Complex Numbers: Tutorial Sheet 2

1. Compute real and imaginary part of z where

$$z = \frac{i-4}{2i-3}.$$

2. Compute the absolute value and the conjugate of

   (a) $z = (1+i)^6$

   (b) $z = (i)^{17}$

3. Write in algebraic form $(a + ib)$ the following complex numbers

   (a) $z = i^5 + i + 1$ (b) $z = (3+3i)^8$ (c) $w = (i)^{17}$

4. Write in trigonometric form $(a(\cos\theta + i\sin\theta))$ the following complex numbers

   (a) 8 (b) $6i$ (c) $\cos(\frac{\pi}{3}) - i\sin(\frac{\pi}{3})^7$

5. Simplify the following expressions

   (a)

$$\frac{1+i}{1-i} \; - \; (1+2i)(2+2i) \; + \; \frac{3-i}{1+i}$$

   (b)

$$(2i(i-1) \; + \; (\sqrt{3}+1)^2 + \; (1+1)(1+i$$

6. Compute the square roots of $z = -1 - i$.

7. Compute the cube roots of $z = -8$.

8. Prove that there is no complex number such that $|z| - z = i$.

9. Find $z \in \mathbb{C}$ such that

   (a) $\bar{z} = i(z-1)$ (b) $z^2 \cdot \bar{z} = z$ (c) $|z + 3i| = 3|z|$:

10. Find $z \in \mathbb{C}$ such that $z^2 \in \mathbb{R}$.

11. Find $z \in \mathbb{C}$ such that

    (a) $Re(z(1+i)) + z\bar{z} = 0$

    (b) $Re(z(1+i)) + z\bar{z} + i\,\mathrm{Im}(\bar{z}(1+2i)) = -3$

    (c) $\mathrm{Im}((2-i)z) = 1.$

12. Find $a \in R$ such that $z = -i$ is a root for the polynomial $P(z) = z3 - z2 + z + 1 + a$. Furthermore, for

    such value of a find the factors of $P(z)$ in $\mathbb{R}$ and in $\mathbb{C}$.