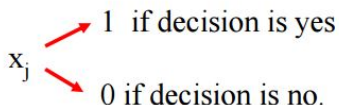


Integer Programming for Decision Making

Encode “Yes or no” decisions with **binary variables**:



Binary Integer Programming (BIP):

- Binary variables + linear constraints.

Binary Integer Programming

Problem:

1. Cal wants to expand:
 - Build new factory in either Los Angeles, San Francisco, both or neither.
 - Build new warehouse (at most one).
 - Warehouse must be built close to the city of a new factory.
2. Available capital: \$10,000,000
3. Cal wants to maximize “total net present value” (profitability vs. time value of money)

		<u>NPV</u>	<u>Price</u>
1	Build a factory in L.A.?	\$9m	\$6m
2	Build a factory in S.F.?	\$5m	\$3m
3	Build a warehouse in L.A.?	\$6m	\$5m
4	Build a warehouse in S.F.?	\$4m	\$2m

Binary Integer Programming

Cal wants to expand:

Build new factory in Los Angeles, San Francisco, both or neither.

Build new warehouse (at most one).

Warehouse must be built close to the city of a new factory.

What decisions are to be made?

1. Build factory in LA
2. Build factory in SFO
3. Build warehouse in LA
4. Build warehouse in SFO

Introduce 4 binary variables x_i

1 if decision i is yes

0 if decision i is no

Binary Integer Programming

1. Cal wants to expand
2. Available capital: \$10,000,000
3. Cal wants to maximize “total net present value” (profitability vs. time value of money)

	<u>NPV</u>	<u>Price</u>
1 Build a factory in L.A.?	\$9m	\$6m
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What is the objective?

- Maximize NPV:

$$Z = 9x_1 + 5x_2 + 6x_3 + 4x_4$$

What are the constraints on capital?

- Don't go beyond means:

$$6x_1 + 3x_2 + 5x_3 + 2x_4 \leq 10$$

Binary Integer Programming

LA factory(x_1), SFO factory(x_2), LA warehouse(x_3), SFO warehouse (x_4)

- Build new factory in Los Angeles, San Francisco, both or neither.
- Build new warehouse (at most one).
- Warehouse must be built close to city of a new factory.

What are the constraints between decisions?

1. No more than one warehouse:

Most 1 of $\{x_3, x_4\}$

2. Warehouse in LA only if Factory is in LA:

x_3 implies x_1

3. Warehouse in SFO only if Factory is in SFO:

x_4 implies x_2

Binary Integer Programming

Exclusive choices

- Example: at most 2 decisions in a group can be yes:

LP Encoding:

$$x_1 + \dots + x_k \leq 2.$$

Logical implications

- x_1 implies x_2 : (x_1 requires x_2)

LP Encoding:

$$x_1 - x_2 \leq 0.$$

Binary Integer Programming

LA factory(x_1), SFO factory(x_2), LA warehouse(x_3), SFO warehouse (x_4)

- Build new factory in Los Angeles, San Francisco, or both.
- Build new warehouse (only one).
- Warehouse must be built close to city of a new factory.

What are the constraints between decisions?

1. No more than one warehouse:

Most 1 of $\{x_3, x_4\}$

$$x_3 + x_4 \leq 1$$

2. Warehouse in LA only if Factory is in LA:

x_3 implies x_1

$$x_3 - x_1 \leq 0$$

3. Warehouse in SFO only if Factory is in SFO:

x_4 implies x_2

$$x_4 - x_2 \leq 0$$

Binary Integer Programming

Complete binary integer program:

$$\text{Maximize } Z = 9x_1 + 5x_2 + 6x_3 + 4x_4$$

$$\text{Subject to: } 6x_1 + 3x_2 + 5x_3 + 2x_4 \leq 10$$

$$x_3 + x_4 \leq 1$$

$$x_3 - x_1 \leq 0$$

$$x_4 - x_2 \leq 0$$

$$x_j \leq 1$$

$$x_j \geq 0$$

$$x_j = \{0,1\}, j=1,2,3,4$$

Binary Integer Programming

Review

- ▶ Be able to state a program for a BIP problem with the appropriate set of constraints (i.e. be able to state a problem just like the previous slide).
- ▶ Remember to state what x_1, x_2 etc mean.