**This paper is not to be removed from the Examination Halls**

**UNIVERSITY OF LONDON** **ST104A ZA**

**BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diplomas in Economics and Social Sciences and Access Route**

**Statistics 1**

Friday, 03 May 2013 : 10.00am to 12.00pm

Candidates should answer **THREE** of the following **FOUR** questions: **QUESTION 1** of Section A (50 marks) and **TWO** questions from Section B (25 marks each). **Candidates are strongly advised to divide their time accordingly.**

A list of formulae and extracts from statistical tables are provided after the final question on this paper.

Graph paper is provided at the end of this question paper. If used, it must be detached and fastened securely inside the answer book.

A calculator may be used when answering questions on this paper and it must comply in all respects with the specification given with your Admission Notice. The make and type of machine must be clearly stated on the front cover of the answer book.

PLEASE TURN OVER

## SECTION A

Answer **all** parts of Question 1 (50 marks in total).

1. (a) Classify each one of the following variables as measurable (continuous) or categorical. If a variable is categorical, further classify it as nominal or ordinal. Justify your answer. (*Note that no marks will be awarded without justification.*)

   i. Country of birth.

   ii. Favourite brand of soft drink.

   iii. Rank of country by academic quality according to ratings given by educational specialists.

   iv. Temperature in degrees Celsius.

   [8 marks]

   (b) The table below contains the ages of the volunteers for a project in two different years:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2011 | 20 | 18 | 38 | 18 | 20 | 18 | | | |
| 2012 | 20 | 22 | 18 | 22 | 20 | 22 | 24 | 22 | 20 |

   i. Find the mean mark and the median mark for each year.

   ii. Calculate the range of the marks for each year and give an explanation for any differences you find.

   iii. Calculate the standard deviation of the marks for each year and give an explanation for any differences you find.

   iv. Comment on the differences in the mean and median for the two years that you found in part i. For this data set, which do you think would give a better description of the difference in marks: the mean or the median? Explain briefly.

   [12 marks]

   (c) Monthly household expenditure in country A is normally distributed with a mean of £1200 per week and a standard deviation of £400 per week. In country B it is also normally distributed but with a mean of £960 per week and a standard deviation of £200 per week. Which country has a higher proportion of households spending less than £800? [4 marks]

   (d) We would like to design a survey to estimate the average number of hours university students spend studying per week. How many students must we randomly select to be 95 percent confident that the sample mean is within 2 hours of the population mean? Assume that a previous survey has shown that the standard deviation of hours spent studying is 6.95 hours. [3 marks]

(e) Suppose that $x_1 = 4$, $x_2 = -3$, $x_3 = 5$, $x_4 = 0$, $x_5 = 3$, and $y_1 = 3$, $y_2 = 2$, $y_3 = 1$, $y_4 = 0$, $y_5 = 1$. Calculate the following quantities:

i. $\displaystyle\sum_{i=1}^{5} x_i$ ii. $\displaystyle\sum_{i=2}^{5} 2x_i(y_i + 1)$ iii. $\displaystyle x_2^2 + \sum_{i=1}^{3}(x_i + y_i^3)$

[6 marks]

(f) In an introductory economics class, the numbers of males and females are 16 and 24, respectively.

i. A student is selected randomly from the class. What is the probability the student is female?

ii. A student is selected at random and removed from the class. A second student is then selected. What is the probability that one of the students is male and the other is female?

iii. What is the probability that the second student is male, given that the first student is female and removed from the class?

iv. In previous years it was found that 80% of males pass the exam and 85% of females pass the examination. Based on the available information, find the probability that a student who passes the exam is female.

[8 marks]

(g) State whether the following are true or false and give a brief explanation. (*Note that no marks will be awarded for a simple true/false answer.*)

i. In an observational study, a control group provides an essential tool to establish causal relationships.

ii. If two variables are correlated we can conclude that one causes the other.

iii. The mean income of British households can be expected to be larger than the median income of British households.

[6 marks]

(h) In the context of sampling, explain the difference between item non-response and unit non-response. [3 marks]

## SECTION B

Answer **two** questions from this section (25 marks each).

2. (a) A social survey in the United States asked subjects, 'Would you say that homeopathy is very scientific, sort of scientific, or not at all scientific?' The table below cross-classifies their responses with their highest level of education.

| | *Homeopathy is scientific* | | | |
|---|---|---|---|---|
| *Highest degree* | Very | Sort of | Not at all | Total |
| Less than High school | 46 (11%) | 168 (41%) | 196 (48%) | 410 (100%) |
| High school | 100(5%) | 572 (31%) | 1148 (63%) | 1820 (100%) |
| College or higher | 32 (2%) | 248 (18%) | 1076 (79%) | 1356 (100%) |
| Total | 178(5%) | 988 (28%) | 2420 (67%) | 3586 (100%) |

i. Based on the data in the table, and *without doing a significance test*, how would you describe the relationship between education and opinion on whether or not homeopathy is scientific? [4 marks]

ii. Calculate the $\chi^2$ statistic and use it to test for independence, using a 1% significance level. What do you conclude? [9 marks]

(b) i. Define each of the following:

- Simple random sampling
- Stratified random sampling.

[4 marks]

ii. Why might a researcher prefer to take a stratified random sample rather than a simple random sample? Give two reasons. [3 marks]

iii. You have been asked to design a nation-wide survey in your country to find out about the smoking habits of adults. Give two stratification factors you might use, and explain why you have chosen them. [5 marks]

3. The level of infant mortality ($y$) is represented by the number of baby deaths for every 1000 births. For 12 areas these are shown in the following table. For each area, the percentage ($x$) of babies born into families earning at least £25,000 is also shown.

| Area | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Percentage ($x$) | 20 | 6 | 10 | 21 | 12 | 36 | 6 | 19 | 26 | 13 | 21 | 16 |
| Infant mortality ($y$) | 5 | 17 | 16 | 8 | 15 | 5 | 25 | 12 | 11 | 11 | 7 | 12 |

The summary statistics for these data are:

| | |
|---|---|
| Sum of $x$ data: 206 | Sum of the squares of $x$ data: 4356 |
| Sum of $y$ data: 144 | Sum of the squares of $y$ data: 2088 |
| Sum of the products of $x$ and $y$ data: 2036 | |

(a) i. Draw a scatter diagram of these data on the graph paper provided. Label the diagram carefully. [4 marks]

ii. Calculate the sample correlation coefficient. Interpret your findings. [3 marks]

iii. Calculate the least squares line of $y$ on $x$ and draw the line on the scatter diagram. [4 marks]

iv. Using the equation you found in iii., obtain the predicted infant mortality for an area where 38% of babies are born into families earning at least £25,000. Do you think this value is realistic? Justify your answer. [2 marks]

(b) A survey is conducted to compare public local attitudes towards environmental policies. A number of people in two areas of interest are sampled, and asked if they are satisfied with their local environmental policy. The results of this survey are shown in the following table.

| | Sample size | Number satisfied |
|---|---|---|
| Area A | 168 | 127 |
| Area B | 207 | 132 |

i. You are asked to consider an appropriate hypothesis test to determine whether there is a difference between the two areas in the proportion who are satisfied. Test at two appropriate significance levels and comment on your findings. Specify the test statistic you use and its distribution under the null hypothesis. [7 marks]

ii. State clearly any other assumptions you make. [2 marks]

iii. Give a 98% confidence interval for the proportion of people in Areas A and B combined who are satisfied, assuming the respective sample sizes are proportional to population sizes. [3 marks]

4. (a) i. Carefully construct a box plot on the graph paper provided to display the following yearly incomes of a group of people, measured in £1000:

9 6 12 24 21 57 6 15 9 12 30 36

[8 marks]

ii. Based on the shape of the box plot you have drawn, describe the distribution of the data. [2 marks]

iii. Name two other types of graphical displays that would be suitable to represent the data. Briefly explain your choices. [3 marks]

(b) A new treatment has been devised with the aim of reducing blood pressure for people with high blood pressure. Each participant's blood pressure was measured before and after the program to see if the treatment is effective. The following data were obtained:

| Before | After |
|---|---|
| 177 | 174 |
| 142 | 146 |
| 146 | 144 |
| 162 | 159 |
| 145 | 145 |
| 162 | 163 |
| 152 | 156 |
| 154 | 150 |
| 171 | 172 |

i. Carry out an appropriate hypothesis test to determine whether the treatment is effective for reducing blood pressure. State the test hypotheses, and specify your test statistic and its distribution under the null hypothesis. Comment on your findings. [6 marks]

ii. State any assumptions you made. [2 marks]

iii. Give a 90% confidence interval for the difference in means. [2 marks]

iv. On the basis of the data alone, would you recommend the programme to a friend who suffers from high blood pressure? Explain why or why not. [2 marks]

END OF PAPER

# ST104a Statistics 1
## Examination Formula Sheet

Expected value of a discrete random variable:

$$\mu = E[X] = \sum_{i=1}^{N} p_i x_i$$

Standard deviation of a discrete random variable:

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{i=1}^{N} p_i (x_i - \mu)^2}$$

The transformation formula:

$$Z = \frac{X - \mu}{\sigma}$$

Finding $Z$ for the sampling distribution of the sample mean:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Finding $Z$ for the sampling distribution of the sample proportion:

$$Z = \frac{P - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$$

Confidence interval endpoints for a single mean ($\sigma$ known):

$$\bar{x} \pm z \frac{\sigma}{\sqrt{n}}$$

Confidence interval endpoints for a single mean ($\sigma$ unknown):

$$\bar{x} \pm t_{n-1} \frac{s}{\sqrt{n}}$$

Confidence interval endpoints for a single proportion:

$$p \pm z \sqrt{\frac{p(1-p)}{n}}$$

Sample size determination for a mean:

$$n \geq \frac{Z^2 \sigma^2}{e^2}$$

Sample size determination for a proportion:

$$n \geq \frac{Z^2 p(1-p)}{e^2}$$

$Z$-test of hypothesis for a single mean ($\sigma$ known):

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

$t$-test of hypothesis for a single mean ($\sigma$ unknown):

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

$Z$-test of hypothesis for a single proportion:

$$Z \cong \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$$

$Z$-test for the difference between two means (variances known):

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$t$-test for the difference between two means (variances unknown):

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Confidence interval endpoints for the difference between two means:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{n_1+n_2-2}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Pooled variance estimator:

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

$t$-test for the difference in means in paired samples:

$$t = \frac{\bar{X}_d - \mu_d}{S_d/\sqrt{n}}$$

Confidence interval endpoints for the difference in means in paired samples:

$$\bar{x}_d \pm t_{n-1}\frac{s_d}{\sqrt{n}}$$

$Z$-test for the difference between two proportions:

$$Z = \frac{(P_1 - P_2) - (\pi_1 - \pi_2)}{\sqrt{P(1-P)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Pooled proportion estimator:

$$P = \frac{R_1 + R_2}{n_1 + n_2}$$

Confidence interval endpoints for the difference between two proportions:

$$(p_1 - p_2) \pm z\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

$\chi^2$ test of association:

$$\sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Sample correlation coefficient:

$$r = \frac{\sum_{i=1}^n x_i y_i - n\bar{x}\bar{y}}{\sqrt{\left(\sum_{i=1}^n x_i^2 - n\bar{x}^2\right)\left(\sum_{i=1}^n y_i^2 - n\bar{y}^2\right)}}$$

Spearman rank correlation:

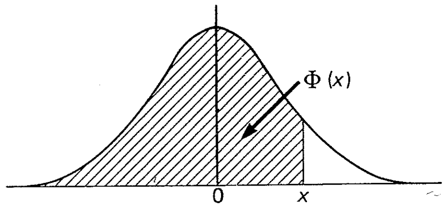
$$r_s = 1 - \frac{6\sum_{i=1}^n d_i^2}{n(n^2-1)}$$

Simple linear regression line estimates:

$$\begin{aligned} b &= \frac{\sum_{i=1}^n x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^n x_i^2 - n\bar{x}^2} \\ a &= \bar{y} - b\bar{x} \end{aligned}$$

# TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to $x$. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.




| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0·00 | 0·5000 | 0·40 | 0·6554 | 0·80 | 0·7881 | 1·20 | 0·8849 | 1·60 | 0·9452 | 2·00 | 0·97725 |
| ·01 | ·5040 | ·41 | ·6591 | ·81 | ·7910 | ·21 | ·8869 | ·61 | ·9463 | ·01 | ·97778 |
| ·02 | ·5080 | ·42 | ·6628 | ·82 | ·7939 | ·22 | ·8888 | ·62 | ·9474 | ·02 | ·97831 |
| ·03 | ·5120 | ·43 | ·6664 | ·83 | ·7967 | ·23 | ·8907 | ·63 | ·9484 | ·03 | ·97882 |
| ·04 | ·5160 | ·44 | ·6700 | ·84 | ·7995 | ·24 | ·8925 | ·64 | ·9495 | ·04 | ·97932 |
| 0·05 | 0·5199 | 0·45 | 0·6736 | 0·85 | 0·8023 | 1·25 | 0·8944 | 1·65 | 0·9505 | 2·05 | 0·97982 |
| ·06 | ·5239 | ·46 | ·6772 | ·86 | ·8051 | ·26 | ·8962 | ·66 | ·9515 | ·06 | ·98030 |
| ·07 | ·5279 | ·47 | ·6808 | ·87 | ·8078 | ·27 | ·8980 | ·67 | ·9525 | ·07 | ·98077 |
| ·08 | ·5319 | ·48 | ·6844 | ·88 | ·8106 | ·28 | ·8997 | ·68 | ·9535 | ·08 | ·98124 |
| ·09 | ·5359 | ·49 | ·6879 | ·89 | ·8133 | ·29 | ·9015 | ·69 | ·9545 | ·09 | ·98169 |
| 0·10 | 0·5398 | 0·50 | 0·6915 | 0·90 | 0·8159 | 1·30 | 0·9032 | 1·70 | 0·9554 | 2·10 | 0·98214 |
| ·11 | ·5438 | ·51 | ·6950 | ·91 | ·8186 | ·31 | ·9049 | ·71 | ·9564 | ·11 | ·98257 |
| ·12 | ·5478 | ·52 | ·6985 | ·92 | ·8212 | ·32 | ·9066 | ·72 | ·9573 | ·12 | ·98300 |
| ·13 | ·5517 | ·53 | ·7019 | ·93 | ·8238 | ·33 | ·9082 | ·73 | ·9582 | ·13 | ·98341 |
| ·14 | ·5557 | ·54 | ·7054 | ·94 | ·8264 | ·34 | ·9099 | ·74 | ·9591 | ·14 | ·98382 |
| 0·15 | 0·5596 | 0·55 | 0·7088 | 0·95 | 0·8289 | 1·35 | 0·9115 | 1·75 | 0·9599 | 2·15 | 0·98422 |
| ·16 | ·5636 | ·56 | ·7123 | ·96 | ·8315 | ·36 | ·9131 | ·76 | ·9608 | ·16 | ·98461 |
| ·17 | ·5675 | ·57 | ·7157 | ·97 | ·8340 | ·37 | ·9147 | ·77 | ·9616 | ·17 | ·98500 |
| ·18 | ·5714 | ·58 | ·7190 | ·98 | ·8365 | ·38 | ·9162 | ·78 | ·9625 | ·18 | ·98537 |
| ·19 | ·5753 | ·59 | ·7224 | ·99 | ·8389 | ·39 | ·9177 | ·79 | ·9633 | ·19 | ·98574 |
| 0·20 | 0·5793 | 0·60 | 0·7257 | 1·00 | 0·8413 | 1·40 | 0·9192 | 1·80 | 0·9641 | 2·20 | 0·98610 |
| ·21 | ·5832 | ·61 | ·7291 | ·01 | ·8438 | ·41 | ·9207 | ·81 | ·9649 | ·21 | ·98645 |
| ·22 | ·5871 | ·62 | ·7324 | ·02 | ·8461 | ·42 | ·9222 | ·82 | ·9656 | ·22 | ·98679 |
| ·23 | ·5910 | ·63 | ·7357 | ·03 | ·8485 | ·43 | ·9236 | ·83 | ·9664 | ·23 | ·98713 |
| ·24 | ·5948 | ·64 | ·7389 | ·04 | ·8508 | ·44 | ·9251 | ·84 | ·9671 | ·24 | ·98745 |
| 0·25 | 0·5987 | 0·65 | 0·7422 | 1·05 | 0·8531 | 1·45 | 0·9265 | 1·85 | 0·9678 | 2·25 | 0·98778 |
| ·26 | ·6026 | ·66 | ·7454 | ·06 | ·8554 | ·46 | ·9279 | ·86 | ·9686 | ·26 | ·98809 |
| ·27 | ·6064 | ·67 | ·7486 | ·07 | ·8577 | ·47 | ·9292 | ·87 | ·9693 | ·27 | ·98840 |
| ·28 | ·6103 | ·68 | ·7517 | ·08 | ·8599 | ·48 | ·9306 | ·88 | ·9699 | ·28 | ·98870 |
| ·29 | ·6141 | ·69 | ·7549 | ·09 | ·8621 | ·49 | ·9319 | ·89 | ·9706 | ·29 | ·98899 |
| 0·30 | 0·6179 | 0·70 | 0·7580 | 1·10 | 0·8643 | 1·50 | 0·9332 | 1·90 | 0·9713 | 2·30 | 0·98928 |
| ·31 | ·6217 | ·71 | ·7611 | ·11 | ·8665 | ·51 | ·9345 | ·91 | ·9719 | ·31 | ·98956 |
| ·32 | ·6255 | ·72 | ·7642 | ·12 | ·8686 | ·52 | ·9357 | ·92 | ·9726 | ·32 | ·98983 |
| ·33 | ·6293 | ·73 | ·7673 | ·13 | ·8708 | ·53 | ·9370 | ·93 | ·9732 | ·33 | ·99010 |
| ·34 | ·6331 | ·74 | ·7704 | ·14 | ·8729 | ·54 | ·9382 | ·94 | ·9738 | ·34 | ·99036 |
| 0·35 | 0·6368 | 0·75 | 0·7734 | 1·15 | 0·8749 | 1·55 | 0·9394 | 1·95 | 0·9744 | 2·35 | 0·99061 |
| ·36 | ·6406 | ·76 | ·7764 | ·16 | ·8770 | ·56 | ·9406 | ·96 | ·9750 | ·36 | ·99086 |
| ·37 | ·6443 | ·77 | ·7794 | ·17 | ·8790 | ·57 | ·9418 | ·97 | ·9756 | ·37 | ·99111 |
| ·38 | ·6480 | ·78 | ·7823 | ·18 | ·8810 | ·58 | ·9429 | ·98 | ·9761 | ·38 | ·99134 |
| ·39 | ·6517 | ·79 | ·7852 | ·19 | ·8830 | ·59 | ·9441 | ·99 | ·9767 | ·39 | ·99158 |
| 0·40 | 0·6554 | 0·80 | 0·7881 | 1·20 | 0·8849 | 1·60 | 0·9452 | 2·00 | 0·9772 | 2·40 | 0·99180 |

# TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2·40 | 0·99180 | 2·55 | 0·99461 | 2·70 | 0·99653 | 2·85 | 0·99781 | 3·00 | 0·99865 | 3·15 | 0·99918 |
| ·41 | ·99202 | ·56 | ·99477 | ·71 | ·99664 | ·86 | ·99788 | ·01 | ·99869 | ·16 | ·99921 |
| ·42 | ·99224 | ·57 | ·99492 | ·72 | ·99674 | ·87 | ·99795 | ·02 | ·99874 | ·17 | ·99924 |
| ·43 | ·99245 | ·58 | ·99506 | ·73 | ·99683 | ·88 | ·99801 | ·03 | ·99878 | ·18 | ·99926 |
| ·44 | ·99266 | ·59 | ·99520 | ·74 | ·99693 | ·89 | ·99807 | ·04 | ·99882 | ·19 | ·99929 |
| 2·45 | 0·99286 | 2·60 | 0·99534 | 2·75 | 0·99702 | 2·90 | 0·99813 | 3·05 | 0·99886 | 3·20 | 0·99931 |
| ·46 | ·99305 | ·61 | ·99547 | ·76 | ·99711 | ·91 | ·99819 | ·06 | ·99889 | ·21 | ·99934 |
| ·47 | ·99324 | ·62 | ·99560 | ·77 | ·99720 | ·92 | ·99825 | ·07 | ·99893 | ·22 | ·99936 |
| ·48 | ·99343 | ·63 | ·99573 | ·78 | ·99728 | ·93 | ·99831 | ·08 | ·99896 | ·23 | ·99938 |
| ·49 | ·99361 | ·64 | ·99585 | ·79 | ·99736 | ·94 | ·99836 | ·09 | ·99900 | ·24 | ·99940 |
| 2·50 | 0·99379 | 2·65 | 0·99598 | 2·80 | 0·99744 | 2·95 | 0·99841 | 3·10 | 0·99903 | 3·25 | 0·99942 |
| ·51 | ·99396 | ·66 | ·99609 | ·81 | ·99752 | ·96 | ·99846 | ·11 | ·99906 | ·26 | ·99944 |
| ·52 | ·99413 | ·67 | ·99621 | ·82 | ·99760 | ·97 | ·99851 | ·12 | ·99910 | ·27 | ·99946 |
| ·53 | ·99430 | ·68 | ·99632 | ·83 | ·99767 | ·98 | ·99856 | ·13 | ·99913 | ·28 | ·99948 |
| ·54 | ·99446 | ·69 | ·99643 | ·84 | ·99774 | ·99 | ·99861 | ·14 | ·99916 | ·29 | ·99950 |
| 2·55 | 0·99461 | 2·70 | 0·99653 | 2·85 | 0·99781 | 3·00 | 0·99865 | 3·15 | 0·99918 | 3·30 | 0·99952 |

The critical table below gives on the left the range of values of $x$ for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|---|---|
| 3·075 | | | 0·9994 | | 0·99990 | | 0·99995 |
| | 0·9990 | 3·263 | | 3·731 | | 3·916 | |
| 3·105 | | | 0·9995 | | 0·99991 | | 0·99996 |
| | 0·9991 | 3·320 | | 3·759 | | 3·976 | |
| 3·138 | | | 0·9996 | | 0·99992 | | 0·99997 |
| | 0·9992 | 3·389 | | 3·791 | | 4·055 | |
| 3·174 | | | 0·9997 | | 0·99993 | | 0·99998 |
| | 0·9993 | 3·480 | | 3·826 | | 4·173 | |
| 3·215 | | | 0·9998 | | 0·99994 | | 0·99999 |
| | 0·9994 | 3·615 | | 3·867 | | 4·417 | |
| | | | 0·9999 | | 0·99995 | | 1·00000 |

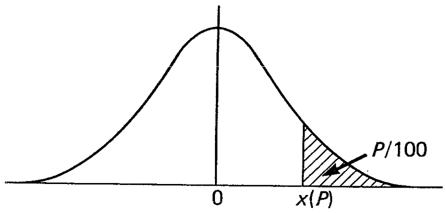
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \dfrac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\dfrac{1}{x^2}+\dfrac{3}{x^4}-\dfrac{15}{x^6}+\dfrac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

# TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If $X$ is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower $P$ per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.




| $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 0·0000 | 5·0 | 1·6449 | 3·0 | 1·8808 | 2·0 | 2·0537 | 1·0 | 2·3263 | 0·10 | 3·0902 |
| 45 | 0·1257 | 4·8 | 1·6646 | 2·9 | 1·8957 | 1·9 | 2·0749 | 0·9 | 2·3656 | 0·09 | 3·1214 |
| 40 | 0·2533 | 4·6 | 1·6849 | 2·8 | 1·9110 | 1·8 | 2·0969 | 0·8 | 2·4089 | 0·08 | 3·1559 |
| 35 | 0·3853 | 4·4 | 1·7060 | 2·7 | 1·9268 | 1·7 | 2·1201 | 0·7 | 2·4573 | 0·07 | 3·1947 |
| 30 | 0·5244 | 4·2 | 1·7279 | 2·6 | 1·9431 | 1·6 | 2·1444 | 0·6 | 2·5121 | 0·06 | 3·2389 |
| 25 | 0·6745 | 4·0 | 1·7507 | 2·5 | 1·9600 | 1·5 | 2·1701 | 0·5 | 2·5758 | 0·05 | 3·2905 |
| 20 | 0·8416 | 3·8 | 1·7744 | 2·4 | 1·9774 | 1·4 | 2·1973 | 0·4 | 2·6521 | 0·01 | 3·7190 |
| 15 | 1·0364 | 3·6 | 1·7991 | 2·3 | 1·9954 | 1·3 | 2·2262 | 0·3 | 2·7478 | 0·005 | 3·8906 |
| 10 | 1·2816 | 3·4 | 1·8250 | 2·2 | 2·0141 | 1·2 | 2·2571 | 0·2 | 2·8782 | 0·001 | 4·2649 |
| 5 | 1·6449 | 3·2 | 1·8522 | 2·1 | 2·0335 | 1·1 | 2·2904 | 0·1 | 3·0902 | 0·0005 | 4·4172 |

# TABLE 7. THE $\chi^2$-DISTRIBUTION FUNCTION

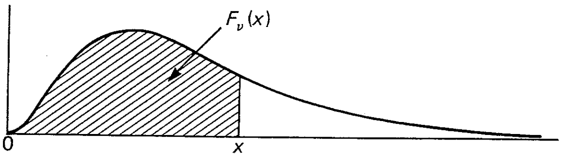
The function tabulated is

$$F_\nu(x) = \frac{1}{2^{\nu/2}\Gamma(\frac{\nu}{2})}\int_0^x t^{\frac{1}{2}\nu-1}e^{-\frac{1}{2}t}\,dt$$

for integer $\nu \leqslant 25$. $F_\nu(x)$ is the probability that a random variable $X$, distributed as $\chi^2$ with $\nu$ degrees of freedom, will be less than or equal to $x$. Note that $F_1(x) = 2\Phi(x^{\frac{1}{2}}) - 1$ (cf. Table 4). For certain values of $x$ and $\nu > 25$ use may be made of the following relation between the $\chi^2$- and Poisson distributions:

$$F_\nu(x) = 1 - F(\tfrac{1}{2}\nu - 1\,|\,\tfrac{1}{2}x)$$

where $F(r|\mu)$ is the Poisson distribution function (see Table 2). If $\nu > 25$, $X$ is approximately normally distributed




(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

with mean $\nu$ and variance $2\nu$. A better approximation is usually obtained by using the formula

$$F_\nu(x) \doteqdot \Phi(\sqrt{2x} - \sqrt{2\nu - 1})$$

where $\Phi(s)$ is the normal distribution function (see Table 4).

Omitted entries to the left and right of tabulated values are 1 and 0 respectively (to four decimal places).

