Channel Capacity

- In information theory, channel capacity is the most conservative upper bound on the amount of information that can be reliably transmitted over a communications channel.
- It is given by the maximum of the mutual information between the input and output of the channel (maximum in respect to input probabilities).

Channel Capacity

A. Channel Capacity per Symbol C:

The channel capacity per symbol of a DMC is defined as

$$C_s = \max_{(P(x_i))} I(X;Y) \text{ b/symbol}$$

where the maximization is over all possible input probability distributions $P(x_i)$ on X. Note that the channel capacity C_s is a function of only the channel transition probabilities that define the channel.

B. Channel Capacity per Second :

If r symbols are being transmitted per second, then the maximum rate of transmission of information per second is rC_s.

This is the channel capacity per second and is denoted by C (b/sec).

$$C = rC_s \text{ b/sec}$$

Capacities of special channels

Lossless Channel

- For a lossless channel, the mutual information (information transfer) is equal to the input (source) entropy), and no source information is lost in transmission.
- It can be shown that $H(X|Y) = 0$ (If y_i is the output, there is certainty about the input). Also $I(X;Y) = H(X)$.
- Consequently, the channel capacity per symbol is

$$C_s = \max_{P(x_i)} H(X) = \log_2 m$$

  where m is the number of symbols in X.
- For example, if there are $m = 4$ input channels, then $C = \log_2 4 = 2$ b/symbol

Capacities of special channels

Deterministic Channel:

- The mutual information (information transfer) is equal to the output entropy.
- It can be shown that $H(Y|X) = 0$ (If x_i is the input, there is certainty about the output). Also $I(X;Y) = H(Y)$.
- The channel capacity per symbol is

$$C_s = \max_{P(x_i)} H(Y) = \log_2 n$$

where n is the number of symbols in Y.

Capacities of special channels

Noiseless Channel:

- Since a noiseless channel is both lossless and deterministic , we can say that $I(X;Y) = H(X) = H(Y)$. The mutual information (information transfer) is equal to the output entropy).
- The channel capacity per symbol is

$$C_s = \log_2 m = \log_2 n$$

Capacities of special channels

Binary Symmetric Channel:

- It can be shown that, for a binary symmetric channel, the the channel capacity per symbol is

$$C_s = 1 + p\log_2 p + 1 - p\log_2(1-p)$$