Kraft-McMillan Inequality

Let C be a code with N codewords with lengths

$$l_1, l_2, \ldots, l_N$$

. If C is uniquely decodable, then

$$\sum_{i=1}^{N} 2^{-l_i} \leq 1$$

This inequality is known as the Kraft-McMillan inequality.

Kraft-McMillan Inequality

$$\sum_{i=1}^{N} 2^{-l_i} \leq 1$$

- N is the number of codewords in a code
- l_i is the length of the $i-$th codeword.

Kraft-McMillan Inequality

Example

Given an alphabet of 4 symbols (A, B, C, D), would it be possible to find a uniquely decodable code in which a codeword of length 2 is assigned to A, codewords of length 1 to B and C, and a codeword of length 3 to D?

Kraft-McMillan Inequality

Solution

Here we have $l_1 = 2$, $l_2 = l_3 = 1$, and $l_4 = 3$.

$$\sum_{i=1}^{4} 2^{-l_i} = \frac{1}{2^2} + \frac{1}{2^1} + \frac{1}{2^1} + \frac{1}{2^3} = 1.375$$

$$\sum_{i=1}^{4} 2^{-l_i} > 1 \quad \therefore \text{NO}$$