1 Discrete Maths - Logic Tutorial Sheet

1.1 Notation - Listing Method

By convention, a set can be written by **listing** its elements, separated by commas, between braces.

Using the sets defined above:

- A = $\{1, 2, 3\}$
- B = $\{red, green, blue\}$
- C = $\{\}$

This is impractical for large sets (D), and impossible for infinite ones (E).

1.2 Notation - Building Method

Thus a set can also be described by naming a particular property that is shared by all its elements and only by them. A common notation uses a bar (—) to separate a variable name (e.g. "x") from a property of the variable that elements of the set must have. For example:

- D = $\{x|$ x is a book and x is in the British Library $\}$
- E = $\{x|$ x is a positive integer$\}$

A simple translation of this notation is that "*D is the set of all x, where x is a book and x is in the British Library*" or "*E is the set of all x, where x is a positive integer*".

Rules of Inclusion, Listing and Cardinality

For each of the following sets, a set is specified by the rules of inclusion method and listing method respectively. Also stated is the cardinality of that data set.

Worked example 1

- $\{x : x$ is an odd integer $5 \leq x \leq 17\}$
- $x = \{5, 7, 9, 11, 13, 15, 17\}$
- The cardinality of set x is 7.

Worked example 2

- $\{y : y$ is an even integer $6 \leq y < 18\}$
- $y = \{6, 8, 10, 12, 14, 16\}$
- The cardinality of set y is 6.

Worked example 3

A perfect square is a number that has a integer value as a square root. 4 and 9 are perfect squares ($\sqrt{4} = 2$, $\sqrt{9} = 3$).

- $\{z : z$ is an perfect square $1 < z < 100\}$
- $z = \{4, 9, 16, 25, 36, 49, 64, 81\}$
- The cardinality of set z is 8.

2 Set Theory : Listing Method

Describe the following sets using the Listing Method

(i) $\{10^m : -2 \leq m \leq 4, m \in \mathbb{Z}\}$

(ii) $\{\frac{1}{n} : 1 < n < 4, n \in \mathbb{Z}\}$

Part 1: $\{10^m : -2 \leq m \leq 4, m \in \mathbb{Z}\}$

(i) Describe the following set by the listing method

$$\{2r + 1 : r \in Z^{+} and r \leq 5\}$$

(ii) Let A,B be subsets of the universal set U.

Question 1

- $\{s :$ s is an odd integer and $2 \leq s \leq 10\}$
- $\{2m : m \in Z$ and $5 \leq m \leq 10\}$
- $\{2^t : t \in Z$ and $0 \leq t \leq 5\}$

Question 2

- $\{12, 13, 14, 15, 16, 17\}$
- $\{0, 5, -5, 10, -10, 15, -15, ...\}$
- $\{6, 8, 10, 12, 14, 16, 18\}$

Exercises

For each of the following sets, write out the set using the listing method. Also write down the cardinality of each set.

- $\{s : s \text{ is an negative integer } -10 \leq s \leq 0\}$
- $\{t : t \text{ is an even number } 1 \leq t \leq 20\}$
- $\{u : u \text{ is a prime number } 1 \leq u \leq 20\}$
- $\{v : v \text{ is a multiple of 3 } 1 \leq v \leq 20\}$

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- Let $A = \{2n + 1 : n \in Z^+\}$ be a set of numbers.
- Describe the set A by the listing method.

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3 Rules of Inclusion, Listing and Cardinality

For each of the following sets, a set is specified by the rules of inclusion method and listing method respectively. Also stated is the cardinality of that data set.

3.1 Worked example 1

$x : xisanoddinteger 5 \leq x \leq 17$ x = 5,7,9,11,13,15,17 The cardinality of set x is 7.

3.2 Worked example 2

$y : yisaneveninteger 6 \leq y < 18$ y = 6,8,10,12,14,16 The cardinality of set y is 6.

3.3 Worked example 3

A perfect square is a number that has a integer value as a square root. For example, 4 and 9 are perfect squares $\sqrt{4} = 2$, $\sqrt{9} = 3$ z : z is an perfect square 1 ¡ z ¡ 100 z = 4,9,16,25,36,49,64,81 The cardinality of set z is 8.

3.4 Exercises

For each of the following sets, write out the set using the listing method. Also write down the cardinality of each set.

- $s : sisannegativeinteger, -10 \leq s \leq 0$
- $t : tisanevennumber, 1 \leq t \leq 20$
- $u : uisaprimenumber, 1 \leq u \leq 20$
- $v : visamultipleof 3, 1 \leq v \leq 20$

Roster or Tabular Form The set is represented by listing all the elements comprising it. The elements are enclosed within braces and separated by commas.

