1 Number Sets

The font that the symbols are written in (i.e. $\mathbb{N}$, $\mathbb{R}$) is known as ***blackboard font***.

- $\mathbb{N}$ Natural Numbers $(0, 1, 2, 3)$ (Not used in the CIS102 Syllabus)
- $\mathbb{Z}$ Integers $(-3, -2, -1, 0, 1, 2, 3, \ldots)$
  - $\mathbb{Z}^+$ Positive Integers
  - $\mathbb{Z}^-$ Negative Integers
- $\mathbb{Q}$ Rational Numbers
- $\mathbb{R}$ Real Numbers

2 Numbers and Set Theory

Suppose we have the sets $\boldsymbol{A}$ and $\boldsymbol{B}$ defined as follows:

$$\boldsymbol{A} = \{\ \sqrt{2},\ \frac{3}{2},\ 2\ \}$$

$$\boldsymbol{B} = \{x \in \mathbb{R} : x \notin \mathbb{Q}\}$$

1 $\boldsymbol{A} \cap \mathbb{Q}$

2 $\boldsymbol{A} \cap \boldsymbol{B}$

3 $\boldsymbol{B} \cup \mathbb{Q}$

- $\mathbb{R}$ Set of all real numbers.
- $\mathbb{Q}$ Set of all quotient numbers.

$$\boldsymbol{B} = \{x \in \mathbb{R} : x \notin \mathbb{Q}\}$$

Set of all real numbers that are not quotients (i.e. numbers that can not be expressed as a division of one integer by another).

$$\boldsymbol{A} \cap \mathbb{Q}$$

Part D: Natural, Rational and Real Numbers

- $\mathbb{N}$: natural numbers (or positive integers) $\{1, 2, 3, \ldots\}$

- $\mathbb{Z}$: integers $\{-3,-2,-1,0,1,2,3,\ldots\}$
  - (The letter $\mathbb{Z}$ comes from the word *Zahlen* which means "numbers" in German.)
- $\mathbb{Q}$: rational numbers
- $\mathbb{R}$: real numbers
- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
  - (All natural numbers are integers. All integers are rational numbers. All rational numbers are real numbers.)

Number Sets

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- $\mathbb{Z}$ Integers $(-3,-2,-1,0,1,2,3,\ldots)$
  - $\mathbb{Z}^+$ Positive Integers
  - $\mathbb{Z}^-$ Negative Integers
  - 0 is not considered as either positive or negative.
- $\mathbb{Q}$ Rational Numbers
- $\mathbb{R}$ Real Numbers
- $\mathbb{C}$ Complex Numbers

3 Special sets of numbers

It is convenient to denote certain key sets of numbers by a standard letter. Definition 2.2 The symbol Z is used to denote the set of integers," denotes the set of positive integers; R denotes the set of real numbers; Q denotes the set of rational numbers the letter Q stands for "quotient"). Of these sets, we can specify the set of positive integers and the set of integers by the listing method using elipses, as follows:

$$Z^+ = 1, 2, s, ...$$

$$2 = o, 1, -1, 2, -2, 3, -2, ...$$

Note that Z includes 0 and the negative whole numbers as well as the positive ones. Similarly, $\mathbb{R}$ contains O and the negative reals and Q contains 0 and the negative rationals, as well as the positive ones.

3.1 Some Important Sets

- N : the set of all natural numbers = 1,2,3,4,.....
- Z : the set of all integers = $\{....., -3, -2, -1, 0, 1, 2, 3, ...\}$
- Z+ : the set of all positive integers
- Q : the set of all rational numbers
- R : the set of all real numbers
- W : the set of all whole numbers

3.2 Number Sets

The font that the symbols are written in (i.e. $\mathbb{N}$, $\mathbb{R}$) is known as ***blackboard font***.

- $\mathbb{N}$ Natural Numbers $(0, 1, 2, 3)$ (Not used in the CIS102 Syllabus)
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  - $\mathbb{Z}^+$ Positive Integers
  - $\mathbb{Z}^-$ Negative Integers
- $\mathbb{Q}$ Rational Numbers
- $\mathbb{R}$ Real Numbers

Set Operations Let $U = \{1, 2, \ldots, 9\}$ be the universal set, and let

- A = $\{1, 2, 3, 4, 5\}$,
- B = $\{4, 5, 6, 7\}$,
- C = $\{5, 6, 7, 8, 9\}$
- D = $\{1, 3, 5, 7, 9\}$,
- E = $\{2, 4, 6, 8\}$,
- F = $\{1, 5, 9\}$.