| $\nu =$ | 1 | $\nu =$ | 1 | $\nu =$ | 2 | $\nu =$ | 2 | $\nu =$ | 3 | $\nu =$ | 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $x =$ 0·0 | 0·0000 | $x =$ 4·0 | 0·9545 | $x =$ 0·0 | 0·0000 | $x =$ 4·0 | 0·8647 | $x =$ 0·0 | 0·0000 | $x =$ 4·0 | 0·7385 |
| ·1 | ·2482 | ·1 | ·9571 | ·1 | ·0488 | ·1 | ·8713 | ·1 | ·0082 | ·2 | ·7593 |
| ·2 | ·3453 | ·2 | ·9596 | ·2 | ·0952 | ·2 | ·8775 | ·2 | ·0224 | ·4 | ·7786 |
| ·3 | ·4161 | ·3 | ·9619 | ·3 | ·1393 | ·3 | ·8835 | ·3 | ·0400 | ·6 | ·7965 |
| ·4 | ·4729 | ·4 | ·9641 | ·4 | ·1813 | ·4 | ·8892 | ·4 | ·0598 | ·8 | ·8130 |
| 0·5 | 0·5205 | 4·5 | 0·9661 | 0·5 | 0·2212 | 4·5 | 0·8946 | 0·5 | 0·0811 | 5·0 | 0·8282 |
| ·6 | ·5614 | ·6 | ·9680 | ·6 | ·2592 | ·6 | ·8997 | ·6 | ·1036 | ·2 | ·8423 |
| ·7 | ·5972 | ·7 | ·9698 | ·7 | ·2953 | ·7 | ·9046 | ·7 | ·1268 | ·4 | ·8553 |
| ·8 | ·6289 | ·8 | ·9715 | ·8 | ·3297 | ·8 | ·9093 | ·8 | ·1505 | ·6 | ·8672 |
| ·9 | ·6572 | ·9 | ·9731 | ·9 | ·3624 | ·9 | ·9137 | ·9 | ·1746 | ·8 | ·8782 |
| 1·0 | 0·6827 | 5·0 | 0·9747 | 1·0 | 0·3935 | 5·0 | 0·9179 | 1·0 | 0·1987 | 6·0 | 0·8884 |
| ·1 | ·7057 | ·1 | ·9761 | ·1 | ·4231 | ·1 | ·9219 | ·1 | ·2229 | ·2 | ·8977 |
| ·2 | ·7267 | ·2 | ·9774 | ·2 | ·4512 | ·2 | ·9257 | ·2 | ·2470 | ·4 | ·9063 |
| ·3 | ·7458 | ·3 | ·9787 | ·3 | ·4780 | ·3 | ·9293 | ·3 | ·2709 | ·6 | ·9142 |
| ·4 | ·7633 | ·4 | ·9799 | ·4 | ·5034 | ·4 | ·9328 | ·4 | ·2945 | ·8 | ·9214 |
| 1·5 | 0·7793 | 5·5 | 0·9810 | 1·5 | 0·5276 | 5·5 | 0·9361 | 1·5 | 0·3177 | 7·0 | 0·9281 |
| ·6 | ·7941 | ·6 | ·9820 | ·6 | ·5507 | ·6 | ·9392 | ·6 | ·3406 | ·2 | ·9342 |
| ·7 | ·8077 | ·7 | ·9830 | ·7 | ·5726 | ·7 | ·9422 | ·7 | ·3631 | ·4 | ·9398 |
| ·8 | ·8203 | ·8 | ·9840 | ·8 | ·5934 | ·8 | ·9450 | ·8 | ·3851 | ·6 | ·9450 |
| ·9 | ·8319 | ·9 | ·9849 | ·9 | ·6133 | ·9 | ·9477 | ·9 | ·4066 | ·8 | ·9497 |
| 2·0 | 0·8427 | 6·0 | 0·9857 | 2·0 | 0·6321 | 6·0 | 0·9502 | 2·0 | 0·4276 | 8·0 | 0·9540 |
| ·1 | ·8527 | ·1 | ·9865 | ·1 | ·6501 | ·2 | ·9550 | ·1 | ·4481 | ·2 | ·9579 |
| ·2 | ·8620 | ·2 | ·9872 | ·2 | ·6671 | ·4 | ·9592 | ·2 | ·4681 | ·4 | ·9616 |
| ·3 | ·8706 | ·3 | ·9879 | ·3 | ·6834 | ·6 | ·9631 | ·3 | ·4875 | ·6 | ·9649 |
| ·4 | ·8787 | ·4 | ·9886 | ·4 | ·6988 | ·8 | ·9666 | ·4 | ·5064 | ·8 | ·9679 |
| 2·5 | 0·8862 | 6·5 | 0·9892 | 2·5 | 0·7135 | 7·0 | 0·9698 | 2·5 | 0·5247 | 9·0 | 0·9707 |
| ·6 | ·8931 | ·6 | ·9898 | ·6 | ·7275 | ·2 | ·9727 | ·6 | ·5425 | ·2 | ·9733 |
| ·7 | ·8997 | ·7 | ·9904 | ·7 | ·7408 | ·4 | ·9753 | ·7 | ·5598 | ·4 | ·9756 |
| ·8 | ·9057 | ·8 | ·9909 | ·8 | ·7534 | ·6 | ·9776 | ·8 | ·5765 | ·6 | ·9777 |
| ·9 | ·9114 | ·9 | ·9914 | ·9 | ·7654 | ·8 | ·9798 | ·9 | ·5927 | ·8 | ·9797 |
| 3·0 | 0·9167 | 7·0 | 0·9918 | 3·0 | 0·7769 | 8·0 | 0·9817 | 3·0 | 0·6084 | 10·0 | 0·9814 |
| ·1 | ·9217 | ·1 | ·9923 | ·1 | ·7878 | ·2 | ·9834 | ·1 | ·6235 | ·2 | ·9831 |
| ·2 | ·9264 | ·2 | ·9927 | ·2 | ·7981 | ·4 | ·9850 | ·2 | ·6382 | ·4 | ·9845 |
| ·3 | ·9307 | ·3 | ·9931 | ·3 | ·8080 | ·6 | ·9864 | ·3 | ·6524 | ·6 | ·9859 |
| ·4 | ·9348 | ·4 | ·9935 | ·4 | ·8173 | ·8 | ·9877 | ·4 | ·6660 | ·8 | ·9871 |
| 3·5 | 0·9386 | 7·5 | 0·9938 | 3·5 | 0·8262 | 9·0 | 0·9889 | 3·5 | 0·6792 | 11·0 | 0·9883 |
| ·6 | ·9422 | ·6 | ·9942 | ·6 | ·8347 | ·2 | ·9899 | ·6 | ·6920 | ·2 | ·9893 |
| ·7 | ·9456 | ·7 | ·9945 | ·7 | ·8428 | ·4 | ·9909 | ·7 | ·7043 | ·4 | ·9903 |
| ·8 | ·9487 | ·8 | ·9948 | ·8 | ·8504 | ·6 | ·9918 | ·8 | ·7161 | ·6 | ·9911 |
| ·9 | ·9517 | ·9 | ·9951 | ·9 | ·8577 | ·8 | ·9926 | ·9 | ·7275 | ·8 | ·9919 |
| 4·0 | 0·9545 | 8·0 | 0·9953 | 4·0 | 0·8647 | 10·0 | 0·9933 | 4·0 | 0·7385 | 12·0 | 0·9926 |

# TABLE 7. THE $\chi^2$-DISTRIBUTION FUNCTION

| $\nu =$ | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $x =$ 0·5 | 0·0265 | 0·0079 | 0·0022 | 0·0006 | 0·0001 | | | | | | |
| 1·0 | ·0902 | ·0374 | ·0144 | ·0052 | ·0018 | 0·0006 | 0·0002 | 0·0001 | | | |
| 1·5 | ·1734 | ·0869 | ·0405 | ·0177 | ·0073 | ·0029 | ·0011 | ·0004 | 0·0001 | | |
| 2·0 | ·2642 | ·1509 | ·0803 | ·0402 | ·0190 | ·0085 | ·0037 | ·0015 | ·0006 | 0·0002 | 0·0001 |
| 2·5 | 0·3554 | 0·2235 | 0·1315 | 0·0729 | 0·0383 | 0·0191 | 0·0091 | 0·0042 | 0·0018 | 0·0008 | 0·0003 |
| 3·0 | ·4422 | ·3000 | ·1912 | ·1150 | ·0656 | ·0357 | ·0186 | ·0093 | ·0045 | ·0021 | ·0009 |
| 3·5 | ·5221 | ·3766 | ·2560 | ·1648 | ·1008 | ·0589 | ·0329 | ·0177 | ·0091 | ·0046 | ·0022 |
| 4·0 | ·5940 | ·4506 | ·3233 | ·2202 | ·1429 | ·0886 | ·0527 | ·0301 | ·0166 | ·0088 | ·0045 |
| 4·5 | ·6575 | ·5201 | ·3907 | ·2793 | ·1906 | ·1245 | ·0780 | ·0471 | ·0274 | ·0154 | ·0084 |
| 5·0 | 0·7127 | 0·5841 | 0·4562 | 0·3400 | 0·2424 | 0·1657 | 0·1088 | 0·0688 | 0·0420 | 0·0248 | 0·0142 |
| 5·5 | ·7603 | ·6421 | ·5185 | ·4008 | ·2970 | ·2113 | ·1446 | ·0954 | ·0608 | ·0375 | ·0224 |
| 6·0 | ·8009 | ·6938 | ·5768 | ·4603 | ·3528 | ·2601 | ·1847 | ·1266 | ·0839 | ·0538 | ·0335 |
| 6·5 | ·8352 | ·7394 | ·6304 | ·5173 | ·4086 | ·3110 | ·2283 | ·1620 | ·1112 | ·0739 | ·0477 |
| 7·0 | ·8641 | ·7794 | ·6792 | ·5711 | ·4634 | ·3629 | ·2746 | ·2009 | ·1424 | ·0978 | ·0653 |
| 7·5 | 0·8883 | 0·8140 | 0·7229 | 0·6213 | 0·5162 | 0·4148 | 0·3225 | 0·2427 | 0·1771 | 0·1254 | 0·0863 |
| 8·0 | ·9084 | ·8438 | ·7619 | ·6674 | ·5665 | ·4659 | ·3712 | ·2867 | ·2149 | ·1564 | ·1107 |
| 8·5 | ·9251 | ·8693 | ·7963 | ·7094 | ·6138 | ·5154 | ·4199 | ·3321 | ·2551 | ·1904 | ·1383 |
| 9·0 | ·9389 | ·8909 | ·8264 | ·7473 | ·6577 | ·5627 | ·4679 | ·3781 | ·2971 | ·2271 | ·1689 |
| 9·5 | ·9503 | ·9093 | ·8527 | ·7813 | ·6981 | ·6075 | ·5146 | ·4242 | ·3403 | ·2658 | ·2022 |
| 10·0 | 0·9596 | 0·9248 | 0·8753 | 0·8114 | 0·7350 | 0·6495 | 0·5595 | 0·4696 | 0·3840 | 0·3061 | 0·2378 |
| 10·5 | ·9672 | ·9378 | ·8949 | ·8380 | ·7683 | ·6885 | ·6022 | ·5140 | ·4278 | ·3474 | ·2752 |
| 11·0 | ·9734 | ·9486 | ·9116 | ·8614 | ·7983 | ·7243 | ·6425 | ·5567 | ·4711 | ·3892 | ·3140 |
| 11·5 | ·9785 | ·9577 | ·9259 | ·8818 | ·8251 | ·7570 | ·6801 | ·5976 | ·5134 | ·4310 | ·3536 |
| 12·0 | ·9826 | ·9652 | ·9380 | ·8994 | ·8488 | ·7867 | ·7149 | ·6364 | ·5543 | ·4724 | ·3937 |
| 12·5 | 0·9860 | 0·9715 | 0·9483 | 0·9147 | 0·8697 | 0·8134 | 0·7470 | 0·6727 | 0·5936 | 0·5129 | 0·4338 |
| 13·0 | ·9887 | ·9766 | ·9570 | ·9279 | ·8882 | ·8374 | ·7763 | ·7067 | ·6310 | ·5522 | ·4735 |
| 13·5 | ·9909 | ·9809 | ·9643 | ·9392 | ·9042 | ·8587 | ·8030 | ·7381 | ·6662 | ·5900 | ·5124 |
| 14·0 | ·9927 | ·9844 | ·9704 | ·9488 | ·9182 | ·8777 | ·8270 | ·7670 | ·6993 | ·6262 | ·5503 |
| 14·5 | ·9941 | ·9873 | ·9755 | ·9570 | ·9304 | ·8944 | ·8486 | ·7935 | ·7301 | ·6604 | ·5868 |
| 15·0 | 0·9953 | 0·9896 | 0·9797 | 0·9640 | 0·9409 | 0·9091 | 0·8679 | 0·8175 | 0·7586 | 0·6926 | 0·6218 |
| 15·5 | ·9962 | ·9916 | ·9833 | ·9699 | ·9499 | ·9219 | ·8851 | ·8393 | ·7848 | ·7228 | ·6551 |
| 16·0 | ·9970 | ·9932 | ·9862 | ·9749 | ·9576 | ·9331 | ·9004 | ·8589 | ·8088 | ·7509 | ·6866 |
| 16·5 | ·9976 | ·9944 | ·9887 | ·9791 | ·9642 | ·9429 | ·9138 | ·8764 | ·8306 | ·7768 | ·7162 |
| 17·0 | ·9981 | ·9955 | ·9907 | ·9826 | ·9699 | ·9513 | ·9256 | ·8921 | ·8504 | ·8007 | ·7438 |
| 17·5 | 0·9985 | 0·9964 | 0·9924 | 0·9856 | 0·9747 | 0·9586 | 0·9360 | 0·9061 | 0·8683 | 0·8226 | 0·7695 |
| 18·0 | ·9988 | ·9971 | ·9938 | ·9880 | ·9788 | ·9648 | ·9450 | ·9184 | ·8843 | ·8425 | ·7932 |
| 18·5 | ·9990 | ·9976 | ·9949 | ·9901 | ·9822 | ·9702 | ·9529 | ·9293 | ·8987 | ·8606 | ·8151 |
| 19·0 | ·9992 | ·9981 | ·9958 | ·9918 | ·9851 | ·9748 | ·9597 | ·9389 | ·9115 | ·8769 | ·8351 |
| 19·5 | ·9994 | ·9984 | ·9966 | ·9932 | ·9876 | ·9787 | ·9656 | ·9473 | ·9228 | ·8916 | ·8533 |
| 20 | 0·9995 | 0·9988 | 0·9972 | 0·9944 | 0·9897 | 0·9821 | 0·9707 | 0·9547 | 0·9329 | 0·9048 | 0·8699 |
| 21 | ·9997 | ·9992 | ·9982 | ·9962 | ·9929 | ·9873 | ·9789 | ·9666 | ·9496 | ·9271 | ·8984 |
| 22 | ·9998 | ·9995 | ·9988 | ·9975 | ·9951 | ·9911 | ·9849 | ·9756 | ·9625 | ·9446 | ·9214 |
| 23 | ·9999 | ·9997 | ·9992 | ·9983 | ·9966 | ·9938 | ·9893 | ·9823 | ·9723 | ·9583 | ·9397 |
| 24 | ·9999 | ·9998 | ·9995 | ·9989 | ·9977 | ·9957 | ·9924 | ·9873 | ·9797 | ·9689 | ·9542 |
| 25 | 0·9999 | 0·9999 | 0·9997 | 0·9992 | 0·9984 | 0·9970 | 0·9947 | 0·9909 | 0·9852 | 0·9769 | 0·9654 |
| 26 | | ·9999 | ·9998 | ·9995 | ·9989 | ·9980 | ·9963 | ·9935 | ·9893 | ·9830 | ·9741 |
| 27 | | ·9999 | ·9999 | ·9997 | ·9993 | ·9986 | ·9974 | ·9954 | ·9923 | ·9876 | ·9807 |
| 28 | | | ·9999 | ·9998 | ·9995 | ·9990 | ·9982 | ·9968 | ·9945 | ·9910 | ·9858 |
| 29 | | | ·9999 | ·9999 | ·9997 | ·9994 | ·9988 | ·9977 | ·9961 | ·9935 | ·9895 |
| 30 | | | | 0·9999 | 0·9998 | 0·9996 | 0·9991 | 0·9984 | 0·9972 | 0·9953 | 0·9924 |