Example 1 : Set of vowels in English alphabet, A=a,e,i,o,u Example 2 : Set of odd numbers less than 10, B=1,3,5,7,9

3.5 Set Builder Notation

The set is defined by specifying a property that elements of the set have in common. The set is described as A=x:p(x)A=x:p(x) Example 1 : The set a,e,i,o,ua,e,i,o,u is written as :

$$A = x : xisavowelinEnglishalphabet$$

- Specifying Sets
- Listing Method
- Rules of Inclusion method

4 2.1 Specifying sets

When you have completed your study of this section, you should be able to:

- use set notation for specifying sets by the listing method and rules of inclusion method;
- use and interpret the standard symbols for special sets of numbers and for the empty set.

4.1 2.1.1 Listing method

We usually use an upper case letter to denote a set and a lower case letter to denote a member of the set. To specify a set, we must describe its members in an unambiguous way. One way of doing this is to list the members of the set, separated by commas, and enclose the list in a pair of brace brackets.

- The set D of decimal digits can be expressed as

$$D = 0, l, 2, 3, 4, 5, 6, 7, 8, 9$$

- The set B of bits can be expressed as

$$B = 0, 1$$

.

4.2 2.1.2 Rules of inclusion method

4.3 Rules of Inclusion

Another way of specifying a set is by giving rules of inclusion that distinguish members of the set from objects not in the set.

Definition 2.3 The context of the problem in which u set arises determines an underlying set, called the uni-vet-sul set for the problem, from which the elements of the set will be drawn. For example, if our subject is a set of leopards, the universal set, explicitly stated or implied by the context, might be all wild animals in Africa or all animals in London Zoo or all animals belonging to the cat family.

Example 2.4 To specify the set H of Example 2.3, we could write

$$H = n \in Z : 1 \leq n \leq 100\}.$$

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(i) Describe the following set by the listing method

$$\{2r + 1 : r \in Z^{+} and r \leq 5\}$$

(ii) Let A,B be subsets of the universal set U.

5 Specifying Sets

1. Listing Method
2. Rules of Inclusion

Part A : Builder Method

The following sets have been defined using the **Building Method** of notation. Re-write them by listing **some** of the elements.

1. $\{p|p$ is a capital city, p is in Europe$\}$
2. $\{x|x = 2n - 5$, x and n are natural numbers$\}$
3. $\{y|2y^2 = 50$, y is an integer$\}$
4. $\{z|z = n^3$, z and n are natural numbers$\}$

Part B : Sets

U = natural numbers; $A = \{2, 4, 6, 8, 10\}$; $B = \{1, 3, 6, 7, 8\}$. State whether each of the following is true or false:

(i) $A \subset U$

(ii) $B \subseteq A$

(iii) $\emptyset \subset U$

NOTATIONS FOR A SET:

A set can be represented by two methods: 1.ROSTER METHOD 2.BUILDER METHOD ROSTER METHOD:

In this method the elements of a set are separated by commas and are enclosed within curly brackets . For example:

$A = 1, 2, 3, 4, 5, 6$ is a set of numbers.

B = Sunny, Joy, Kartik, Harish, Girish is a set of names.

C = a, e, i, o, u is a set of vowels.

D = apple, banana, guava, orange, pear is a set of fruits.

Listing the elements in this way is called Roster method. In this method, it is not necessary for the elements to be listed in a particular order. The elements of the set can be written just plainly, separated by commas and in any order.

BUILDER METHOD:

This method is also called Property method. In Builder method, a set is represented by stating all the properties which are satisfied by the elements of that particular set only.

For example:

If A is a set of elements less than 0, then in Builder method it will be written as

$A = x : x < 0$, this statement is read as "the set of all x such that x is less than 0"

If A is a set of all real numbers less than 7, then in Builder method it is written as $A = xR : x < 7$

Similarly,

$A = 2i : iisaninteger$ is a set of all even integers.

$A = xR : x \neq 2$ is a set of all real numbers except 2.

$A = xR : x > 3andx < 7$ is a set of real numbers greater than 3 but less than 7.

$A = xZ : x > 6$ is a set of integers greater than 6.

$A = xZ : 2x + 1$ is a set of all odd integers.

A $\{2n : n \in \mathbb{Z}^+\}$

B $\{3, 6, 9, 12, 15, 18, \ldots\}$

Questions

(i) **A** is described by the rules of inclusion. Describe **A** with the listing method.

(ii) **B** is described by the listing method. Describe **B** with the rules of inclusion.

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