TEMPLATE EXAM

IMPORTANT NOTES

- Topics **NOT** on the final exam:
  - Counting Techniques - Lecture5
  - Chi-Squared Test - Lecture17
- Students who do well in the final exam will be graded as:
  - Best midterm (15%) + project (10%) + final exam (75%).
    (otherwise the original scheme stands: 30% + 10% + 60%)
  - Thus, you can still do very well in the module (even if the midterms didn't go your way).
  - In order to do well in the final exam, you must set everything out logically using correct symbols etc. (see points in red below).

MODULE CODE: MA4413

SEMESTER: Autumn 2014

MODULE TITLE: Statistics for Computing

DURATION OF EXAM: 2.5 hours

LECTURER: Dr. Kevin Burke

GRADING SCHEME: 100 marks
(60% of module)

INSTRUCTIONS TO CANDIDATE

- **Attempt four** of the six questions (each one carries 25 marks).
- All work must be shown *clearly and logically* using appropriate symbols and probability notation. Failure to do so will *lose marks*.
- Write down the formula you intend to use at each stage *before* filling it in with numbers.
- Formula sheets are provided at the back of this exam paper.
- Statistical tables are available from the invigilators.

Question 1 (25 Marks)

a) **Boxplots:**

- Lecture2
- Tutorial1 Q3, Q4(d)(e)(f)

b) **Data types:**

- Lecture1
- Tutorial1 Q2

c) **Identify parameter & statistic / calculate confidence interval:**

- Lecture1
- Lecture13
- Tutorial1 Q1, Q2
- Tutorial7 Q1, Q2, Q3, Q4, Q5

Question 2 **(25 Marks)**

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a) **Histogram:**

- Lecture1
- Tutorial1 Q4
- Lecture2-Q1

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b) **Inference concerning difference in means ($\mu_1 - \mu_2$):**

- Lecture13
- Lecture14
- Lecture16
- Tutorial7 Q5
- Tutorial8 Q2
- Tutorial10 Q1, Q2, Q3

Question 3 (25 Marks)

a) **Inference concerning one mean (μ):**

- Lecture13
- Lecture14
- Lecture15
- Tutorial7 Q2, Q3, Q6, Q7
- Tutorial9 Q1, Q2, Q3, Q4

b) **Basic probability rules:**

- Lecture3
- Lecture4
- Tutorial1 Q6, Q7

c) **Law of total probability:**

- Lecture4
- Tutorial2 Q2, Q3
- Tutorial5 Q4
- Tutorial6 Q2
- Note: For the exam question you will be told that
  • $X \mid A_1 \sim \text{Normal}(\mu_1, \sigma_1)$ and
  • $X \mid A_2 \sim \text{Normal}(\mu_2, \sigma_2)$.
  $\Rightarrow$ $\Pr(X > x \mid A_1)$ and $\Pr(X > x \mid A_2)$ must be calculated using normal tables.

Question 4 **(25 Marks)**

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a) **RAID:**

- Lecture3 Q3, Q5
- Tutorial1 Q8
- Note: When finding the number of cheap disks required to meet a desired failure probability, you must use logs to solve.
  (see Lecture3 Solutions "Q5(e) - alternative method")

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b) **Binomial:**

- Lecture7
- Tutorial3 Q4, Q5, Q6

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c) **Poisson:**

- Lecture8
- Lecture9 (knowledge of exponential distribution is also needed)
- Tutorial4 Q2

Question 5 (25 Marks)

a) **Random variable basics:**

- Lecture6
- Tutorial3 Q1, Q3(a)

b) **Inference concerning one proportion (p):**

- Lecture13
- Lecture15
- Tutorial7 Q1, Q4(d)
- Tutorial9 Q5, Q6, Q7, Q8

c) **Normal distribution:**

- Lecture10
- Lecture12
- Tutorial5 Q5, Q6
- Tutorial6 Q1(a)(b), Q3(c), Q4(a)(b)

Question 6 (25 Marks)

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a) **Queueing theory:**

- Lecture9
- Tutorial4 Q5, Q6, Q7
- Tutorial5 Q1, Q2, Q3

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b) **Huffman coding:**

- Lecture18
- Tutorial11 Q1, Q2, Q3.

