Complements

- The Complements of A and B are the elements of the universal set not contained in A and B.
- The complements are denoted $\mathcal{A}^c$ and $\mathcal{B}^c$

$$\mathcal{A}^c = \{4, 6, 8, 9\},$$

$$\mathcal{B}^c = \{1, 3, 5, 7, 9\},$$

Intersection

- Intersection of two sets describes the elements that are members of both the specified Sets
- The intersection is denoted $\mathcal{A} \cap \mathcal{B}$

$$\mathcal{A} \cap \mathcal{B} = \{2\}$$

- only one element is a member of both A and B.

Set Difference

- The Set Difference of A with regard to B are list of elements of A not contained by B.
- The complements are denoted $\mathcal{A} - \mathcal{B}$ and $\mathcal{B} - \mathcal{A}$

$$\mathcal{A} - \mathcal{B} = \{1, 3, 5, 7\},$$

$$\mathcal{B} - \mathcal{A} = \{4, 6, 8\},$$

Union and intersection of sets

- The **union** of two sets A and B is a set containing all the elements in either A or B (or both) i.e.
$$A \cup B = x/x \in A \text{ or } x \in B.$$
- The **intersection** of two sets A and B is a set containing all the elements that are both in A and B i.e.
$$A \cap B = x/x \in A \text{ and } x \in B$$
.

- If sets A and B have no elements in common, i.e. $A \cap B = \emptyset$,then A and B are termed **disjoint sets**.

Complement of a Set

Consider the universal set U such that

$$U = \{2, 4, 6, 8, 10, 12, 15\}$$

For each of the sets A,B,C and D, specify the complement sets.

Set	Complement
$A = \{4, 6, 12, 15\}$	$A^c = \{2, 8, 10\}$
$B = \{4, 8, 10, 15\}$	
$C = \{2, 6, 12, 15\}$	
$D = \{8, 10, 15\}$	

0.1 Set operations

Suppose X and Y are sets. Various operations allow us to build new sets from them.

Union The union of X and Y, written $X \cup Y$, contains all the elements in X and all those in Y. Thus $A \cup B = \{1, 2, 3, red, green, blue\}$. As A is a subset of E, the set $A \cup E$ is just E.

Intersection The intersection of X and Y, written $X \cap Y$, contains all the elements that are common to both X and Y. Thus $1, 2, 3, red, green, blue \cap 2, 4, 6, 8, 10 = 2$.

Set difference The difference X minus Y, written X-Y or $X \| Y$, contains all those elements in X that are not also in Y. For example, E-A contains all integers greater than 3. A-B is just A; red, green and blue were not elements of A, so no difference is made by excluding them.

1 Set Theory Operations

1. The Universal Set $\mathcal{U}$
2. Union
3. Intersection
4. Set Difference
5. Relative Difference

Set Operations

- Union ($\cup$) - also known as the **OR** operator. A union signifies a bringing together. The union of the sets A and B consists of the elements that are in either A or B.
- Intersection ($\cap$) - also known as the **AND** operator. An intersection is where two things meet. The intersection of the sets A and B consists of the elements that in both A and B.
- Complement (A' or A^c) - The complement of the set A consists of all of the elements in the universal set that are not elements of A.

Complements

- The Complements of A and B are the elements of the universal set not contained in A and B.
- The complements are denoted $\mathcal{A}'$ and $\mathcal{B}'$

$$\mathcal{A}' = \{4, 6, 8, 9\},$$

$$\mathcal{B}' = \{1, 3, 5, 7, 9\},$$

Complement of a Set

Consider the universal set U such that

$$U = \{2, 4, 6, 8, 10, 12, 15\}$$

For each of the sets A,B,C and D, specify the complement sets.

