Question 1

You develop a random number generater which assigns a value to the random variable X according to the following probability distribution:

x	0.0	0.5	1.0	2.0	3.0
$\Pr(X = x)$	0.4	0.2	0.15	0.15	?

(a) What is value the value of $\Pr(X = 3.0)$? **(b)** Calculate $E(X)$ and $Sd(X)$. **(c)** You produce a gambling game where the player wins (in euro) the value of X generated, e.g., if a 2.0 appears, €2 is won. How much should you charge for a play of this game so that that *you* (the programmer) make a profit of €0.10 on average per game? (i.e., the player *loses* €0.10 on average) **(d)** Using your answer to part (c), determine the probability that you make a profit when somebody plays this game. **(e)** If 10 people play this game, what is the probability that you make a profit 8 times?

Question 2

You flip three coins. Let $X =$ "the number of heads" and $Y =$ "the number of unique faces".

(a) What is the sample space for this experiment? **(b)** Construct the *joint distribution* for X and Y. **(c)** Based on this joint distribution, construct the *marginal* distribution for X and for Y. **(d)** Are X and Y independent? **(e)** Calculate $E(Y)$ and $Sd(Y)$. **(f)** Calculate $\Pr(Y = 2 \mid X = 2)$ and interpret its value (compare with $\Pr(Y = 2)$).

Question 3

Let $X =$ "the attack power of player 1" and let $Y =$ "the attack power of player 2".

Let the probability distributions for X and Y be:

x	0	100	300
$\Pr(X = x)$	0.2	0.75	0.05

y	0	80	200
$\Pr(Y = y)$	0.1	0.6	0.3

(e.g., p1 misses 20% of the time, deals 100 points of damage 75% of the time and performs a critical blow 5% of the time.)

(a) What is the average attack power of each player? **(b)** If both players have 1000 hit-points, how many attacks does it take for player 1 to defeat player 2 and vice versa? Which player will win on average? **(c)** Let's now assume that player 1 uses his/her *first* turn to cast a spell (and therefore does not attack in this turn). The result of the spell is that player 2 can no longer perform a critical blow, i.e., $\Pr(Y = 200) = 0$, *from turn two onwards.* Since setting $p(200) = 0$ leads to $\sum p(y) \neq 1$, assume that the remaining probability ($= 0.3$) is distributed evenly between $p(0)$ and $p(80)$. What is the outcome of the battle now?

Question 4

You flip a coin 10 times - let $X =$ "the number of heads". Using the binomial probability function, calculate the following:

(a) $\Pr(X = 2)$. **(b)** $\Pr(X = 0)$. **(c)** $\Pr(X > 2)$. **(d)** $\Pr(X \leq 3)$. **(e)** $\Pr(5 \leq X \leq 7)$. **(f)** $E(X)$ and $Sd(X)$. **(g)** Using the binomial tables, calculate $\Pr(X \leq 10)$ in the case where the coin is flipped 20 times. **(h)** If the coin is flipped 50 times, what is $E(X)$?

Question 5

Repeat Question 4 (a) - (e) but now using the binomial tables.

Question 6

Let's assume that a sequence of bits (binary numbers) is transmitted and, at the other end, decoded; the decoder has a 10% chance reading a bit incorrectly (i.e., reading a 0 as 1 or vice versa). Let X be the number of errors in the sequence received (i.e., the decoded sequence). Calculate the probability that there are:

(a) *No* errors in a 20-bit string. **(b)** Less than three errors in a 10-bit string. **(c)** More than 10 errors in (i) a 50-bit string and (ii) a 100-bit string (hint: use tables). **(d)** Calculate the average number of errors in a 100-bit string. Calculate the standard deviation also.

Question 7

We follow on from Question 6 but now consider the case where, to reduce the probability of error, each bit is sent *three* times and then a "majority vote" approach is used to determine the value of each received bit. The following example explains the situation:

Sent	$\overbrace{000}^{0}$	$\overbrace{111}^{1}$	$\overbrace{111}^{1}$	$\overbrace{000}^{0}$
Received	$\underbrace{001}_{0}$	$\underbrace{111}_{1}$	$\underbrace{010}_{0}$	$\underbrace{000}_{0}$

$\Rightarrow$ there is one error in decoding the first 000, but since the majority result is taken, this bit is correctly identified as a 0. There are two errors in decoding the second 111, so this bit is misread as a 0. It is clear that a character is misread if the decoder makes *two or three errors* in these blocks of three replicates.

(a) Show that sending each bit 3 times reduces the error probability from 10% to 2.8%.
(b) Using this reduced value, $p = 0.028$, calculate the probability that there are no errors in a 20-bit string. Compare this result to Q6(a). **(c)** Now assume that each bit is sent 5 times and, again, the majority vote approach is used. Calculate the probability that there are no errors in a 20-bit string in this case.