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UNIVERSITY OF LONDON **ST104A ZB (279 004A)**

BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diplomas in Economics and Social Sciences and Access Route

Statistics 1 (half unit)

Friday, 4 May 2012 : 10.00am to 12.00pm

Candidates should answer **THREE** of the following **FOUR** questions: **QUESTION 1** of Section A (50 marks) and **TWO** questions from Section B (25 marks each). **Candidates are strongly advised to divide their time accordingly.**

A list of formulae and extracts from statistical tables are given after the final question on this paper.

Graph paper is provided at the end of this question paper. If used, it must be detached and fastened securely inside the answer book.

A calculator may be used when answering questions on this paper and it must comply in all respects with the specification given with your Admission Notice. The make and type of machine must be clearly stated on the front cover of the answer book.

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SECTION A

Answer **all** parts of Question 1 (50 marks in total).

1. (a) The following data represent different types of variables. Classify each one of them as measurable (continuous) or categorical. If a variable is categorical, further classify it as nominal or ordinal. Justify your answer. (*Note that no marks will be awarded without justification.*)

    i. The amount of time it takes each of 15 telephone installers to hook up a wall phone.

    ii. The style of music preferred by each of 30 randomly selected radio listeners.

    iii. The lengths of 50 randomly selected cars.

    iv. The classification of a student (First, Upper Second, Lower Second, Third, Pass, Fail) in the course 04a: Statistics 1.

(8 marks)

(b) The number of raisins in each of 16 mini boxes for two brands are shown below:

Brand A: 22	27	20	29	24	31	25	26
Brand B: 26	29	25	33	24	35	31	27

i. Find the mean and the mode for each brand.

ii. Find the upper quartile of Brand A and the lower quartile of Brand B.

iii. The mini boxes were made in 8 different machines corresponding to each column in the table above. Calculate the Spearman rank correlation coefficient and interpret its value.

(13 marks)

(c) A test is taken by some students, their marks are recorded and we are interested in the properties of the sample mean. Under the assumption that the marks follow a Normal distribution with exact mean 65 and variance 144, calculate the probability that the mark of a randomly selected student

i. is greater than 67.5 exactly; and

ii. lies between 63 and 67 exactly.

(4 marks)

(d) A sample of 160 students was taken and each student was questioned regarding their preferences for a number of courses. The course in Economics was chosen by 75 students. Calculate a 95% confidence interval for the proportion of students in favour of Economics in the population.

(3 marks)

(e) Suppose that $x_1 = 3$, $x_2 = 2$, $x_3 = 0$, $x_4 = 4$, $x_5 = 1$, and $y_1 = 1$, $y_2 = 0$, $y_3 = 2$, $y_4 = 3$, $y_5 = 2$. Calculate the following quantities:

i. $\sum_{i=1}^{i=4} 2(x_i - 2)$ ii. $\sum_{i=3}^{i=5} (x_i + y_i)$ iii. $\sum_{i=4}^{i=5} x_i(y_i - 3)$

(6 marks)

(f) The probability distribution of a variable X is given below.

x	1	3	4	6
$p_X(x)$	.2	.3	.4	.1

i. Find the probability that X is an odd number.
ii. Find the expected value of X, $E(X)$.

(4 marks)

(g) Two fair dice are thrown.

i. Suppose that D is the absolute difference between the scores on the two dice. State the probability distribution of D.
ii. You are told that the sum of the scores on the two dice is at least 10. What is the probability of at least one score being 6?

(4 marks)

(h) State whether the following are true or false and give a brief explanation. (*Note that no marks will be awarded for a simple true/false answer.*)

i. A 95% confidence interval for the mean is wider than a 99% one when obtained from the same data.
ii. A p-value is the probability of not rejecting the null hypothesis.
iii. As the value of a chi-squared test statistic becomes larger, the associated p-value becomes smaller.

(6 marks)

(i) Provide an example where selection bias may occur. Be brief in explaining why selection bias may occur.

(2 marks)

SECTION B

Answer **two** questions from this section (25 marks each).

2. (a) An experiment was conducted in order to determine whether contacting people by phone or by letter before sending them a survey will increase the response rate. Specifically, one group of people received a letter before getting the survey; one group received a phone call before receiving the survey; and one group did not receive any information before the survey arrived. For this study, a response was defined as returning the survey within 2 weeks.

	no contact	letter	phone
Number of people who responded	10	17	37
Number of people who did not respond	31	22	12

   i. Test for an association between the method of contact prior to the survey and response at two appropriate significance levels. State the null and alternative hypotheses clearly.

   ii. Comment on your results describing potential associations in detail. Discuss the potential differences in response rates for different methods of contact.

(13 marks)

(b) You work for a market research company and your boss has asked you to carry out a random sample survey for a mobile phone company to identify whether a recently launched mobile phone is attractive to younger people. Limited time and money resources are available at your disposal. You are being asked to prepare a brief summary containing the items below. (*Note you are not supposed to provide a lengthy answer. You are in danger of losing marks should you do so.*)

   i. Choose an appropriate probability sampling scheme. Provide a brief justification for your answer.

   ii. Describe the sampling frame and the method of contact you will use. Briefly explain the reasons for your choices.

   iii. Provide an example in which response bias may occur. State an action that you would take to address this issue.

   iv. State the main research question of the survey. Identify the variables associated with this question.

(12 marks)

3. (a) We are interested in assessing the potential impact of the growth rate (X) of the Gross National Product (GNP) on the birth rate (Y) of a country. The table below provides data for these quantities for 12 countries:

Country	Birth rate (y)	GNP growth rate (x)
Brazil	30	5.1
Colombia	29	3.2
Costa Rica	30	3.0
India	35	1.4
Mexico	36	3.8
Peru	36	1.0
Philippines	34	2.8
Senegal	48	-0.3
South Korea	24	6.9
Sri Lanka	27	2.5
Taiwan	21	6.2
Thailand	30	4.6

The summary statistics for these data are:

Sum of x data: 40.2	Sum of the squares of x data: 184.04
Sum of y data: 380	Sum of the squares of y data: 12,564
Sum of the products of x and y data: 1,139.7	

i. Draw a scatter diagram of these data on the graph paper provided. Label the diagram carefully.

ii. Calculate the correlation coefficient. Interpret your findings.

iii. Calculate the least squares line of y on x and draw the line on the scatter diagram.

iv. Obtain the predicted birth rate value of a country with a GNP growth rate of 5.0 according to the equation in (iii.). Would you use this value to predict the birth rate of this country? Justify your answer.

