

1 Number Sets

The font that the symbols are written in (i.e. \mathbb{N} , \mathbb{R}) is known as ***blackboard font***.

- \mathbb{N} Natural Numbers $(0, 1, 2, 3)$ (Not used in the CIS102 Syllabus)
- \mathbb{Z} Integers $(-3, -2, -1, 0, 1, 2, 3, \dots)$
 - * \mathbb{Z}^+ Positive Integers
 - * \mathbb{Z}^- Negative Integers
- \mathbb{Q} Rational Numbers
- \mathbb{R} Real Numbers

2 Numbers and Set Theory

Suppose we have the sets A and B defined as follows:

$$A = \{ \sqrt{2}, \frac{3}{2}, 2 \}$$

$$B = \{x \in \mathbb{R} : x \notin \mathbb{Q}\}$$

1 $A \cap \mathbb{Q}$

2 $A \cap B$

3 $B \cup \mathbb{Q}$

- \mathbb{R} Set of all real numbers.
- \mathbb{Q} Set of all quotient numbers.

$$B = \{x \in \mathbb{R} : x \notin \mathbb{Q}\}$$

Set of all real numbers that are not quotients (i.e. numbers that can not be expressed as a division of one integer by another).

$$A \cap \mathbb{Q}$$

Part D: Natural, Rational and Real Numbers

- \mathbb{N} : natural numbers (or positive integers) $\{1, 2, 3, \dots\}$

- \mathbb{Z} : integers $\{-3, -2, -1, 0, 1, 2, 3, \dots\}$
 - * (The letter \mathbb{Z} comes from the word *Zahlen* which means “numbers” in German.)
- \mathbb{Q} : rational numbers
- \mathbb{R} : real numbers
- $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
 - * (All natural numbers are integers. All integers are rational numbers. All rational numbers are real numbers.)

1. State which of the following sets the following numbers belong to.

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|------------------|-------------------|-----------|-----------------|
| 1) 18 | 3) π | 5) $17/4$ | 7) $\sqrt{\pi}$ |
| 2) $8.2347\dots$ | 4) $1.33333\dots$ | 6) 4.25 | 8) $\sqrt{25}$ |

The possible answers are

- a) Natural number : $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
- b) Integer : $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
- c) Rational Number : $\mathbb{Q} \subseteq \mathbb{R}$
- d) Real Number \mathbb{R}

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 - 0 is not considered as either positive or negative.
- \mathbb{Q} Rational Numbers
- \mathbb{R} Real Numbers
- \mathbb{C} Complex Numbers

3 Special sets of numbers

It is convenient to denote certain key sets of numbers by a standard letter. Definition 2.2 The symbol \mathbb{Z} is used to denote the set of integers,” denotes the set of positive integers; \mathbb{R} denotes the set of real numbers; \mathbb{Q} denotes the set of rational numbers the letter \mathbb{Q} stands for “quotient”). Of these sets, we can specify the set of positive integers and the set of integers by the listing method using ellipses, as follows:

$$\mathbb{Z}^+ = 1, 2, 3, \dots$$

$$\mathbb{Z} = 0, 1, -1, 2, -2, 3, -3, \dots$$

Note that \mathbb{Z} includes 0 and the negative whole numbers as well as the positive ones. Similarly, \mathbb{R} contains 0 and the negative reals and \mathbb{Q} contains 0 and the negative rationals, as well as the positive ones.

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3.1 Some Important Sets

- \mathbb{N} : the set of all natural numbers = $1, 2, 3, 4, \dots$
- \mathbb{Z} : the set of all integers = $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- \mathbb{Z}^+ : the set of all positive integers
- \mathbb{Q} : the set of all rational numbers
- \mathbb{R} : the set of all real numbers
- \mathbb{W} : the set of all whole numbers

3.2 Number Sets

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- \mathbb{Q} Rational Numbers
- \mathbb{R} Real Numbers

Set Operations Let $U = \{1, 2, \dots, 9\}$ be the universal set, and let

- $A = \{1, 2, 3, 4, 5\}$,
- $B = \{4, 5, 6, 7\}$,
- $C = \{5, 6, 7, 8, 9\}$
- $D = \{1, 3, 5, 7, 9\}$,
- $E = \{2, 4, 6, 8\}$,
- $F = \{1, 5, 9\}$.

Set Operations

Find:

- (a) $A \cup B$ and $A \cap B$,
- (b) $C \cup D$ and $C \cap D$,
- (c) $E \cup F$ and $E \cap F$.

Set Operations

Find:

- (a) $A \cup B$ and $A \cap B$,
- $A = \{1, 2, 3, 4, 5\}$
- $B = \{4, 5, 6, 7\}$

Set Operations

Find:

- (b) $C \cup D$ and $C \cap D$,
- $C = \{5, 6, 7, 8, 9\}$
- $D = \{1, 3, 5, 7, 9\}$

Set Operations

Find:

- (c) $E \cup F$ and $E \cap F$.
- $E = \{2, 4, 6, 8\}$
- $F = \{1, 5, 9\}$

Finite Sets

Determine which of the following sets are finite:

- (a) Set of Prime numbers
- (b) Set of two digit Prime numbers
- (c) Letters in the English alphabet.
- (d) Integers which are multiples of 5.
- (e) Days of the week

Set Theory

Given the set \mathbf{A} is constructed as follows

$$[\{a, b\}, \{c\}, \{d, e, f\}].$$

- (a) List the elements of \mathbf{A} .
- (b) Find the cardinality of $\mathbf{A} : n(\mathbf{A})$.
- (c) Find the power set of \mathbf{A} .

