0.1 Cardinality of a set

A set in called finite when it contains a finite number of elements, and otherwise it is called infinite.

The number of distinct elements in a finite set is called its cardinality.

- The cardinality of a set A is the number of elements in A, which is written as $|A|$.
- An element of a set is any one of the **distinct** objects that make up that set.
- Note that this vertical-bar notation looks the same as absolute value notation, but the meaning of cardinality is different from absolute value.
- In particular, absolute value operates on numbers (e.g., $|-4| = 4$) while cardinality operates on sets (e.g., $|\{-4\}| = 1$).

0.2 Cardinality of a Set

Cardinality of a set S, denoted by $|S|$, is the number of elements of the set. The number is also referred as the cardinal number. If a set has an infinite number of elements, its cardinality is ∞.

Example $|1, 4, 3, 5| = 4$,$|1, 2, 3, 4, 5, \ldots| = \infty$

If there are two sets X and Y,

—X—=—Y——X—=—Y— denotes two sets X and Y having same cardinality. It occurs when the number of elements in X is exactly equal to the number of elements in Y. In this case, there exists a bijective function 'f' from X to Y.

$|X| \leq |Y|$ denotes that set X's cardinality is less than or equal to set Y's cardinality. It occurs when number of elements in X is less than or equal to that of Y. Here, there exists an injective function "f" from X to Y.

$|X| < |Y|$ denotes that set X's cardinality is less than set Y's cardinality. It occurs when number of elements in X is less than that of Y. Here, the function 'f' from X to Y is injective function but not bijective.

If $|X| \leq |Y|$ and $|X| \geq |Y|$ then $|X| = |Y|$. The sets X and Y are commonly referred as equivalent sets.

Cardinality

The cardinality $\|X\|$ of X is, roughly speaking, its size. The empty set has size 0, while B = $\{red, green, blue\}$ has size 3.

This method of expressing cardinality works for finite sets but is not helpful for infinite ones. A more useful notion is that of two sets having the same size: if a direct one-to-one correspondence can be found between the elements of X and those of Y, they have the same size.

Consider the sets, both infinite, of positive integers $\{1, 2, 3, 4, \ldots\}$ and of even positive integers $\{2, 4, 6, 8, \ldots\}$.

One is a subset of the other. Nevertheless they have the same cardinality, as is shown by the correspondence mapping n in the former set to 2n in the latter. We can express this by writing $\|X\| = \|Y\|$, or by saying that X and Y are "equinumerous".

Examples

(i) $|\{2, 6, 7\}|$

(ii) $|\{5, 6, 5, 2, 2, 6, 5, 1, 1, 1\}|$

(iii) $|\{\ \}| = 0$.

(iv) $|\{\{1, 2\}, \{3, 4\}\}| = 2$.

Examples

(i) $|\{2, 6, 7\}|$

(ii) $|\{5, 6, 5, 2, 2, 6, 5, 1, 1, 1\}|$

(i) $|\{2, 6, 7\}| = 3$

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Examples

(iv) $|\{\{1, 2\}, \{3, 4\}\}| = 2$.
In this case the two elements of $\{\{1, 2\}, \{3, 4\}\}$ are themselves sets: $\{1, 2\}$ and $\{3, 4\}$.

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- The cardinality of a set A is the number of elements in A, which is written as $|A|$.
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The empty set has no elements.

(iv) $|\{\{1, 2\}, \{3, 4\}\}| = 2$.

In this case the two elements of $\{\{1, 2\}, \{3, 4\}\}$ are themselves sets: $\{1, 2\}$ and $\{3, 4\}$.

0.3 Cardinality

The cardinality $\|X\|$ of X is, roughly speaking, its size. The empty set has size 0, while B = $\{red, green, blue\}$ has size 3.

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