Statistics for Computing MA4413

Midterm Examination 1

Type D

- Do not turn over the page until instructed to do so.
- Rough work pages are provided within.
- Useful formulae and Binomial tables are provided at the back.
- **Enter your answers (using an "X") in the table on the last page.**
- There are 15 questions in total: each correct answer = 1% (*there are no negative marks*).
- For each question, only *one* answer is correct.
- Scientific calculators approved by the University of Limerick can be used.

Questions 1 - 5

A company have developed a new type of CPU (in total 50,000 have been manufactured). It is believed this CPU can perform a particular benchmark task in 1 second. In order to test this hypothesis, 45 of these CPUs were randomly selected. It was found that the average time to complete the task was 0.91 seconds.

Q1 What type of data was collected?

(a) numeric discrete **(b)** random **(c)** numeric continuous **(d)** categorical

Q2 What is the parameter here?

(a) $\mu =$ unknown **(b)** $p =$ unknown **(c)** $\mu = 1$ **(d)** $n = 50,000$

Q3 What is the statistic here?

(a) $\mu = 1$ **(b)** $\hat{p} = \frac{45}{50{,}000}$ **(c)** $\hat{p} = \frac{0.91}{45}$ **(d)** $\bar{x} = 0.91$

Consider the following sample of ages of mechanical components:

3	3	4	2	1	8

Q4 What is the value of the standard deviation for this sample?

(a) $s = 2.43$ years **(b)** $s^2 = 5.90$ years^2 **(c)** $s = 1.83$ years **(d)** $\sigma = 2.43$ years

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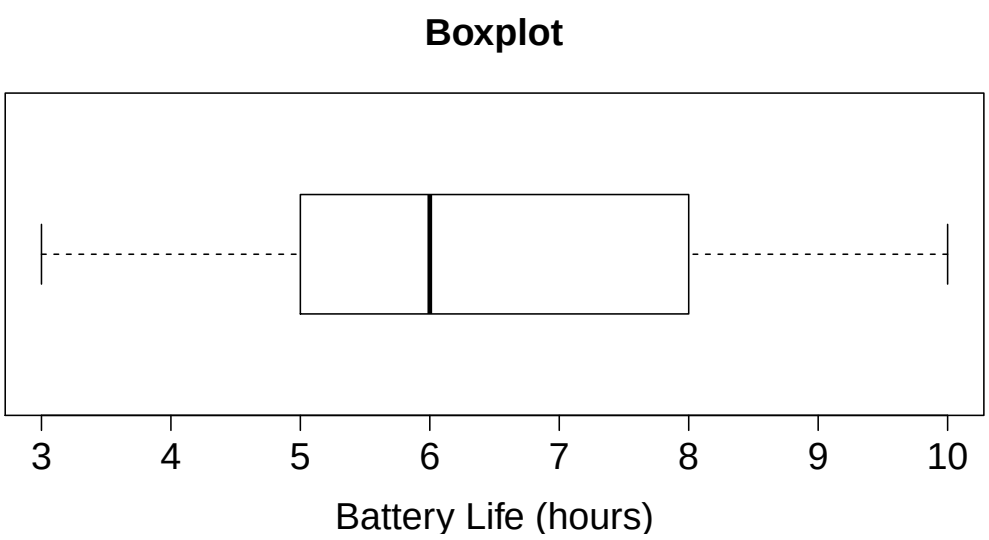


Q5 Based on the above boxplot, what is the value of the IQR?

(a) 4 hours **(b)** 7 hours **(c)** 3 hours **(d)** 1 hour

Rough Work

Next page: Questions 6 - 10

Questions 6 - 10

Consider the following set of numbers:

