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- Explain briefly why the following strategy for the solution of I.P.s is not useful:

  "Solve the L.P. relaxation then round off the components of the solution to the nearest integers".

Integer Programming by Branch and Bound

Consider the Integer Linear Program (IP):

$$
\begin{aligned}
\max\ 5x_1 + 2x_2 & \\
5x_1 + x_2 &\leq 5/2 \\
10x_1 + 6x_2 &\leq 45 \\
x_1, x_2 \leq 0 \text{ and integer.} &
\end{aligned}
$$

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The corresponding Simplex Tableau (transforming the max problem into a min problem) is:

0	-5	-2	0	0
5/2	5	1	1	0
45	10	6	0	1

N.B. The Simplex Method and the Dual Simplex Method are stated on the last page of this paper.

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(i) Apply one iteration of the Simplex Method and show that the Simplex Tableau now takes the form:

2.5	0	-1	1	0
0.5	1	0.2	0.2	0
40	0	4	-2	1

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Initial Tableau

0	-5	-2	0	0
5/2	5	1	1	0
45	10	6	0	1

- Indicator Row at top of tableau. Pick the lowest (i.e most negative) value. (i.e.-5)
- This is our Pivot Column.
- To pick the Pivot Row, compute the ratio of value in the pivot column to the "*barrier column*" (i.e. the first column)
  (barrier column value is numerator)

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- Pick the row with the lowest ratio.
  - $\frac{5/2}{5} = 0.5$ Lowest - Pick this Row
  - $\frac{45}{10} = 4.5$
- Pivot Point is intersection of pivot column and pivot row.

Elementary Row Operations

- Divide Pivot Row by value of Pivot Point (Pivot Point should become 1)
- Perform EROs such that other values on pivot column become 0.

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- You should end up with this tableau

2.5	0	-1	1	0
0.5	1	0.2	0.2	0
40	0	4	-2	1

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Next Question

(ii) After a second iteration of the Simplex Method the Simplex Tableau now takes the form: (N.B.do not perform the arithmetic!)
Explain why this Tableau is optimal.

5	5	0	2	0
2.5	5	1	1	0
30	-20	0	-6	1

Look at the top row (i.e. the indicator row.) All values on indicator row are positive. This is our main stopping condition.

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Important: When the two-phase Simplex method stops and all the artificial variables have value $= 0$, we can remove the artificial variables and remaining variables will form a feasible solution for the original LP problem

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Which Columns have 0 or 1 only in their column?

- x_1 : 5 and -20 N0
- x_2 : Yes
- s_1 : 1 and -6 NO
- s_2 : Yes

Set values to zero, if this condition is not met. $x_1 = 0, s_1 = 0$.

Solve for the other values.
Pivot Row: $2.5 = 5x_1 + x_2 + s_1 + 0s_2$
Necessarily x_1 and $s_1 = 0$: $2.5 = x_2 + 0s_2$
By Inspection $x_2 = 2.5$
Solution : (0,2.5)

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(iii) Explain why the solution to the LP Relaxation of the IP is $x_1 = 0, x_2 = 2.5$ and why we must now branch on x_2 and what are the two branches?

Remarks Solution already provided.
x_1 has integer solution.
x_2 has none-integer solution.
Apply constraints using floor and ceiling function of x_2.

Branches: $x_2 \leq 2$ and $x_2 \geq 3$.

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A. First show that the basic variable x2 may be expressed in terms of the non-basic variables x_1 and x_3 (also known as s1) as:

$$x2 = 2.5 - 5x1 - x3.$$

B. Substitute this expression for x2 into the S_0 branch constraint and show that it takes the form

$$-5x_1 - x_3 + s = -0.5.$$

(Discussed Previously, but slack variable denoted with s_1 rather than x_3)

(The variable s is the slack variable for the constraint $x_2 \leq 2$.)

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C. Show that the Simplex Tableau with the addition of this constraint takes the form:

5	5	0	2	0	0
2.5	5	1	1	0	0
30	-20	0	-6	1	0
-0.5	-5	0	-1	0	1

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(v) Explain why this tableau is optimal but infeasible.

5	5	0	2	0	0
2.5	5	1	1	0	0
30	-20	0	-6	1	0
-0.5	-5	0	-1	0	1

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Important If an LP is infeasible, then the two-phase Simplex method will stop with a solution where some slack variable has a negative value

Key Indicator: Infeasibility is evident when there is a negative value in the barrier column when the tableau meets the stopping condition (i.e. the indicator row shows all positive)

Solve for all values on tableau of previous slide.

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Dual Simplex Method

(vi) Apply one iteration of the Dual Simplex Method (DSM) to this tableau and show that the Simplex Tableau now takes the form:

4.5	0	0	1	0	1
2	0	1	0	0	1
32	0	0	-2	1	-4
0.1	1	0	0.20	0	-0.2

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(vii) Is this tableau LP optimal? Is it integer feasible? Explain. What is the solution to this LP relaxation? 1

(viii) What branching should you now make?

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(ix) Having chosen a suitable branching, explain why the new tableau for the left-hand branch ($x_1 = 0$) is:

5	5	0	2	0	0
2.5	5	1	1	0	0
30	-20	0	-6	1	0
-0.5	-5	0	-1	0	1

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A. What row and column is selected for pivoting with DSM?

B. Check that, after pivoting with DSM, x1 and x2 remain basic with $x1 = 0$ and $x2 = 2$. (*No need to update the full tableau, just the top left-hand element and the second-last element in the the left-hand column.*) and hence find the integer optimal value of z?