0.1 Independent Events

Two events, A and B, are independent if the fact that A occurs does not affect the probability of B occurring.

Some other examples of independent events are:

- Landing on heads after tossing a coin AND rolling a 5 on a single 6-sided die.
- Choosing a marble from a jar AND landing on heads after tossing a coin.
- Choosing a 3 from a deck of cards, replacing it, AND then choosing an ace as the second card.
- Rolling a 4 on a single 6-sided die, AND then rolling a 1 on a second roll of the die.

Multiplication Rules

When two events, A and B, are independent, the probability of both occurring is:

$$P(A \cap B) = P(A) \times P(B)$$

Similarly for events A,B and C, the probability of all three events occurring is:

$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$

0.2 Independent Events

Events A and B in a probability space S are said to be independent if the occurrence of one of them does not influence the occurrence of the other.

More specifically, B is independent of A if $P(B)$ is the same as $P(B|A)$. Now substituting $P(B)$ for $P(B|A)$ in the multiplication theorem from the previous slide yields.

$$P(A \cap B) = P(A) \times P(B)$$

We formally use the above equation as our definition of independence.

Independent Events

- Two events are independent if the occurrence of one does not change the probability of the other occurring.
- An example would be rolling a 2 on a die and flipping a head on a coin. Rolling the 2 does not affect the probability of flipping the head.
- If events are independent, then the probability of them both occurring is the product of the probabilities of each occurring.

$$P(A \cap B) = P(A) \times P(B)$$

Independent events

- Two events are independent when the occurrence or nonoccurrence of one event has no effect on the probability of occurrence of the other event.
- Two events are dependent when the occurrence or nonoccurrence of one event does affect the probability of occurrence of the other event.

0.2.1 Independent Events

- Suppose that a man and a woman each have a pack of 52 playing cards.
- Each draws a card from his/her pack. Find the probability that they each draw a Queen.
- We define the events:
  - A = probability that man draws a Queen = 4/52 = 1/13
  - B = probability that woman draws a Queen = 1/13
- Clearly events A and B are independent so:

$$P(A \cap B) = 1/13 \times 1/13 = 0.005917$$

0.2.2 Independent Events

Events A and B in a probability space S are said to be independent if the occurrence of one of them does not influence the occurrence of the other.

More specifically, B is independent of A if $P(B)$ is the same as $P(B|A)$. Now substituting $P(B)$ for $P(B|A)$ in the multiplication theorem from the previous slide yields.

$$P(A \cap B) = P(A) \times P(B)$$

We formally use the above equation as our definition of independence.

0.3 Mutually exclusive events

Two or more events are mutually exclusive, or disjoint, if they cannot occur together. That is, the occurrence of one event automatically precludes the occurrence of the other event (or events). For instance, suppose we consider the two possible events “ace” and “king” with respect to a card being drawn from a deck of playing cards. These two events are mutually exclusive, because any given card cannot be both an ace and a king. Two or more events are nonexclusive when it is possible for them to occur together.
Note that this definition does not indicate that such events must necessarily always occur jointly. For instance, suppose we consider the two possible events “ace” and “spade”. These events are not mutually exclusive, because a given card can be both an ace and a spade; however, it does not follow that every ace is a spade or every spade is an ace.

- Two events are mutually exclusive if they cannot both happen.
- A simple example is the set of outcomes of a single coin toss, which can result in either heads or tails, but not both.
- Two events are mutually exclusive (or disjoint) if it is impossible for them to occur together.
- Formally, two events A and B are mutually exclusive if and only if $A \cap B = \varnothing$

Consider our die example

- Event A = 'observe an odd number' = $\{1, 3, 5\}$
- Event B = 'observe an even number' = $\{2, 4, 6\}$
- $A \cap B = \varnothing$ (i.e. the empty set), so A and B are mutually exclusive.

0.4 Mutually Exclusive Events

Mutually exclusive events are events that cannot happen at the same time.

$$P(AandB) = P(A) + P(B)$$

Mutually Exclusive Events

- Two events are mutually exclusive (or disjoint) if it is impossible for them to occur together.
- Formally, two events A and B are mutually exclusive if and only if $A \cap B = \varnothing$

Consider our die example

- Event A = 'observe an odd number' = $\{1, 3, 5\}$
- Event B = 'observe an even number' = $\{2, 4, 6\}$
- $A \cap B = \varnothing$ (i.e. the empty set), so A and B are mutually exclusive.
- In a given observation or experiment, an event must either occur or not occur.
- Therefore, the sum of the probability of occurrence plus the probability of nonoccurrence always equals 1.
- Thus, where A' indicates the nonoccurrence of event A, we have $P(A) + P(A^c) = 1$
- Two or more events are mutually exclusive, or disjoint, if they cannot occur together. That is, the occurrence of one event automatically precludes the occurrence of the other event (or events).
- For instance, suppose we consider the two possible events "ace" and "king" with respect to a card being drawn from a deck of playing cards.

- These two events are mutually exclusive, because any given card cannot be both an ace and a king.
- Two or more events are nonexclusive when it is possible for them to occur together.
- Note that this definition does not indicate that such events must necessarily always occur jointly. For instance, suppose we consider the two possible events "ace" and "spade".
- These events are not mutually exclusive, because a given card can be both an ace and a spade; however, it does not follow that every ace is a spade or every spade is an ace.

Mutually Exclusive Events

- Suppose you roll a 6 sided die.
- Let **A** be the event that the number is odd and **B** be the event that the number is even.
- Since the die is only rolled once, it is impossible for the number that lands face up is both odd and even.
- The events **A** and **B** are said to be mutually exclusive events.
- If two events cannot happen at the same time, they are said to be mutually exclusive.

Mutually Exclusive Events

- Two events are **mutually exclusive** if they cannot occur together.
- Another way of expressing mutually events is **disjoint** events.
- If two events are mutually exclusive, then the probability of them both occurring at the same time is 0. Disjoint:
$$P(A \cap B) = 0$$
- If two events are mutually exclusive, then the probability of either occurring is the sum of the probabilities of each occurring.
- Specific Addition Rule: Only valid when the events are mutually exclusive.
$$P(A \cup B) = P(A) + P(B)$$

0.5 Independence Vs Mutual Exclusion

Independence Vs Mutual Exclusion

Do not mix up the ideas of independence and mutual exclusion.

- **Independent events**
  - Have *no effect* on each other.
  - *Can* happen at the same time (but work *independently* of each other).

  - Allow us to simplify the **multiplication rule**.

- **Mutually exclusive events**
  - *Cannot* happen at the same time.
  - Certainly *affect* each other since the presence of one excludes the presence of the other.
  - Allow us to simplify the **addition rule**.

Bottom line: If events are independent they are not mutually exclusive. If events are mutually exclusive they are not independent.

Classify the following pairs of events as being mutually exclusive, independent or dependent (but not mutually exclusive).

	Event A	Event B
a)	A coin shows a head	The same coin shows a tail
b)	You work hard	You get promoted
c)	You are Irish	It rains in Japan
d)	Anti-virus out of date	Laptop is virus-free
e)	You are in this lecture hall	You are in Scholars
f)	You are in this lecture	You are on Facebook
g)	An individual is not wealthy	He/she drives an expensive car
h)	One coin shows a head	Another coin shows a head

Addition Rule (Continued) For mutually exclusive events, that is events which cannot occur together: $P(A \cap B) = 0$.
The addition rule therefore reduces to

$$P(A \cup B) = P(A) + P(B)$$