Tutorial Sheet 2: Solving Logarithm Equations

1. Solve the equation
$$2^{2x} - 10(2^x) + 16 = 0.$$

2. Find two numbers the sum of which is 25 and the sum of whose logarithms to the base 10 is 2.

3. If $\log_{10}(x) = 1 + p$ and $\log_{10}(y) = 1 - p$, show that $xy = 100$.

4. If $\log_2(a) = m$ and $\log_3(v) = n$, show that
$$n \log_6(a) + m \log_6(b) = mn.$$

5. Solve the equation $7^x + 7^{x+2} = 3000$ giving the answer correct to one decimal place.

6. Solve the equation $6^{x-1} + 6^{x-3} = 3417$ giving the answer correct to one decimal place.

7. If $\log_2(x) = \log_5(y)$ and $\log_{10}(x) + \log_{10}(y) = 3/2$ find the values of x and y.

8. If $\log_2(y) + \log_5(y) = k$, prove that $\log_2(y) = k \log_{10}(5)$.

9. Prove that $[\log_5(x) \times \log_2(y)]$ - $[\log_{10}(x) \times \log_5(y)] = [\log_2(x) \times \log_{10}(y)]$.

10. If $x^2 + 1 = 3x$, show that $\log(x + 1) - 1/2 \log(x) = 1/2 \log(5)$.

11. If $\log_8(9) = p$ and $\log_3(5) = q$, show that
$$p = \frac{2 \log_{10}(3)}{3 \log_{10}(2)}.$$
Furthermore express $\log_{10}(2)$ in terms of p and q.

12. If $\log_3\left(\frac{10}{a}\right) = p$ and $\log_{3a}(9) = q$, show that
$$p = \frac{1 - \log_{10}(a)}{\log_{10}(3)}.$$
Express pq in terms of $\log_{10}(a)$.

13. If $ab = x^2$, show that $\log_a(x) + \log_b(x) = 2 \log_a(x) \log_b(x)$.

14. If $a^x = 2$ and $\log_{10}(a) = 0.04$, find the value of x, correct to two significant figures.

15. Solve the simultaneous equations:
$$x - y = 2$$
$$\log_{10} x - \log_{10} y = 0.2553.$$

16. Without using the tables, simplify the following expression:
$$\frac{1}{\sqrt{12}} \times 18^{3/2} \times (54)^{-1/2}.$$

17. If $\log_2(x) = a$ and $\log_4(8x) = b$, show that

$$a - 2b + 3 = 0$$

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18. Use the tables to evaluate $\log_{2.73}(10,000)$.

19. Express in a form free from logarithms the equation

$$2\log_{10}(9x - 5) = 2 + \log_{10}(3x + 1)$$

and find the integral value of x which satisfies the equation.

20. Find the solution set of $2\log(x^2) - 3\log(x) = 1.2$.

21. Solve the following equation

$$\frac{2}{3 - \log_{10}(x)} + \frac{5}{4 + \log_{10}(x)} = 2.$$

22. Solve the equation

$$\log_{10}(3x + 7) + \log_{10}(2x + 8) - \log_{10}(x + 1) = 2$$

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23. By using $2^3 < 10$ and $3^2 < 10$ show that $\log_{10}(2) < 1/3$ and $\log_{10}(3) < 0.5$, show that $2^{10} > 10^3$ and $3^4 > 2^3 \times 10$, and deduce that $\log_{10}(2) > 0.3$ and $\log_{10}(3) > 0.475$.
Hence show that $\log_{10}(360)$ lies between 2.55 and 2.67.