Useful Formulae: Page 1

Histogram:

- class width $= \dfrac{\max(x) - \min(x)}{\text{number of classes}}$

Numerical Summaries:

- $\bar{x} = \dfrac{\sum x_i}{n}$
- $s^2 = \dfrac{\sum x_i^2 - n\,\bar{x}^2}{n-1}$
- Position of Q_k : $\dfrac{n+1}{4} \times k$
- $IQR = Q_3 - Q_1$
- $LF = Q_1 - 1.5 \times IQR$
- $UF = Q_3 + 1.5 \times IQR$

Probability:

- $\Pr(A^c) = 1 - \Pr(A)$
- $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
- $\Pr(E_1 \cup E_2 \cup \cdots \cup E_k) = \Pr(E_1) + \Pr(E_2) + \cdots + \Pr(E_k)$ (if mutually exclusive)
- $\Pr(A \cap B) = \Pr(A)\ \Pr(B \mid A) = \Pr(B)\ \Pr(A \mid B)$
- $\Pr(E_1 \cap E_2 \cap \cdots \cap E_k) = \Pr(E_1)\ \Pr(E_2)\ \cdots\ \Pr(E_k)$ (if independent)
- $\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)} = \dfrac{\Pr(A)\ \Pr(B \mid A)}{\Pr(B)}$
- $$\begin{aligned}\Pr(B) &= \Pr(B \cap E_1) + \Pr(B \cap E_2) + \cdots + \Pr(B \cap E_k) \\ &= \Pr(E_1)\ \Pr(B \mid E_1) + \Pr(E_2)\ \Pr(B \mid E_2) + \cdots + \Pr(E_k)\ \Pr(B \mid E_k)\end{aligned}$$

  (if $E_1, \ldots, E_k$ are mutually exclusive & exhaustive)

Useful Formulae: Page 2

Counting Techniques:

- $n\,! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$

- $\displaystyle \binom{n}{k} = \frac{n\,!}{k\,!\,(n-k)\,!}$

Random Variables:

- $E(X) = \sum x_i \; p(x_i)$

- $E(X^2) = \sum x_i^2 \; p(x_i)$

- $Var(X) = E(X^2) - [E(X)]^2$

- $Sd(X) = \sqrt{Var(X)}$

Distributions:

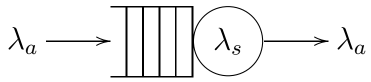
• $X \sim \text{Binomial}(n, p)$	• $X \sim \text{Poisson}(\lambda)$	• $T \sim \text{Exponential}(\lambda)$
• $\Pr(X = x) = \binom{n}{x} p^x \, (1-p)^{n-x}$	• $\Pr(X = x) = \frac{\lambda^x}{x\,!} \, e^{-\lambda}$	• $\Pr(T > t) = e^{-\lambda\, t}$
• $x \in \{0, 1, 2, \ldots, n\}$	• $x \in \{0, 1, 2, \ldots, \infty\}$	• $t \in [0, \, \infty)$
• $E(X) = n\,p$	• $E(X) = \lambda$	• $E(T) = \frac{1}{\lambda}$
• $Var(X) = n\,p\,(1-p)$	• $Var(X) = \lambda$	• $Var(T) = \frac{1}{\lambda^2}$

Note: the normal distribution is shown on the next page

Useful Formulae: Page 3

Queueing Theory:

- $E(N) = \lambda_a \, E(T)$
- $\rho = \dfrac{\lambda_a}{\lambda_s}$
- $M/M/1$ System:



$$\Rightarrow \; T \sim \text{Exponential}(\lambda_s - \lambda_a)$$

(where T is the total time in the system)

Normal Distribution:

