1 Question 5 (Sample Variant 2)[25 marks]

(a) **Binary Channels (5 Marks)**
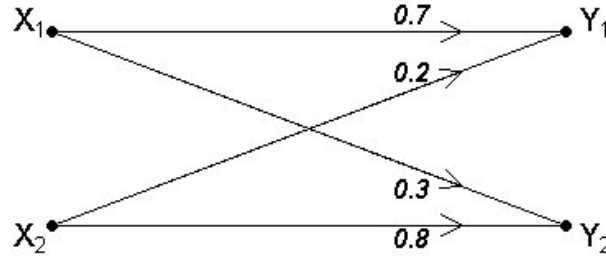
Consider the binary channel in the figure below.

Figure 1:

    (i) (1 Marks) Determine the channel matrix of the channel

    (ii) (2 Marks) Find $P(Y_1)$ and $P(Y_2)$ when $P(X_1) = 0.7$ and $P(X_2) = 0.3$.

    (iii) (2 Marks) Find the joint probabilities $P(X_1, Y_1)$ and $P(X_2, Y_2)$.

(b) **Huffman Coding (10 Marks)**
A discrete memoryless source X has five symbols $\{x_1, x_2, x_3, x_4, x_5\}$ with probabilities $P(x_1) = 0.45$, $P(x_2) = 0.20$, $P(x_3) = 0.16$, $P(x_4) = 0.14$ and $P(x_5) = 0.05$.

    (i) (5 Marks) Construct a Huffman code for X.

    (ii) (4 Marks) Calculate the efficiency of the code.

    (iii) (1 marks) Calculate the redundancy of the code.

- (c)

(b) **Shannon Coding (6 Marks)**
A discrete memoryless source X has five symbols $\{x_1; x_2; x_3; x_4; x_5\}$ with prob- abilities P(x1) = 0:40 , P(x2) = 0:25, P(x3) = 0:15, P(x4) = 0:12 and P(x5) = 0:08.

    – (i) (4 marks) Construct a Shanno code for X.

    – (ii) (2 marks) Calculate the entropy of the code.

(d) **Communication Channels (4 Marks)**

The input source to a noisy communication channel is a random variable X over the four symbols $\{a, b, c, d\}$. The output from this channel is a random variable Y over these same four symbols.

The joint distribution of these two random variables is as follows:

y=a	0.125	0.03125	0	0.015625
y=b	0	0.1875	0.125	0
y=c	0	0.015625	0.1875	0
y=d	0.0625	0	0	0.25

(i) (2 Marks) Write down the marginal distribution for X and compute the marginal entropy $H(X)$.

(ii) (2 Marks) Write down the marginal distribution for Y and compute the marginal entropy $H(Y)$.