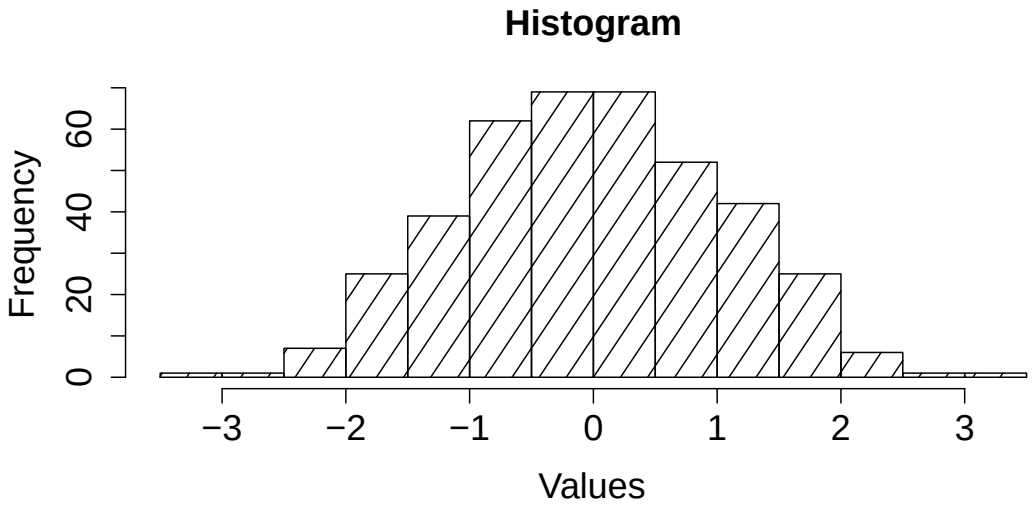
8	4	14	23	11	13	15	13	12

Q6 What is the value of the median?

(a) 12.5 **(b)** 5 **(c)** 13 **(d)** 11

Q7 How many outliers are there?

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3



Q8 Based on the above histogram, which of the following is likely to be true?

(a) $\bar{x} > Q_2$ **(b)** $\bar{x} < Q_2$ **(c)** $\bar{x} \approx Q_2$ **(d)** $\bar{x} \approx \mu$

A laptop manufacturer uses two types of keyboard: Type-1 is used 80% of the time and Type-2 is used 20% of the time. It is known that 10% of Type-1 keyboards are faulty and 30% of Type-2 keyboards are faulty.

Q9 What is the probability that a randomly selected keyboard will be faulty?

(a) 0.26 **(b)** 0.19 **(c)** 0.4 **(d)** 0.14

Q10 A keyboard is tested and found to be *working*; what is the probability that it is Type-2?

(a) 0.84 **(b)** 0.43 **(c)** 0.16 **(d)** 0.7

Rough Work

Next page: Questions 11 - 15

Questions 11 - 15

Q11 Let $\Pr(A) = 0.2$, $\Pr(B) = 0.4$ and $\Pr(A \cap B) = 0.15$. What is the value of $\Pr(A^c \cap B^c)$?

(a) 0.4 **(b)** 0.85 **(c)** 0.55 **(d)** 0.48

Q12 You have 11 t-shirts. You're going on holidays and can only bring 5 of them. How many possible groups of 5 t-shirts are there if one of them is your "lucky" t-shirt and you *must* bring it?

(a) 210 **(b)** 5040 **(c)** 330 **(d)** 462

Consider a random number generator which assigns a value to X according to the following probability distribution:

x	0	1	3	8
$\Pr(X = x)$	0.2	0.6	?	0.1

Q13 What is the value of $E(X)$?

(a) 1.4 **(b)** 0.1 **(c)** 1.7 **(d)** 2.2

There is an 8% chance of pressing the wrong button on a keyboard and thus make a typographical error. Assuming these errors occur independently, the number of errors in typing n letters is $X \sim \text{Binomial}(n, p)$.

Q14 You type 14 letters. What is the probability of making *less than* 2 errors?

(a) 0.3788 **(b)** 0.2141 **(c)** 0.9042 **(d)** 0.6900

Q15 You type 50 letters. What is $\Pr(X > 9)$?

(a) 0.0056 **(b)** 0.0110 **(c)** 0.0167 **(d)** 0.0039

Rough Work

Don't forget to enter your answers on the last page!

Useful Formulae: Page 1

Histogram:

- class width $= \dfrac{\max(x) - \min(x)}{\text{number of classes}}$

Numerical Summaries:

- $\bar{x} = \dfrac{\sum x_i}{n}$
- $s^2 = \dfrac{\sum x_i^2 - n\,\bar{x}^2}{n-1}$
- Position of Q_k : $\dfrac{n+1}{4} \times k$
- $IQR = Q_3 - Q_1$
- $LF = Q_1 - 1.5 \times IQR$
- $UF = Q_3 + 1.5 \times IQR$

Probability:

- $\Pr(A^c) = 1 - \Pr(A)$
- $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
- $\Pr(E_1 \cup E_2 \cup \cdots \cup E_k) = \Pr(E_1) + \Pr(E_2) + \cdots + \Pr(E_k)$ (if mutually exclusive)
- $\Pr(A \cap B) = \Pr(A)\ \Pr(B \mid A) = \Pr(B)\ \Pr(A \mid B)$
- $\Pr(E_1 \cap E_2 \cap \cdots \cap E_k) = \Pr(E_1)\ \Pr(E_2) \cdots \Pr(E_k)$ (if independent)
- $\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)} = \dfrac{\Pr(A)\ \Pr(B \mid A)}{\Pr(B)}$
- If $E_1, \ldots, E_k$ are mutually exclusive & exhaustive

