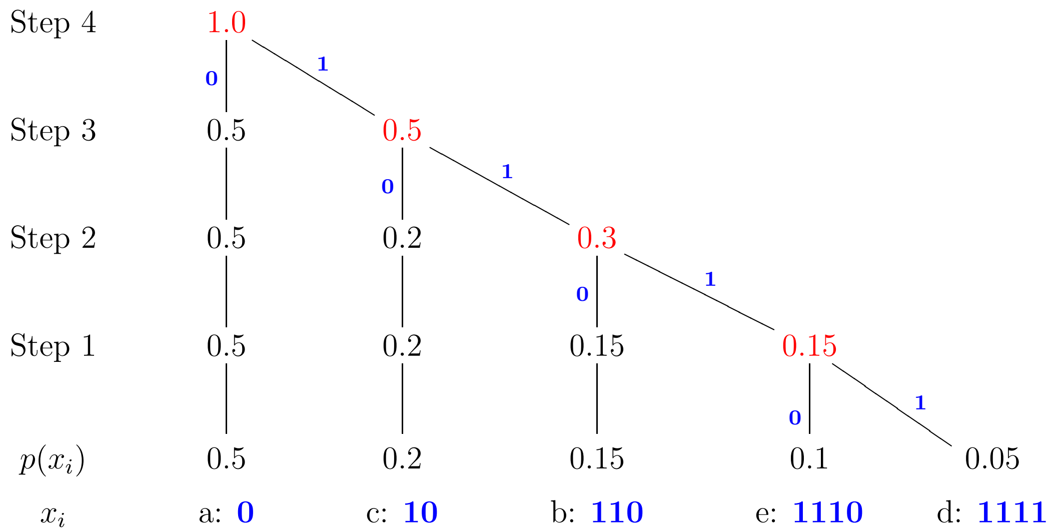
Question 1

a) First we must calculate the information contents: $h_{(}x_i) = -\log_2 p(x_i)$. Note: the table has been reordered for the purpose of constructing the Huffman code.

x_i	a	c	b	e	d
$p(x_i)$	0.5	0.2	0.15	0.1	0.05
$h(x_i)$	1.000	2.322	2.737	3.322	4.322

$$
\begin{aligned}
H(X) = E[h(X)] = \sum h(x_i)\,p(x_i) &= 1.000(0.5) + 2.322(0.2) + 2.737(0.15) + 3.3229(0.1) + 4.322(0.05) \\
&= 0.500 + 0.464 + 0.411 + 0.332 + 0.216 = 1.923 \text{ bits.}
\end{aligned}
$$

b)



c)

x_i	a	c	b	e	d
$p(x_i)$	0.5	0.2	0.15	0.1	0.05
$h(x_i)$	1.000	2.322	2.737	3.322	4.322
$c(x_i)$	0	10	110	1110	1111
$\ell(x_i)$	1	2	3	4	4

$$
\begin{aligned}
E(L) = E[\ell(X)] = \sum \ell(x_i)\,p(x_i) &= 1(0.5) + 2(0.2) + 3(0.15) + 4(0.1) + 4(0.05) \\
&= 0.50 + 0.40 + 0.45 + 0.40 + 0.20 = 1.95 \text{ bits.}
\end{aligned}
$$

d)

$$
e = \frac{H(X)}{E(L)} = \frac{1.923}{1.95} = 0.986.
$$

$\Rightarrow$ This Huffman code is 98.6% efficient.

