Question 1

a) $X \sim \text{Binomial}(n = 20, p = 0.1)$.

$$\Pr(X = 2) = \binom{20}{2} (0.1^2)\,(0.9^{18}) = 0.2852.$$

$$\lambda = n\,p = 20(0.1) = 2$$

$$\Pr(X = 2) \approx \frac{2^2}{2!}\, e^{-2} = 0.2707.$$

b) $X \sim \text{Binomial}(n = 100, p = 0.02)$.

$$\Pr(X = 5) = \binom{100}{5} (0.02^5)\,(0.98^{95}) = 0.0353.$$

$$\lambda = n\,p = 100(0.02) = 2$$

$$\Pr(X = 5) \approx \frac{2^5}{5!}\, e^{-2} = 0.0361.$$

c) $X \sim \text{Binomial}(n = 1000, p = 0.005)$.

$$\begin{aligned}\Pr(X = 3) &= \binom{1000}{3} (0.005^3)\,(0.995^{997})\\ &= 0.1403.\end{aligned}$$

$$\lambda = n\,p = 1000(0.005) = 5$$

$$\Pr(X = 3) \approx \frac{5^3}{3!}\, e^{-5} = 0.1404.$$

d) $X \sim \text{Binomial}(n = 10000, p = 0.0001)$.

$$\begin{aligned}\Pr(X = 1) &= \binom{10000}{1} (0.0001^1)\,(0.9999^{9999})\\ &= 0.3679.\end{aligned}$$

$$\lambda = n\,p = 10000(0.0001) = 1$$

$$\Pr(X = 1) \approx \frac{1^1}{1!}\, e^{-1} = 0.3679.$$

Question 2

Here we have $X \sim \text{Poisson}(\lambda = 2)$.

$\Rightarrow \Pr(X = x) = \frac{\lambda^x}{x!}\, e^{-\lambda} = \frac{2^x}{x!}\, e^{-2}$ in one hour.

a)
$$\Pr(X = 6) = \frac{2^6}{6\,!}\, e^{-2} = 0.0120.$$

b)
$$\begin{aligned}\Pr(X < 3) = \Pr(X \le 2) &= p(0) + p(1) + p(2)\\ &= \frac{2^0}{0\,!}\, e^{-2} + \frac{2^1}{1\,!}\, e^{-2} + \frac{2^2}{2\,!}\, e^{-2}\\ &= 0.1353 + 0.2707 + 0.2707\\ &= 0.6767.\end{aligned}$$

c) Average is $2(2) = 4$ per two hours.

$$\Rightarrow \Pr(X = 0) = \frac{4^0}{0\,!}\, e^{-4} = 0.0183.$$

d) Again the average is 4 per two hours.

$$\begin{aligned}\Pr(X > 4) &= 1 - \Pr(X \le 4)\\ &= 1 - [p(0) + p(1) + (2) + p(3) + p(4)]\\ &= 1 - \left(\frac{4^0}{0\,!}\, e^{-4} + \frac{4^1}{1\,!}\, e^{-4} + \frac{4^2}{2\,!}\, e^{-4} + \right.\\ &\qquad \left. \frac{4^3}{3\,!}\, e^{-4} + \frac{4^4}{4\,!}\, e^{-4}\right)\\ &= 1 - (0.0183 + 0.0733 + 0.1465 +\\ &\qquad 0.1954 + 0.1954)\\ &= 1 - 0.6289 = 0.3711.\end{aligned}$$

e) The average is $2(0.5) = 1$ per 0.5 hours.

$$\begin{aligned}\Pr(X \ge 1) &= 1 - \Pr(X = 0)\\ &= 1 - \frac{1^0}{0\,!}\, e^{-1}\\ &= 1 - 0.3679 = 0.6321.\end{aligned}$$

f) $E(X) = \lambda = 2$ emails per hour.

$Sd(X) = \sqrt{Var(X)} = \sqrt{\lambda} = \sqrt{2} = 1.414$ emails per hour.

Question 3

Same as Q2 but now using the Poisson tables. We must rework the questions in terms of *greater than or equal to* probabilities.

a)
$$\begin{aligned}\Pr(X = 6) &= \Pr(X \geq 6) - \Pr(X \geq 7)\\ &= 0.0166 - 0.0045 = 0.0121.\end{aligned}$$

b)
$$\begin{aligned}\Pr(X < 3) &= 1 - \Pr(X \geq 3)\\ &= 1 - 0.3233 = 0.6767.\end{aligned}$$

c) Here $\lambda = 2(2) = 4$.

$$\begin{aligned}\Pr(X = 0) &= \Pr(X \geq 0) - \Pr(X \geq 1)\\ &= 1.000 - 0.9817 = 0.0183.\end{aligned}$$

d) Again $\lambda = 4$.

$$\Pr(X > 4) = \Pr(X \geq 5) = 0.3712.$$

e) Here $\lambda = 2(0.5) = 1$.

$$\Pr(X \geq 1) = 0.6321.$$

We can see that these are the same as above apart from small differences due to rounding.