Find:

(a) A $\cup$ B and A $\cap$ B,

(b) C $\cup$ D and C $\cap$ D,

(c) E $\cup$ F and E $\cap$ F.

Find:

(a) A $\cup$ B and A $\cap$ B,

- A = $\{1, 2, 3, 4, 5\}$
- B = $\{4, 5, 6, 7\}$

Find:

(b) C $\cup$ D and C $\cap$ D,

- C = $\{5, 6, 7, 8, 9\}$
- D = $\{1, 3, 5, 7, 9\}$

Find:

(c) E $\cup$ F and E $\cap$ F.

- E = $\{2, 4, 6, 8\}$
- F = $\{1, 5, 9\}$

Finite Sets
Determine which of the following sets are finite:

(a) Set of Prime numbers

(b) Set of two digit Prime numbers

(c) Letters in the English alphabet.

(d) Integers which are multiples of 5.

(e) Days of the week

Given the set **A** is contructed as follows

$$[\{a, b\}, \{c\}, \{d, e, f\}].$$

(a) List the elements of **A**.

(b) Find the cardinality of **A** : $n(\boldsymbol{A})$.

(c) Find the power set of **A**.

Set Theory

$$\boldsymbol{A} = [\{a, b\}, \{c\}, \{d, e, f\}].$$

(a) List the elements of **A**.

$$\boldsymbol{A} = [\{a, b\}, \{c\}, \{d, e, f\}].$$

(b) Find the cardinality of **A** : $n(\boldsymbol{A})$.

$$\boldsymbol{A} = [\{a, b\}, \{c\}, \{d, e, f\}].$$

(c) Find the power set of **A**.

Consider the set **A**, which is a subset of the the universal set of real numbers $\mathbb{R}$

$$\boldsymbol{A} = \{4, \sqrt{2}, 2/3, -2.5, -5, 33, \sqrt{9}, \pi\}$$

Using formal set notation, write the sets of:

(a) natural numbers in **A**

(b) integers in **A**

(c) rational numbers in **A**

(d) irrational numbers in **A**

$$\boldsymbol{A} = \{4,\ \sqrt{2},\ 2/3,\ -2.5,\ -5,\ 33,\ \sqrt{9},\ \pi\}$$

(a) natural numbers in **A**

(b) integers in **A**

(c) rational numbers in **A**

(d) irrational numbers in **A**

$$\boldsymbol{A} = \{4,\ \sqrt{2},\ 2/3,\ -2.5,\ -5,\ 33,\ \sqrt{9},\ \pi\}$$

(a) natural numbers in **A**
Answer: $\{4,\ 33,\ \sqrt{9}\}$

(b) integers in **A**

(c) rational numbers in **A**

(d) irrational numbers in **A**

$$\boldsymbol{A} = \{4,\ \sqrt{2},\ 2/3,\ -2.5,\ -5,\ 33,\ \sqrt{9},\ \pi\}$$

(a) natural numbers in **A**
Answer: $\{4,\ 33,\ \sqrt{9}\}$

(b) integers in **A**
Answer: $\{4,\ -5,\ 33,\ \sqrt{9}\}$

(c) rational numbers in **A**

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Answer: $\{4,\ 33,\ \sqrt{9}\}$

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Answer: $\{4,\ -5,\ 33,\ \sqrt{9}\}$

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Answer: $\{4,\ 2/3,\ -2.5,\ -5,\ 33,\ \sqrt{9}\}$

(d) irrational numbers in **A**

$$\boldsymbol{A} = \{4,\ \sqrt{2},\ 2/3,\ -2.5,\ -5,\ 33,\ \sqrt{9},\ \pi\}$$

(a) natural numbers in **A**
Answer: $\{4,\ 33,\ \sqrt{9}\}$

(b) integers in **A**
Answer: $\{4,\ -5,\ 33,\ \sqrt{9}\}$

(c) rational numbers in **A**
Answer: $\{4,\ 2/3,\ -2.5,\ -5,\ 33,\ \sqrt{9}\}$

(d) irrational numbers in **A**
Answer: $\{\sqrt{2},\ \pi\}$

3.3 Number Sets

Blackboard Bold Typeface

- Conventionally the symbols for numbers sets are written in a special typeface, known as **blackboard bold**.
- Examples : $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{R}$.

