Question 1

a) $X \sim \text{Normal}(\mu = 60, \sigma = 10)$

$$\Rightarrow \overline{X} \sim \text{Normal}\left(\mu = 60,\, \sigma(\overline{X}) = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{n}}\right).$$

b) Here $\overline{X} \sim \text{Normal}(60, \frac{10}{\sqrt{30}} = 1.826)$.

$$\begin{aligned}\Pr(\overline{X} > 66) &= \Pr(Z > \tfrac{66-60}{1.826}) \\ &= \Pr(Z > 3.29) \\ &= 0.0005.\end{aligned}$$

c) 99% limits $\Rightarrow \alpha = 0.1 \Rightarrow \alpha/2 = 0.005$.

$$\begin{aligned}&60 \pm z_{0.005}(1.826) \\ &60 \pm 2.58(1.826) \\ &60 \pm 4.71\end{aligned}$$

$$[55.29,\, 64.71].$$

99% of the time, the sample mean for 30 individuals will lie in the above interval.

d) Here $\sigma(\overline{X}) = \frac{10}{\sqrt{50}} = 1.414$

$$\begin{aligned}&60 \pm 2.58(1.414) \\ &60 \pm 3.65\end{aligned}$$

$$[56.35,\, 63.65].$$

Note that for $n = 50$ the interval is tighter (around μ) than for $n = 30$, i.e., $\overline{X}$ varies less in the bigger sample as we would expect.