration tree on an otherwise blank page.

$$\begin{aligned} \max\ & 16x_1 + 7x_2 \\ & 7x_1 + 3x_2 \le 26 \\ & 5x_1 + 2x_2 \le 22 \\ & x_1, x_2 \ge 0 \text{ and integer.} \end{aligned}$$

The corresponding **Simplex Tableau** (transforming the **max** problem into a **min** problem) is:

$$T_0 = \begin{array}{|c|cccc|} \hline 0 & -16 & -7 & 0 & 0 \\ \hline 26 & 7 & 3 & 1 & 0 \\ 22 & 5 & 2 & 0 & 1 \\ \hline \end{array}$$

(ii) After a second iteration of the Simplex Method the Simplex Tableau now takes the form: (N.B. Do not perform the arithmetic!)

$$T_2 = \begin{array}{|c|cccc|}\hline 60\frac{2}{3} & \frac{1}{3} & 0 & 2\frac{1}{3} & 0 \\ \hline 8\frac{2}{3} & 2\frac{1}{3} & 1 & \frac{1}{3} & 0 \\ 4\frac{2}{3} & \frac{1}{3} & 0 & -0\frac{2}{3} & 1 \\ \hline \end{array} \equiv$$

$$\equiv \begin{array}{|c|cccc|}\hline 60.67 & 0.33 & 0.00 & 2.33 & 0.00 \\ \hline 8.67 & 2.33 & 1.00 & 0.33 & 0.00 \\ 4.67 & 0.33 & 0.00 & -0.67 & 1.00 \\ \hline \end{array}.$$

Explain why this Tableau is LP optimal. 1/2%

(iii) What is the solution to the LP Relaxation of the IP? 1/2%

(iv) Why must we now branch on x2 and what are the two branches? 1%

(v) Consider the right branch corresponding to a lower bound on x2 ($x2 \geq ?$).

A. Amend the LP-optimal tableau T2 by adding an extra row & column corresponding to the extra constraint creating a new tableau T3 with 4 rows and 6 columns. (2%)

B. Perform one pivot on the appropriate element of T3 to eliminate x2 from the new row.
*N.B. Just update the last row, not the full tableau. (1%)*

C. Explain why the resulting tableau is infeasible. Update your enumeration tree. (1%)

(vi) Starting at T2, the left branch, $x_2 \leq 8$ (T4, say) (when the new constraint is added and two pivots are performed) has the LP optimal tableau T4: (N.B. Do not perform the arithmetic!)

$$T_4 = \begin{array}{|c|ccccc|}\hline 60\frac{4}{7} & 0 & 0 & 2\frac{2}{7} & 0 & \frac{1}{7} \\ \hline 8 & 0 & 1 & 0 & 0 & 1 \\ 4\frac{4}{7} & 0 & 0 & -\frac{5}{7} & 1 & \frac{1}{7} \\ \frac{2}{7} & 1 & -0 & \frac{1}{7} & -0 & -\frac{3}{7} \\ \hline \end{array}$$

$$\equiv \begin{array}{|c|ccccc|}\hline 60.57 & 0.00 & 0.00 & 2.29 & 0.00 & 0.14 \\ \hline 8.00 & 0.00 & 1.00 & 0.00 & 0.00 & 1.00 \\ 4.57 & 0.00 & 0.00 & -0.71 & 1.00 & 0.14 \\ 0.29 & 1.00 & -0.00 & 0.14 & -0.00 & -0.43 \\ \hline \end{array}.$$

Update your enumeration tree including the new upper bound on the overall problem. Why must we now branch on x1 and what are the two branches?

(vii) Starting with T4, add a row corresponding to the left branch. Why may we treat the new constraint as an equality constraint? 1

(viii) Explain why we must pivot on the entry in row 4 and column 2 of the resulting tableau. 1

(ix) The tableau (T5 say) resulting from the extra row and the pivot is:

N.B. Do not perform the arithmetic!

Perform one step of the Dual Simplex Method and show that the resulting tableau is

$\equiv$

| | | | | | |
|---|---|---|---|---|---|
| 60.57 | 0.00 | 0.00 | 2.29 | 0.00 | 0.14 |
| 8.00 | 0.00 | 1.00 | 0.00 | 0.00 | 1.00 |
| 4.57 | 0.00 | 0.00 | −0.71 | 1.00 | 0.14 |
| 0.29 | 1.00 | −0.00 | 0.14 | −0.00 | −0.43 |
| −0.29 | 0.00 | 0.00 | −0.14 | 0.00 | 0.43 |

Figure:

. (Just calculate the first and fourth columns, columns 2 & 3 do not change. Columns 5 & 6 are not needed.) 2

(x) Finally; answer the following questions related to T6, briefly explaining your answer to each.

A. What is the solution? 1/2%

B. Is it IP optimal? 1/2%

C. If possible update the global upper or lower bound on z? Explain whicch can be updated. 1%

D. Is the IP solved? If not, what other branch or branches must be searched/fathomed/pruned? 1%