Question 1

a) $$\Pr(Z > 0.83) = 0.2033.$$

b) $$\begin{aligned}\Pr(Z < 1.05) &= 1 - \Pr(Z > 1.05)\\ &= 1 - 0.1469\\ &= 0.8531.\end{aligned}$$

c) $$\begin{aligned}\Pr(1 < Z < 2) &= \Pr(Z > 1) - \Pr(Z > 2)\\ &= 0.1587 - 0.02275\\ &= 0.13595.\end{aligned}$$

d) $$\begin{aligned}\Pr(Z < -1.8) &= \Pr(Z > 1.8)\\ &= 0.0359.\end{aligned}$$

e) $$\begin{aligned}\Pr(-1 < Z < 1) &= \Pr(Z > -1) - \Pr(Z > 1)\\ &= \Pr(Z < 1) - \Pr(Z > 1)\\ &= 1 - \Pr(Z > 1) - \Pr(Z > 1)\\ &= 1 - 2\,\Pr(Z > 1)\\ &= 1 - 2\,(0.1587)\\ &= 0.6826.\end{aligned}$$

f) For this we look in the body of the table and then find the z value corresponding to $\Pr(Z > z) = 0.1$. We see that:

$$\Pr(Z > 1.28) = 0.1003$$
and
$$\Pr(Z > 1.29) = 0.0985$$

Thus, since 0.1003 is close to 0.1, we may say that $z = 1.28$ leads to $\Pr(Z > z) = 0.1$.

Alternatively we may say that the value lies between 1.28 and 1.29 and so choose the midpoint: $\frac{1.28+1.29}{2} = 1.285$.

Question 2

For this we will need to convert to a Z score using the formula $Z = \frac{X-\mu}{\sigma} = \frac{X-12}{0.1}$

a) $$\begin{aligned}\Pr(X > 12.15) &= \Pr(Z > \tfrac{12.15-12}{0.1})\\ &= \Pr(Z > 1.5)\\ &= 0.0668.\end{aligned}$$

b) $$\begin{aligned}\Pr(X < 12.38) &= \Pr(Z < \tfrac{12.38-12}{0.1})\\ &= \Pr(Z < 3.8)\\ &= 1 - \Pr(Z > 3.8)\\ &= 1 - 0.000072\\ &= 0.999928.\end{aligned}$$

c) $$\begin{aligned}&\Pr(11.85 < X < 12.15)\\ &= \Pr(X > 11.85) - \Pr(X > 12.15)\\ &= \Pr(Z > \tfrac{11.85-12}{0.1}) - 0.0668\\ &= \Pr(Z > -1.5) - 0.0668\\ &= \Pr(Z < 1.5) - 0.0668\\ &= 1 - \Pr(Z > 1.5) - 0.0668\\ &= 1 - 0.0668 - 0.0668\\ &= 0.8664.\end{aligned}$$

d) $$\begin{aligned}\Pr(X < x) &= 0.9\\ \Pr(Z < \tfrac{x-12}{0.1}) &= 0.9\\ 1 - \Pr(Z < \tfrac{x-12}{0.1}) &= 1 - 0.9\\ \Pr(Z > \tfrac{x-12}{0.1}) &= 0.1.\end{aligned}$$

From the tables we see $\Pr(Z > 1.28) = 0.1003$.

$$\begin{aligned}\Rightarrow \tfrac{x-12}{0.1} &= 1.28\\ x - 12 &= 1.28(0.1)\\ x &= 1.28(0.1) + 12\\ x &= 12.128.\end{aligned}$$

e) $$\begin{aligned}\Pr(X < x) &= 0.1\\ \Pr(Z < \tfrac{x-12}{0.1}) &= 0.1\\ \Pr(Z > -\tfrac{x-12}{0.1}) &= 0.1.\end{aligned}$$

Since $\Pr(Z > 1.28) = 0.1003$, we have

$$\begin{aligned}-\tfrac{x-12}{0.1} &= 1.28\\ \tfrac{x-12}{0.1} &= -1.28\\ x - 12 &= -1.28(0.1)\\ x &= -1.28(0.1) + 12\\ x &= 11.872.\end{aligned}$$