(13 marks)

(b) A transport company operates two types of trucks (A and B) and wants to compare them in terms of fuel consumption. An experiment is conducted and the kilometers per litre (kpl) rates of various type A and type B trucks are recorded and summarised in the following table:

	Sample size	Average kpl	Sample standard deviation
Type A	33	31.0	7.6
Type B	40	32.2	1.8

i. You are asked to consider an appropriate hypothesis test to determine whether the mean distances per litre, covered by each of the two types of trucks, are different. Test at two appropriate significance levels and comment on your findings. Specify the test statistic you use and its distribution under the null hypothesis.

ii. State clearly any other assumptions you make.

iii. Give a 98% confidence interval for the mean kpl rate for the type A trucks.

(12 marks)

4. (a) The following figures are the hottest daily temperatures (in degrees Celsius) for 15 days of June at two coastal resorts:

19	20	21	21	22
22	22	22	23	23
23	23	23	23	24
24	24	24	24	25
25	25	25	25	26
26	26	27	27	28

i. Carefully construct, draw and label a histogram of these data on the graph paper provided.

ii. Find the mean, the median, the interquartile range and the modal group.

iii. Comment on the data given the shape of the histogram and the measures you have calculated.

(13 marks)

(b) i. A pharmaceutical company is conducting an experiment to test whether a new type of pain reliever is effective. The pain reliever was given to 30 patients and it reduced the pain for 16 of them. You are asked to use an appropriate hypothesis test to determine whether the pain reliever is effective. State the test hypotheses, and specify your test statistic and its distribution under the null hypothesis. Comment on your findings.

ii. A second experiment followed where a placebo pill was given to another group of 40 patients. A placebo pill contains no medication and is prescribed so that the patient will expect to get well. In some situations, this expectation is enough for the patient to recover. This effect, also known as the placebo effect, occurred to some extent in the second experiment where the pain was reduced for 13 of the patients. You are asked to consider an appropriate hypothesis test to incorporate this new evidence with the previous data and re-assess the effectiveness of the pain reliever.

(12 marks)

END OF PAPER

ST104a **Statistics 1**

Examination Formula Sheet

Expected value of a discrete random variable:

$$\mu = E[X] = \sum_{i=1}^{N} p_i x_i$$

Standard deviation of a discrete random variable:

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{i=1}^{N} p_i (x_i - \mu)^2}$$

The transformation formula:

$$Z = \frac{X - \mu}{\sigma}$$

Finding Z for the sampling distribution of the sample mean:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Finding Z for the sampling distribution of the sample proportion:

$$Z = \frac{P - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$$

Confidence interval endpoints for a single mean (σ known):

$$\bar{x} \pm z \frac{\sigma}{\sqrt{n}}$$

Confidence interval endpoints for a single mean (σ unknown):

$$\bar{x} \pm t_{n-1} \frac{s}{\sqrt{n}}$$

Confidence interval endpoints for a single proportion:

$$p \pm z \sqrt{\frac{p(1-p)}{n}}$$

Sample size determination for a mean:

$$n \geq \frac{Z^2 \sigma^2}{e^2}$$

Sample size determination for a proportion:

$$n \geq \frac{Z^2 p(1-p)}{e^2}$$

Z-test of hypothesis for a single mean (σ known):

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

t-test of hypothesis for a single mean (σ unknown):

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

Z-test of hypothesis for a single proportion:

$$Z \cong \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$$

Z-test for the difference between two means (variances known):

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

t-test for the difference between two means (variances unknown):

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Confidence interval endpoints for the difference between two means:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{n_1+n_2-2} \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Pooled variance estimator:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

t-test for the difference in means in paired samples:

$$t = \frac{\bar{X}_d - \mu_d}{S_d/\sqrt{n}}$$

Confidence interval endpoints for the difference in means in paired samples:

$$\bar{x}_d \pm t_{n-1} \frac{s_d}{\sqrt{n}}$$

Z-test for the difference between two proportions:

$$Z = \frac{(P_1 - P_2) - (\pi_1 - \pi_2)}{\sqrt{P(1-P)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Pooled proportion estimator:

$$P = \frac{R_1 + R_2}{n_1 + n_2}$$

Confidence interval endpoints for the difference between two proportions:

$$(p_1 - p_2) \pm z \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

χ^2 test of association:

$$\sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Sample correlation coefficient:

$$r = \frac{\sum_{i=1}^n x_i y_i - n\bar{x}\bar{y}}{\sqrt{\left(\sum_{i=1}^n x_i^2 - n\bar{x}^2\right)\left(\sum_{i=1}^n y_i^2 - n\bar{y}^2\right)}}$$

Spearman rank correlation:

$$r_s = 1 - \frac{6\sum_{i=1}^n d_i^2}{n(n^2-1)}$$

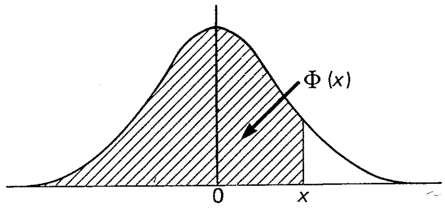
Simple linear regression line estimates:

$$b = \frac{\sum_{i=1}^n x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^n x_i^2 - n\bar{x}^2}$$

$$a = \bar{y} - b\bar{x}$$

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

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TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

			0·9994		0·99990		0·99995
3·075		3·263		3·731		3·916	
	0·9990		0·9995		0·99991		0·99996
3·105		3·320		3·759		3·976	
	0·9991		0·9996		0·99992		0·99997
3·138		3·389		3·791		4·055	
	0·9992		0·9997		0·99993		0·99998
3·174		3·480		3·826		4·173	
	0·9993		0·9998		0·99994		0·99999
3·215		3·615		3·867		4·417	
	0·9994		0·9999		0·99995		1·00000

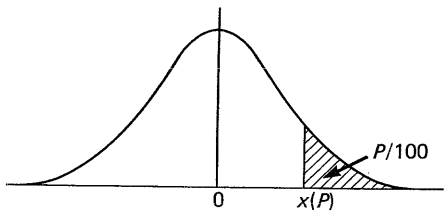
When $x > 3·3$ the formula $1-\Phi(x) \doteqdot \dfrac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\dfrac{1}{x^2}+\dfrac{3}{x^4}-\dfrac{15}{x^6}+\dfrac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

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TABLE 7. THE χ^2-DISTRIBUTION FUNCTION

