FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS AND STATISTICS

MID-TERM ASSESSMENT EXAMINATION 1

MODULE CODE: MA4603

SEMESTER: Autumn 2015

MODULE TITLE: Science Mathematics 3

DURATION OF EXAM: 45 minutes

LECTURER: Mr. Kevin O'Brien

GRADING SCHEME: 20 marks
20% of total module marks

INSTRUCTIONS TO CANDIDATES

- Attempt all questions.
- Show your working clearly.
- Please write your name and student number on each page you submit.

Attempt ALL questions

Q1. Theory for Inference Procedures (3 Marks)

Answer the three short questions. Each correct answer will be awarded 1 mark.

i. Briefly describe how $p-$value is used in hypothesis testing

ii. What is meant by a Type I error?

iii. What is meant by a Type II error?

Q2. Normal Distribution (6 Marks)

Assume that the diameter of a critical component is normally distributed with a Mean of 50mm and a Standard Deviation of 2mm. You are required to estimate the approximate probability of the following measurements occurring on an individual component.

i. (2 Marks) Between 50 and 51.2mm

ii. (2 Marks) Less than 48.5 mm

iii. (2 Marks) Between 48.2 and 51.6 mm

Use the normal tables to determine the probabilities for the above exercises. You are required to show all of your workings.

Q3. Dixon Q Test For Outliers (4 Marks)

The typing speeds for one group of 12 Engineering students were recorded both at the beginning of year 1 of their studies. The results (in words per minute) are given below:

121	146	150	149	142	170	153
137	161	156	165	137	178	159

Use the Dixon Q-test to determine if the lowest value (121) is an outlier. You may assume a significance level of 5%.

i. (1 Mark) Formally state the null hypothesis and the alternative hypothesis.

ii. (1 Mark) Compute the Test Statistic.

iii. (2 Mark) By comparing the Test Statistic to the appropriate Critical Value, state your conclusion for this test.

Q4. Testing Normality (2 Marks)

A graphical procedure was carried out to assess whether or not this assumption of normality is valid for data set `Y`. Consider the Q-Q plot in the figure below.

i. (1 Mark) Provide a brief description on how to interpret this plot.

ii. (1 Mark) What is your conclusion for this procedure? Justify your answer.

Q5. Testing Normality (3 Marks)

Consider the following inference procedure performed on data set X.

```
> shapiro.test(X)

Shapiro-Wilk normality test

data:  X
W = 0.9619, p-value = 0.6671
```

i. (1 Mark) Describe what is the purpose of this procedure.

ii. (1 Mark) What is the null and alternative hypothesis?

iii. (1 Mark) Write the conclusion that follows from it.

Q6. Testing For Outliers (6 Marks)

(i) (3 Marks) Provide a brief description for three tests from the family of Grubb's Outliers Tests. Include in your description a statement of the null and alternative hypothesis for each test, any required assumptions and the limitations of these tests.

(ii) (3 Marks) Showing your working, use the Dixon Q Test to test the hypothesis that the maximum value of the following data set is an outlier.

$$19, 22, 23, 24, 25, 26, 29, 38$$

Q2. Normal Distribution (5 Marks)

Assume that the diameter of a critical component is normally distributed with a Mean of 100mm and a Standard Deviation of 5mm. You are required to estimate the approximate probability of the following measurements occurring on an individual component.

i. (1 Mark) Greater than 103mm

ii. (2 Marks) Less than 94.2 mm

iii. (2 Marks)[$*$] Between 94.2 and 103 mm

Use the normal tables to determine the probabilities for the above exercises. You are required to show all of your workings.

(Write Your Answers Here)

Q3. Chi-Square Test (8 Marks)

A market research survey was carried out to assess preferences for three brands of chocolate bar, A, B, and C. The study group was categorised by gender to determine any difference in preferences.

	A	B	C	Total
Females	50	70	80	200
Males	90	50	20	160
Total	140	120	100	360

i. (1 Mark)[∗] Formally state the null and alternative hypotheses.

ii. (2 Marks) Compute the cell values expected under the null hypothesis. Show your workings for two cells.

iii. (3 Marks) Compute the Test Statistic.

iv. (1 Mark)[∗] State the appropriate Critical Value for this hypothesis test.

v. (1 Mark)[∗] Discuss your conclusion to this test, supporting your statement with reference to appropriate values.