# TABLE 7. THE $\chi^2$-DISTRIBUTION FUNCTION

| $\nu =$ | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 3$ | 0·0004 | 0·0002 | 0·0001 | | | | | | | | |
| 4 | ·0023 | ·0011 | ·0005 | 0·0002 | 0·0001 | | | | | | |
| 5 | 0·0079 | 0·0042 | 0·0022 | 0·0011 | 0·0006 | 0·0003 | 0·0001 | 0·0001 | | | |
| 6 | ·0203 | ·0119 | ·0068 | ·0038 | ·0021 | ·0011 | ·0006 | ·0003 | 0·0001 | 0·0001 | |
| 7 | ·0424 | ·0267 | ·0165 | ·0099 | ·0058 | ·0033 | ·0019 | ·0010 | ·0005 | ·0003 | 0·0001 |
| 8 | ·0762 | ·0511 | ·0335 | ·0214 | ·0133 | ·0081 | ·0049 | ·0028 | ·0016 | ·0009 | ·0005 |
| 9 | ·1225 | ·0866 | ·0597 | ·0403 | ·0265 | ·0171 | ·0108 | ·0067 | ·0040 | ·0024 | ·0014 |
| 10 | 0·1803 | 0·1334 | 0·0964 | 0·0681 | 0·0471 | 0·0318 | 0·0211 | 0·0137 | 0·0087 | 0·0055 | 0·0033 |
| 11 | ·2474 | ·1905 | ·1434 | ·1056 | ·0762 | ·0538 | ·0372 | ·0253 | ·0168 | ·0110 | ·0071 |
| 12 | ·3210 | ·2560 | ·1999 | ·1528 | ·1144 | ·0839 | ·0604 | ·0426 | ·0295 | ·0201 | ·0134 |
| 13 | ·3977 | ·3272 | ·2638 | ·2084 | ·1614 | ·1226 | ·0914 | ·0668 | ·0480 | ·0339 | ·0235 |
| 14 | ·4745 | ·4013 | ·3329 | ·2709 | ·2163 | ·1695 | ·1304 | ·0985 | ·0731 | ·0533 | ·0383 |
| 15 | 0·5486 | 0·4754 | 0·4045 | 0·3380 | 0·2774 | 0·2236 | 0·1770 | 0·1378 | 0·1054 | 0·0792 | 0·0586 |
| 16 | ·6179 | ·5470 | ·4762 | ·4075 | ·3427 | ·2834 | ·2303 | ·1841 | ·1447 | ·1119 | ·0852 |
| 17 | ·6811 | ·6144 | ·5456 | ·4769 | ·4101 | ·3470 | ·2889 | ·2366 | ·1907 | ·1513 | ·1182 |
| 18 | ·7373 | ·6761 | ·6112 | ·5443 | ·4776 | ·4126 | ·3510 | ·2940 | ·2425 | ·1970 | ·1576 |
| 19 | ·7863 | ·7313 | ·6715 | ·6082 | ·5432 | ·4782 | ·4149 | ·3547 | ·2988 | ·2480 | ·2029 |
| 20 | 0·8281 | 0·7798 | 0·7258 | 0·6672 | 0·6054 | 0·5421 | 0·4787 | 0·4170 | 0·3581 | 0·3032 | 0·2532 |
| 21 | ·8632 | ·8215 | ·7737 | ·7206 | ·6632 | ·6029 | ·5411 | ·4793 | ·4189 | ·3613 | ·3074 |
| 22 | ·8922 | ·8568 | ·8153 | ·7680 | ·7157 | ·6595 | ·6005 | ·5401 | ·4797 | ·4207 | ·3643 |
| 23 | ·9159 | ·8863 | ·8507 | ·8094 | ·7627 | ·7112 | ·6560 | ·5983 | ·5392 | ·4802 | ·4224 |
| 24 | ·9349 | ·9105 | ·8806 | ·8450 | ·8038 | ·7576 | ·7069 | ·6528 | ·5962 | ·5384 | ·4806 |
| 25 | 0·9501 | 0·9302 | 0·9053 | 0·8751 | 0·8395 | 0·7986 | 0·7528 | 0·7029 | 0·6497 | 0·5942 | 0·5376 |
| 26 | ·9620 | ·9460 | ·9255 | ·9002 | ·8698 | ·8342 | ·7936 | ·7483 | ·6991 | ·6468 | ·5924 |
| 27 | ·9713 | ·9585 | ·9419 | ·9210 | ·8953 | ·8647 | ·8291 | ·7888 | ·7440 | ·6955 | ·6441 |
| 28 | ·9784 | ·9684 | ·9551 | ·9379 | ·9166 | ·8906 | ·8598 | ·8243 | ·7842 | ·7400 | ·6921 |
| 29 | ·9839 | ·9761 | ·9655 | ·9516 | ·9340 | ·9122 | ·8860 | ·8551 | ·8197 | ·7799 | ·7361 |
| 30 | 0·9881 | 0·9820 | 0·9737 | 0·9626 | 0·9482 | 0·9301 | 0·9080 | 0·8815 | 0·8506 | 0·8152 | 0·7757 |
| 31 | ·9912 | ·9865 | ·9800 | ·9712 | ·9596 | ·9448 | ·9263 | ·9039 | ·8772 | ·8462 | ·8110 |
| 32 | ·9936 | ·9900 | ·9850 | ·9780 | ·9687 | ·9567 | ·9414 | ·9226 | ·8999 | ·8730 | ·8420 |
| 33 | ·9953 | ·9926 | ·9887 | ·9833 | ·9760 | ·9663 | ·9538 | ·9381 | ·9189 | ·8959 | ·8689 |
| 34 | ·9966 | ·9946 | ·9916 | ·9874 | ·9816 | ·9739 | ·9638 | ·9509 | ·9348 | ·9153 | ·8921 |
| 35 | 0·9975 | 0·9960 | 0·9938 | 0·9905 | 0·9860 | 0·9799 | 0·9718 | 0·9613 | 0·9480 | 0·9316 | 0·9118 |
| 36 | ·9982 | ·9971 | ·9954 | ·9929 | ·9894 | ·9846 | ·9781 | ·9696 | ·9587 | ·9451 | ·9284 |
| 37 | ·9987 | ·9979 | ·9966 | ·9948 | ·9921 | ·9883 | ·9832 | ·9763 | ·9675 | ·9562 | ·9423 |
| 38 | ·9991 | ·9985 | ·9975 | ·9961 | ·9941 | ·9911 | ·9871 | ·9817 | ·9745 | ·9653 | ·9537 |
| 39 | ·9994 | ·9989 | ·9982 | ·9972 | ·9956 | ·9933 | ·9902 | ·9859 | ·9802 | ·9727 | ·9632 |
| 40 | 0·9995 | 0·9992 | 0·9987 | 0·9979 | 0·9967 | 0·9950 | 0·9926 | 0·9892 | 0·9846 | 0·9786 | 0·9708 |
| 41 | ·9997 | ·9994 | ·9991 | ·9985 | ·9976 | ·9963 | ·9944 | ·9918 | ·9882 | ·9833 | ·9770 |
| 42 | ·9998 | ·9996 | ·9993 | ·9989 | ·9982 | ·9972 | ·9958 | ·9937 | ·9909 | ·9871 | ·9820 |
| 43 | ·9998 | ·9997 | ·9995 | ·9992 | ·9987 | ·9980 | ·9969 | ·9953 | ·9931 | ·9901 | ·9860 |
| 44 | ·9999 | ·9998 | ·9997 | ·9994 | ·9991 | ·9985 | ·9977 | ·9965 | ·9947 | ·9924 | ·9892 |
| 45 | 0·9999 | 0·9999 | 0·9998 | 0·9996 | 0·9993 | 0·9989 | 0·9983 | 0·9973 | 0·9960 | 0·9942 | 0·9916 |
| 46 | ·9999 | ·9999 | ·9998 | ·9997 | ·9995 | ·9992 | ·9987 | ·9980 | ·9970 | ·9956 | ·9936 |
| 47 | | ·9999 | ·9999 | ·9998 | ·9996 | ·9994 | ·9991 | ·9985 | ·9978 | ·9967 | ·9951 |
| 48 | | | ·9999 | ·9998 | ·9997 | ·9996 | ·9993 | ·9989 | ·9983 | ·9975 | ·9963 |
| 49 | | | ·9999 | ·9999 | ·9998 | ·9997 | ·9995 | ·9992 | ·9988 | ·9981 | ·9972 |
| 50 | | | | 0·9999 | 0·9999 | 0·9998 | 0·9996 | 0·9994 | 0·9991 | 0·9986 | 0·9979 |

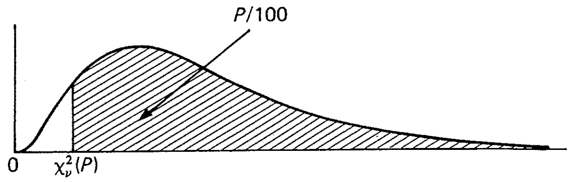
# TABLE 8. PERCENTAGE POINTS OF THE $\chi^2$-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})} \int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1}\, e^{-\frac{1}{2}x}\, dx.$$

If $X$ is a variable distributed as $\chi^2$ with $\nu$ degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu - 1}$ and unit variance.




(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

| $P$ | 99·95 | 99·9 | 99·5 | 99 | 97·5 | 95 | 90 | 80 | 70 | 60 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\nu = 1$ | $0{\cdot}0^{6}3927$ | $0{\cdot}0^{5}1571$ | $0{\cdot}0^{4}3927$ | $0{\cdot}0^{3}1571$ | $0{\cdot}0^{3}9821$ | 0·003932 | 0·01579 | 0·06418 | 0·1485 | 0·2750 |
| 2 | 0·001000 | 0·002001 | 0·01003 | 0·02010 | 0·05064 | 0·1026 | 0·2107 | 0·4463 | 0·7133 | 1·022 |
| 3 | 0·01528 | 0·02430 | 0·07172 | 0·1148 | 0·2158 | 0·3518 | 0·5844 | 1·005 | 1·424 | 1·869 |
| 4 | 0·06392 | 0·09080 | 0·2070 | 0·2971 | 0·4844 | 0·7107 | 1·064 | 1·649 | 2·195 | 2·753 |
| 5 | 0·1581 | 0·2102 | 0·4117 | 0·5543 | 0·8312 | 1·145 | 1·610 | 2·343 | 3·000 | 3·655 |
| 6 | 0·2994 | 0·3811 | 0·6757 | 0·8721 | 1·237 | 1·635 | 2·204 | 3·070 | 3·828 | 4·570 |
| 7 | 0·4849 | 0·5985 | 0·9893 | 1·239 | 1·690 | 2·167 | 2·833 | 3·822 | 4·671 | 5·493 |
| 8 | 0·7104 | 0·8571 | 1·344 | 1·646 | 2·180 | 2·733 | 3·490 | 4·594 | 5·527 | 6·423 |
| 9 | 0·9717 | 1·152 | 1·735 | 2·088 | 2·700 | 3·325 | 4·168 | 5·380 | 6·393 | 7·357 |
| 10 | 1·265 | 1·479 | 2·156 | 2·558 | 3·247 | 3·940 | 4·865 | 6·179 | 7·267 | 8·295 |
| 11 | 1·587 | 1·834 | 2·603 | 3·053 | 3·816 | 4·575 | 5·578 | 6·989 | 8·148 | 9·237 |
| 12 | 1·934 | 2·214 | 3·074 | 3·571 | 4·404 | 5·226 | 6·304 | 7·807 | 9·034 | 10·18 |
| 13 | 2·305 | 2·617 | 3·565 | 4·107 | 5·009 | 5·892 | 7·042 | 8·634 | 9·926 | 11·13 |
| 14 | 2·697 | 3·041 | 4·075 | 4·660 | 5·629 | 6·571 | 7·790 | 9·467 | 10·82 | 12·08 |
| 15 | 3·108 | 3·483 | 4·601 | 5·229 | 6·262 | 7·261 | 8·547 | 10·31 | 11·72 | 13·03 |
| 16 | 3·536 | 3·942 | 5·142 | 5·812 | 6·908 | 7·962 | 9·312 | 11·15 | 12·62 | 13·98 |
| 17 | 3·980 | 4·416 | 5·697 | 6·408 | 7·564 | 8·672 | 10·09 | 12·00 | 13·53 | 14·94 |
| 18 | 4·439 | 4·905 | 6·265 | 7·015 | 8·231 | 9·390 | 10·86 | 12·86 | 14·44 | 15·89 |
| 19 | 4·912 | 5·407 | 6·844 | 7·633 | 8·907 | 10·12 | 11·65 | 13·72 | 15·35 | 16·85 |
| 20 | 5·398 | 5·921 | 7·434 | 8·260 | 9·591 | 10·85 | 12·44 | 14·58 | 16·27 | 17·81 |
| 21 | 5·896 | 6·447 | 8·034 | 8·897 | 10·28 | 11·59 | 13·24 | 15·44 | 17·18 | 18·77 |
| 22 | 6·404 | 6·983 | 8·643 | 9·542 | 10·98 | 12·34 | 14·04 | 16·31 | 18·10 | 19·73 |
| 23 | 6·924 | 7·529 | 9·260 | 10·20 | 11·69 | 13·09 | 14·85 | 17·19 | 19·02 | 20·69 |
| 24 | 7·453 | 8·085 | 9·886 | 10·86 | 12·40 | 13·85 | 15·66 | 18·06 | 19·94 | 21·65 |
| 25 | 7·991 | 8·649 | 10·52 | 11·52 | 13·12 | 14·61 | 16·47 | 18·94 | 20·87 | 22·62 |
| 26 | 8·538 | 9·222 | 11·16 | 12·20 | 13·84 | 15·38 | 17·29 | 19·82 | 21·79 | 23·58 |
| 27 | 9·093 | 9·803 | 11·81 | 12·88 | 14·57 | 16·15 | 18·11 | 20·70 | 22·72 | 24·54 |
| 28 | 9·656 | 10·39 | 12·46 | 13·56 | 15·31 | 16·93 | 18·94 | 21·59 | 23·65 | 25·51 |
| 29 | 10·23 | 10·99 | 13·12 | 14·26 | 16·05 | 17·71 | 19·77 | 22·48 | 24·58 | 26·48 |
| 30 | 10·80 | 11·59 | 13·79 | 14·95 | 16·79 | 18·49 | 20·60 | 23·36 | 25·51 | 27·44 |
| 32 | 11·98 | 12·81 | 15·13 | 16·36 | 18·29 | 20·07 | 22·27 | 25·15 | 27·37 | 29·38 |
| 34 | 13·18 | 14·06 | 16·50 | 17·79 | 19·81 | 21·66 | 23·95 | 26·94 | 29·24 | 31·31 |
| 36 | 14·40 | 15·32 | 17·89 | 19·23 | 21·34 | 23·27 | 25·64 | 28·73 | 31·12 | 33·25 |
| 38 | 15·64 | 16·61 | 19·29 | 20·69 | 22·88 | 24·88 | 27·34 | 30·54 | 32·99 | 35·19 |
| 40 | 16·91 | 17·92 | 20·71 | 22·16 | 24·43 | 26·51 | 29·05 | 32·34 | 34·87 | 37·13 |
| 50 | 23·46 | 24·67 | 27·99 | 29·71 | 32·36 | 34·76 | 37·69 | 41·45 | 44·31 | 46·86 |
| 60 | 30·34 | 31·74 | 35·53 | 37·48 | 40·48 | 43·19 | 46·46 | 50·64 | 53·81 | 56·62 |
| 70 | 37·47 | 39·04 | 43·28 | 45·44 | 48·76 | 51·74 | 55·33 | 59·90 | 63·35 | 66·40 |
| 80 | 44·79 | 46·52 | 51·17 | 53·54 | 57·15 | 60·39 | 64·28 | 69·21 | 72·92 | 76·19 |
| 90 | 52·28 | 54·16 | 59·20 | 61·75 | 65·65 | 69·13 | 73·29 | 78·56 | 82·51 | 85·99 |
| 100 | 59·90 | 61·92 | 67·33 | 70·06 | 74·22 | 77·93 | 82·36 | 87·95 | 92·13 | 95·81 |

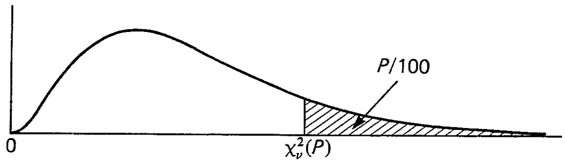
# TABLE 8. PERCENTAGE POINTS OF THE $\chi^2$-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})} \int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1}\, e^{-\frac{1}{2}x}\, dx.$$

If $X$ is a variable distributed as $\chi^2$ with $\nu$ degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu-1}$ and unit variance.