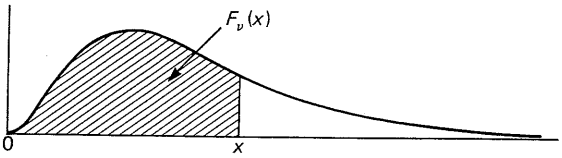
The function tabulated is

$$F_\nu(x) = \frac{1}{2^{\nu/2}\Gamma(\frac{\nu}{2})}\int_0^x t^{\frac{1}{2}\nu-1}\, e^{-\frac{1}{2}t}\, dt$$

for integer $\nu \leqslant 25$. $F_\nu(x)$ is the probability that a random variable X, distributed as χ^2 with ν degrees of freedom, will be less than or equal to x. Note that $F_1(x) = 2\Phi(x^{\frac{1}{2}}) - 1$ (cf. Table 4). For certain values of x and $\nu > 25$ use may be made of the following relation between the χ^2- and Poisson distributions:

$$F_\nu(x) = 1 - F(\tfrac{1}{2}\nu - 1 \,|\, \tfrac{1}{2}x)$$

where $F(r|\mu)$ is the Poisson distribution function (see Table 2). If $\nu > 25$, X is approximately normally distributed



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

with mean ν and variance 2ν. A better approximation is usually obtained by using the formula

$$F_\nu(x) \doteqdot \Phi(\sqrt{2x} - \sqrt{2\nu - 1})$$

where $\Phi(s)$ is the normal distribution function (see Table 4).

Omitted entries to the left and right of tabulated values are 1 and 0 respectively (to four decimal places).

$\nu =$	1	$\nu =$	1	$\nu =$	2	$\nu =$	2	$\nu =$	3	$\nu =$	3
$x =$ 0·0	0·0000	$x =$ 4·0	0·9545	$x =$ 0·0	0·0000	$x =$ 4·0	0·8647	$x =$ 0·0	0·0000	$x =$ 4·0	0·7385
·1	·2482	·1	·9571	·1	·0488	·1	·8713	·1	·0082	·2	·7593
·2	·3453	·2	·9596	·2	·0952	·2	·8775	·2	·0224	·4	·7786
·3	·4161	·3	·9619	·3	·1393	·3	·8835	·3	·0400	·6	·7965
·4	·4729	·4	·9641	·4	·1813	·4	·8892	·4	·0598	·8	·8130
0·5	0·5205	4·5	0·9661	0·5	0·2212	4·5	0·8946	0·5	0·0811	5·0	0·8282
·6	·5614	·6	·9680	·6	·2592	·6	·8997	·6	·1036	·2	·8423
·7	·5972	·7	·9698	·7	·2953	·7	·9046	·7	·1268	·4	·8553
·8	·6289	·8	·9715	·8	·3297	·8	·9093	·8	·1505	·6	·8672
·9	·6572	·9	·9731	·9	·3624	·9	·9137	·9	·1746	·8	·8782
1·0	0·6827	5·0	0·9747	1·0	0·3935	5·0	0·9179	1·0	0·1987	6·0	0·8884
·1	·7057	·1	·9761	·1	·4231	·1	·9219	·1	·2229	·2	·8977
·2	·7267	·2	·9774	·2	·4512	·2	·9257	·2	·2470	·4	·9063
·3	·7458	·3	·9787	·3	·4780	·3	·9293	·3	·2709	·6	·9142
·4	·7633	·4	·9799	·4	·5034	·4	·9328	·4	·2945	·8	·9214
1·5	0·7793	5·5	0·9810	1·5	0·5276	5·5	0·9361	1·5	0·3177	7·0	0·9281
·6	·7941	·6	·9820	·6	·5507	·6	·9392	·6	·3406	·2	·9342
·7	·8077	·7	·9830	·7	·5726	·7	·9422	·7	·3631	·4	·9398
·8	·8203	·8	·9840	·8	·5934	·8	·9450	·8	·3851	·6	·9450
·9	·8319	·9	·9849	·9	·6133	·9	·9477	·9	·4066	·8	·9497
2·0	0·8427	6·0	0·9857	2·0	0·6321	6·0	0·9502	2·0	0·4276	8·0	0·9540
·1	·8527	·1	·9865	·1	·6501	·2	·9550	·1	·4481	·2	·9579
·2	·8620	·2	·9872	·2	·6671	·4	·9592	·2	·4681	·4	·9616
·3	·8706	·3	·9879	·3	·6834	·6	·9631	·3	·4875	·6	·9649
·4	·8787	·4	·9886	·4	·6988	·8	·9666	·4	·5064	·8	·9679
2·5	0·8862	6·5	0·9892	2·5	0·7135	7·0	0·9698	2·5	0·5247	9·0	0·9707
·6	·8931	·6	·9898	·6	·7275	·2	·9727	·6	·5425	·2	·9733
·7	·8997	·7	·9904	·7	·7408	·4	·9753	·7	·5598	·4	·9756
·8	·9057	·8	·9909	·8	·7534	·6	·9776	·8	·5765	·6	·9777
·9	·9114	·9	·9914	·9	·7654	·8	·9798	·9	·5927	·8	·9797
3·0	0·9167	7·0	0·9918	3·0	0·7769	8·0	0·9817	3·0	0·6084	10·0	0·9814
·1	·9217	·1	·9923	·1	·7878	·2	·9834	·1	·6235	·2	·9831
·2	·9264	·2	·9927	·2	·7981	·4	·9850	·2	·6382	·4	·9845
·3	·9307	·3	·9931	·3	·8080	·6	·9864	·3	·6524	·6	·9859
·4	·9348	·4	·9935	·4	·8173	·8	·9877	·4	·6660	·8	·9871
3·5	0·9386	7·5	0·9938	3·5	0·8262	9·0	0·9889	3·5	0·6792	11·0	0·9883
·6	·9422	·6	·9942	·6	·8347	·2	·9899	·6	·6920	·2	·9893
·7	·9456	·7	·9945	·7	·8428	·4	·9909	·7	·7043	·4	·9903
·8	·9487	·8	·9948	·8	·8504	·6	·9918	·8	·7161	·6	·9911
·9	·9517	·9	·9951	·9	·8577	·8	·9926	·9	·7275	·8	·9919
4·0	0·9545	8·0	0·9953	4·0	0·8647	10·0	0·9933	4·0	0·7385	12·0	0·9926

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TABLE 7. THE χ^2-DISTRIBUTION FUNCTION

