Binary Integer Programming

Integer Programming for Decision Making

Encode "Yes or no" decisions with binary variables:

$$x_j \begin{cases} 1 & \text{if decision is yes} \\ 0 & \text{if decision is no.} \end{cases}$$

Binary Integer Programming (BIP):

- Binary variables + linear constraints.

Binary Integer Programming

Problem:

1. Cal wants to expand:
   - Build new factory in either Los Angeles, San Francisco, both or neither.
   - Build new warehouse (at most one).
   - Warehouse must be built close to the city of a new factory.

2. Available capital: $10,000,000

3. Cal wants to maximize "total net present value" (profitability vs. time value of money)

		NPV	Price
1	Build a factory in L.A.?	$9m	$6m
2	Build a factory in S.F.?	$5m	$3m
3	Build a warehouse in L.A.?	$6m	$5m
4	Build a warehouse in S.F.?	$4m	$2m

Binary Integer Programming

Cal wants to expand:

Build new factory in Los Angeles, San Francisco, both or neither.

Build new warehouse (at most one).

Warehouse must be built close to the city of a new factory.

What decisions are to be made?

1. Build factory in LA
2. Build factory in SFO
3. Build warehouse in LA
4. Build warehouse in SFO

Introduce 4 binary variables $x_i = \begin{cases} 1 & \text{if decision } i \text{ is yes} \\ 0 & \text{if decision } i \text{ is no} \end{cases}$

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What is the objective?

- Maximize NPV:

$$Z = 9x_1 + 5x_2 + 6x_3 + 4x_4$$

What are the constraints on capital?

- Don't go beyond means:

$$6x_1 + 3x_2 + 5x_3 + 2x_4 \leq 10$$

Binary Integer Programming

LA factory(x_1), SFO factory(x_2), LA warehouse(x_3),SFO warehouse (x_4)

- Build new factory in Los Angeles, San Francisco, both or neither.
- Build new warehouse (at most one).
- Warehouse must be built close to city of a new factory.

What are the constraints between decisions?

1. No more than one warehouse:

   Most 1 of $\{x_3, x_4\}$

2. Warehouse in LA only if Factory is in LA:

   x_3 implies x_1

3. Warehouse in SFO only if Factory is in SFO:

   x_4 implies x_2

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Exclusive choices

- Example: at most 2 decisions in a group can be yes:

LP Encoding:

$$x_1 + \ldots + x_k \leq 2.$$

Logical implications

- x_1 implies x_2: (x_1 requires x_2)

LP Encoding:

$$x_1 - x_2 \leq 0.$$

Binary Integer Programming

LA factory(x1), SFO factory(x2), LA warehouse(x3),SFO warehouse (x4)

- Build new factory in Los Angeles, San Francisco, or both.
- Build new warehouse (only one).
- Warehouse must be built close to city of a new factory.

What are the constraints between decisions?

1. No more than one warehouse:

   Most 1 of $\{x_3, x_4\}$

   $\mathbf{x_3 + x_4 \leq 1}$

2. Warehouse in LA only if Factory is in LA:

   x_3 implies x_1

   $\mathbf{x_3 - x_1 \leq 0}$

3. Warehouse in SFO only if Factory is in SFO:

   x_4 implies x_2

   $\mathbf{x_4 - x_2 \leq 0}$

Binary Integer Programming

<u>Complete binary integer program:</u>

Maximize $Z = 9x_1 + 5x_2 + 6x_3 + 4x_4$

Subject to: $6x_1 + 3x_2 + 5x_3 + 2x_4 \leq 10$

$$x_3 + x_4 \leq 1$$
$$x_3 - x_1 \leq 0$$
$$x_4 - x_2 \leq 0$$
$$x_j \leq 1$$
$$x_j = \{0,1\}, j=1,2,3,4$$
$$x_j \geq 0$$

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Review

- Be able to state a program for a BIP problme with the appropriate set of constraints (i.e. be able to state a problem just like the previous slide).
- Remember to state what x_1,x_2 etc mean.