Probability : Medical Diagnosis Worked Example

- A new test has been developed to diagnose a particular disease. If a person has the disease, the test has a 95% chance of identifying them as having the disease.
- If a person does not have the disease, the test has a 1% chance of identifying them as having the disease.
- Suppose that 5% of the population have this disease. Suppose we select a person at random from the population.

Questions

Q1 - What is the probability that the test will identify them as having the disease?

Q2 - What is the probability that the person has the disease given that the test identifies them as having the disease?

Solution: State Each of the Events

- Let **P** signify that a test will give a positive result.
- Let **N** signify that a test will give a negative result.
- Let **D** signify that the person in question has the disease.
- Let **H** signify that the person doesnt have the disease (or in other words , is healthy) .

We are asked to determine the following

Q1 The probability of a positive test - $\Pr(P)$

Q2 The probability that they have the disease given that they have tested positive $\Pr(D|P)$

Solution: What Information are we given?

We are told that 5% of the population have this disease We know that D and H are complementary events, so we can work out the probabilities of both.

$$\Pr(D) = 1 - \Pr(H)$$

$$\Pr(D) = 0.05 \qquad \therefore \quad \Pr(H) = 0.95$$

(P and N are complements also)

People who test positive are made up of two groups

- People who test positive and who do have the disease (P and D)
- People who test positive and who dont have the disease (P and H)

$$\Pr(P) \; = \; \Pr(P \text{ and } D) \; + \; \Pr(P \text{ and } H)$$

- A new test has been developed to diagnose a particular disease. If a person has the disease, the test has a 95% chance of identifying them as having the disease.

$$\Pr(P|D) = 0.95$$

- If a person does not have the disease, the test has a 1% chance of identifying them as having the disease.

$$\Pr(P|H) = 0.01$$

The conditional probability is useful here

$$\Pr(A|B) = \frac{\Pr(A \text{ and } B)}{\Pr(B)}$$

We can rearrange it as follows

$$\Pr(A \text{ and } B) = \Pr(A|B) \times \Pr(B)$$

We can now write our equation in terms of all the information we have :

- $\Pr(P \text{ and } D) = \Pr(P|D) \times \Pr(D)$

$$\Pr(P \text{ and } D) = 0.95 \times 0.05 = 0.0475$$

- $\Pr(P \text{ and } H) = \Pr(P|H) \times \Pr(H)$

$$\Pr(P \text{ and } H) = 0.01 \times 0.95 = 0.0095$$

$$\Pr(P) = \Pr(P \text{ and } D) + \Pr(P \text{ and } H)$$
$$\Pr(P) = 0.0475 + 0.0095 = \mathbf{0.057}$$

Solution: Answer to Question 2
The answer to the first question is $\Pr(P) = 0.057$. We still have to compute $\Pr(D|P)$. Now that we have all the information we need, we simply use the Conditional Probability Formula again.

$$\Pr(D|P) = \frac{\Pr(P \text{ and } D)}{\Pr(P)} = \frac{0.0475}{0.057} = \mathbf{0.8333}$$