$\nu =$	4	5	6	7	8	9	10	11	12	13	14
$x =$ 0·5	0·0265	0·0079	0·0022	0·0006	0·0001						
1·0	·0902	·0374	·0144	·0052	·0018	0·0006	0·0002	0·0001			
1·5	·1734	·0869	·0405	·0177	·0073	·0029	·0011	·0004	0·0001		
2·0	·2642	·1509	·0803	·0402	·0190	·0085	·0037	·0015	·0006	0·0002	0·0001
2·5	0·3554	0·2235	0·1315	0·0729	0·0383	0·0191	0·0091	0·0042	0·0018	0·0008	0·0003
3·0	·4422	·3000	·1912	·1150	·0656	·0357	·0186	·0093	·0045	·0021	·0009
3·5	·5221	·3766	·2560	·1648	·1008	·0589	·0329	·0177	·0091	·0046	·0022
4·0	·5940	·4506	·3233	·2202	·1429	·0886	·0527	·0301	·0166	·0088	·0045
4·5	·6575	·5201	·3907	·2793	·1906	·1245	·0780	·0471	·0274	·0154	·0084
5·0	0·7127	0·5841	0·4562	0·3400	0·2424	0·1657	0·1088	0·0688	0·0420	0·0248	0·0142
5·5	·7603	·6421	·5185	·4008	·2970	·2113	·1446	·0954	·0608	·0375	·0224
6·0	·8009	·6938	·5768	·4603	·3528	·2601	·1847	·1266	·0839	·0538	·0335
6·5	·8352	·7394	·6304	·5173	·4086	·3110	·2283	·1620	·1112	·0739	·0477
7·0	·8641	·7794	·6792	·5711	·4634	·3629	·2746	·2009	·1424	·0978	·0653
7·5	0·8883	0·8140	0·7229	0·6213	0·5162	0·4148	0·3225	0·2427	0·1771	0·1254	0·0863
8·0	·9084	·8438	·7619	·6674	·5665	·4659	·3712	·2867	·2149	·1564	·1107
8·5	·9251	·8693	·7963	·7094	·6138	·5154	·4199	·3321	·2551	·1904	·1383
9·0	·9389	·8909	·8264	·7473	·6577	·5627	·4679	·3781	·2971	·2271	·1689
9·5	·9503	·9093	·8527	·7813	·6981	·6075	·5146	·4242	·3403	·2658	·2022
10·0	0·9596	0·9248	0·8753	0·8114	0·7350	0·6495	0·5595	0·4696	0·3840	0·3061	0·2378
10·5	·9672	·9378	·8949	·8380	·7683	·6885	·6022	·5140	·4278	·3474	·2752
11·0	·9734	·9486	·9116	·8614	·7983	·7243	·6425	·5567	·4711	·3892	·3140
11·5	·9785	·9577	·9259	·8818	·8251	·7570	·6801	·5976	·5134	·4310	·3536
12·0	·9826	·9652	·9380	·8994	·8488	·7867	·7149	·6364	·5543	·4724	·3937
12·5	0·9860	0·9715	0·9483	0·9147	0·8697	0·8134	0·7470	0·6727	0·5936	0·5129	0·4338
13·0	·9887	·9766	·9570	·9279	·8882	·8374	·7763	·7067	·6310	·5522	·4735
13·5	·9909	·9809	·9643	·9392	·9042	·8587	·8030	·7381	·6662	·5900	·5124
14·0	·9927	·9844	·9704	·9488	·9182	·8777	·8270	·7670	·6993	·6262	·5503
14·5	·9941	·9873	·9755	·9570	·9304	·8944	·8486	·7935	·7301	·6604	·5868
15·0	0·9953	0·9896	0·9797	0·9640	0·9409	0·9091	0·8679	0·8175	0·7586	0·6926	0·6218
15·5	·9962	·9916	·9833	·9699	·9499	·9219	·8851	·8393	·7848	·7228	·6551
16·0	·9970	·9932	·9862	·9749	·9576	·9331	·9004	·8589	·8088	·7509	·6866
16·5	·9976	·9944	·9887	·9791	·9642	·9429	·9138	·8764	·8306	·7768	·7162
17·0	·9981	·9955	·9907	·9826	·9699	·9513	·9256	·8921	·8504	·8007	·7438
17·5	0·9985	0·9964	0·9924	0·9856	0·9747	0·9586	0·9360	0·9061	0·8683	0·8226	0·7695
18·0	·9988	·9971	·9938	·9880	·9788	·9648	·9450	·9184	·8843	·8425	·7932
18·5	·9990	·9976	·9949	·9901	·9822	·9702	·9529	·9293	·8987	·8606	·8151
19·0	·9992	·9981	·9958	·9918	·9851	·9748	·9597	·9389	·9115	·8769	·8351
19·5	·9994	·9984	·9966	·9932	·9876	·9787	·9656	·9473	·9228	·8916	·8533
20	0·9995	0·9988	0·9972	0·9944	0·9897	0·9821	0·9707	0·9547	0·9329	0·9048	0·8699
21	·9997	·9992	·9982	·9962	·9929	·9873	·9789	·9666	·9496	·9271	·8984
22	·9998	·9995	·9988	·9975	·9951	·9911	·9849	·9756	·9625	·9446	·9214
23	·9999	·9997	·9992	·9983	·9966	·9938	·9893	·9823	·9723	·9583	·9397
24	·9999	·9998	·9995	·9989	·9977	·9957	·9924	·9873	·9797	·9689	·9542
25	0·9999	0·9999	0·9997	0·9992	0·9984	0·9970	0·9947	0·9909	0·9852	0·9769	0·9654
26		·9999	·9998	·9995	·9989	·9980	·9963	·9935	·9893	·9830	·9741
27		·9999	·9999	·9997	·9993	·9986	·9974	·9954	·9923	·9876	·9807
28			·9999	·9998	·9995	·9990	·9982	·9968	·9945	·9910	·9858
29			·9999	·9999	·9997	·9994	·9988	·9977	·9961	·9935	·9895
30				0·9999	0·9998	0·9996	0·9991	0·9984	0·9972	0·9953	0·9924

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TABLE 7. THE χ^2-DISTRIBUTION FUNCTION