  $\Rightarrow \Pr(B) = \Pr(B \cap E_1) + \Pr(B \cap E_2) + \cdots + \Pr(B \cap E_k)$

  $= \Pr(E_1)\ \Pr(B \mid E_1) + \Pr(E_2)\ \Pr(B \mid E_2) + \cdots + \Pr(E_k)\ \Pr(B \mid E_k)$

Useful Formulae: Page 2

Counting Techniques:

- $n\,! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$
- $\binom{n}{k} = \dfrac{n\,!}{k\,!(n-k)\,!}$

Random Variables:

- $E(X) = \sum x_i \; p(x_i)$
- $E(X^2) = \sum x_i^2 \; p(x_i)$
- $Var(X) = E(X^2) - [E(X)]^2$
- $Sd(X) = \sqrt{Var(X)}$

Binomial Distribution:

- $X \sim \text{Binomial}(n, p)$
- $\Pr(X = x) = \binom{n}{x} p^x \, (1-p)^{n-x}$
- $x \in \{0, 1, 2, \ldots, n\}$
- $E(X) = n\,p$
- $Var(X) = n\,p\,(1-p)$

Table 1 Cumulative Binomial Probabilities

p = probability of success in a single trial; n = number of trials. The table gives the probability of obtaining r *or more* successes in n independent trials. That is

$$\sum_{x=r}^{n} \binom{n}{x} p^x (1-p)^{n-x}$$

When there is no entry for a particular pair of values of r and p, this indicates that the appropriate probability is less than 0.000 05. Similarly, except for the case $r = 0$, when the entry is exact, a tabulated value of 1.0000 represents a probability greater than 0.999 95.

	p =	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$n = 2$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.0199	.0396	.0591	.0784	.0975	.1164	.1351	.1536	.1719
	2	.0001	.0004	.0009	.0016	.0025	.0036	.0049	.0064	.0081
$n = 5$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.0490	.0961	.1413	.1846	.2262	.2661	.3043	.3409	.3760
	2	.0010	.0038	.0085	.0148	.0226	.0319	.0425	.0544	.0674
	3		.0001	.0003	.0006	.0012	.0020	.0031	.0045	.0063
	4						.0001	.0001	.0002	.0003
$n = 10$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.0956	.1829	.2626	.3352	.4013	.4614	.5160	.5656	.6106
	2	.0043	.0162	.0345	.0582	.0861	.1176	.1517	.1879	.2254
	3	.0001	.0009	.0028	.0062	.0115	.0188	.0283	.0401	.0540
	4			.0001	.0004	.0010	.0020	.0036	.0058	.0088
	5					.0001	.0002	.0003	.0006	.0010
	6									.0001
$n = 20$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.1821	.3324	.4562	.5580	.6415	.7099	.7658	.8113	.8484
	2	.0169	.0599	.1198	.1897	.2642	.3395	.4131	.4831	.5484
	3	.0010	.0071	.0210	.0439	.0755	.1150	.1610	.2121	.2666
	4		.0006	.0027	.0074	.0159	.0290	.0471	.0706	.0993
	5			.0003	.0010	.0026	.0056	.0107	.0183	.0290
	6				.0001	.0003	.0009	.0019	.0038	.0068
	7						.0001	.0003	.0006	.0013
	8								.0001	.0002
$n = 50$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.3950	.6358	.7819	.8701	.9231	.9547	.9734	.9845	.9910
	2	.0894	.2642	.4447	.5995	.7206	.8100	.8735	.9173	.9468
	3	.0138	.0784	.1892	.3233	.4595	.5838	.6892	.7740	.8395
	4	.0016	.0178	.0628	.1391	.2396	.3527	.4673	.5747	.6697
	5	.0001	.0032	.0168	.0490	.1036	.1794	.2710	.3710	.4723
	6		.0005	.0037	.0144	.0378	.0776	.1350	.2081	.2928
	7		.0001	.0007	.0036	.0118	.0289	.0583	.1019	.1596
	8			.0001	.0008	.0032	.0094	.0220	.0438	.0768
	9				.0001	.0008	.0027	.0073	.0167	.0328
	10					.0002	.0007	.0022	.0056	.0125
	11						.0002	.0006	.0017	.0043
	12							.0001	.0005	.0013
	13								.0001	.0004
	14									.0001