(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

| $P$ | 50 | 40 | 30 | 20 | 10 | 5 | 2·5 | 1 | 0·5 | 0·1 | 0·05 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu = 1$ | 0·4549 | 0·7083 | 1·074 | 1·642 | 2·706 | 3·841 | 5·024 | 6·635 | 7·879 | 10·83 | 12·12 |
| 2 | 1·386 | 1·833 | 2·408 | 3·219 | 4·605 | 5·991 | 7·378 | 9·210 | 10·60 | 13·82 | 15·20 |
| 3 | 2·366 | 2·946 | 3·665 | 4·642 | 6·251 | 7·815 | 9·348 | 11·34 | 12·84 | 16·27 | 17·73 |
| 4 | 3·357 | 4·045 | 4·878 | 5·989 | 7·779 | 9·488 | 11·14 | 13·28 | 14·86 | 18·47 | 20·00 |
| 5 | 4·351 | 5·132 | 6·064 | 7·289 | 9·236 | 11·07 | 12·83 | 15·09 | 16·75 | 20·52 | 22·11 |
| 6 | 5·348 | 6·211 | 7·231 | 8·558 | 10·64 | 12·59 | 14·45 | 16·81 | 18·55 | 22·46 | 24·10 |
| 7 | 6·346 | 7·283 | 8·383 | 9·803 | 12·02 | 14·07 | 16·01 | 18·48 | 20·28 | 24·32 | 26·02 |
| 8 | 7·344 | 8·351 | 9·524 | 11·03 | 13·36 | 15·51 | 17·53 | 20·09 | 21·95 | 26·12 | 27·87 |
| 9 | 8·343 | 9·414 | 10·66 | 12·24 | 14·68 | 16·92 | 19·02 | 21·67 | 23·59 | 27·88 | 29·67 |
| 10 | 9·342 | 10·47 | 11·78 | 13·44 | 15·99 | 18·31 | 20·48 | 23·21 | 25·19 | 29·59 | 31·42 |
| 11 | 10·34 | 11·53 | 12·90 | 14·63 | 17·28 | 19·68 | 21·92 | 24·72 | 26·76 | 31·26 | 33·14 |
| 12 | 11·34 | 12·58 | 14·01 | 15·81 | 18·55 | 21·03 | 23·34 | 26·22 | 28·30 | 32·91 | 34·82 |
| 13 | 12·34 | 13·64 | 15·12 | 16·98 | 19·81 | 22·36 | 24·74 | 27·69 | 29·82 | 34·53 | 36·48 |
| 14 | 13·34 | 14·69 | 16·22 | 18·15 | 21·06 | 23·68 | 26·12 | 29·14 | 31·32 | 36·12 | 38·11 |
| 15 | 14·34 | 15·73 | 17·32 | 19·31 | 22·31 | 25·00 | 27·49 | 30·58 | 32·80 | 37·70 | 39·72 |
| 16 | 15·34 | 16·78 | 18·42 | 20·47 | 23·54 | 26·30 | 28·85 | 32·00 | 34·27 | 39·25 | 41·31 |
| 17 | 16·34 | 17·82 | 19·51 | 21·61 | 24·77 | 27·59 | 30·19 | 33·41 | 35·72 | 40·79 | 42·88 |
| 18 | 17·34 | 18·87 | 20·60 | 22·76 | 25·99 | 28·87 | 31·53 | 34·81 | 37·16 | 42·31 | 44·43 |
| 19 | 18·34 | 19·91 | 21·69 | 23·90 | 27·20 | 30·14 | 32·85 | 36·19 | 38·58 | 43·82 | 45·97 |
| 20 | 19·34 | 20·95 | 22·77 | 25·04 | 28·41 | 31·41 | 34·17 | 37·57 | 40·00 | 45·31 | 47·50 |
| 21 | 20·34 | 21·99 | 23·86 | 26·17 | 29·62 | 32·67 | 35·48 | 38·93 | 41·40 | 46·80 | 49·01 |
| 22 | 21·34 | 23·03 | 24·94 | 27·30 | 30·81 | 33·92 | 36·78 | 40·29 | 42·80 | 48·27 | 50·51 |
| 23 | 22·34 | 24·07 | 26·02 | 28·43 | 32·01 | 35·17 | 38·08 | 41·64 | 44·18 | 49·73 | 52·00 |
| 24 | 23·34 | 25·11 | 27·10 | 29·55 | 33·20 | 36·42 | 39·36 | 42·98 | 45·56 | 51·18 | 53·48 |
| 25 | 24·34 | 26·14 | 28·17 | 30·68 | 34·38 | 37·65 | 40·65 | 44·31 | 46·93 | 52·62 | 54·95 |
| 26 | 25·34 | 27·18 | 29·25 | 31·79 | 35·56 | 38·89 | 41·92 | 45·64 | 48·29 | 54·05 | 56·41 |
| 27 | 26·34 | 28·21 | 30·32 | 32·91 | 36·74 | 40·11 | 43·19 | 46·96 | 49·64 | 55·48 | 57·86 |
| 28 | 27·34 | 29·25 | 31·39 | 34·03 | 37·92 | 41·34 | 44·46 | 48·28 | 50·99 | 56·89 | 59·30 |
| 29 | 28·34 | 30·28 | 32·46 | 35·14 | 39·09 | 42·56 | 45·72 | 49·59 | 52·34 | 58·30 | 60·73 |
| 30 | 29·34 | 31·32 | 33·53 | 36·25 | 40·26 | 43·77 | 46·98 | 50·89 | 53·67 | 59·70 | 62·16 |
| 32 | 31·34 | 33·38 | 35·66 | 38·47 | 42·58 | 46·19 | 49·48 | 53·49 | 56·33 | 62·49 | 65·00 |
| 34 | 33·34 | 35·44 | 37·80 | 40·68 | 44·90 | 48·60 | 51·97 | 56·06 | 58·96 | 65·25 | 67·80 |
| 36 | 35·34 | 37·50 | 39·92 | 42·88 | 47·21 | 51·00 | 54·44 | 58·62 | 61·58 | 67·99 | 70·59 |
| 38 | 37·34 | 39·56 | 42·05 | 45·08 | 49·51 | 53·38 | 56·90 | 61·16 | 64·18 | 70·70 | 73·35 |
| 40 | 39·34 | 41·62 | 44·16 | 47·27 | 51·81 | 55·76 | 59·34 | 63·69 | 66·77 | 73·40 | 76·09 |
| 50 | 49·33 | 51·89 | 54·72 | 58·16 | 63·17 | 67·50 | 71·42 | 76·15 | 79·49 | 86·66 | 89·56 |
| 60 | 59·33 | 62·13 | 65·23 | 68·97 | 74·40 | 79·08 | 83·30 | 88·38 | 91·95 | 99·61 | 102·7 |
| 70 | 69·33 | 72·36 | 75·69 | 79·71 | 85·53 | 90·53 | 95·02 | 100·4 | 104·2 | 112·3 | 115·6 |
| 80 | 79·33 | 82·57 | 86·12 | 90·41 | 96·58 | 101·9 | 106·6 | 112·3 | 116·3 | 124·8 | 128·3 |
| 90 | 89·33 | 92·76 | 96·52 | 101·1 | 107·6 | 113·1 | 118·1 | 124·1 | 128·3 | 137·2 | 140·8 |
| 100 | 99·33 | 102·9 | 106·9 | 111·7 | 118·5 | 124·3 | 129·6 | 135·8 | 140·2 | 149·4 | 153·2 |

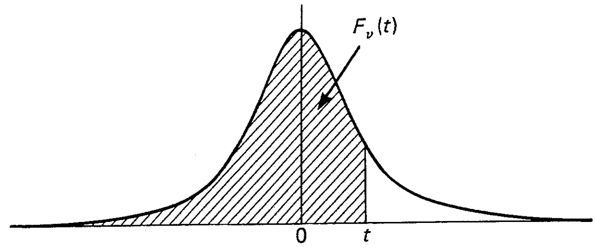
# TABLE 9. THE $t$-DISTRIBUTION FUNCTION

The function tabulated is

$$F_\nu(t) = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{-\infty}^{t} \frac{ds}{(1+s^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

$F_\nu(t)$ is the probability that a random variable, distributed as $t$ with $\nu$ degrees of freedom, will be less than or equal to $t$. When $t < 0$ use $F_\nu(t) = 1 - F_\nu(-t)$, the $t$ distribution being symmetric about zero.

The limiting distribution of $t$ as $\nu$ tends to infinity is the normal distribution with zero mean and unit variance (see Table 4). When $\nu$ is large interpolation in $\nu$ should be harmonic.




Omitted entries to the right of tabulated values are 1 (to four decimal places).

| $\nu =$ | 1 | $\nu =$ | 1 | $\nu =$ | 2 | $\nu =$ | 2 | $\nu =$ | 3 | $\nu =$ | 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $t =$ 0·0 | 0·5000 | $t =$ 4·0 | 0·9220 | $t =$ 0·0 | 0·5000 | $t =$ 4·0 | 0·9714 | $t =$ 0·0 | 0·5000 | $t =$ 4·0 | 0·9860 |
| ·1 | ·5317 | 4·2 | ·9256 | ·1 | ·5353 | ·1 | ·9727 | ·1 | ·5367 | ·1 | ·9869 |
| ·2 | ·5628 | 4·4 | ·9289 | ·2 | ·5700 | ·2 | ·9739 | ·2 | ·5729 | ·2 | ·9877 |
| ·3 | ·5928 | 4·6 | ·9319 | ·3 | ·6038 | ·3 | ·9750 | ·3 | ·6081 | ·3 | ·9884 |
| ·4 | ·6211 | 4·8 | ·9346 | ·4 | ·6361 | ·4 | ·9760 | ·4 | ·6420 | ·4 | ·9891 |
| 0·5 | 0·6476 | 5·0 | 0·9372 | 0·5 | 0·6667 | 4·5 | 0·9770 | 0·5 | 0·6743 | 4·5 | 0·9898 |
| ·6 | ·6720 | 5·5 | ·9428 | ·6 | ·6953 | ·6 | ·9779 | ·6 | ·7046 | ·6 | ·9903 |
| ·7 | ·6944 | 6·0 | ·9474 | ·7 | ·7218 | ·7 | ·9788 | ·7 | ·7328 | ·7 | ·9909 |
| ·8 | ·7148 | 6·5 | ·9514 | ·8 | ·7462 | ·8 | ·9796 | ·8 | ·7589 | ·8 | ·9914 |
| ·9 | ·7333 | 7·0 | ·9548 | ·9 | ·7684 | ·9 | ·9804 | ·9 | ·7828 | ·9 | ·9919 |
| 1·0 | 0·7500 | 7·5 | 0·9578 | 1·0 | 0·7887 | 5·0 | 0·9811 | 1·0 | 0·8045 | 5·0 | 0·9923 |
| ·1 | ·7651 | 8·0 | ·9604 | ·1 | ·8070 | ·1 | ·9818 | ·1 | ·8242 | ·1 | ·9927 |
| ·2 | ·7789 | 8·5 | ·9627 | ·2 | ·8235 | ·2 | ·9825 | ·2 | ·8419 | ·2 | ·9931 |
| ·3 | ·7913 | 9·0 | ·9648 | ·3 | ·8384 | ·3 | ·9831 | ·3 | ·8578 | ·3 | ·9934 |
| ·4 | ·8026 | 9·5 | ·9666 | ·4 | ·8518 | ·4 | ·9837 | ·4 | ·8720 | ·4 | ·9938 |
| 1·5 | 0·8128 | 10·0 | 0·9683 | 1·5 | 0·8638 | 5·5 | 0·9842 | 1·5 | 0·8847 | 5·5 | 0·9941 |
| ·6 | ·8222 | 10·5 | ·9698 | ·6 | ·8746 | ·6 | ·9848 | ·6 | ·8960 | ·6 | ·9944 |
| ·7 | ·8307 | 11·0 | ·9711 | ·7 | ·8844 | ·7 | ·9853 | ·7 | ·9062 | ·7 | ·9946 |
| ·8 | ·8386 | 11·5 | ·9724 | ·8 | ·8932 | ·8 | ·9858 | ·8 | ·9152 | ·8 | ·9949 |
| ·9 | ·8458 | 12·0 | ·9735 | ·9 | ·9011 | ·9 | ·9862 | ·9 | ·9232 | ·9 | ·9951 |
| 2·0 | 0·8524 | 12·5 | 0·9746 | 2·0 | 0·9082 | 6·0 | 0·9867 | 2·0 | 0·9303 | 6·0 | 0·9954 |
| ·1 | ·8585 | 13·0 | ·9756 | ·1 | ·9147 | ·1 | ·9871 | ·1 | ·9367 | ·1 | ·9956 |
| ·2 | ·8642 | 13·5 | ·9765 | ·2 | ·9206 | ·2 | ·9875 | ·2 | ·9424 | ·2 | ·9958 |
| ·3 | ·8695 | 14·0 | ·9773 | ·3 | ·9259 | ·3 | ·9879 | ·3 | ·9475 | ·3 | ·9960 |
| ·4 | ·8743 | 14·5 | ·9781 | ·4 | ·9308 | ·4 | ·9882 | ·4 | ·9521 | ·4 | ·9961 |
| 2·5 | 0·8789 | 15 | 0·9788 | 2·5 | 0·9352 | 6·5 | 0·9886 | 2·5 | 0·9561 | 6·5 | 0·9963 |
| ·6 | ·8831 | 16 | ·9801 | ·6 | ·9392 | ·6 | ·9889 | ·6 | ·9598 | ·6 | ·9965 |
| ·7 | ·8871 | 17 | ·9813 | ·7 | ·9429 | ·7 | ·9892 | ·7 | ·9631 | ·7 | ·9966 |
| ·8 | ·8908 | 18 | ·9823 | ·8 | ·9463 | ·8 | ·9895 | ·8 | ·9661 | ·8 | ·9967 |
| ·9 | ·8943 | 19 | ·9833 | ·9 | ·9494 | ·9 | ·9898 | ·9 | ·9687 | ·9 | ·9969 |
| 3·0 | 0·8976 | 20 | 0·9841 | 3·0 | 0·9523 | 7·0 | 0·9901 | 3·0 | 0·9712 | 7·0 | 0·9970 |
| ·1 | ·9007 | 21 | ·9849 | ·1 | ·9549 | ·1 | ·9904 | ·1 | ·9734 | ·1 | ·9971 |
| ·2 | ·9036 | 22 | ·9855 | ·2 | ·9573 | ·2 | ·9906 | ·2 | ·9753 | ·2 | ·9972 |
| ·3 | ·9063 | 23 | ·9862 | ·3 | ·9596 | ·3 | ·9909 | ·3 | ·9771 | ·3 | ·9973 |
| ·4 | ·9089 | 24 | ·9867 | ·4 | ·9617 | ·4 | ·9911 | ·4 | ·9788 | ·4 | ·9974 |
| 3·5 | 0·9114 | 25 | 0·9873 | 3·5 | 0·9636 | 7·5 | 0·9913 | 3·5 | 0·9803 | 7·5 | 0·9975 |
| ·6 | ·9138 | 30 | ·9894 | ·6 | ·9654 | ·6 | ·9916 | ·6 | ·9816 | ·6 | ·9976 |
| ·7 | ·9160 | 35 | ·9909 | ·7 | ·9670 | ·7 | ·9918 | ·7 | ·9829 | ·7 | ·9977 |
| ·8 | ·9181 | 40 | ·9920 | ·8 | ·9686 | ·8 | ·9920 | ·8 | ·9840 | ·8 | ·9978 |
| ·9 | ·9201 | 45 | ·9929 | ·9 | ·9701 | ·9 | ·9922 | ·9 | ·9850 | ·9 | ·9979 |
| 4·0 | 0·9220 | 50 | 0·9936 | 4·0 | 0·9714 | 8·0 | 0·9924 | 4·0 | 0·9860 | 8·0 | 0·9980 |