$\nu =$	15	16	17	18	19	20	21	22	23	24	25
$x = 3$	0·0004	0·0002	0·0001								
4	·0023	·0011	·0005	0·0002	0·0001						
5	0·0079	0·0042	0·0022	0·0011	0·0006	0·0003	0·0001	0·0001			
6	·0203	·0119	·0068	·0038	·0021	·0011	·0006	·0003	0·0001	0·0001	
7	·0424	·0267	·0165	·0099	·0058	·0033	·0019	·0010	·0005	·0003	0·0001
8	·0762	·0511	·0335	·0214	·0133	·0081	·0049	·0028	·0016	·0009	·0005
9	·1225	·0866	·0597	·0403	·0265	·0171	·0108	·0067	·0040	·0024	·0014
10	0·1803	0·1334	0·0964	0·0681	0·0471	0·0318	0·0211	0·0137	0·0087	0·0055	0·0033
11	·2474	·1905	·1434	·1056	·0762	·0538	·0372	·0253	·0168	·0110	·0071
12	·3210	·2560	·1999	·1528	·1144	·0839	·0604	·0426	·0295	·0201	·0134
13	·3977	·3272	·2638	·2084	·1614	·1226	·0914	·0668	·0480	·0339	·0235
14	·4745	·4013	·3329	·2709	·2163	·1695	·1304	·0985	·0731	·0533	·0383
15	0·5486	0·4754	0·4045	0·3380	0·2774	0·2236	0·1770	0·1378	0·1054	0·0792	0·0586
16	·6179	·5470	·4762	·4075	·3427	·2834	·2303	·1841	·1447	·1119	·0852
17	·6811	·6144	·5456	·4769	·4101	·3470	·2889	·2366	·1907	·1513	·1182
18	·7373	·6761	·6112	·5443	·4776	·4126	·3510	·2940	·2425	·1970	·1576
19	·7863	·7313	·6715	·6082	·5432	·4782	·4149	·3547	·2988	·2480	·2029
20	0·8281	0·7798	0·7258	0·6672	0·6054	0·5421	0·4787	0·4170	0·3581	0·3032	0·2532
21	·8632	·8215	·7737	·7206	·6632	·6029	·5411	·4793	·4189	·3613	·3074
22	·8922	·8568	·8153	·7680	·7157	·6595	·6005	·5401	·4797	·4207	·3643
23	·9159	·8863	·8507	·8094	·7627	·7112	·6560	·5983	·5392	·4802	·4224
24	·9349	·9105	·8806	·8450	·8038	·7576	·7069	·6528	·5962	·5384	·4806
25	0·9501	0·9302	0·9053	0·8751	0·8395	0·7986	0·7528	0·7029	0·6497	0·5942	0·5376
26	·9620	·9460	·9255	·9002	·8698	·8342	·7936	·7483	·6991	·6468	·5924
27	·9713	·9585	·9419	·9210	·8953	·8647	·8291	·7888	·7440	·6955	·6441
28	·9784	·9684	·9551	·9379	·9166	·8906	·8598	·8243	·7842	·7400	·6921
29	·9839	·9761	·9655	·9516	·9340	·9122	·8860	·8551	·8197	·7799	·7361
30	0·9881	0·9820	0·9737	0·9626	0·9482	0·9301	0·9080	0·8815	0·8506	0·8152	0·7757
31	·9912	·9865	·9800	·9712	·9596	·9448	·9263	·9039	·8772	·8462	·8110
32	·9936	·9900	·9850	·9780	·9687	·9567	·9414	·9226	·8999	·8730	·8420
33	·9953	·9926	·9887	·9833	·9760	·9663	·9538	·9381	·9189	·8959	·8689
34	·9966	·9946	·9916	·9874	·9816	·9739	·9638	·9509	·9348	·9153	·8921
35	0·9975	0·9960	0·9938	0·9905	0·9860	0·9799	0·9718	0·9613	0·9480	0·9316	0·9118
36	·9982	·9971	·9954	·9929	·9894	·9846	·9781	·9696	·9587	·9451	·9284
37	·9987	·9979	·9966	·9948	·9921	·9883	·9832	·9763	·9675	·9562	·9423
38	·9991	·9985	·9975	·9961	·9941	·9911	·9871	·9817	·9745	·9653	·9537
39	·9994	·9989	·9982	·9972	·9956	·9933	·9902	·9859	·9802	·9727	·9632
40	0·9995	0·9992	0·9987	0·9979	0·9967	0·9950	0·9926	0·9892	0·9846	0·9786	0·9708
41	·9997	·9994	·9991	·9985	·9976	·9963	·9944	·9918	·9882	·9833	·9770
42	·9998	·9996	·9993	·9989	·9982	·9972	·9958	·9937	·9909	·9871	·9820
43	·9998	·9997	·9995	·9992	·9987	·9980	·9969	·9953	·9931	·9901	·9860
44	·9999	·9998	·9997	·9994	·9991	·9985	·9977	·9965	·9947	·9924	·9892
45	0·9999	0·9999	0·9998	0·9996	0·9993	0·9989	0·9983	0·9973	0·9960	0·9942	0·9916
46	·9999	·9999	·9998	·9997	·9995	·9992	·9987	·9980	·9970	·9956	·9936
47		·9999	·9999	·9998	·9996	·9994	·9991	·9985	·9978	·9967	·9951
48			·9999	·9998	·9997	·9996	·9993	·9989	·9983	·9975	·9963
49			·9999	·9999	·9998	·9997	·9995	·9992	·9988	·9981	·9972
50				0·9999	0·9999	0·9998	0·9996	0·9994	0·9991	0·9986	0·9979

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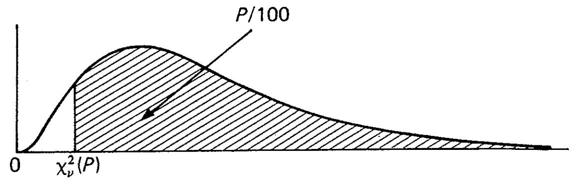
TABLE 8. PERCENTAGE POINTS OF THE χ^2-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})} \int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1}\, e^{-\frac{1}{2}x}\, dx.$$

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu-1}$ and unit variance.



