Power set

The power set of X, $P(X)$, is the set whose elements are all the subsets of X. Thus

$$P(A) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

. The power set of the empty set $P(\{\}) = \{\{\}\}$.

Note that in both cases the cardinality of the power set is strictly greater than that of base set: No one-to-one correspondence exists between the set and its power set.

Power Sets

- Consider the set A where $A = \{w, x, y, z\}$
- There are 4 elements in set A.
- The power set of A contains 16 element data sets.
- $$\mathcal{P}(A) = \{\{x\}, \{y\}\}$$
- (i.e. 1 null set, 4 single element sets, 6 two -elemnts sets, 4 three lement set and one 4- element set.)

Power Sets

Worked Example

Consider the set Z:

$$Z = \{a, b, c\}$$

Q1 How many sets are in the power set of Z?

Q2 Write out the power set of Z.

Q3 How many elements are in each element set?

Solutions to Worked Example

Q1 There are 3 elements in Z. So there is $2^3 = 8$ element sets contained in the power set.

Q2 Write out the power set of Z.

$$\mathcal{P}(Z) = \{\{0\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$$

Q3
* One element set is the null set - i.e. containing no elements
- Three element sets have only elements
- Three element sets have two elements
- One element set contains all three elements
- 1+3+3+1=8

Exercise

For the set $Y = \{u, v, w, x\}$, answer the questions from the previous exercise

Power Set Power set of a set S is the set of all subsets of S including the empty set. The cardinality of a power set of a set S of cardinality n is 2n2n. Power set is denoted as P(S)P(S).

Example

For a set $S = a, b, c, d$ let us calculate the subsets

- Subsets with 0 elements : $\{\varnothing\}$ (the empty set)
- Subsets with 1 element : a,b,c,d
- Subsets with 2 elements : a,b,a,c,a,d,b,c,b,d,c,d
- Subsets with 3 elements : a,b,c,a,b,d,a,c,d,b,c,d
- Subsets with 4 elements : a,b,c,d

Hence,

$$P(S) = \varnothing, a, b, c, d, a, b, a, c, a, d, b, c, b, d, c, d, a, b, c, a, b, d, a, c, d, b, c, d, a, b, c, d$$

—P(S)—=24=16 Note: The power set of an empty set is also an empty set.

$|P(\varnothing)| = 2^0 = 1$

0.1 Partitioning of a Set

Partition of a set, say S, is a collection of n disjoint subsets, say $P1, P2 \ldots Pn$ that satisfies the following three conditions :

1. Pi does not contain the empty set.

   $[Pi \neq \varnothing forall 0 < i \leq n]$

2. The union of the subsets must equal the entire original set.

   FIX

3. The ntersection of any two distinct sets is empty.

   $[Pa \cap Pb = \varnothing, fora \neq bwheren \geq a, b \geq 0]$

Example

Let $S = a, b, c, d, e, f, g, h$

- One probable partitioning is a, b, c, d, e, f, g, h
- Another probable partitioning is a, b, c, d, e, f, g, h

Power Sets

Worked Example

Consider the set Z:

$$Z = \{a, b, c\}$$

Q1 How many sets are in the power set of Z?

Q2 Write out the power set of Z.

Q3 How many elements are in each element set?

Solutions to Worked Example

Q1 There are 3 elements in Z. So there is $2^3 = 8$ element sets contained in the power set.

Q2 Write out the power set of Z.

$$\mathcal{P}(Z) = \{\{0\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$$

Q3
* One element set is the null set - i.e. containing no elements
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* One element set contains all three elements
* 1+3+3+1=8

Exercise

For the set $Y = \{u, v, w, x\}$, answer the questions from the previous exercise

Worked Example

Consider the set Z:

$$Z = \{a, b, c\}$$

(i) How many sets are in the power set of Z?

(ii) Write out the power set of Z.

(iii) How many elements are in each element set?

Solutions to Worked Example

(i) There are 3 elements in Z. So there is $2^3 = 8$ element sets contained in the power set.

(ii) Write out the power set of Z.

$$\mathcal{P}(Z) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

(iii)
- One element set is the null set - i.e. containing no elements
- Three element sets have only elements
- Three element sets have two elements
- One element set contains all three elements
- 1+3+3+1=8

Exercise

For the set $Y = \{u, v, w, x\}$, answer the questions from the previous exercise

1 Power Sets

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