Part 1. Theory for Inference Procedures (4 Marks)

Answer the four short questions. Each correct answer will be awarded 1 mark. Reasonably short answers will suffice, i.e. your answers should not exceed a paragraph.

(i) (1 Mark) What is a $p-$value?

(ii) (1 Mark) Briefly describe how $p-$value is used in hypothesis testing

(iii) (1 Mark) What is meant by a Type I error?

(iv) (1 Mark) What is meant by a Type II error?

(Once you have completed this section, **please turn over.**)

Part 2. Normal Distribution (5 Marks)

The strength of an analogue signal received at a detector (measured in microvolts : μV), is normally distributed with a mean of 100 μV and a variance of 100 μV^2 .
With the signal strength denoted as X, we can say:

$$X \sim N(100, 100)$$

(i) (1 Mark) What is the Z-score for 115 μV ?

(ii) (1 Mark) What is the Z-score for 80 μV ?

(iii) (1 Mark) What is the probability that the signal will exceed 115 μV ?

(iv) (1 Mark) Estimate the value of z_o, which is an observation from the Standard Normal (Z) distribution, given that:

$$P(Z \geq z_o) = 0.04.$$

(Hint: you may use a reasonable close value)

(v) (1 Mark) What is the micro-voltage threshold x_0 below which 96% of the signals will be?

$$P(X \leq x_o) = 0.96,$$

$$P(X \geq x_o) = 0.04.$$

(Once you have completed this section, **please turn over.**)

(**Room for Answers**)

Part 3. Confidence Interval (3 Marks)

Suppose that the mean weight of a sample of 16 items is 160g, and the sample standard deviation is 32g.

(ii) (1 Mark) Compute the standard error that would correspond to the sample mean.

(ii) (1 Mark) State the appropropriate quantile from the $t-$distribution that would be used to compute the 95% confidence interval for the mean.

(iii) (1 Mark) Determine the 95% confidence interval for the mean.

(Once you have completed this section, **please turn over.**)

Part 4. Hypothesis Testing (3 Marks)

A sample of 500 voters was taken by a political pollster to estimate the proportion of first preference votes a particular candidate will obtain in a forthcoming election.

It was found that 280 out of these 500 voters would give the candidate their first preference.

$$\hat{p} = 56\% \text{ (i.e. } 0.56)$$

Using a significance level of 5%, test the hypothesis that the percentage of voters who will give this particular candidate their first preference in the election is 60%.

(i) (1 Mark) Formally state the null and alternative hypotheses. (You may work on the basis that this is a two-tailed hypothesis test.)

(ii) (1 Mark) Compute the Test Statistic for this hypothesis test.

(iii) (1 Mark) Given that the critical value is 1.96, state your conclusion for this test.

(**Room for Answers**)

Formulas for Standard Errors

Confidence Intervals

One sample

$$S.E.(\bar{X}) = \frac{\sigma}{\sqrt{n}}.$$

$$S.E.(\hat{P}) = \sqrt{\frac{\hat{p} \times (100 - \hat{p})}{n}}.$$

Hypothesis tests

One sample

$$S.E.(\bar{X}) = \frac{\sigma}{\sqrt{n}}.$$

$$S.E.(\pi) = \sqrt{\frac{\pi \times (100 - \pi)}{n}}$$