The Universal Set and the Empty Set

- The first is the ***universal set***, typically denoted U. This set is all of the elements that we may choose from. This set may be different from one setting to the next.
- For example one universal set may be the set of all real numbers, denoted $\mathbb{R}$, whereas for another problem the universal set may be the whole numbers $\{0, 1, 2, \ldots\}$.
- The other set that requires consideration is called the ***empty set***. The empty set is the unique set is the set with no elements. We write this as $\{\}$ and denote this set by $\emptyset$.

Number Sets

The font that the following symbols are written in (i.e. $\mathbb{N}$, $\mathbb{R}$) is known as ***blackboard font***.

- $\mathbb{N}$ Natural Numbers $(1, 2, 3, \ldots)$
- $\mathbb{Z}$ Integers $(-3, -2, -1, 0, 1, 2, 3, \ldots)$
  - $\mathbb{Z}^+$ Positive Integers
  - $\mathbb{Z}^-$ Negative Integers
  - 0 is not considered as either positive or negative.
- $\mathbb{Q}$ Rational Numbers
- $\mathbb{R}$ Real Numbers
- $\mathbb{C}$ Complex Numbers

Rules of Inclusion, Listing and Cardinality

For each of the following sets, a set is specified by the rules of inclusion method and listing method respectively. Also stated is the cardinality of that data set.

Worked example 1

- $\{x : x \text{ is an odd integer } 5 \leq x \leq 17\}$
- $x = \{5, 7, 9, 11, 13, 15, 17\}$
- The cardinality of set x is 7.

Worked example 2

- $\{y : y \text{ is an even integer } 6 \leq y < 18\}$
- $y = \{6, 8, 10, 12, 14, 16\}$
- The cardinality of set y is 6.

Worked example 3

A perfect square is a number that has a integer value as a square root. 4 and 9 are perfect squares ($\sqrt{4} = 2$, $\sqrt{9} = 3$).

- $\{z : z \text{ is an perfect square } 1 < z < 100\}$
- $z = \{4, 9, 16, 25, 36, 49, 64, 81\}$
- The cardinality of set z is 8.

Exercises

For each of the following sets, write out the set using the listing method. Also write down the cardinality of each set.

- $\{s : s \text{ is an negative integer } -10 \leq s \leq 0\}$
- $\{t : t \text{ is an even number } 1 \leq t \leq 20\}$
- $\{u : u \text{ is a prime number } 1 \leq u \leq 20\}$
- $\{v : v \text{ is a multiple of 3 } 1 \leq v \leq 20\}$

Power Sets

Worked Example

Consider the set Z:

$$Z = \{a, b, c\}$$

(i) How many sets are in the power set of Z?

(ii) Write out the power set of Z.

(iii) How many elements are in each element set?

Solutions to Worked Example

(i) There are 3 elements in Z. So there is $2^3 = 8$ element sets contained in the power set.

(ii) Write out the power set of Z.

$$\mathcal{P}(Z) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

(iii)
- One element set is the null set - i.e. containing no elements
- Three element sets have only elements
- Three element sets have two elements
- One element set contains all three elements
- 1+3+3+1=8

Exercise

For the set $Y = \{u, v, w, x\}$, answer the questions from the previous exercise

Complement of a Set

Consider the universal set U such that

$$U = \{2, 4, 6, 8, 10, 12, 15\}$$

For each of the sets A,B,C and D, specify the complement sets.

Set	Complement
$A = \{4, 6, 12, 15\}$	$A' = \{2, 8, 10\}$
$B = \{4, 8, 10, 15\}$	
$C = \{2, 6, 12, 15\}$	
$D = \{8, 10, 15\}$	

Set Operations

- Union ($\cup$) - also known as the **OR** operator. A union signifies a bringing together. The union of the sets A and B consists of the elements that are in either A or B.
- Intersection ($\cap$) - also known as the **AND** operator. An intersection is where two things meet. The intersection of the sets A and B consists of the elements that in both A and B.
- Complement (A' or A^c) - The complement of the set A consists of all of the elements in the universal set that are not elements of A.

Exercise

Consider the universal set U such that

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

and the sets

$$A = \{2, 5, 7, 9\}$$
$$B = \{2, 4, 6, 8, 9\}$$

(a) $A - B$

(b) $A \otimes B$

(c) $A \cap B$

(d) $A \cup B$

(e) $A' \cap B'$

(f) $A' \cup B'$

Venn Diagrams

Draw a Venn Diagram to represent the universal set $\mathcal{U} = \{0, 1, 2, 3, 4, 5, 6\}$ with subsets:
$A = \{2, 4, 5\}$
$B = \{1, 4, 5, 6\}$

Find each of the following

(a) $A \cup B$

(b) $A \cap B$

(c) $A - B$

(d) $B - A$

(e) $A \otimes B$

(i) Describe the following set by the listing method

$$\{2r+1 : r \in Z^{+} and r \leq 5\}$$

(ii) Let A,B be subsets of the universal set U.

Question 1

- $\{s :$ s is an odd integer and $2 \leq s \leq 10\}$
- $\{2m : m \in Z$ and $5 \leq m \leq 10\}$
- $\{2^t : t \in Z$ and $0 \leq t \leq 5\}$

Question 2

- $\{12, 13, 14, 15, 16, 17\}$
- $\{0, 5, -5, 10, -10, 15, -15, ...\}$
- $\{6, 8, 10, 12, 14, 16, 18\}$

Question 7 : Membership Tables

Using membership tables

A	B	C	x	y	z
0	0	0	1	1	1
0	0	1	0	0	1
0	1	0	0	0	1
0	1	1	0	0	1
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	0	0	1
1	1	1	1	0	1

(i) Draw a venn diagram to show three subsets A,B and C of a universal set U intersecting in the most general way?

(ii) How are sets X and Z related?

(iii) Can you describe each of the subsets X,Y and Z in terms of the sets A,B,C using the operations union intersection and set complement.

Question 8

(i)

(ii)

(iii)