Q7. Testing for Outliers (3 Marks)

The following statistical procedure is based on this dataset.

6.98	8.49	7.97	6.64
8.80	8.48	5.94	6.94
6.89	7.47	7.32	4.01

```
> grubbs.test(x, two.sided=T)

Grubbs test for one outlier

data:  x
G = 2.4093, U = 0.4243, p-value = 0.05069
alternative hypothesis: lowest value 4.01 is an outlier
```

i. (1 Mark) Describe what is the purpose of this procedure. State the null and alternative hypothesis.

ii. (1 Mark) Write the conclusion that follows from it.

iii. (1 Mark) State any relevant assumptions for this procedure.

Q8. Chi-Square Test (9 Marks)

Suppose you conducted a drug trial on a group of animals and you hypothesized that the animals receiving the drug would show increased heart rates compared to those that did not receive the drug. You conduct the study and collect the following data:

	Heart Rate Increased	No Heart Rate Increase	Increase
Treated	36	14	50
Not Treated	30	25	55
	66	39	105

i. (1 Mark) Formally state the null and alternative hypotheses.

ii. (2 Marks) Compute the cell values expected under the null hypothesis. Show your workings for two cells.

iii. (3 Marks) Compute the Test Statistic.

iv. (1 Mark) State the appropriate Critical Value for this hypothesis test.

v. (2 Marks) Discuss your conclusion to this test, supporting your statement with reference to appropriate values.

Formulae and Tables

Critical Values for Dixon Q Test

N	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
3	0.941	0.97	0.994
4	0.765	0.829	0.926
5	0.642	0.71	0.821
6	0.56	0.625	0.74
7	0.507	0.568	0.68
8	0.468	0.526	0.634
9	0.437	0.493	0.598
10	0.412	0.466	0.568
11	0.392	0.444	0.542
12	0.376	0.426	0.522
13	0.361	0.41	0.503
14	0.349	0.396	0.488
15	0.338	0.384	0.475
16	0.329	0.374	0.463

Critical Values for Chi Square Test

n	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.001$
1	2.705	3.841	6.634	10.827
2	4.605	5.991	7.378	9.21
3	6.251	7.815	9.348	11.345
4	7.779	9.488	11.143	13.277
5	9.236	11.07	12.833	15.086
6	10.645	12.592	14.449	16.812
7	12.017	14.067	16.013	18.475
8	13.362	15.507	17.535	20.09
9	14.684	16.919	19.023	21.666
10	15.987	18.307	20.483	23.209

Test Statistic for Chi-Square Test

$$\chi^2_{TS} = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

Confidence Intervals

One sample

$$S.E.(\bar{X}) = \frac{\sigma}{\sqrt{n}}.$$

$$S.E.(\hat{P}) = \sqrt{\frac{\hat{p} \times (100 - \hat{p})}{n}}.$$

Two samples

$$S.E.(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$$

$$S.E.(\hat{P}_1 - \hat{P}_2) = \sqrt{\frac{\hat{p}_1 \times (100 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2 \times (100 - \hat{p}_2)}{n_2}}.$$

Hypothesis tests

One sample

$$S.E.(\bar{X}) = \frac{\sigma}{\sqrt{n}}.$$

$$S.E.(\pi) = \sqrt{\frac{\pi \times (100 - \pi)}{n}}$$

Two large independent samples

$$S.E.(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$$

$$S.E.(\hat{P}_1 - \hat{P}_2) = \sqrt{(\bar{p} \times (100 - \bar{p}))\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}.$$

Two small independent samples

$$S.E.(\bar{X}_1 - \bar{X}_2) = \sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}.$$

$$s_p^2 = \frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}.$$

Paired sample

$$S.E.(\bar{d}) = \frac{s_d}{\sqrt{n}}.$$

Standard Deviation of case-wise differences (computational formula)

$$s_d = \sqrt{\frac{\sum d_i^2 - n\bar{d}^2}{n-1}}.$$

Q1. Theory for Inference Procedures (2 Marks)

Answer the following short questions. Each correct answer will be awarded 1 mark.

i. (1 Marks)[∗] What is meant by a Type I error?

ii. (1 Marks)[∗] What is meant by a Type II error?