Set	Complement
$A = \{4, 6, 12, 15\}$	$A' = \{2, 8, 10\}$
$B = \{4, 8, 10, 15\}$	
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- The Set Difference of A with regard to B are list of elements of A not contained by B.
- The complements are denoted $\mathcal{A} - \mathcal{B}$ and $\mathcal{B} - \mathcal{A}$

$$\mathcal{A} - \mathcal{B} = \{1, 3, 5, 7\},$$

$$\mathcal{B} - \mathcal{A} = \{4, 6, 8\},$$

Relative Difference

- $A \oplus B$

1.1 Important Operations in Set Theory

- Union ($\cup$) - also known as the OR operator. A union signifies a bringing together. The union of the sets A and B consists of the elements that are in either A or B.
- Intersection ($\cap$) - also known as the AND operator. An intersection is where two things meet. The intersection of the sets A and B consists of the elements that in both A and B.
- Complement (^{c}) - The complement of the set A consists of all of the elements in the universal set that are not elements of A.

Dice Rolls

Consider rolls of a die. What is the universal set?

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6\}$$

Worked Example

Suppose that the Universal Set $\mathcal{U}$ is the set of integers from 1 to 9.

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\},$$

and that the set $\mathcal{A}$ contains the prime numbers between 1 to 9 inclusive.

$$\mathcal{A} = \{1, 2, 3, 5, 7\},$$

and that the set $\mathcal{B}$ contains the even numbers between 1 to 9 inclusive.

$$\mathcal{B} = \{2, 4, 6, 8\}.$$

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$$\mathcal{A} - \mathcal{B} = \{1, 3, 5, 7\},$$

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symbols

$\varnothing$, $\forall$, $\in$, $\notin$, $\cup$

2 Set Operations

Set Operations include Set Union, Set Intersection, Set Difference, Complement of Set, and Cartesian Product.

2.1 Set Union

The union of sets A and B (denoted by $A \cup B$) is the set of elements which are in A, in B, or in both A and B. Hence, $A \cup B = x|x \in$ A OR x $\in B$.

Example - $IfA = 10, 11, 12, 13$ and $B = 13, 14, 15$, then $A \cup B = 10, 11, 12, 13, 14, 15$. (The common element occurs only once)

2.2 Set Intersection

The intersection of sets A and B (denoted by AnB) is the set of elements which are in both A and B. Hence, $A \cap B = x|x \in AANDx \in B$.

Example - If $A = 11, 12, 13$ and $B = 13, 14, 15$, then $A \cap B = 13$.

2.3 Set Difference/ Relative Complement

The set difference of sets A and B (denoted by $A–B$) is the set of elements which are only in A but not in B. Hence, $A - B = x|x \in AANDx \notin B$. Example: If $A = 10, 11, 12, 13$ and $B = 13, 14, 15$, then $(A - B) = 10, 11, 12$ and $(B - A) = 14, 15$. Here, we can see (A-B)?(B-A)(A-B)?(B-A) Set Difference

2.4 Complement of a Set

The complement of a set A (denoted by A^c) is the set of elements which are not in set A. Hence, $A^c = x|x \notin A$. More specifically, $A^c = (U - A)$ where UU is a universal set which contains all objects.

2.5 Example

- If A=x—x belongstosetofoddintegersA=x—x belongstosetofoddintegers then $A^c = y|y$does not belong toset

Complement Set

Set Operations

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- Intersection ($\cap$) - also known as the **AND** operator. An intersection is where two things meet. The intersection of the sets A and B consists of the elements that in both A and B.
- Complement (A^c or A^c) - The complement of the set A consists of all of the elements in the universal set that are not elements of A.

Exercise

Consider the universal set U such that

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

and the sets

$$A = \{2, 5, 7, 9\}$$

$$B = \{2, 4, 6, 8, 9\}$$

(a) $A - B$

(b) $A \otimes B$

(c) $A \cap B$

(d) $A \cup B$

(e) $A^c \cap B^c$

(f) $A^c \cup B^c$

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$$\mathcal{B} - \mathcal{A} = \{4, 6, 8\},$$

symbols

$\varnothing$, $\forall$, $\in$, $\notin$, $\cup$

Relative Difference

- $A \otimes B$

Power Sets

- Consider the set A where $A = \{w, x, y, z\}$
- There are 4 elements in set A.
- The power set of A contains 16 element data sets.
-
$$\mathcal{P}(A) = \{\{x\}, \{y\}\}$$
- (i.e. 1 null set, 4 single element sets, 6 two -elemnts sets, 4 three lement set and one 4- element set.)