

1. Large and small bottles of water are manufactured in a factory. The volume of water in the large bottles follows a Normal distribution with mean 1.5 litres and standard deviation 0.01. The volume of water in the small bottles follows an independent Normal distribution with mean 0.5 litres and standard deviation 0.008.
 - (a) One large bottle and three small bottles are chosen at random. What is the probability that the volume in the large bottle is greater than the total volume in the three small bottles?
 - (b) Find the distribution of the total volume of 10 large bottles and 3 small bottles.
2. In a certain large population, the heights X in cm of adult males have a Normal distribution with mean 170 and variance 100.

Find the probability that a randomly chosen man is within 5 cm of the population average height.

 - (ii) Find the probability that a randomly chosen man is more than 185 cm tall.
 - (iii) Given that a randomly chosen man is more than 185 cm tall, find the probability that he is more than 189.6 cm tall.

Four men are chosen at random from this population.

 - (a)
 - (i) Find the probability that all four are within 5 cm of the population average height.
 - (ii) Find the probability that exactly two of the four are within 5 cm of the population average height.
 - (iii) Find the probability that the average height of the sample of four is within 5 cm of the population average height.
 - (b) You are given that $P(X > 189.6) = 0.025$. Given a random sample of 100 men from the population, let Y denote the number of men in the sample who are over 189.6 cm tall. State
 - (i) the exact distribution of Y ,
 - (ii) a suitable approximation to the exact distribution of Y . Use this approximate distribution to find the value of $P(Y > 3)$.
3. A manufacturer produces bags of sweets. The weights of the bags are independently distributed with mean 30 g and standard deviation 5 g. A random sample of 50 bags of sweets is taken.
 - (a) State the approximate distribution of the mean weight of the bags in the sample, quoting any results you use.
 - (b) Use this distribution to find the probability that the mean weight of the bags in the sample is less than 29 g.
 - (c) Suppose instead a random sample of size n bags is taken. It is desired that the probability that the mean weight of bags in the sample is below 29 g is less than 0.05. Find the minimum required value of n .
4. The random variables $X_i, i = \{1, \dots, n\}$, are independent and are Normally distributed with means μ_i and variances σ_i^2 . Their total is given by $T = \sum_{i=1}^n X_i$

- (a) Write down $E(T)$.
 - (b) Write down $\text{Var}(T)$.
 - (c) What is the distribution of T ?
5. A company packs parcels of books. Suppose the weights of hardback books are Normally distributed with mean 1.5 kg and standard deviation 0.5 kg and the weights of paperback books are Normally distributed with mean 0.8 kg and standard deviation 0.3 kg. Assume the weights are all independent of each other.
- (a) A parcel has three hardback books and six paperback books. Find the probability that the parcel weighs more than 11 kg.
 - (b) Find the probability that three hardback books weigh more than six paperback books.
 - (c) A pallet of hardback books is to be moved by a fork-lift truck. Show that the maximum number of books which can be put on the pallet, so that there is a probability of no more than 0.01 that the combined weight of the books exceeds 100 kg, is given by the largest integer n that satisfies the inequality $1.5n + 1.1632\sqrt{n} - 100 < 0$.