1. A pair of dice is thrown. Let X denote the minimum of the two numbers which occur. Find the distributions and expected value of X.

2. A fair coin is tossed four times. Let X denote the longest string of heads.

   Find the distribution and expectation of X.

3. A pair of dice is thrown. Let X denote the minimum of the two numbers which occur. Find the distributions and expected value of X.

4. A fair coin is tossed four times. Let X denote the longest string of heads. Find the distribution and expectation of X.

5. A fair coin is tossed until a head or five tails occurs. Find the expected number E of tosses of the coin.

6. A coin is weighted so that $P(H) = 0.75$ and $P(T) = 0.25$

   The coin is tossed three times. Let X denote the number of heads that appear.

   (a) Find the distribution f of X.

   (b) Find the expectation E(X).

7. A box contains two gold balls and three silver balls.

   - You are allowed to choose successively balls from the box at random.
   - You win 1 dollar each time you draw a gold ball and lose 1 dollar each time you draw a silver ball.
   - After a draw, the ball is not replaced.

   Show that, if you draw until you are ahead by 1 dollar or until there are no more gold balls, this is a favorable game.

8. A pair of dice is thrown. Let X denote the minimum of the two numbers which occur. Find the distributions and expected value of X.

9. A fair coin is tossed four times. Let X denote the longest string of heads. Find the distribution and expectation of X.

10. A fair coin is tossed until a head or five tails occurs. Find the expected number E of tosses of the coin.

11. A player tosses two fair coins. He wins \$2 if two heads occur, and \$1 if one head occurs. On the other hand, he loses \$3 if no heads occur.

    Find the expected value $E(X)$ of the game. Is the game fair?

12. Suppose X has the following probability mass function: $p(0) = 0.2$, $p(1) = 0.5$, $p(2) = 0.3$. Calculate $E[X]$ and $E[X^2]$

13. A coin is weighted so that P(H) = 0.75 and $P(T) = 0.25$.

    The coin is tossed three times. Let X denote the number of heads that appear.

(a) Find the distribution f of X.

(b) Find the expectation $E(X)$.

14. A fair coin is tossed until a head or five tails occurs. Find the expected number E of tosses of the coin.

15. On a roulette wheel there are 37 numbers $\{0, 1, \ldots, 36\}$. 18 numbers are black. If I bet 1 on black, I win 1 if a black number comes up, otherwise I lose my stake. Let X denote my winnings on one bet.

    (i) Calculate E(X) and Var(X)

    Suppose I make 6 such bets. Let Y denote my total winnings.

    (ii) Derive the distribution of Y.

    (iii) Calculate E(Y) and Var(Y)

16. The probability distribution of discrete random variable X is tabulated below. There are 6 possible outcome of X, i.e. 0, 1, 2, 4 ,8 and 10.

x_i	0	1	2	4	8	10
$P(x_i)$	0.25	0.15	0.25	0.15	k	0.10

    (a) Compute the value for k.

    (b) Determine the expected value $E(X)$.

    (c) Evaluate $E(X^2)$.

    (d) Compute the variance of random variable X.

    [(a)]

17. The probability distribute of discrete random variable X is tabulated below. There are 5 possible outcome of X, i.e. 1, 2, 3, 4 and 5.

x_i	1	2	3	4	5
$p(x_i)$	0.30	0.20	k	0.10	0.20

    (a) Compute the value of k.

    (b) What is the expected value of X?

    (c) Compute the value of $E(X^2)$

    (d) Given that $E(X^2) = 9.5$, compute the variance of X.

18. Suppose X has the following probability mass function: $p(0) = 0.2$, $p(1) = 0.5$, $p(2) = 0.3$. Calculate $E[X]$ and $E[X^2]$

19. Consider the random variables X and Y. Both X and Y take the values 0, 1 and 2. The joint probabilities for each pair are given by the following table.

	$X = 0$	$X = 1$	$X = 2$
$Y = 0$	0.1	0.15	0.1
$Y = 1$	0.1	0.1	0.1
$Y = 2$	0.2	0.05	0.1

Compute the $E(U)$ expected value of U, where $U = X - Y$.

20. Suppose X is a random variable with

- $E(X^2) = 3.6$
- $P(X = 2) = 0.6$
- $P(X = 3) = 0.1$

(a) The random variable takes just one other value besides 2 and 3. This value is greater than 0. What is this value?

(b) What is the variance of X?

21. For a particular Java assembler interface, the operand stack size has the following probabilities:

Stack Size	0	1	2	3	4
Probability	0.15	0.05	0.10	0.20	0.50

(i) Calculate the expected stack size.

(ii) Calculate the variance of the stack size.

22. The probability distribute of discrete random variable X is tabulated below. There are 5 possible outcome of X, i.e. 1, 2, 3, 4 and 5.

x_i	1	2	3	4	5
$p(x_i)$	0.30	0.20	k	0.10	0.20

(a) Compute the value of k.

(b) What is the expected value of X?

(c) Compute the value of $E(X^2)$

(d) Given that $E(X^2) = 9.5$, compute the variance of X.

23. The probability distribution of discrete random variable X is tabulated below. There are 6 possible outcome of X, i.e. 0, 1, 2, 4 ,8 and 10.

x_i	0	1	2	4	8	10
$P(x_i)$	0.25	0.15	0.25	0.15	k	0.10

(a) Compute the value for k.

(b) Determine the expected value $E(X)$.

(c) Evaluate $E(X^2)$.

(d) Compute the variance of random variable X.

24. Suppose X is a random variable with

    (a) $E(X^2) = 3.6$
    (b) $P(X = 2) = 0.6$
    (c) $P(X = 3) = 0.1$

    (a) The random variable takes just one other value besides 2 and 3. This value is greater than 0. What is this value?
    (b) What is the variance of X?