# TABLE 9. THE $t$-DISTRIBUTION FUNCTION

| $\nu =$ | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $t =$ 0·0 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 |
| ·1 | ·5374 | ·5379 | ·5382 | ·5384 | ·5386 | ·5387 | ·5388 | ·5389 | ·5390 | ·5391 | ·5391 |
| ·2 | ·5744 | ·5753 | ·5760 | ·5764 | ·5768 | ·5770 | ·5773 | ·5774 | ·5776 | ·5777 | ·5778 |
| ·3 | ·6104 | ·6119 | ·6129 | ·6136 | ·6141 | ·6145 | ·6148 | ·6151 | ·6153 | ·6155 | ·6157 |
| ·4 | ·6452 | ·6472 | ·6485 | ·6495 | ·6502 | ·6508 | ·6512 | ·6516 | ·6519 | ·6522 | ·6524 |
| 0·5 | 0·6783 | 0·6809 | 0·6826 | 0·6838 | 0·6847 | 0·6855 | 0·6861 | 0·6865 | 0·6869 | 0·6873 | 0·6876 |
| ·6 | ·7096 | ·7127 | ·7148 | ·7163 | ·7174 | ·7183 | ·7191 | ·7197 | ·7202 | ·7206 | ·7210 |
| ·7 | ·7387 | ·7424 | ·7449 | ·7467 | ·7481 | ·7492 | ·7501 | ·7508 | ·7514 | ·7519 | ·7523 |
| ·8 | ·7657 | ·7700 | ·7729 | ·7750 | ·7766 | ·7778 | ·7788 | ·7797 | ·7804 | ·7810 | ·7815 |
| ·9 | ·7905 | ·7953 | ·7986 | ·8010 | ·8028 | ·8042 | ·8054 | ·8063 | ·8071 | ·8078 | ·8083 |
| 1·0 | 0·8130 | 0·8184 | 0·8220 | 0·8247 | 0·8267 | 0·8283 | 0·8296 | 0·8306 | 0·8315 | 0·8322 | 0·8329 |
| ·1 | ·8335 | ·8393 | ·8433 | ·8461 | ·8483 | ·8501 | ·8514 | ·8526 | ·8535 | ·8544 | ·8551 |
| ·2 | ·8518 | ·8581 | ·8623 | ·8654 | ·8678 | ·8696 | ·8711 | ·8723 | ·8734 | ·8742 | ·8750 |
| ·3 | ·8683 | ·8748 | ·8793 | ·8826 | ·8851 | ·8870 | ·8886 | ·8899 | ·8910 | ·8919 | ·8927 |
| ·4 | ·8829 | ·8898 | ·8945 | ·8979 | ·9005 | ·9025 | ·9041 | ·9055 | ·9066 | ·9075 | ·9084 |
| 1·5 | 0·8960 | 0·9030 | 0·9079 | 0·9114 | 0·9140 | 0·9161 | 0·9177 | 0·9191 | 0·9203 | 0·9212 | 0·9221 |
| ·6 | ·9076 | ·9148 | ·9196 | ·9232 | ·9259 | ·9280 | ·9297 | ·9310 | ·9322 | ·9332 | ·9340 |
| ·7 | ·9178 | ·9251 | ·9300 | ·9335 | ·9362 | ·9383 | ·9400 | ·9414 | ·9426 | ·9435 | ·9444 |
| ·8 | ·9269 | ·9341 | ·9390 | ·9426 | ·9452 | ·9473 | ·9490 | ·9503 | ·9515 | ·9525 | ·9533 |
| ·9 | ·9349 | ·9421 | ·9469 | ·9504 | ·9530 | ·9551 | ·9567 | ·9580 | ·9591 | ·9601 | ·9609 |
| 2·0 | 0·9419 | 0·9490 | 0·9538 | 0·9572 | 0·9597 | 0·9617 | 0·9633 | 0·9646 | 0·9657 | 0·9666 | 0·9674 |
| ·1 | ·9482 | ·9551 | ·9598 | ·9631 | ·9655 | ·9674 | ·9690 | ·9702 | ·9712 | ·9721 | ·9728 |
| ·2 | ·9537 | ·9605 | ·9649 | ·9681 | ·9705 | ·9723 | ·9738 | ·9750 | ·9759 | ·9768 | ·9774 |
| ·3 | ·9585 | ·9651 | ·9694 | ·9725 | ·9748 | ·9765 | ·9779 | ·9790 | ·9799 | ·9807 | ·9813 |
| ·4 | ·9628 | ·9692 | ·9734 | ·9763 | ·9784 | ·9801 | ·9813 | ·9824 | ·9832 | ·9840 | ·9846 |
| 2·5 | 0·9666 | 0·9728 | 0·9767 | 0·9795 | 0·9815 | 0·9831 | 0·9843 | 0·9852 | 0·9860 | 0·9867 | 0·9873 |
| ·6 | ·9700 | ·9759 | ·9797 | ·9823 | ·9842 | ·9856 | ·9868 | ·9877 | ·9884 | ·9890 | ·9895 |
| ·7 | ·9730 | ·9786 | ·9822 | ·9847 | ·9865 | ·9878 | ·9888 | ·9897 | ·9903 | ·9909 | ·9914 |
| ·8 | ·9756 | ·9810 | ·9844 | ·9867 | ·9884 | ·9896 | ·9906 | ·9914 | ·9920 | ·9925 | ·9929 |
| ·9 | ·9779 | ·9831 | ·9863 | ·9885 | ·9901 | ·9912 | ·9921 | ·9928 | ·9933 | ·9938 | ·9942 |
| 3·0 | 0·9800 | 0·9850 | 0·9880 | 0·9900 | 0·9915 | 0·9925 | 0·9933 | 0·9940 | 0·9945 | 0·9949 | 0·9952 |
| ·1 | ·9819 | ·9866 | ·9894 | ·9913 | ·9927 | ·9936 | ·9944 | ·9949 | ·9954 | ·9958 | ·9961 |
| ·2 | ·9835 | ·9880 | ·9907 | ·9925 | ·9937 | ·9946 | ·9953 | ·9958 | ·9962 | ·9965 | ·9968 |
| ·3 | ·9850 | ·9893 | ·9918 | ·9934 | ·9946 | ·9954 | ·9960 | ·9965 | ·9968 | ·9971 | ·9974 |
| ·4 | ·9864 | ·9904 | ·9928 | ·9943 | ·9953 | ·9961 | ·9966 | ·9970 | ·9974 | ·9976 | ·9978 |
| 3·5 | 0·9876 | 0·9914 | 0·9936 | 0·9950 | 0·9960 | 0·9966 | 0·9971 | 0·9975 | 0·9978 | 0·9980 | 0·9982 |
| ·6 | ·9886 | ·9922 | ·9943 | ·9956 | ·9965 | ·9971 | ·9976 | ·9979 | ·9982 | ·9984 | ·9986 |
| ·7 | ·9896 | ·9930 | ·9950 | ·9962 | ·9970 | ·9975 | ·9979 | ·9982 | ·9985 | ·9987 | ·9988 |
| ·8 | ·9904 | ·9937 | ·9955 | ·9966 | ·9974 | ·9979 | ·9983 | ·9985 | ·9987 | ·9989 | ·9990 |
| ·9 | ·9912 | ·9943 | ·9960 | ·9971 | ·9977 | ·9982 | ·9985 | ·9988 | ·9989 | ·9991 | ·9992 |
| 4·0 | 0·9919 | 0·9948 | 0·9964 | 0·9974 | 0·9980 | 0·9984 | 0·9987 | 0·9990 | 0·9991 | 0·9992 | 0·9993 |
| ·1 | ·9926 | ·9953 | ·9968 | ·9977 | ·9983 | ·9987 | ·9989 | ·9991 | ·9993 | ·9994 | ·9995 |
| ·2 | ·9932 | ·9958 | ·9972 | ·9980 | ·9985 | ·9988 | ·9991 | ·9993 | ·9994 | ·9995 | ·9996 |
| ·3 | ·9937 | ·9961 | ·9975 | ·9982 | ·9987 | ·9990 | ·9992 | ·9994 | ·9995 | ·9996 | ·9996 |
| ·4 | ·9942 | ·9965 | ·9977 | ·9984 | ·9989 | ·9991 | ·9993 | ·9995 | ·9996 | ·9996 | ·9997 |
| 4·5 | 0·9946 | 0·9968 | 0·9979 | 0·9986 | 0·9990 | 0·9993 | 0·9994 | 0·9995 | 0·9996 | 0·9997 | 0·9998 |
| ·6 | ·9950 | ·9971 | ·9982 | ·9988 | ·9991 | ·9994 | ·9995 | ·9996 | ·9997 | ·9998 | ·9998 |
| ·7 | ·9953 | ·9973 | ·9983 | ·9989 | ·9992 | ·9994 | ·9996 | ·9997 | ·9997 | ·9998 | ·9998 |
| ·8 | ·9957 | ·9976 | ·9985 | ·9990 | ·9993 | ·9995 | ·9996 | ·9997 | ·9998 | ·9998 | ·9999 |
| ·9 | ·9960 | ·9978 | ·9986 | ·9991 | ·9994 | ·9996 | ·9997 | ·9998 | ·9998 | ·9999 | ·9999 |
| 5·0 | 0·9963 | 0·9979 | 0·9988 | 0·9992 | 0·9995 | 0·9996 | 0·9997 | 0·9998 | 0·9998 | 0·9999 | 0·9999 |

# TABLE 9. THE $t$-DISTRIBUTION FUNCTION

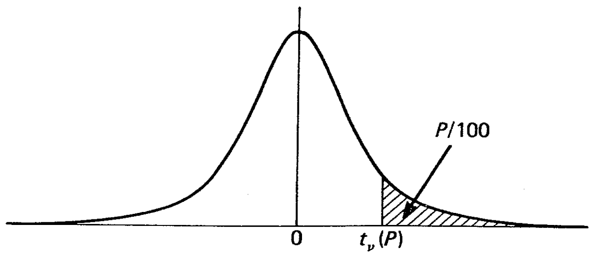
| $\nu =$ | 15 | 16 | 17 | 18 | 19 | 20 | 24 | 30 | 40 | 60 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $t =$ 0·0 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 | 0·5000 |
| ·1 | ·5392 | ·5392 | ·5392 | ·5393 | ·5393 | ·5393 | ·5394 | ·5395 | ·5396 | ·5397 | ·5398 |
| ·2 | ·5779 | ·5780 | ·5781 | ·5781 | ·5782 | ·5782 | ·5784 | ·5786 | ·5788 | ·5789 | ·5793 |
| ·3 | ·6159 | ·6160 | ·6161 | ·6162 | ·6163 | ·6164 | ·6166 | ·6169 | ·6171 | ·6174 | ·6179 |
| ·4 | ·6526 | ·6528 | ·6529 | ·6531 | ·6532 | ·6533 | ·6537 | ·6540 | ·6544 | ·6547 | ·6554 |
| 0·5 | 0·6878 | 0·6881 | 0·6883 | 0·6884 | 0·6886 | 0·6887 | 0·6892 | 0·6896 | 0·6901 | 0·6905 | 0·6915 |
| ·6 | ·7213 | ·7215 | ·7218 | ·7220 | ·7222 | ·7224 | ·7229 | ·7235 | ·7241 | ·7246 | ·7257 |
| ·7 | ·7527 | ·7530 | ·7533 | ·7536 | ·7538 | ·7540 | ·7547 | ·7553 | ·7560 | ·7567 | ·7580 |
| ·8 | ·7819 | ·7823 | ·7826 | ·7829 | ·7832 | ·7834 | ·7842 | ·7850 | ·7858 | ·7866 | ·7881 |
| ·9 | ·8088 | ·8093 | ·8097 | ·8100 | ·8103 | ·8106 | ·8115 | ·8124 | ·8132 | ·8141 | ·8159 |
| 1·0 | 0·8334 | 0·8339 | 0·8343 | 0·8347 | 0·8351 | 0·8354 | 0·8364 | 0·8373 | 0·8383 | 0·8393 | 0·8413 |
| ·1 | ·8557 | ·8562 | ·8567 | ·8571 | ·8575 | ·8578 | ·8589 | ·8600 | ·8610 | ·8621 | ·8643 |
| ·2 | ·8756 | ·8762 | ·8767 | ·8772 | ·8776 | ·8779 | ·8791 | ·8802 | ·8814 | ·8826 | ·8849 |
| ·3 | ·8934 | ·8940 | ·8945 | ·8950 | ·8954 | ·8958 | ·8970 | ·8982 | ·8995 | ·9007 | ·9032 |
| ·4 | ·9091 | ·9097 | ·9103 | ·9107 | ·9112 | ·9116 | ·9128 | ·9141 | ·9154 | ·9167 | ·9192 |
| 1·5 | 0·9228 | 0·9235 | 0·9240 | 0·9245 | 0·9250 | 0·9254 | 0·9267 | 0·9280 | 0·9293 | 0·9306 | 0·9332 |
| ·6 | ·9348 | ·9354 | ·9360 | ·9365 | ·9370 | ·9374 | ·9387 | ·9400 | ·9413 | ·9426 | ·9452 |
| ·7 | ·9451 | ·9458 | ·9463 | ·9468 | ·9473 | ·9477 | ·9490 | ·9503 | ·9516 | ·9528 | ·9554 |
| ·8 | ·9540 | ·9546 | ·9552 | ·9557 | ·9561 | ·9565 | ·9578 | ·9590 | ·9603 | ·9616 | ·9641 |
| ·9 | ·9616 | ·9622 | ·9627 | ·9632 | ·9636 | ·9640 | ·9652 | ·9665 | ·9677 | ·9689 | ·9713 |
| 2·0 | 0·9680 | 0·9686 | 0·9691 | 0·9696 | 0·9700 | 0·9704 | 0·9715 | 0·9727 | 0·9738 | 0·9750 | 0·9772 |
| ·1 | ·9735 | ·9740 | ·9745 | ·9750 | ·9753 | ·9757 | ·9768 | ·9779 | ·9790 | ·9800 | ·9821 |
| ·2 | ·9781 | ·9786 | ·9790 | ·9794 | ·9798 | ·9801 | ·9812 | ·9822 | ·9832 | ·9842 | ·9861 |
| ·3 | ·9819 | ·9824 | ·9828 | ·9832 | ·9835 | ·9838 | ·9848 | ·9857 | ·9866 | ·9875 | ·9893 |
| ·4 | ·9851 | ·9855 | ·9859 | ·9863 | ·9866 | ·9869 | ·9877 | ·9886 | ·9894 | ·9902 | ·9918 |
| 2·5 | 0·9877 | 0·9882 | 0·9885 | 0·9888 | 0·9891 | 0·9894 | 0·9902 | 0·9909 | 0·9917 | 0·9924 | 0·9938 |
| ·6 | ·9900 | ·9903 | ·9907 | ·9910 | ·9912 | ·9914 | ·9921 | ·9928 | ·9935 | ·9941 | ·9953 |
| ·7 | ·9918 | ·9921 | ·9924 | ·9927 | ·9929 | ·9931 | ·9937 | ·9944 | ·9949 | ·9955 | ·9965 |
| ·8 | ·9933 | ·9936 | ·9938 | ·9941 | ·9943 | ·9945 | ·9950 | ·9956 | ·9961 | ·9966 | ·9974 |
| ·9 | ·9945 | ·9948 | ·9950 | ·9952 | ·9954 | ·9956 | ·9961 | ·9965 | ·9970 | ·9974 | ·9981 |
| 3·0 | 0·9955 | 0·9958 | 0·9960 | 0·9962 | 0·9963 | 0·9965 | 0·9969 | 0·9973 | 0·9977 | 0·9980 | 0·9987 |
| ·1 | ·9963 | ·9966 | ·9967 | ·9969 | ·9971 | ·9972 | ·9976 | ·9979 | ·9982 | ·9985 | ·9990 |
| ·2 | ·9970 | ·9972 | ·9974 | ·9975 | ·9976 | ·9978 | ·9981 | ·9984 | ·9987 | ·9989 | ·9993 |
| ·3 | ·9976 | ·9977 | ·9979 | ·9980 | ·9981 | ·9982 | ·9985 | ·9988 | ·9990 | ·9992 | ·9995 |
| ·4 | ·9980 | ·9982 | ·9983 | ·9984 | ·9985 | ·9986 | ·9988 | ·9990 | ·9992 | ·9994 | ·9997 |
| 3·5 | 0·9984 | 0·9985 | 0·9986 | 0·9987 | 0·9988 | 0·9989 | 0·9991 | 0·9993 | 0·9994 | 0·9996 | 0·9998 |
| ·6 | ·9987 | ·9988 | ·9989 | ·9990 | ·9990 | ·9991 | ·9993 | ·9994 | ·9996 | ·9997 | ·9998 |
| ·7 | ·9989 | ·9990 | ·9991 | ·9992 | ·9992 | ·9993 | ·9994 | ·9996 | ·9997 | ·9998 | ·9999 |
| ·8 | ·9991 | ·9992 | ·9993 | ·9993 | ·9994 | ·9994 | ·9996 | ·9997 | ·9998 | ·9998 | ·9999 |
| ·9 | ·9993 | ·9994 | ·9994 | ·9995 | ·9995 | ·9996 | ·9997 | ·9997 | ·9998 | ·9999 | |
| 4·0 | 0·9994 | 0·9995 | 0·9995 | 0·9996 | 0·9996 | 0·9996 | 0·9997 | 0·9998 | 0·9999 | 0·9999 | |
| ·1 | ·9995 | ·9996 | ·9996 | ·9997 | ·9997 | ·9997 | ·9998 | ·9999 | ·9999 | ·9999 | |
| ·2 | ·9996 | ·9997 | ·9997 | ·9997 | ·9998 | ·9998 | ·9998 | ·9999 | ·9999 | | |
| ·3 | ·9997 | ·9997 | ·9998 | ·9998 | ·9998 | ·9998 | ·9999 | ·9999 | ·9999 | | |
| ·4 | ·9997 | ·9998 | ·9998 | ·9998 | ·9998 | ·9999 | ·9999 | ·9999 | | | |
| 4·5 | 0·9998 | 0·9998 | 0·9998 | 0·9999 | 0·9999 | 0·9999 | 0·9999 | | | | |