Natural Numbers ($\mathbb{N}$)

- The whole numbers from 1 upwards.
- The set of natural numbers is

$$\{1, 2, 3, 4, 5, 6, \ldots\}$$

- In some branches of mathematics, 0 might be counted as a natural number.

$$\{0, 1, 2, 3, 4, 5, 6, \ldots\}$$

3.4 Number Sets

Integers ($\mathbb{Z}$)

- The integers are all the whole numbers, all the negative whole numbers and zero.
- The set of integers is

$$\{\ldots, -4, -3, -2, -1,\ 0,\ 1,\ 2,\ 3, \ldots\}$$

- The notation $\mathbb{Z}$ is from the German word for numbers: *Zahlen.*
- All natural numbers are integers.

$$\mathbb{Q} \subset \mathbb{Z}$$

3.5 Number Sets

Integers ($\mathbb{Z}$)

- Natural numbers may also be referred to as positive integers, denoted $\mathbb{Z}^+$.
  (note the superscript)
- Negative integers are denoted $\mathbb{Z}^-$.

$$\{\ldots, -4, -3, -2, -1\}$$

3.6 Number Sets

Integers ($\mathbb{Z}$)

- 0 is neither positive nor negative. The following set of non-negative numbers

$$\{0, 1, 2, 3, 4, 5, 6, \ldots\}$$

  might be denoted $0 \cup \mathbb{Z}^+$
- $\cup$ is the mathematical symbol for **union**.

3.7 Number Sets

Rational Numbers ($\mathbb{Q}$)

- Rational numbers, also known as quotients, are numbers you can make by dividing one integer by another (but not dividing by zero).
- If a number can be expressed as one integer divided by another, it is a rational number.

$$\mathbb{Q} = \left\{ \frac{p}{q} \,\middle|\, p \in \mathbb{Z},\ q \in \mathbb{Z},\ q \neq 0 \right\}$$

3.8 Number Sets

Rational Numbers ($\mathbb{Q}$)

- All integers are rational numbers
$$\mathbb{Z} \subset \mathbb{Q}$$
(and by extension all natural numbers are rational numbers too)

- Examples of rational numbers
$$9500,\ 7,\ \frac{1}{2},\ \frac{3}{7},\ -2.6,\ 0.001$$

3.9 Number Sets

Irrational Numbers

- A number that can not be written as the ratio of two integers is known as an irrational number.

- Two famous examples of irrational numbers are π and $\sqrt{2}$.
$$\pi = 3.141592\ldots$$
$$\sqrt{2} = 1.41421\ldots$$

Real Numbers ($\mathbb{R}$)

- Irrational numbers are types of real numbers.

- Rational numbers are real numbers too.
$$\mathbb{Q} \subset \mathbb{R}$$

- A real number is simply any point anywhere on the number line.

Real Numbers ($\mathbb{R}$)

- There are numbers that are not real numbers, for example **imaginary numbers**, but we will not cover them in this presentation.

Natural Numbers ($\mathbb{N}$)

- The whole numbers from 1 upwards.

- The set of natural numbers is
$$\{1, 2, 3, 4, 5, 6, \ldots\}$$

- In some branches of mathematics, 0 might be counted as a natural number.

$$\{0, 1, 2, 3, 4, 5, 6, \ldots\}$$

Integers ($\mathbb{Z}$)

- Natural numbers may also be referred to as positive integers, denoted $\mathbb{Z}^+$. (note the superscript)

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Integers ($\mathbb{Z}$)

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- If a number can be expressed as one integer divided by another, it is a rational number.

$$\mathbb{Q} = \left\{ \frac{p}{q} \middle| p \in \mathbb{Z},\ q \in \mathbb{Z},\ q \neq 0 \right\}$$

Rational Numbers ($\mathbb{Q}$)

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  (and by extension all natural numbers are rational numbers too)

- Examples of rational numbers

$$9500,\ 7,\ \frac{1}{2},\ \frac{3}{7},\ -2.6,\ 0.001$$

3.10 Real Numbers

($\mathbb{R}$)

- There are numbers that are not real numbers, for example **imaginary numbers**, but we will not cover them in this presentation.

1. State which of the following sets the following numbers belong to.

    1) 18
    2) 8.2347...
    3) π
    4) 1.33333...
    5) 17/4
    6) 4.25
    7) $\sqrt{\pi}$
    8) $\sqrt{25}$

    The possible answers are

    a) Natural number : $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
    b) Integer : $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
    c) Rational Number : $\mathbb{Q} \subseteq \mathbb{R}$
    d) Real Number $\mathbb{R}$