

0.0.1 CI for Difference in Two Means

A research company is comparing computers from two different companies, X-Cel and Yellow, on the basis of energy consumption per hour. Given the following data, compute a 95% confidence interval for the difference in energy consumption.

Type	sample size	mean	variance
X-cel	17	5.353	2.743
Yellow	17	3.882	2.985

Remark: It is reasonable to believe that the variances of both groups is the same. Be mindful of this.

0.0.2 CI for Difference in Two Means (Example)

From the previous example (comparing X-cel and Yellow) lets compute a 95% confidence interval when the sample sizes are $n_x = 10$ and $n_y = 12$ respectively. (Lets assume the other values remain as they are.)

Type	sample size	mean	variance
X-cel	10	5.353	2.743
Yellow	12	3.882	2.985

The point estimate $\bar{x} - \bar{y}$ remains as 1.469. Also we require that both samples have equal variance. As both X and Y have variances at a similar level, we will assume equal variance.

0.0.3 Computing the Confidence Interval

- Pooled variance s_p^2 is computed as:

$$s_p^2 = \frac{(10-1)2.743 + (12-1)2.985}{(10-1) + (12-1)} = \frac{57.52}{20} = 2.87$$

- Standard error for difference of two means is therefore

$$S.E.(\bar{x} - \bar{y}) = \sqrt{2.87 \left(\frac{1}{10} + \frac{1}{12} \right)} = 0.726$$

- The aggregate sample size is small i.e. 22. The degrees of freedom is $n_x + n_y - 2 = 20$. From Murdoch Barnes tables 7, the quantile for a 95% confidence interval is 2.086.
- The confidence interval is therefore

$$1.469 \pm (2.086 \times 0.726) = 1.4699 \pm 1.514 = (-0.044, 2.984)$$