Table 1 Cumulative Binomial Probabilities – continued

	p =	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$n = 100$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.6340	.8674	.9524	.9831	.9941	.9979	.9993	.9998	.9999
	2	.2642	.5967	.8054	.9128	.9629	.9848	.9940	.9977	.9991
	3	.0794	.3233	.5802	.7679	.8817	.9434	.9742	.9887	.9952
	4	.0184	.1410	.3528	.5705	.7422	.8570	.9256	.9633	.9827
	5	.0034	.0508	.1821	.3711	.5640	.7232	.8368	.9097	.9526
	6	.0005	.0155	.0808	.2116	.3840	.5593	.7086	.8201	.8955
	7	.0001	.0041	.0312	.1064	.2340	.3936	.5557	.6968	.8060
	8		.0009	.0106	.0475	.1280	.2517	.4012	.5529	.6872
	9		.0002	.0032	.0190	.0631	.1463	.2660	.4074	.5506
	10			.0009	.0068	.0282	.0775	.1620	.2780	.4125
	11			.0002	.0022	.0115	.0376	.0908	.1757	.2882
	12				.0007	.0043	.0168	.0469	.1028	.1876
	13				.0002	.0015	.0069	.0224	.0559	.1138
	14					.0005	.0026	.0099	.0282	.0645
	15					.0001	.0009	.0041	.0133	.0341
	16						.0003	.0016	.0058	.0169
	17						.0001	.0006	.0024	.0078
	18							.0002	.0009	.0034
	19							.0001	.0003	.0014
	20								.0001	.0005
	21									.0002
	22									.0001

	p =	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 2$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.1900	.2775	.3600	.4375	.5100	.5775	.6400	.6975	.7500
	2	.0100	.0225	.0400	.0625	.0900	.1225	.1600	.2025	.2500
$n = 5$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.4095	.5563	.6723	.7627	.8319	.8840	.9222	.9497	.9688
	2	.0815	.1648	.2627	.3672	.4718	.5716	.6630	.7438	.8125
	3	.0086	.0266	.0579	.1035	.1631	.2352	.3174	.4069	.5000
	4	.0005	.0022	.0067	.0156	.0308	.0540	.0870	.1312	.1875
	5		.0001	.0003	.0010	.0024	.0053	.0102	.0185	.0313
$n = 10$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.6513	.8031	.8926	.9437	.9718	.9865	.9940	.9975	.9990
	2	.2639	.4557	.6242	.7560	.8507	.9140	.9536	.9767	.9893
	3	.0702	.1798	.3222	.4744	.6172	.7384	.8327	.9004	.9453
	4	.0128	.0500	.1209	.2241	.3504	.4862	.6177	.7430	.8281
	5	.0016	.0099	.0328	.0781	.1503	.2485	.3669	.4956	.6230
	6	.0001	.0014	.0064	.0197	.0473	.0949	.1662	.2616	.3770
	7		.0001	.0009	.0035	.0106	.0260	.0548	.1020	.1719
	8			.0001	.0004	.0016	.0048	.0123	.0274	.0547
	9					.0001	.0005	.0017	.0045	.0107
	10							.0001	.0003	.0010
$n = 20$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.8784	.9612	.9885	.9968	.9992	.9998	1.0000	1.0000	1.0000
	2	.6083	.8244	.9308	.9757	.9924	.9979	.9995	.9999	1.0000
	3	.3231	.5951	.7939	.9087	.9645	.9879	.9964	.9991	.9998
	4	.1330	.3523	.5886	.7748	.8929	.9556	.9840	.9951	.9987
	5	.0432	.1702	.3704	.5852	.7625	.8818	.9490	.9811	.9941
	6	.0113	.0673	.1958	.3828	.5836	.7546	.8744	.9447	.9793
	7	.0024	.0219	.0867	.2142	.3920	.5834	.7500	.8701	.9423
	8	.0004	.0059	.0321	.1018	.2277	.3990	.5841	.7480	.8684
	9	.0001	.0013	.0100	.0409	.1133	.2376	.4044	.5857	.7483
	10		.0002	.0026	.0139	.0480	.1218	.2447	.4086	.5881
	11			.0006	.0039	.0171	.0532	.1275	.2493	.4119
	12			.0001	.0009	.0051	.0196	.0565	.1308	.2517
	13				.0002	.0013	.0060	.0210	.0580	.1316
	14					.0003	.0015	.0065	.0214	.0577
	15						.0003	.0016	.0064	.0207
	16							.0003	.0015	.0059
	17								.0003	.0013
	18									.0002

Answer Sheet

Name: _____

ID Number: _____

Enter your answers with an "X' in the table below.

Do not enter the "X" until you have made your *final decision* to avoid scribbling out.

	A	B	C	D
Q1				
Q2				
Q3				
Q4				
Q5				

Q6				
Q7				
Q8				
Q9				
Q10				

Q11				
Q12				
Q13				
Q14				
Q15				