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

P	99·95	99·9	99·5	99	97·5	95	90	80	70	60
$\nu = 1$	$0{\cdot}0^{6}3927$	$0{\cdot}0^{5}1571$	$0{\cdot}0^{4}3927$	$0{\cdot}0^{3}1571$	$0{\cdot}0^{3}9821$	0·003932	0·01579	0·06418	0·1485	0·2750
2	0·001000	0·002001	0·01003	0·02010	0·05064	0·1026	0·2107	0·4463	0·7133	1·022
3	0·01528	0·02430	0·07172	0·1148	0·2158	0·3518	0·5844	1·005	1·424	1·869
4	0·06392	0·09080	0·2070	0·2971	0·4844	0·7107	1·064	1·649	2·195	2·753
5	0·1581	0·2102	0·4117	0·5543	0·8312	1·145	1·610	2·343	3·000	3·655
6	0·2994	0·3811	0·6757	0·8721	1·237	1·635	2·204	3·070	3·828	4·570
7	0·4849	0·5985	0·9893	1·239	1·690	2·167	2·833	3·822	4·671	5·493
8	0·7104	0·8571	1·344	1·646	2·180	2·733	3·490	4·594	5·527	6·423
9	0·9717	1·152	1·735	2·088	2·700	3·325	4·168	5·380	6·393	7·357
10	1·265	1·479	2·156	2·558	3·247	3·940	4·865	6·179	7·267	8·295
11	1·587	1·834	2·603	3·053	3·816	4·575	5·578	6·989	8·148	9·237
12	1·934	2·214	3·074	3·571	4·404	5·226	6·304	7·807	9·034	10·18
13	2·305	2·617	3·565	4·107	5·009	5·892	7·042	8·634	9·926	11·13
14	2·697	3·041	4·075	4·660	5·629	6·571	7·790	9·467	10·82	12·08
15	3·108	3·483	4·601	5·229	6·262	7·261	8·547	10·31	11·72	13·03
16	3·536	3·942	5·142	5·812	6·908	7·962	9·312	11·15	12·62	13·98
17	3·980	4·416	5·697	6·408	7·564	8·672	10·09	12·00	13·53	14·94
18	4·439	4·905	6·265	7·015	8·231	9·390	10·86	12·86	14·44	15·89
19	4·912	5·407	6·844	7·633	8·907	10·12	11·65	13·72	15·35	16·85
20	5·398	5·921	7·434	8·260	9·591	10·85	12·44	14·58	16·27	17·81
21	5·896	6·447	8·034	8·897	10·28	11·59	13·24	15·44	17·18	18·77
22	6·404	6·983	8·643	9·542	10·98	12·34	14·04	16·31	18·10	19·73
23	6·924	7·529	9·260	10·20	11·69	13·09	14·85	17·19	19·02	20·69
24	7·453	8·085	9·886	10·86	12·40	13·85	15·66	18·06	19·94	21·65
25	7·991	8·649	10·52	11·52	13·12	14·61	16·47	18·94	20·87	22·62
26	8·538	9·222	11·16	12·20	13·84	15·38	17·29	19·82	21·79	23·58
27	9·093	9·803	11·81	12·88	14·57	16·15	18·11	20·70	22·72	24·54
28	9·656	10·39	12·46	13·56	15·31	16·93	18·94	21·59	23·65	25·51
29	10·23	10·99	13·12	14·26	16·05	17·71	19·77	22·48	24·58	26·48
30	10·80	11·59	13·79	14·95	16·79	18·49	20·60	23·36	25·51	27·44
32	11·98	12·81	15·13	16·36	18·29	20·07	22·27	25·15	27·37	29·38
34	13·18	14·06	16·50	17·79	19·81	21·66	23·95	26·94	29·24	31·31
36	14·40	15·32	17·89	19·23	21·34	23·27	25·64	28·73	31·12	33·25
38	15·64	16·61	19·29	20·69	22·88	24·88	27·34	30·54	32·99	35·19
40	16·91	17·92	20·71	22·16	24·43	26·51	29·05	32·34	34·87	37·13
50	23·46	24·67	27·99	29·71	32·36	34·76	37·69	41·45	44·31	46·86
60	30·34	31·74	35·53	37·48	40·48	43·19	46·46	50·64	53·81	56·62
70	37·47	39·04	43·28	45·44	48·76	51·74	55·33	59·90	63·35	66·40
80	44·79	46·52	51·17	53·54	57·15	60·39	64·28	69·21	72·92	76·19
90	52·28	54·16	59·20	61·75	65·65	69·13	73·29	78·56	82·51	85·99
100	59·90	61·92	67·33	70·06	74·22	77·93	82·36	87·95	92·13	95·81

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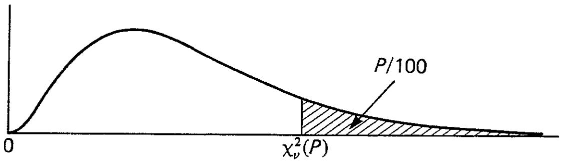
TABLE 8. PERCENTAGE POINTS OF THE χ^2-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})}\int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1}\,e^{-\frac{1}{2}x}\,dx.$$

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu-1}$ and unit variance.



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

P	50	40	30	20	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·4549	0·7083	1·074	1·642	2·706	3·841	5·024	6·635	7·879	10·83	12·12
2	1·386	1·833	2·408	3·219	4·605	5·991	7·378	9·210	10·60	13·82	15·20
3	2·366	2·946	3·665	4·642	6·251	7·815	9·348	11·34	12·84	16·27	17·73
4	3·357	4·045	4·878	5·989	7·779	9·488	11·14	13·28	14·86	18·47	20·00
5	4·351	5·132	6·064	7·289	9·236	11·07	12·83	15·09	16·75	20·52	22·11
6	5·348	6·211	7·231	8·558	10·64	12·59	14·45	16·81	18·55	22·46	24·10
7	6·346	7·283	8·383	9·803	12·02	14·07	16·01	18·48	20·28	24·32	26·02
8	7·344	8·351	9·524	11·03	13·36	15·51	17·53	20·09	21·95	26·12	27·87
9	8·343	9·414	10·66	12·24	14·68	16·92	19·02	21·67	23·59	27·88	29·67
10	9·342	10·47	11·78	13·44	15·99	18·31	20·48	23·21	25·19	29·59	31·42
11	10·34	11·53	12·90	14·63	17·28	19·68	21·92	24·72	26·76	31·26	33·14
12	11·34	12·58	14·01	15·81	18·55	21·03	23·34	26·22	28·30	32·91	34·82
13	12·34	13·64	15·12	16·98	19·81	22·36	24·74	27·69	29·82	34·53	36·48
14	13·34	14·69	16·22	18·15	21·06	23·68	26·12	29·14	31·32	36·12	38·11
15	14·34	15·73	17·32	19·31	22·31	25·00	27·49	30·58	32·80	37·70	39·72
16	15·34	16·78	18·42	20·47	23·54	26·30	28·85	32·00	34·27	39·25	41·31
17	16·34	17·82	19·51	21·61	24·77	27·59	30·19	33·41	35·72	40·79	42·88
18	17·34	18·87	20·60	22·76	25·99	28·87	31·53	34·81	37·16	42·31	44·43
19	18·34	19·91	21·69	23·90	27·20	30·14	32·85	36·19	38·58	43·82	45·97
20	19·34	20·95	22·77	25·04	28·41	31·41	34·17	37·57	40·00	45·31	47·50
21	20·34	21·99	23·86	26·17	29·62	32·67	35·48	38·93	41·40	46·80	49·01
22	21·34	23·03	24·94	27·30	30·81	33·92	36·78	40·29	42·80	48·27	50·51
23	22·34	24·07	26·02	28·43	32·01	35·17	38·08	41·64	44·18	49·73	52·00
24	23·34	25·11	27·10	29·55	33·20	36·42	39·36	42·98	45·56	51·18	53·48
25	24·34	26·14	28·17	30·68	34·38	37·65	40·65	44·31	46·93	52·62	54·95
26	25·34	27·18	29·25	31·79	35·56	38·89	41·92	45·64	48·29	54·05	56·41
27	26·34	28·21	30·32	32·91	36·74	40·11	43·19	46·96	49·64	55·48	57·86
28	27·34	29·25	31·39	34·03	37·92	41·34	44·46	48·28	50·99	56·89	59·30
29	28·34	30·28	32·46	35·14	39·09	42·56	45·72	49·59	52·34	58·30	60·73
30	29·34	31·32	33·53	36·25	40·26	43·77	46·98	50·89	53·67	59·70	62·16
32	31·34	33·38	35·66	38·47	42·58	46·19	49·48	53·49	56·33	62·49	65·00
34	33·34	35·44	37·80	40·68	44·90	48·60	51·97	56·06	58·96	65·25	67·80
36	35·34	37·50	39·92	42·88	47·21	51·00	54·44	58·62	61·58	67·99	70·59
38	37·34	39·56	42·05	45·08	49·51	53·38	56·90	61·16	64·18	70·70	73·35
40	39·34	41·62	44·16	47·27	51·81	55·76	59·34	63·69	66·77	73·40	76·09
50	49·33	51·89	54·72	58·16	63·17	67·50	71·42	76·15	79·49	86·66	89·56
60	59·33	62·13	65·23	68·97	74·40	79·08	83·30	88·38	91·95	99·61	102·7
70	69·33	72·36	75·69	79·71	85·53	90·53	95·02	100·4	104·2	112·3	115·6
80	79·33	82·57	86·12	90·41	96·58	101·9	106·6	112·3	116·3	124·8	128·3
90	89·33	92·76	96·52	101·1	107·6	113·1	118·1	124·1	128·3	137·2	140·8
100	99·33	102·9	106·9	111·7	118·5	124·3	129·6	135·8	140·2	149·4	153·2

