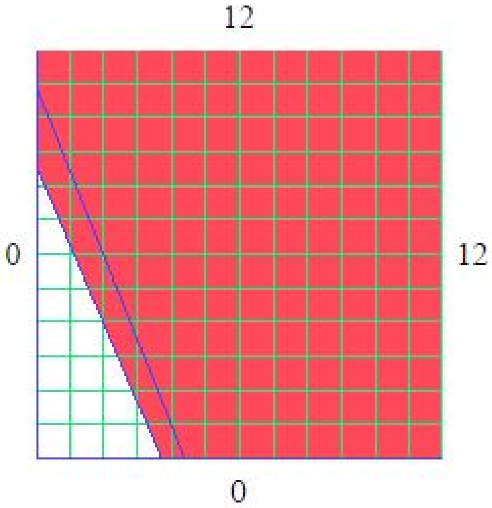
Consider the IP:

$$\begin{aligned} \max\ 16x_1 + 7x_2 & \\ 7x_1 + 3x_2 & \leq 26 \\ 5x_1 + 2x_2 & \leq 22 \\ x_1, x_2 \geq 0 \text{ and integer.} & \end{aligned}$$




The feasible region is shown in white.

- You will be asked to partly solve the IP, using the Branch and Bound Method.
- You should draw an enumeration tree/diagram to keep track of your progress draw the enumeration tree on an otherwise blank page.

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The corresponding Simplex Tableau (transforming the max problem into a min problem) is:

$$T_0 = \begin{array}{|c|cccc|}
\hline
0 & -16 & -7 & 0 & 0 \\
\hline
26 & 7 & 3 & 1 & 0 \\
22 & 5 & 2 & 0 & 1 \\
\hline
\end{array}$$

The columns are for $x_1$,$x_2$, $s_1$ and $s_2$. $S_1$ and $s_2$ are called slack variables.

- $7x1 + 3x2 \leq 26$
- $5x1 + 2x2 \leq 22$

- In contrast to the material used in Week 11 , the indicator row is listed at the top of the tableau
- The $b$ column is listed on the left hand side of the tableau

*N.B. The Simplex Method and the Dual Simplex Method are stated on the last page of this paper. You will partly solve the IP, following the steps on the next pages remember to draw and fill in an enumeration tree.*

(i) Apply one iteration of the Simplex Method to the starting tableau (T0) and show that the Simplex Tableau now takes the form (just calculate the first 3 columns): 3%

$$T_1 = \begin{array}{|c|cccc|}
\hline
59\frac{3}{7} & 0 & -\frac{1}{7} & 2\frac{2}{7} & 0 \\
\hline
3\frac{5}{7} & 1 & \frac{3}{7} & \frac{1}{7} & 0 \\
3\frac{3}{7} & 0 & -\frac{1}{7} & -\frac{5}{7} & 1 \\
\hline
\end{array}$$

$$= \begin{array}{|c|cccc|}
\hline
59.43 & 0.00 & -0.14 & 2.29 & 0.00 \\
\hline
3.71 & 1.00 & 0.43 & 0.14 & 0.00 \\
3.43 & 0.00 & -0.14 & -0.71 & 1.00 \\
\hline
\end{array}$$

(ii) After a second iteration of the Simplex Method the Simplex Tableau now takes the form: (N.B. Do not perform the arithmetic!)

$$T_2 = \begin{array}{|c|cccc|} \hline 60\frac{2}{3} & \frac{1}{3} & 0 & 2\frac{1}{3} & 0 \\ \hline 8\frac{2}{3} & 2\frac{1}{3} & 1 & \frac{1}{3} & 0 \\ 4\frac{2}{3} & \frac{1}{3} & 0 & -0\frac{2}{3} & 1 \\ \hline \end{array}$$

$$\equiv \begin{array}{|c|cccc|} \hline 60.67 & 0.33 & 0.00 & 2.33 & 0.00 \\ \hline 8.67 & 2.33 & 1.00 & 0.33 & 0.00 \\ 4.67 & 0.33 & 0.00 & -0.67 & 1.00 \\ \hline \end{array}.$$

Explain why this Tableau is LP optimal. 1/2%

(iii) What is the solution to the LP Relaxation of the IP? 1/2%

(iv) Why must we now branch on x2 and what are the two branches? 1%

(v) Consider the right branch corresponding to a lower bound on x2 ($x2 \geq ?$).

A. Amend the LP-optimal tableau T2 by adding an extra row & column corresponding to the extra constraint creating a new tableau T3 with 4 rows and 6 columns. (2%)

B. Perform one pivot on the appropriate element of T3 to eliminate x2 from the new row.
*N.B. Just update the last row, not the full tableau. (1%)*

C. Explain why the resulting tableau is infeasible. Update your enumeration tree. (1%)

(vi) Starting at T2, the left branch, $x_2 \leq 8$ (T4, say) (when the new constraint is added and two pivots are performed) has the LP optimal tableau T4: (N.B. Do not perform the arithmetic!)

$$T_4 = \begin{array}{|c|ccccc|}
\hline
60\frac{4}{7} & 0 & 0 & 2\frac{2}{7} & 0 & \frac{1}{7} \\
\hline
8 & 0 & 1 & 0 & 0 & 1 \\
4\frac{4}{7} & 0 & 0 & -\frac{5}{7} & 1 & \frac{1}{7} \\
\frac{2}{7} & 1 & -0 & \frac{1}{7} & -0 & -\frac{3}{7} \\
\hline
\end{array}$$

$$\equiv \begin{array}{|c|ccccc|}
\hline
60.57 & 0.00 & 0.00 & 2.29 & 0.00 & 0.14 \\
\hline
8.00 & 0.00 & 1.00 & 0.00 & 0.00 & 1.00 \\
4.57 & 0.00 & 0.00 & -0.71 & 1.00 & 0.14 \\
0.29 & 1.00 & -0.00 & 0.14 & -0.00 & -0.43 \\
\hline
\end{array}.$$

Update your enumeration tree including the new upper bound on the overall problem. Why must we now branch on x1 and what are the two branches?

(vii) Starting with T4, add a row corresponding to the left branch. Why may we treat the new constraint as an equality constraint? 1

(viii) Explain why we must pivot on the entry in row 4 and column 2 of the resulting tableau. 1

(ix) The tableau (T5 say) resulting from the extra row and the pivot is:

N.B. Do not perform the arithmetic!

Perform one step of the Dual Simplex Method and show that the resulting tableau is

$$
\equiv \begin{array}{|r|rrrrr|}
\hline
60.57 & 0.00 & 0.00 & 2.29 & 0.00 & 0.14 \\
\hline
8.00 & 0.00 & 1.00 & 0.00 & 0.00 & 1.00 \\
4.57 & 0.00 & 0.00 & -0.71 & 1.00 & 0.14 \\
0.29 & 1.00 & -0.00 & 0.14 & -0.00 & -0.43 \\
-0.29 & 0.00 & 0.00 & -0.14 & 0.00 & 0.43 \\
\hline
\end{array}
$$

Figure:

. (Just calculate the first and fourth columns, columns 2 & 3 do not change. Columns 5 & 6 are not needed.) 2

(x) Finally; answer the following questions related to T6, briefly explaining your answer to each.

A. What is the solution? 1/2%

B. Is it IP optimal? 1/2%

C. If possible update the global upper or lower bound on z? Explain whicch can be updated. 1%

D. Is the IP solved? If not, what other branch or branches must be searched/fathomed/pruned? 1%