Question 1

a) $Y = X_1 - X_2$, i.e., the difference in the time it takes for these two people to complete the task.

$$\mu_Y = \mu_1 - \mu_2 = 45 - 44 = 1$$

$$\sigma_Y = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{1^2 + 1.5^2} = 1.8028$$

$$\Rightarrow Y \sim \text{Normal}(\mu = 1, \sigma = 1.8028)$$

b) Person 1 finishes first $\Rightarrow Y < 0$ since $X_1 < X_2$.

$$\begin{aligned}\Pr(Y < 0) &= \Pr(Z < \tfrac{0-1}{1.8028}) \\ &= \Pr(Z < -0.55) \\ &= \Pr(Z > 0.55) \\ &= 0.2912.\end{aligned}$$

c) Person 2 finishes first $\Rightarrow Y > 0$ since $X_1 > X_2$.

$$\begin{aligned}\Pr(Y > 0) &= \Pr(Z > \tfrac{0-1}{1.8028}) \\ &= \Pr(Z > -0.55) \\ &= \Pr(Z < 0.55) \\ &= 1 - \Pr(Z > 0.55) \\ &= 1 - 0.2912 = 0.7088.\end{aligned}$$

Note: Part (c) can also be done by noticing that $Y > 0$ and $Y < 0$ are complementary events and the fact that we have $\Pr(Y < 0)$ from part (a):
$\Pr(Y > 0) = 1 - \Pr(Y < 0) = 1 - 0.2912 = 0.7088$.

d) We have not stated who the winner is. All we know is that whoever wins finishes at least 2 seconds before the other person.

Person 1 wins:
$\Rightarrow X_2$ is at least 2 seconds more than X_1
$\Rightarrow Y < -2$.

Person 2 wins:
$\Rightarrow X_2$ is at least 2 seconds less than X_1
$\Rightarrow Y > 2$.

This is the same as saying the *absolute* difference is more than 2 seconds, i.e., $|Y| > 2$.

$$\begin{aligned}&\Pr(Y < -2) + \Pr(Y > 2) \\ &= \Pr(Z < \tfrac{-2-1}{1.8028}) + \Pr(Z > \tfrac{2-1}{1.8028}) \\ &= \Pr(Z < -1.66) + \Pr(Z > 0.55) \\ &= \Pr(Z > 1.66) + \Pr(Z > 0.55) \\ &= 0.0485 + 0.2912 \\ &= 0.3403.\end{aligned}$$

Question 2

a) For the 99% limits $\Rightarrow$ 0.99 of the distribution is covered with 0.01 remaining, i.e., 0.005 in the left tail and 0.005 in the right tail:

lower tail	**upper tail**
$\Pr(X < x_1) = 0.005$	$\Pr(X > x_2) = 0.005$
$\Pr(Z < \frac{x_1-30}{4}) = 0.005$	$\Pr(Z > \frac{x_2-30}{4}) = 0.005$
$\Pr(Z > -\frac{x_1-30}{4}) = 0.005$	

We find that the z score which corresponds to $\Pr(Z > z) = 0.005$ is $z = 2.58$ (from the tables):

lower limit	**upper limit**
$-\frac{x_1-30}{4} = 2.58$	$\frac{x_2-30}{4} = 2.58$
$\frac{x_1-30}{4} = -2.58$	
$x_1 - 30 = -2.58(4)$	$x_2 - 30 = 2.58(4)$
$x_1 = 30 - 2.58(4)$	$x_2 = 30 + 2.58(4)$
$x_1 = 19.68$	$x_2 = 40.32$

Notice that these limits are $30 \pm (2.58 \times 4) = \mu \pm z_{0.005}\,\sigma$.