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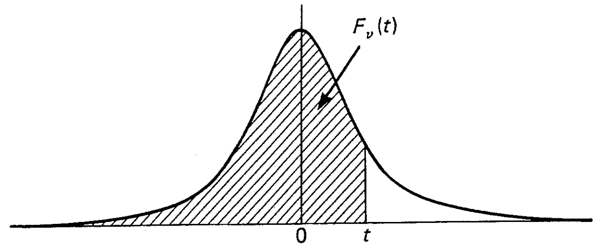
TABLE 9. THE t-DISTRIBUTION FUNCTION

The function tabulated is

$$F_\nu(t) = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{-\infty}^{t} \frac{ds}{(1+s^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

$F_\nu(t)$ is the probability that a random variable, distributed as t with ν degrees of freedom, will be less than or equal to t. When $t < 0$ use $F_\nu(t) = 1-F_\nu(-t)$, the t distribution being symmetric about zero.

The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance (see Table 4). When ν is large interpolation in ν should be harmonic.



Omitted entries to the right of tabulated values are 1 (to four decimal places).

$\nu =$	1	$\nu =$	1	$\nu =$	2	$\nu =$	2	$\nu =$	3	$\nu =$	3
$t =$ 0·0	0·5000	$t =$ 4·0	0·9220	$t =$ 0·0	0·5000	$t =$ 4·0	0·9714	$t =$ 0·0	0·5000	$t =$ 4·0	0·9860
·1	·5317	4·2	·9256	·1	·5353	·1	·9727	·1	·5367	·1	·9869
·2	·5628	4·4	·9289	·2	·5700	·2	·9739	·2	·5729	·2	·9877
·3	·5928	4·6	·9319	·3	·6038	·3	·9750	·3	·6081	·3	·9884
·4	·6211	4·8	·9346	·4	·6361	·4	·9760	·4	·6420	·4	·9891
0·5	0·6476	5·0	0·9372	0·5	0·6667	4·5	0·9770	0·5	0·6743	4·5	0·9898
·6	·6720	5·5	·9428	·6	·6953	·6	·9779	·6	·7046	·6	·9903
·7	·6944	6·0	·9474	·7	·7218	·7	·9788	·7	·7328	·7	·9909
·8	·7148	6·5	·9514	·8	·7462	·8	·9796	·8	·7589	·8	·9914
·9	·7333	7·0	·9548	·9	·7684	·9	·9804	·9	·7828	·9	·9919
1·0	0·7500	7·5	0·9578	1·0	0·7887	5·0	0·9811	1·0	0·8045	5·0	0·9923
·1	·7651	8·0	·9604	·1	·8070	·1	·9818	·1	·8242	·1	·9927
·2	·7789	8·5	·9627	·2	·8235	·2	·9825	·2	·8419	·2	·9931
·3	·7913	9·0	·9648	·3	·8384	·3	·9831	·3	·8578	·3	·9934
·4	·8026	9·5	·9666	·4	·8518	·4	·9837	·4	·8720	·4	·9938
1·5	0·8128	10·0	0·9683	1·5	0·8638	5·5	0·9842	1·5	0·8847	5·5	0·9941
·6	·8222	10·5	·9698	·6	·8746	·6	·9848	·6	·8960	·6	·9944
·7	·8307	11·0	·9711	·7	·8844	·7	·9853	·7	·9062	·7	·9946
·8	·8386	11·5	·9724	·8	·8932	·8	·9858	·8	·9152	·8	·9949
·9	·8458	12·0	·9735	·9	·9011	·9	·9862	·9	·9232	·9	·9951
2·0	0·8524	12·5	0·9746	2·0	0·9082	6·0	0·9867	2·0	0·9303	6·0	0·9954
·1	·8585	13·0	·9756	·1	·9147	·1	·9871	·1	·9367	·1	·9956
·2	·8642	13·5	·9765	·2	·9206	·2	·9875	·2	·9424	·2	·9958
·3	·8695	14·0	·9773	·3	·9259	·3	·9879	·3	·9475	·3	·9960
·4	·8743	14·5	·9781	·4	·9308	·4	·9882	·4	·9521	·4	·9961
2·5	0·8789	15	0·9788	2·5	0·9352	6·5	0·9886	2·5	0·9561	6·5	0·9963
·6	·8831	16	·9801	·6	·9392	·6	·9889	·6	·9598	·6	·9965
·7	·8871	17	·9813	·7	·9429	·7	·9892	·7	·9631	·7	·9966
·8	·8908	18	·9823	·8	·9463	·8	·9895	·8	·9661	·8	·9967
·9	·8943	19	·9833	·9	·9494	·9	·9898	·9	·9687	·9	·9969
3·0	0·8976	20	0·9841	3·0	0·9523	7·0	0·9901	3·0	0·9712	7·0	0·9970
·1	·9007	21	·9849	·1	·9549	·1	·9904	·1	·9734	·1	·9971
·2	·9036	22	·9855	·2	·9573	·2	·9906	·2	·9753	·2	·9972
·3	·9063	23	·9862	·3	·9596	·3	·9909	·3	·9771	·3	·9973
·4	·9089	24	·9867	·4	·9617	·4	·9911	·4	·9788	·4	·9974
3·5	0·9114	25	0·9873	3·5	0·9636	7·5	0·9913	3·5	0·9803	7·5	0·9975
·6	·9138	30	·9894	·6	·9654	·6	·9916	·6	·9816	·6	·9976
·7	·9160	35	·9909	·7	·9670	·7	·9918	·7	·9829	·7	·9977
·8	·9181	40	·9920	·8	·9686	·8	·9920	·8	·9840	·8	·9978
·9	·9201	45	·9929	·9	·9701	·9	·9922	·9	·9850	·9	·9979
4·0	0·9220	50	0·9936	4·0	0·9714	8·0	0·9924	4·0	0·9860	8·0	0·9980

