The Exponential Distribution

- The exponential distribution is often used to model the waiting time X between events occurring randomly and independently in time (or space).
- Because it is a continuous distribution, the height of the exponential curve at any X refers to probability density rather than probability.
- Probability is instead represented by area under the exponential curve.

The Exponential Distribution

- The main assumption of the exponential distribution is that at any point in time, the probability of an event occurring in the next instant does not depend on how much time has already elapsed since the previous event.
- The parameter of the exponential distribution is the rate at which events occur the number of events per unit time.
- The mean of the exponential distribution is 1/rate.

The Exponential Distribution

Waiting time X to the next event has probability density

```
dexp(X, rate=1)
dexp(X, rate=1, log=TRUE)
```

The default value for the rate is 1, so you must alter to fit your circumstance.

Exponential Distribution : Problem

Suppose the mean checkout time of a supermarket cashier is three minutes. Find the probability of a customer checkout being completed by the cashier in less than two minutes, three minutes and four minutes. (i.e. what percentage of “waiting times” are less than two, three and four minutes?)

Solution

- The checkout processing rate is equals to one divided by the mean checkout completion time.
- Hence the processing rate is $1/3$ checkouts per minute.
- We then apply the function `pexp()` of the exponential distribution with rate$=1/3$.

The Exponential Distribution

```
> pexp(2,rate=1/3)
[1] 0.4865829
>
> pexp(3,rate=1/3)
[1] 0.6321206
>
> pexp(4,rate=1/3)
[1] 0.7364029
>
> pexp(5,rate=1/3,lower=FALSE)
[1] 0.1888756
```

The Exponential Distribution

- What is the median waiting time? To answer this question we would use the `qexp()` function.
- Recall that the median is value of x such that $P(X \leq x) = 0.50$.
- Also determine the first and third quartiles Q_1 and Q_3.

```
> qexp(0.5,rate=1/3)
[1] 2.079442
> qexp(0.25,rate=1/3)
[1] 0.8630462
> qexp(0.75,rate=1/3)
[1] 4.158883
```