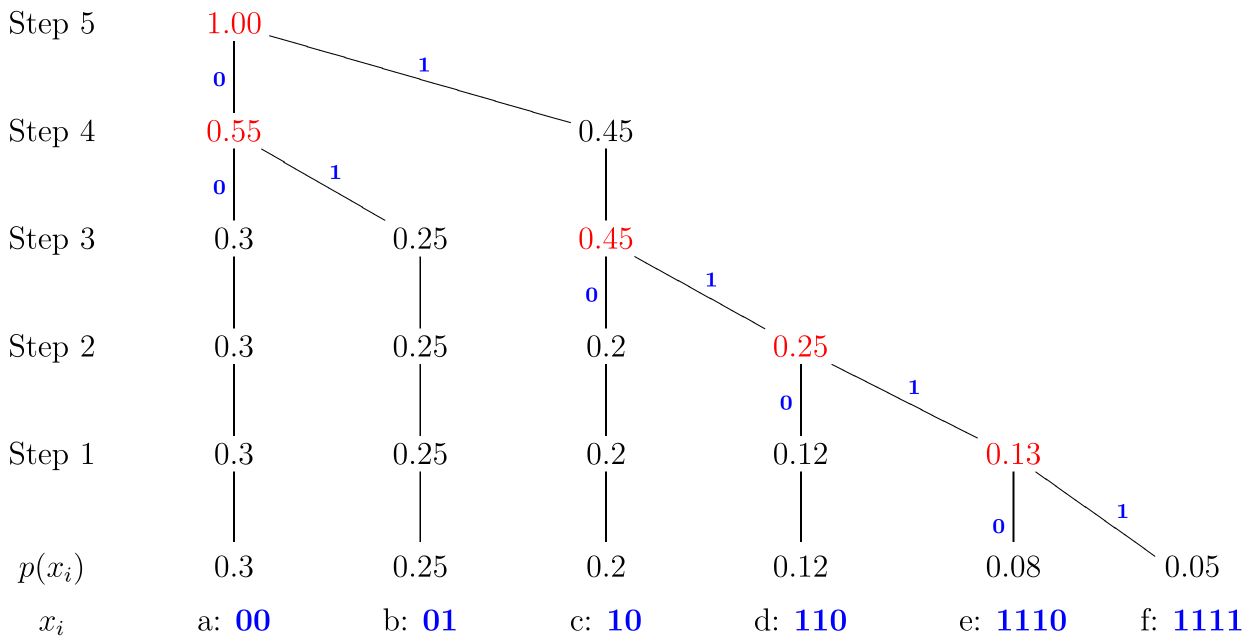
Question 2

a) First we must calculate the information contents: $h_(x_i) = -\log_2 p(x_i)$.

x_i	a	b	c	d	e	f
$p(x_i)$	0.3	0.25	0.2	0.12	0.08	0.05
$h(x_i)$	1.737	2.000	2.322	3.059	3.644	4.322

$$\begin{aligned} H(X) = E[h(X)] &= 1.737(0.3) + 2.000(0.25) + 2.322(0.2) + 3.059(0.12) + 3.644(0.08) + 4.322(0.05) \\ &= 0.521 + 0.500 + 0.464 + 0.367 + 0.292 + 0.216 = 2.36 \text{ bits.} \end{aligned}$$

b)



c)

x_i	a	b	c	d	e	f
$p(x_i)$	0.3	0.25	0.2	0.12	0.08	0.05
$h(x_i)$	1.737	2.000	2.322	3.059	3.644	4.322
$c(x_i)$	00	01	10	110	1110	1111
$\ell(x_i)$	2	2	2	3	4	4

$$\begin{aligned} E(L) = E[\ell(X)] &= 2(0.3) + 2(0.25) + 2(0.2) + 3(0.12) + 4(0.08) + 4(0.05) \\ &= 0.60 + 0.50 + 0.40 + 0.36 + 0.32 + 0.20 = 2.38 \text{ bits.} \end{aligned}$$

d)

$$e = \frac{H(X)}{E(L)} = \frac{2.36}{2.38} = 0.992.$$

$\Rightarrow$ This Huffman code is 99.2% efficient.