Set Theory

$$\mathbf{A} = [\{a, b\}, \{c\}, \{d, e, f\}].$$

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Set Theory

$$\mathbf{A} = [\{a, b\}, \{c\}, \{d, e, f\}].$$

- (c) Find the power set of \mathbf{A} .

Consider the set \mathbf{A} , which is a subset of the the universal set of real numbers \mathbb{R}

$$\mathbf{A} = \{4, \sqrt{2}, 2/3, -2.5, -5, 33, \sqrt{9}, \pi\}$$

Using formal set notation, write the sets of:

- (a) natural numbers in \mathbf{A}

- (b) integers in \mathbf{A}
- (c) rational numbers in \mathbf{A}
- (d) irrational numbers in \mathbf{A}

$$\mathbf{A} = \{4, \sqrt{2}, 2/3, -2.5, -5, 33, \sqrt{9}, \pi\}$$

- (a) natural numbers in \mathbf{A}
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$$\mathbf{A} = \{4, \sqrt{2}, 2/3, -2.5, -5, 33, \sqrt{9}, \pi\}$$

- (a) natural numbers in \mathbf{A}
Answer: $\{4, 33, \sqrt{9}\}$

- (b) integers in \mathbf{A}
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Answer: $\{4, 33, \sqrt{9}\}$
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(a) natural numbers in \mathbf{A}

Answer: $\{4, 33, \sqrt{9}\}$

(b) integers in \mathbf{A}

Answer: $\{4, -5, 33, \sqrt{9}\}$

(c) rational numbers in \mathbf{A}

Answer: $\{4, 2/3, -2.5, -5, 33, \sqrt{9}\}$

(d) irrational numbers in \mathbf{A}

$$\mathbf{A} = \{4, \sqrt{2}, 2/3, -2.5, -5, 33, \sqrt{9}, \pi\}$$

(a) natural numbers in \mathbf{A}

Answer: $\{4, 33, \sqrt{9}\}$

(b) integers in \mathbf{A}

Answer: $\{4, -5, 33, \sqrt{9}\}$

(c) rational numbers in \mathbf{A}

Answer: $\{4, 2/3, -2.5, -5, 33, \sqrt{9}\}$

(d) irrational numbers in \mathbf{A}

Answer: $\{\sqrt{2}, \pi\}$

3.3 Number Sets

Blackboard Bold Typeface

- Conventionally the symbols for numbers sets are written in a special typeface, known as **blackboard bold**.
- Examples : \mathbb{N} , \mathbb{Z} and \mathbb{R} .

Natural Numbers (\mathbb{N})

- The whole numbers from 1 upwards.
- The set of natural numbers is

$$\{1, 2, 3, 4, 5, 6, \dots\}$$

- In some branches of mathematics, 0 might be counted as a natural number.

$$\{0, 1, 2, 3, 4, 5, 6, \dots\}$$

3.4 Number Sets

Integers (\mathbb{Z})

- The integers are all the whole numbers, all the negative whole numbers and zero.
- The set of integers is

$$\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

- The notation \mathbb{Z} is from the German word for numbers: *Zahlen*.
- All natural numbers are integers.

$$\mathbb{Q} \subset \mathbb{Z}$$

3.5 Number Sets

Integers (\mathbb{Z})

- Natural numbers may also be referred to as positive integers, denoted \mathbb{Z}^+ .
(note the superscript)
- Negative integers are denoted \mathbb{Z}^- .

$$\{\dots, -4, -3, -2, -1\}$$

3.6 Number Sets

Integers (\mathbb{Z})

- 0 is neither positive nor negative. The following set of non-negative numbers

$$\{0, 1, 2, 3, 4, 5, 6, \dots\}$$

might be denoted $0 \cup \mathbb{Z}^+$

- \cup is the mathematical symbol for **union**.

3.7 Number Sets

Rational Numbers (\mathbb{Q})

- Rational numbers, also known as quotients, are numbers you can make by dividing one integer by another (but not dividing by zero).
- If a number can be expressed as one integer divided by another, it is a rational number.

$$\mathbb{Q} = \left\{ \frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0 \right\}$$

3.8 Number Sets

Rational Numbers (\mathbb{Q})

- All integers are rational numbers

$$\mathbb{Z} \subset \mathbb{Q}$$

(and by extension all natural numbers are rational numbers too)

- Examples of rational numbers

$$9500, 7, \frac{1}{2}, \frac{3}{7}, -2.6, 0.001$$

3.9 Number Sets

Irrational Numbers

- A number that can not be written as the ratio of two integers is known as an irrational number.
- Two famous examples of irrational numbers are π and $\sqrt{2}$.

$$\pi = 3.141592 \dots$$

$$\sqrt{2} = 1.41421 \dots$$

Real Numbers (\mathbb{R})

- Irrational numbers are types of real numbers.
- Rational numbers are real numbers too.

$$\mathbb{Q} \subset \mathbb{R}$$

- A real number is simply any point anywhere on the number line.

Real Numbers (\mathbb{R})

- There are numbers that are not real numbers, for example **imaginary numbers**, but we will not cover them in this presentation.

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Integers (\mathbb{Z})

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