0.1 Exponential Distribution

Question 3

A particular brand of hard disk is designed to last an average of 2 years. Assume that its lifetime is $T \sim \text{Exponential}(\lambda)$.

(a) What is the value of λ?

(b) What is $Sd(T)$?

(c) Calculate $\Pr(T > 1)$.

(d) Calculate $\Pr(T < 5)$.

(e) Calculate $\Pr(2 < T < 5)$.

(f) Calculate the value of t such that 80% of hard disks fail before this time, i.e., $\Pr(T > t) = 0.2$.

Question 4

Let $X \sim \text{Exponential}(\lambda = 0.02)$. Calculate the following:

(a) $\Pr(\overline{X} > 55)$ in a group of 100.

(b) $\Pr(\overline{X} < 53)$ in a group of 40.

(c) The value of $\bar{x}$ such that $\Pr(\overline{X} > \bar{x}) = 0.1$ when $n = 65$.

(c) The value of n if $\Pr(\overline{X} < 49) = 0.1$.

Question 5

The *average time* between customers arriving to a shop is 5 minutes. We will assume that the time, T, has an exponential distribution. Calculate the following:

(a) The average arrival *rate*, i.e., λ customers per minute.

(b) The probability that we wait more than 15 minutes for the next customer.

(c) The probability that the next customer arrives within 1 minute.

(d) The average *number of customers* in a 1 hour period. What is the standard deviation that goes with this average?

(e) The probability that *15 or more* customers arrive in a 1 hour period.

0.2 Exponential Distribution Problem

$$P(X > 15) = e^{-15\lambda} = e^{-3/2} = 0.22$$

What is the probability that a customer will spend more than 15 minutes in the bank given that he is still in the bank after 10 minutes?

$$P(X > 15|X > 10) = P(X > 5) = e = e^{-3/2} = 0.604$$

0.3 The exponential distribution

The average lifespan ppf a laptop is 5 year. You may assume that the lifespan of laptop computers follows an exponential distribution.

1. What is the probability that the lifespan of the laptop will be at least 6 years.

$$e^{-6/5} = 0.3011942$$

2. What is the probability that the lifespan of the laptop will not exceed 4 years.

$$e^{-4/5} = 0.449329$$

3. What is the probability that the lifespan of the laptop will be between 5 years and 6 years.

$$e^{-5/5} = 0.3678794$$

Question 1

Jobs are sent to a supercomputer at a rate of 10 per hour and take the supercomputer on average 4 minutes to process. We will assume that the number of arrivals is $X_a \sim \text{Poisson}(\lambda_a)$ and the processing (i.e., service) time is $T_s \sim \text{Exponential}(\lambda_s)$. This leads to an $M/M/1$ system.

(a) Let T be the total time in the system - what distribution has T? **(b)** What is the average time spent in the system? Calculate $Sd(T)$ also. **(c)** How many jobs are in the system on average? (hint: Little's law) **(d)** From the time the job is sent, what is the probability that it takes more than 15 minutes to complete? **(e)** From the time the job enters the processor (i.e., service component), what is the probability that it takes more than 15 minutes to complete? **(f)** What is the average number of jobs completed in a 3 hour period of operation? (hint: Burke's theorem) **(g)** What is the probability that more than 40 jobs are completed in a 3 hour period? (hint: Burke's theorem again)

Question 5

The average lifespan of a PC monitor is 6 years. You may assume that the lifespan of monitors follows an exponential probability distribution.

(i) What is the probability that the lifespan of the monitor will be at least 5 years?

(ii) What is the probability that the lifespan of the monitor will not exceed 4 years?

(iii) What is the probability of the lifespan being between 5 years and 7 years?

[For exponential distributions, with mean duration : P(X k) = 1- e(-k/)]