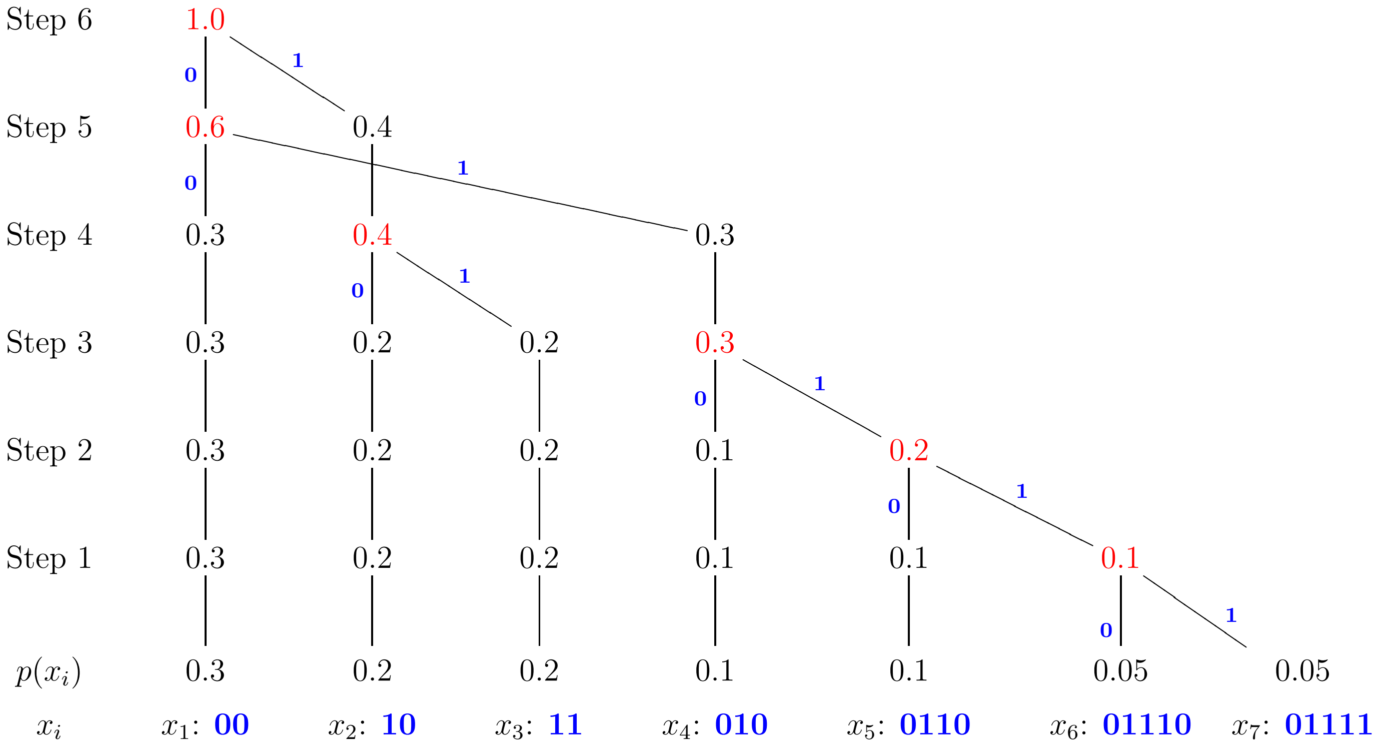
Question 3

a) First we must calculate the information contents: $h_(x_i) = -\log_2 p(x_i)$.

x_i	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$p(x_i)$	0.3	0.2	0.2	0.1	0.1	0.05	0.05
$h(x_i)$	1.737	2.322	2.322	3.322	3.322	4.322	4.322

$$\begin{aligned} H(X) = E[h(X)] &= 1.737(0.3) + 2.322(0.2) + 2.322(0.2) + 3.322(0.1) \\ &\qquad + 3.322(0.1) + 4.322(0.05) + 4.322(0.05) \\ &= 0.521 + 0.464 + 0.464 + 0.332 + 0.332 + 0.216 + 0.216 = 2.545 \text{ bits.} \end{aligned}$$

b)



c)

x_i	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$p(x_i)$	0.3	0.2	0.2	0.1	0.1	0.05	0.05
$h(x_i)$	1.737	2.322	2.322	3.322	3.322	4.322	4.322
$c(x_i)$	00	10	11	010	0110	01110	01111
$\ell(x_i)$	2	2	2	3	4	5	5

$$\begin{aligned} E(L) = E[\ell(X)] &= 2(0.3) + 2(0.2) + 2(0.2) + 3(0.1) + 4(0.1) + 5(0.05) + 5(0.05) \\ &= 0.60 + 0.40 + 0.40 + 0.30 + 0.40 + 0.25 + 0.25 = 2.6 \text{ bits.} \end{aligned}$$

