Question 1

Here we have $X \sim \text{Binomial}(n = 20, p = 0.1)$.

$\Rightarrow$ The *probability function* is $\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{20}{x} 0.1^x\ 0.9^{20-x}$.

a)
$$\Pr(X = 2) = \binom{20}{2} 0.1^2\ 0.9^{20-2} = \binom{20}{2} 0.1^2\ 0.9^{18} = 190(0.01)(0.1501) = 0.2852.$$

b)
$$\Pr(X = 0) = \binom{20}{0} 0.1^0\ 0.9^{20-0} = \binom{20}{0} 0.1^0\ 0.9^{20} = 1(1)(0.1216) = 0.1216.$$

c)
$$\begin{aligned}\Pr(X < 4) = \Pr(X \leq 3) &= p(0) + p(1) + p(2) + p(3) \\ &= \binom{20}{0} 0.1^0\ 0.9^{20} + \binom{20}{1} 0.1^1\ 0.9^{19} + \binom{20}{2} 0.1^2\ 0.9^{18} + \binom{20}{3} 0.1^3\ 0.9^{17} \\ &= 0.1216 + 0.2702 + 0.2852 + 0.1901 \\ &= 0.8671.\end{aligned}$$

d) Note that $\Pr(X \geq 2) = p(2) + p(3) + p(4) + \ldots + p(20)$ but there is less work using the complement rule:

$$\begin{aligned}\Pr(X \geq 2) = 1 - \Pr(X < 2) &= 1 - \Pr(X \leq 1) \\ &= 1 - [p(0) + p(1)] \\ &= 1 - (0.1216 + 0.2702) \\ &= 1 - 0.3918 \\ &= 0.6082.\end{aligned}$$

e) $E(X) = n\,p = 20(0.1) = 2$ defective resistors on average.

f) $Sd(X) = \sqrt{Var(X)} = \sqrt{n\,p\,(1-p)} = \sqrt{20(0.1)(0.9)} = \sqrt{1.8} = 1.34$ defective resistors.

Question 2

Same as above but now using the binomial tables. We must rework the questions in terms of *greater than or equal to* probabilities.

a)
$$\begin{aligned}\Pr(X = 2) &= \Pr(X \geq 2) - \Pr(X \geq 3) \\ &= 0.6083 - 0.3231 \\ &= 0.2852.\end{aligned}$$

b)
$$\begin{aligned}\Pr(X = 0) &= \Pr(X \geq 0) - \Pr(X \geq 1) \\ &= 1.0000 - 0.8784 \\ &= 0.1216.\end{aligned}$$

c)
$$\begin{aligned}\Pr(X < 4) &= 1 - \Pr(X \geq 4) \\ &= 1 - 0.1330 \\ &= 0.8670.\end{aligned}$$

d)
$$\Pr(X \geq 2) = 0.6083.$$

We can see that these are the same as above apart from small differences due to rounding.