# TABLE 10. PERCENTAGE POINTS OF THE *t*-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}}\frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)}\int_{t_\nu(P)}^{\infty}\frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let $X_1$ and $X_2$ be independent random variables having a normal distribution with zero mean and unit variance and a $\chi^2$-distribution with $\nu$ degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's *t*-distribution with $\nu$ degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.




The limiting distribution of $t$ as $\nu$ tends to infinity is the normal distribution with zero mean and unit variance. When $\nu$ is large interpolation in $\nu$ should be harmonic.

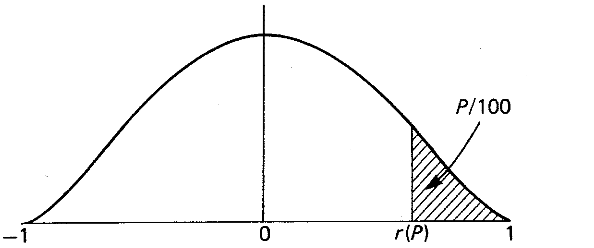
| $P$ | 40 | 30 | 25 | 20 | 15 | 10 | 5 | 2·5 | 1 | 0·5 | 0·1 | 0·05 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu = 1$ | 0·3249 | 0·7265 | 1·0000 | 1·3764 | 1·963 | 3·078 | 6·314 | 12·71 | 31·82 | 63·66 | 318·3 | 636·6 |
| 2 | 0·2887 | 0·6172 | 0·8165 | 1·0607 | 1·386 | 1·886 | 2·920 | 4·303 | 6·965 | 9·925 | 22·33 | 31·60 |
| 3 | 0·2767 | 0·5844 | 0·7649 | 0·9785 | 1·250 | 1·638 | 2·353 | 3·182 | 4·541 | 5·841 | 10·21 | 12·92 |
| 4 | 0·2707 | 0·5686 | 0·7407 | 0·9410 | 1·190 | 1·533 | 2·132 | 2·776 | 3·747 | 4·604 | 7·173 | 8·610 |
| 5 | 0·2672 | 0·5594 | 0·7267 | 0·9195 | 1·156 | 1·476 | 2·015 | 2·571 | 3·365 | 4·032 | 5·893 | 6·869 |
| 6 | 0·2648 | 0·5534 | 0·7176 | 0·9057 | 1·134 | 1·440 | 1·943 | 2·447 | 3·143 | 3·707 | 5·208 | 5·959 |
| 7 | 0·2632 | 0·5491 | 0·7111 | 0·8960 | 1·119 | 1·415 | 1·895 | 2·365 | 2·998 | 3·499 | 4·785 | 5·408 |
| 8 | 0·2619 | 0·5459 | 0·7064 | 0·8889 | 1·108 | 1·397 | 1·860 | 2·306 | 2·896 | 3·355 | 4·501 | 5·041 |
| 9 | 0·2610 | 0·5435 | 0·7027 | 0·8834 | 1·100 | 1·383 | 1·833 | 2·262 | 2·821 | 3·250 | 4·297 | 4·781 |
| 10 | 0·2602 | 0·5415 | 0·6998 | 0·8791 | 1·093 | 1·372 | 1·812 | 2·228 | 2·764 | 3·169 | 4·144 | 4·587 |
| 11 | 0·2596 | 0·5399 | 0·6974 | 0·8755 | 1·088 | 1·363 | 1·796 | 2·201 | 2·718 | 3·106 | 4·025 | 4·437 |
| 12 | 0·2590 | 0·5386 | 0·6955 | 0·8726 | 1·083 | 1·356 | 1·782 | 2·179 | 2·681 | 3·055 | 3·930 | 4·318 |
| 13 | 0·2586 | 0·5375 | 0·6938 | 0·8702 | 1·079 | 1·350 | 1·771 | 2·160 | 2·650 | 3·012 | 3·852 | 4·221 |
| 14 | 0·2582 | 0·5366 | 0·6924 | 0·8681 | 1·076 | 1·345 | 1·761 | 2·145 | 2·624 | 2·977 | 3·787 | 4·140 |
| 15 | 0·2579 | 0·5357 | 0·6912 | 0·8662 | 1·074 | 1·341 | 1·753 | 2·131 | 2·602 | 2·947 | 3·733 | 4·073 |
| 16 | 0·2576 | 0·5350 | 0·6901 | 0·8647 | 1·071 | 1·337 | 1·746 | 2·120 | 2·583 | 2·921 | 3·686 | 4·015 |
| 17 | 0·2573 | 0·5344 | 0·6892 | 0·8633 | 1·069 | 1·333 | 1·740 | 2·110 | 2·567 | 2·898 | 3·646 | 3·965 |
| 18 | 0·2571 | 0·5338 | 0·6884 | 0·8620 | 1·067 | 1·330 | 1·734 | 2·101 | 2·552 | 2·878 | 3·610 | 3·922 |
| 19 | 0·2569 | 0·5333 | 0·6876 | 0·8610 | 1·066 | 1·328 | 1·729 | 2·093 | 2·539 | 2·861 | 3·579 | 3·883 |
| 20 | 0·2567 | 0·5329 | 0·6870 | 0·8600 | 1·064 | 1·325 | 1·725 | 2·086 | 2·528 | 2·845 | 3·552 | 3·850 |
| 21 | 0·2566 | 0·5325 | 0·6864 | 0·8591 | 1·063 | 1·323 | 1·721 | 2·080 | 2·518 | 2·831 | 3·527 | 3·819 |
| 22 | 0·2564 | 0·5321 | 0·6858 | 0·8583 | 1·061 | 1·321 | 1·717 | 2·074 | 2·508 | 2·819 | 3·505 | 3·792 |
| 23 | 0·2563 | 0·5317 | 0·6853 | 0·8575 | 1·060 | 1·319 | 1·714 | 2·069 | 2·500 | 2·807 | 3·485 | 3·768 |
| 24 | 0·2562 | 0·5314 | 0·6848 | 0·8569 | 1·059 | 1·318 | 1·711 | 2·064 | 2·492 | 2·797 | 3·467 | 3·745 |
| 25 | 0·2561 | 0·5312 | 0·6844 | 0·8562 | 1·058 | 1·316 | 1·708 | 2·060 | 2·485 | 2·787 | 3·450 | 3·725 |
| 26 | 0·2560 | 0·5309 | 0·6840 | 0·8557 | 1·058 | 1·315 | 1·706 | 2·056 | 2·479 | 2·779 | 3·435 | 3·707 |
| 27 | 0·2559 | 0·5306 | 0·6837 | 0·8551 | 1·057 | 1·314 | 1·703 | 2·052 | 2·473 | 2·771 | 3·421 | 3·690 |
| 28 | 0·2558 | 0·5304 | 0·6834 | 0·8546 | 1·056 | 1·313 | 1·701 | 2·048 | 2·467 | 2·763 | 3·408 | 3·674 |
| 29 | 0·2557 | 0·5302 | 0·6830 | 0·8542 | 1·055 | 1·311 | 1·699 | 2·045 | 2·462 | 2·756 | 3·396 | 3·659 |
| 30 | 0·2556 | 0·5300 | 0·6828 | 0·8538 | 1·055 | 1·310 | 1·697 | 2·042 | 2·457 | 2·750 | 3·385 | 3·646 |
| 32 | 0·2555 | 0·5297 | 0·6822 | 0·8530 | 1·054 | 1·309 | 1·694 | 2·037 | 2·449 | 2·738 | 3·365 | 3·622 |
| 34 | 0·2553 | 0·5294 | 0·6818 | 0·8523 | 1·052 | 1·307 | 1·691 | 2·032 | 2·441 | 2·728 | 3·348 | 3·601 |
| 36 | 0·2552 | 0·5291 | 0·6814 | 0·8517 | 1·052 | 1·306 | 1·688 | 2·028 | 2·434 | 2·719 | 3·333 | 3·582 |
| 38 | 0·2551 | 0·5288 | 0·6810 | 0·8512 | 1·051 | 1·304 | 1·686 | 2·024 | 2·429 | 2·712 | 3·319 | 3·566 |
| 40 | 0·2550 | 0·5286 | 0·6807 | 0·8507 | 1·050 | 1·303 | 1·684 | 2·021 | 2·423 | 2·704 | 3·307 | 3·551 |
| 50 | 0·2547 | 0·5278 | 0·6794 | 0·8489 | 1·047 | 1·299 | 1·676 | 2·009 | 2·403 | 2·678 | 3·261 | 3·496 |
| 60 | 0·2545 | 0·5272 | 0·6786 | 0·8477 | 1·045 | 1·296 | 1·671 | 2·000 | 2·390 | 2·660 | 3·232 | 3·460 |
| 120 | 0·2539 | 0·5258 | 0·6765 | 0·8446 | 1·041 | 1·289 | 1·658 | 1·980 | 2·358 | 2·617 | 3·160 | 3·373 |
| ∞ | 0·2533 | 0·5244 | 0·6745 | 0·8416 | 1·036 | 1·282 | 1·645 | 1·960 | 2·326 | 2·576 | 3·090 | 3·291 |

# TABLE 13. PERCENTAGE POINTS OF THE CORRELATION COEFFICIENT $r$ WHEN $\rho = 0$

The function tabulated is $r(P) = r(P|\nu)$ defined by the equation

$$\frac{\Gamma\left(\frac{\nu-1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{\nu-2}{2}\right)} \int_{r(P)}^{1} (1-r^2)^{\frac{\nu-4}{2}}\,dr = P/100.$$

Let $r$ be a partial correlation coefficient, after $s$ variables have been eliminated, in a sample of size $n$ from a multivariate normal population with corresponding true partial correlation coefficient $\rho = 0$, and let $\nu = n-s$. This table gives upper $P$ per cent points of $r$; the corresponding lower $P$ per cent points are given by $-r(P)$, and the tabulated values are also upper $2P$ per cent points of $|r|$. For $s = 0$ we have $\nu = n$ and $r$ is the ordinary correlation coefficient. When $\nu > 130$ use the results that $r$ is approximately normally distributed with zero mean and variance $\frac{1}{\nu-1}$, or (more accurately) that $z = \tanh^{-1} r$ is approximately normally distributed with zero mean and variance $\frac{1}{\nu-3}$ (cf. Tables 16 and 17).




(This shape applies for $\nu \geqslant 5$ only. When $\nu = 4$ the distribution is uniform and when $\nu = 3$ the probability density function is U-shaped.)