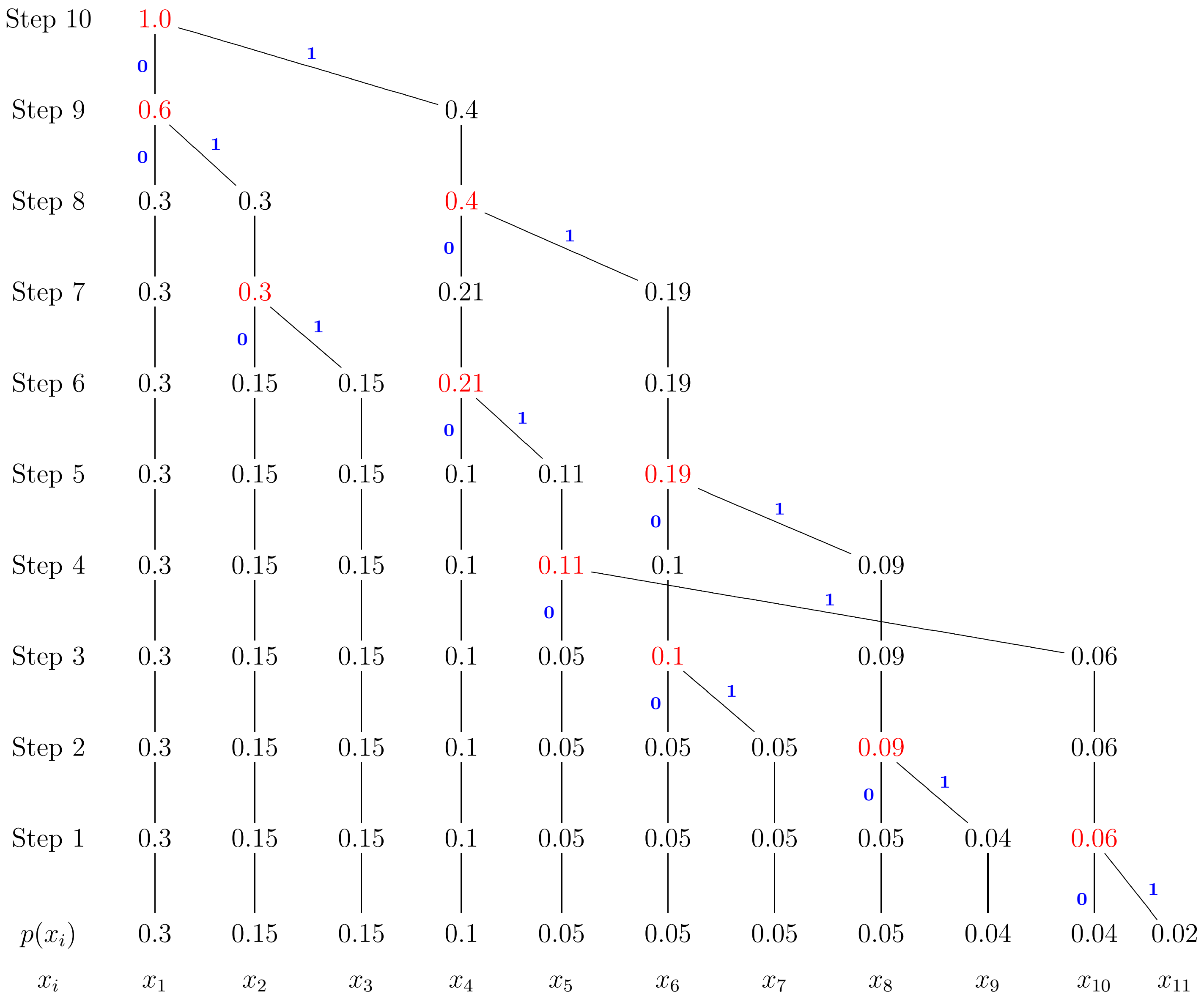
d)

$$e = \frac{H(X)}{E(L)} = \frac{2.545}{2.6} = 0.979.$$

$\Rightarrow$ This Huffman code is 97.9% efficient.

Question 4

a)



- x_1: **00**
- x_2: **010**
- x_3: **011**
- x_4: **100**
- x_5: **1010**
- x_6: **1100**
- x_7: **1101**
- x_8: **1110**
- x_9: **1111**
- x_{10}: **10110**
- x_{11}: **10111**