- $X \sim \text{Normal}(\mu, \sigma)$
- $E(X) = \mu$
- $Var(X) = \sigma^2$
- $(1-\alpha)100\%$ of the Normal(μ, σ) distribution lies in $\mu \pm z_{\alpha/2}\, \sigma$
- $\Pr(X > x) = \Pr\left(Z > \dfrac{x-\mu}{\sigma}\right)$
- $\Pr(Z < -z) = \Pr(Z > z)$
- $\Pr(Z > -z) = \Pr(Z < z) = 1 - \Pr(Z > z)$
- If $X_1 \sim \text{Normal}(\mu_1, \sigma_1)$ and $X_2 \sim \text{Normal}(\mu_2, \sigma_2)$

$$\Rightarrow \quad \text{Sum:} \quad X_1 + X_2 \sim \text{Normal}\left(\mu_1 + \mu_2, \; \sqrt{\sigma_1^2 + \sigma_2^2}\right)$$

$$\Rightarrow \quad \text{Difference:} \quad X_1 - X_2 \sim \text{Normal}\left(\mu_1 - \mu_2, \; \sqrt{\sigma_1^2 + \sigma_2^2}\right)$$

- For $X_1, \ldots, X_n \sim$ any distribution with $\mu = E(X)$ and $\sigma = Sd(X) = \sqrt{Var(X)}$

$$\Rightarrow \quad \text{Sample mean:} \quad \overline{X} \sim \text{Normal}\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \quad \text{if } n > 30$$

Useful Formulae: Page 4

Statistics and Standard Errors:

Parameter	Statistic	Standard Error	Samples	Details
μ	$\bar{x}$	$\dfrac{s}{\sqrt{n}}$	large / small	$\nu = n - 1$
p	$\hat{p}$	$\sqrt{\dfrac{\hat{p}\,(1-\hat{p})}{n}}$	large	confidence interval
		$\sqrt{\dfrac{p_0\,(1-p_0)}{n}}$	large	hypothesis test
$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$	$\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$	large / small	$\nu = \dfrac{(a+b)^2}{\frac{a^2}{n_1-1} + \frac{b^2}{n_2-1}}$ $a = \dfrac{s_1^2}{n_1}, \quad b = \dfrac{s_2^2}{n_2}$
		$\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}$ where $s_p^2 = \dfrac{(n_1-1)\,s_1^2 + (n_2-1)\,s_2^2}{n_1+n_2-2}$	small	$\nu = n_1 + n_2 - 2$ assuming $\sigma_1^2 = \sigma_2^2$
$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$	$\sqrt{\dfrac{\hat{p}_1\,(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2\,(1-\hat{p}_2)}{n_2}}$	large	confidence interval
		$\sqrt{\dfrac{\hat{p}_c\,(1-\hat{p}_c)}{n_1} + \dfrac{\hat{p}_c\,(1-\hat{p}_c)}{n_2}}$ where $\hat{p}_c = \dfrac{x_1 + x_2}{n_1 + n_2}$	large	hypothesis test

Confidence Intervals:

- Large sample: statistic $\pm\ z_{\alpha/2} \times$ standard error
- Small sample: statistic $\pm\ t_{\nu,\alpha/2} \times$ standard error

Useful Formulae: Page 5

Hypothesis Testing:

- $$z = \frac{\text{statistic} - \text{hypothesised value}}{\text{standard error}}$$

- $$\text{p-value } = \begin{cases} 2 \times \Pr(Z > |z|) & \text{if } H_a : \mu \neq \mu_0 \\ \Pr(Z < z) & \text{if } H_a : \mu < \mu_0 \\ \Pr(Z > z) & \text{if } H_a : \mu > \mu_0 \end{cases}$$

- $$F = \frac{\text{larger variance}}{\text{smaller variance}} = \frac{s^2_{\text{larger}}}{s^2_{\text{smaller}}}$$

$$\nu_1 = n_{\text{top}} - 1, \quad \nu_2 = n_{\text{bottom}} - 1$$

- $$\chi^2 = \sum \frac{(o_i - e_i)^2}{e_i}$$

Goodness-of-fit: $e_i = \text{total} \times p(x_i)$, $\quad \nu = n_f - 1 - k$

Independence: $e_{ij} = \dfrac{r_i \times c_j}{\text{total}}$, $\quad \nu = (n_r - 1) \times (n_c - 1)$

Information Theory:

- $h(x) = -\log_2[p(x)]$

- $H(X) = E[h(X)] = \sum h(x_i)\, p(x_i)$

- $l(x_i) = \text{code-length for character } x_i$

- $E(L) = \sum l(x_i)\, p(x_i)$

- $$e = \frac{H(X)}{E(L)}$$

- $\sum 2^{-l(x_i)} \leq 1$