Tables of the distribution of $r$ for various values of $\rho$ are given by, for example, F. N. David, *Tables of the Ordinates and Probability Integral of the Distribution of the Correlation Coefficient in Small Samples*, Cambridge University Press (1954), and R. E. Odeh, 'Critical values of the sample product–moment correlation coefficient in the bivariate normal distribution', *Commun. Statist. – Simula Computa.* **11** (1) (1982), pp. 1–26. The $z$-transformation may also be used (cf. Tables 16 and 17).

| $P$ | 5 | 2·5 | 1 | 0·5 | 0·1 |
|---|---|---|---|---|---|
| $\nu = 3$ | 0·9877 | 0·9969 | 0·9995 | 0·9999 | 0·999995 |
| 4 | ·9000 | ·9500 | ·9800 | ·9900 | ·9980 |
| 5 | 0·8054 | 0·8783 | 0·9343 | 0·9587 | 0·9859 |
| 6 | ·7293 | ·8114 | ·8822 | ·9172 | ·9633 |
| 7 | ·6694 | ·7545 | ·8329 | ·8745 | ·9350 |
| 8 | ·6215 | ·7067 | ·7887 | ·8343 | ·9049 |
| 9 | ·5822 | ·6664 | ·7498 | ·7977 | ·8751 |
| 10 | 0·5494 | 0·6319 | 0·7155 | 0·7646 | 0·8467 |
| 11 | ·5214 | ·6021 | ·6851 | ·7348 | ·8199 |
| 12 | ·4973 | ·5760 | ·6581 | ·7079 | ·7950 |
| 13 | ·4762 | ·5529 | ·6339 | ·6835 | ·7717 |
| 14 | ·4575 | ·5324 | ·6120 | ·6614 | ·7501 |
| 15 | 0·4409 | 0·5140 | 0·5923 | 0·6411 | 0·7301 |
| 16 | ·4259 | ·4973 | ·5742 | ·6226 | ·7114 |
| 17 | ·4124 | ·4821 | ·5577 | ·6055 | ·6940 |
| 18 | ·4000 | ·4683 | ·5425 | ·5897 | ·6777 |
| 19 | ·3887 | ·4555 | ·5285 | ·5751 | ·6624 |
| 20 | 0·3783 | 0·4438 | 0·5155 | 0·5614 | 0·6481 |
| 21 | ·3687 | ·4329 | ·5034 | ·5487 | ·6346 |
| 22 | ·3598 | ·4227 | ·4921 | ·5368 | ·6219 |
| 23 | ·3515 | ·4132 | ·4815 | ·5256 | ·6099 |
| 24 | ·3438 | ·4044 | ·4716 | ·5151 | ·5986 |
| 25 | 0·3365 | 0·3961 | 0·4622 | 0·5052 | 0·5879 |
| 26 | ·3297 | ·3882 | ·4534 | ·4958 | ·5776 |
| 27 | ·3233 | ·3809 | ·4451 | ·4869 | ·5679 |
| 28 | ·3172 | ·3739 | ·4372 | ·4785 | ·5587 |
| 29 | ·3115 | ·3673 | ·4297 | ·4705 | ·5499 |
| 30 | 0·3061 | 0·3610 | 0·4226 | 0·4629 | 0·5415 |
| 31 | ·3009 | ·3550 | ·4158 | ·4556 | ·5334 |
| 32 | ·2960 | ·3494 | ·4093 | ·4487 | ·5257 |
| 33 | ·2913 | ·3440 | ·4032 | ·4421 | ·5184 |
| 34 | ·2869 | ·3388 | ·3972 | ·4357 | ·5113 |
| 35 | 0·2826 | 0·3338 | 0·3916 | 0·4296 | 0·5045 |
| 36 | ·2785 | ·3291 | ·3862 | ·4238 | ·4979 |
| 37 | ·2746 | ·3246 | ·3810 | ·4182 | ·4916 |
| 38 | ·2709 | ·3202 | ·3760 | ·4128 | ·4856 |
| 39 | ·2673 | ·3160 | ·3712 | ·4076 | ·4797 |

| $P$ | 5 | 2·5 | 1 | 0·5 | 0·1 |
|---|---|---|---|---|---|
| $\nu = 40$ | 0·2638 | 0·3120 | 0·3665 | 0·4026 | 0·4741 |
| 42 | ·2573 | ·3044 | ·3578 | ·3932 | ·4633 |
| 44 | ·2512 | ·2973 | ·3496 | ·3843 | ·4533 |
| 46 | ·2455 | ·2907 | ·3420 | ·3761 | ·4439 |
| 48 | ·2403 | ·2845 | ·3348 | ·3683 | ·4351 |
| 50 | 0·2353 | 0·2787 | 0·3281 | 0·3610 | 0·4267 |
| 52 | ·2306 | ·2732 | ·3218 | ·3542 | ·4188 |
| 54 | ·2262 | ·2681 | ·3158 | ·3477 | ·4114 |
| 56 | ·2221 | ·2632 | ·3102 | ·3415 | ·4043 |
| 58 | ·2181 | ·2586 | ·3048 | ·3357 | ·3976 |
| 60 | 0·2144 | 0·2542 | 0·2997 | 0·3301 | 0·3912 |
| 62 | ·2108 | ·2500 | ·2948 | ·3248 | ·3850 |
| 64 | ·2075 | ·2461 | ·2902 | ·3198 | ·3792 |
| 66 | ·2042 | ·2423 | ·2858 | ·3150 | ·3736 |
| 68 | ·2012 | ·2387 | ·2816 | ·3104 | ·3683 |
| 70 | 0·1982 | 0·2352 | 0·2776 | 0·3060 | 0·3632 |
| 72 | ·1954 | ·2319 | ·2737 | ·3017 | ·3583 |
| 74 | ·1927 | ·2287 | ·2700 | ·2977 | ·3536 |
| 76 | ·1901 | ·2257 | ·2664 | ·2938 | ·3490 |
| 78 | ·1876 | ·2227 | ·2630 | ·2900 | ·3447 |
| 80 | 0·1852 | 0·2199 | 0·2597 | 0·2864 | 0·3405 |
| 82 | ·1829 | ·2172 | ·2565 | ·2830 | ·3364 |
| 84 | ·1807 | ·2146 | ·2535 | ·2796 | ·3325 |
| 86 | ·1786 | ·2120 | ·2505 | ·2764 | ·3287 |
| 88 | ·1765 | ·2096 | ·2477 | ·2732 | ·3251 |
| 90 | 0·1745 | 0·2072 | 0·2449 | 0·2702 | 0·3215 |
| 92 | ·1726 | ·2050 | ·2422 | ·2673 | ·3181 |
| 94 | ·1707 | ·2028 | ·2396 | ·2645 | ·3148 |
| 96 | ·1689 | ·2006 | ·2371 | ·2617 | ·3116 |
| 98 | ·1671 | ·1986 | ·2347 | ·2591 | ·3085 |
| 100 | 0·1654 | 0·1966 | 0·2324 | 0·2565 | 0·3054 |
| 105 | ·1614 | ·1918 | ·2268 | ·2504 | ·2983 |
| 110 | ·1576 | ·1874 | ·2216 | ·2446 | ·2915 |
| 115 | ·1541 | ·1832 | ·2167 | ·2393 | ·2853 |
| 120 | ·1509 | ·1793 | ·2122 | ·2343 | ·2794 |
| 125 | 0·1478 | 0·1757 | 0·2079 | 0·2296 | 0·2738 |
| 130 | ·1449 | ·1723 | ·2039 | ·2252 | ·2686 |

# TABLE 14. PERCENTAGE POINTS OF SPEARMAN'S $S$
# TABLE 15. PERCENTAGE POINTS OF KENDALL'S $K$

Spearman's $S$ and Kendall's $K$ are both used to measure the degree of association between two rankings of $n$ objects. Let $d_i$ $(1 \leqslant i \leqslant n)$ be the difference in the ranks of the $i$th object; Spearman's $S$ is defined as $\sum_{i=1}^{n} d_i^2$. To define Kendall's $K$, re-order the pairs of ranks so that the first set is in natural order from left to right, and let $m_i$ $(1 \leqslant i \leqslant n)$ be the number of ranks greater than $i$ in the second ranking which are to the right of rank $i$. Kendall's $K$ is defined as $\sum_{i=1}^{n} m_i$.

For Table 14 the tabulated value $x(P)$ is the lower percentage point, i.e. the largest value $x$ such that, in independent rankings, $\Pr(S \leqslant x) \leqslant P/100$; in Table 15, $K$ replaces $S$ and the upper percentage point is given. A dash indicates that there is no value with the required property. The distributions are symmetric about means $\frac{1}{6}(n^3-n)$ for $S$ and $\frac{1}{4}n(n-1)$ for $K$, with maxima equal to twice the means; hence the upper percentage points of $S$ are $\frac{1}{3}(n^3-n)-x(P)$ and the lower percentage points of $K$ are $\frac{1}{2}n(n-1)-x(P)$. The variances are $\frac{1}{36}n^2(n+1)^2(n-1)$ for $S$ and $\frac{1}{72}n(n-1)(2n+5)$ for $K$, and when $n > 40$ both statistics are approximately normally distributed; more accurately, the distribution function of $X = [S-\frac{1}{6}(n^3-n)]/[\frac{1}{6}n(n+1)\sqrt{n-1}]$ is approximately equal to $\Phi(x) - \frac{\gamma}{24\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(x^3-3x)$, where $\gamma = \frac{-0{\cdot}04(19n^2+5n-36)}{\frac{1}{6}(n^3-n)}$ and $\Phi(x)$ is the normal distribution function (see Table 4).

A test of the null hypothesis of independent rankings is provided by rejecting at the $P$ per cent level if $S \leqslant x(P)$, or $K \geqslant x(P)$, when the alternative is contrary rankings. The other points are similarly used when the alternative is similar rankings. To cover both alternatives reject at the $2P$ per cent level if $S$, or $K$, lies in either tail. Spearman's rank correlation coefficient $r_S$ is defined as $1-6S/(n^3-n)$, and has upper and lower $P$ per cent points $1-6x(P)/(n^3-n)$ and $-[1-6x(P)/(n^3-n)]$ respectively. Kendall's rank correlation coefficient $r_K$ is defined as $4K/[n(n-1)]-1$, and has upper and lower $P$ per cent points $4x(P)/[n(n-1)]-1$ and $-\{4x(P)/[n(n-1)]-1\}$ respectively.

SPEARMAN'S $S$

| $P$ | 5 | 2·5 | 1 | 0·5 | 0·1 | $\frac{1}{6}(n^3-n)$ |
|---|---|---|---|---|---|---|
| $n=4$ | 0 | — | — | — | — | **10** |
| 5 | 2 | 0 | 0 | — | — | **20** |
| 6 | 6 | 4 | 2 | 0 | — | **35** |
| 7 | 16 | 12 | 6 | 4 | 0 | **56** |
| 8 | 30 | 22 | 14 | 10 | 4 | **84** |
| 9 | 48 | 36 | 26 | 20 | 10 | **120** |
| 10 | 72 | 58 | 42 | 34 | 20 | **165** |
| 11 | 102 | 84 | 64 | 54 | 34 | **220** |
| 12 | 142 | 118 | 92 | 78 | 52 | **286** |
| 13 | 188 | 160 | 128 | 108 | 76 | **364** |
| 14 | 244 | 210 | 170 | 146 | 104 | **455** |
| 15 | 310 | 268 | 222 | 194 | 140 | **560** |
| 16 | 388 | 338 | 284 | 248 | 184 | **680** |
| 17 | 478 | 418 | 354 | 312 | 236 | **816** |
| 18 | 580 | 512 | 436 | 388 | 298 | **969** |
| 19 | 694 | 616 | 530 | 474 | 370 | **1140** |
| 20 | 824 | 736 | 636 | 572 | 452 | **1330** |
| 21 | 970 | 868 | 756 | 684 | 544 | **1540** |
| 22 | 1132 | 1018 | 890 | 808 | 650 | **1771** |
| 23 | 1310 | 1182 | 1040 | 948 | 768 | **2024** |
| 24 | 1508 | 1364 | 1206 | 1102 | 900 | **2300** |
| 25 | 1724 | 1566 | 1388 | 1272 | 1048 | **2600** |
| 26 | 1958 | 1784 | 1588 | 1460 | 1210 | **2925** |
| 27 | 2214 | 2022 | 1806 | 1664 | 1388 | **3276** |
| 28 | 2492 | 2282 | 2044 | 1888 | 1584 | **3654** |
| 29 | 2794 | 2564 | 2304 | 2132 | 1796 | **4060** |
| 30 | 3118 | 2866 | 2584 | 2396 | 2028 | **4495** |
| 31 | 3466 | 3194 | 2884 | 2682 | 2280 | **4960** |
| 32 | 3840 | 3544 | 3210 | 2988 | 2552 | **5456** |
| 33 | 4240 | 3920 | 3558 | 3318 | 2844 | **5984** |
| 34 | 4666 | 4322 | 3930 | 3672 | 3160 | **6545** |
| 35 | 5120 | 4750 | 4330 | 4050 | 3498 | **7140** |
| 36 | 5604 | 5206 | 4754 | 4454 | 3858 | **7770** |
| 37 | 6118 | 5692 | 5206 | 4884 | 4244 | **8436** |
| 38 | 6662 | 6206 | 5686 | 5342 | 4656 | **9139** |
| 39 | 7238 | 6750 | 6196 | 5826 | 5092 | **9880** |
| 40 | 7846 | 7326 | 6736 | 6342 | 5556 | **10660** |

KENDALL'S $K$

| $P$ | 5 | 2·5 | 1 | 0·5 | 0·1 | $\frac{1}{4}n(n-1)$ |
|---|---|---|---|---|---|---|
| $n=4$ | 6 | — | — | — | — | **3** |
| 5 | 9 | 10 | 10 | — | — | **5** |
| 6 | 13 | 14 | 14 | 15 | — | **7·5** |
| 7 | 17 | 18 | 19 | 20 | 21 | **10·5** |
| 8 | 22 | 23 | 24 | 25 | 26 | **14** |
| 9 | 27 | 28 | 30 | 31 | 33 | **18** |
| 10 | 33 | 34 | 36 | 37 | 40 | **22·5** |
| 11 | 39 | 41 | 43 | 44 | 47 | **27·5** |
| 12 | 46 | 48 | 51 | 52 | 55 | **33** |
| 13 | 53 | 56 | 59 | 61 | 64 | **39** |
| 14 | 62 | 64 | 67 | 69 | 73 | **45·5** |
| 15 | 70 | 73 | 77 | 79 | 83 | **52·5** |
| 16 | 79 | 83 | 86 | 89 | 94 | **60** |
| 17 | 89 | 93 | 97 | 100 | 105 | **68** |
| 18 | 99 | 103 | 108 | 111 | 117 | **76·5** |
| 19 | 110 | 114 | 119 | 123 | 129 | **85·5** |
| 20 | 121 | 126 | 131 | 135 | 142 | **95** |
| 21 | 133 | 138 | 144 | 148 | 156 | **105** |
| 22 | 146 | 151 | 157 | 161 | 170 | **115·5** |
| 23 | 159 | 164 | 171 | 176 | 184 | **126·5** |
| 24 | 172 | 178 | 185 | 190 | 200 | **138** |
| 25 | 186 | 193 | 200 | 205 | 216 | **150** |
| 26 | 201 | 208 | 216 | 221 | 232 | **162·5** |
| 27 | 216 | 223 | 232 | 238 | 249 | **175·5** |
| 28 | 232 | 239 | 248 | 254 | 267 | **189** |
| 29 | 248 | 256 | 266 | 272 | 285 | **203** |
| 30 | 265 | 273 | 283 | 290 | 303 | **217·5** |
| 31 | 282 | 291 | 301 | 308 | 323 | **232·5** |
| 32 | 300 | 309 | 320 | 328 | 342 | **248** |
| 33 | 318 | 328 | 340 | 347 | 363 | **264** |
| 34 | 337 | 347 | 359 | 368 | 384 | **280·5** |
| 35 | 356 | 367 | 380 | 388 | 405 | **297·5** |
| 36 | 376 | 388 | 401 | 410 | 428 | **315** |
| 37 | 397 | 409 | 422 | 432 | 450 | **333** |
| 38 | 418 | 430 | 444 | 454 | 473 | **351·5** |
| 39 | 440 | 452 | 467 | 477 | 497 | **370·5** |
| 40 | 462 | 475 | 490 | 501 | 522 | **390** |