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TABLE 9. THE t-DISTRIBUTION FUNCTION

$\nu =$	4	5	6	7	8	9	10	11	12	13	14
$t =$ 0·0	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000
·1	·5374	·5379	·5382	·5384	·5386	·5387	·5388	·5389	·5390	·5391	·5391
·2	·5744	·5753	·5760	·5764	·5768	·5770	·5773	·5774	·5776	·5777	·5778
·3	·6104	·6119	·6129	·6136	·6141	·6145	·6148	·6151	·6153	·6155	·6157
·4	·6452	·6472	·6485	·6495	·6502	·6508	·6512	·6516	·6519	·6522	·6524
0·5	0·6783	0·6809	0·6826	0·6838	0·6847	0·6855	0·6861	0·6865	0·6869	0·6873	0·6876
·6	·7096	·7127	·7148	·7163	·7174	·7183	·7191	·7197	·7202	·7206	·7210
·7	·7387	·7424	·7449	·7467	·7481	·7492	·7501	·7508	·7514	·7519	·7523
·8	·7657	·7700	·7729	·7750	·7766	·7778	·7788	·7797	·7804	·7810	·7815
·9	·7905	·7953	·7986	·8010	·8028	·8042	·8054	·8063	·8071	·8078	·8083
1·0	0·8130	0·8184	0·8220	0·8247	0·8267	0·8283	0·8296	0·8306	0·8315	0·8322	0·8329
·1	·8335	·8393	·8433	·8461	·8483	·8501	·8514	·8526	·8535	·8544	·8551
·2	·8518	·8581	·8623	·8654	·8678	·8696	·8711	·8723	·8734	·8742	·8750
·3	·8683	·8748	·8793	·8826	·8851	·8870	·8886	·8899	·8910	·8919	·8927
·4	·8829	·8898	·8945	·8979	·9005	·9025	·9041	·9055	·9066	·9075	·9084
1·5	0·8960	0·9030	0·9079	0·9114	0·9140	0·9161	0·9177	0·9191	0·9203	0·9212	0·9221
·6	·9076	·9148	·9196	·9232	·9259	·9280	·9297	·9310	·9322	·9332	·9340
·7	·9178	·9251	·9300	·9335	·9362	·9383	·9400	·9414	·9426	·9435	·9444
·8	·9269	·9341	·9390	·9426	·9452	·9473	·9490	·9503	·9515	·9525	·9533
·9	·9349	·9421	·9469	·9504	·9530	·9551	·9567	·9580	·9591	·9601	·9609
2·0	0·9419	0·9490	0·9538	0·9572	0·9597	0·9617	0·9633	0·9646	0·9657	0·9666	0·9674
·1	·9482	·9551	·9598	·9631	·9655	·9674	·9690	·9702	·9712	·9721	·9728
·2	·9537	·9605	·9649	·9681	·9705	·9723	·9738	·9750	·9759	·9768	·9774
·3	·9585	·9651	·9694	·9725	·9748	·9765	·9779	·9790	·9799	·9807	·9813
·4	·9628	·9692	·9734	·9763	·9784	·9801	·9813	·9824	·9832	·9840	·9846
2·5	0·9666	0·9728	0·9767	0·9795	0·9815	0·9831	0·9843	0·9852	0·9860	0·9867	0·9873
·6	·9700	·9759	·9797	·9823	·9842	·9856	·9868	·9877	·9884	·9890	·9895
·7	·9730	·9786	·9822	·9847	·9865	·9878	·9888	·9897	·9903	·9909	·9914
·8	·9756	·9810	·9844	·9867	·9884	·9896	·9906	·9914	·9920	·9925	·9929
·9	·9779	·9831	·9863	·9885	·9901	·9912	·9921	·9928	·9933	·9938	·9942
3·0	0·9800	0·9850	0·9880	0·9900	0·9915	0·9925	0·9933	0·9940	0·9945	0·9949	0·9952
·1	·9819	·9866	·9894	·9913	·9927	·9936	·9944	·9949	·9954	·9958	·9961
·2	·9835	·9880	·9907	·9925	·9937	·9946	·9953	·9958	·9962	·9965	·9968
·3	·9850	·9893	·9918	·9934	·9946	·9954	·9960	·9965	·9968	·9971	·9974
·4	·9864	·9904	·9928	·9943	·9953	·9961	·9966	·9970	·9974	·9976	·9978
3·5	0·9876	0·9914	0·9936	0·9950	0·9960	0·9966	0·9971	0·9975	0·9978	0·9980	0·9982
·6	·9886	·9922	·9943	·9956	·9965	·9971	·9976	·9979	·9982	·9984	·9986
·7	·9896	·9930	·9950	·9962	·9970	·9975	·9979	·9982	·9985	·9987	·9988
·8	·9904	·9937	·9955	·9966	·9974	·9979	·9983	·9985	·9987	·9989	·9990
·9	·9912	·9943	·9960	·9971	·9977	·9982	·9985	·9988	·9989	·9991	·9992
4·0	0·9919	0·9948	0·9964	0·9974	0·9980	0·9984	0·9987	0·9990	0·9991	0·9992	0·9993
·1	·9926	·9953	·9968	·9977	·9983	·9987	·9989	·9991	·9993	·9994	·9995
·2	·9932	·9958	·9972	·9980	·9985	·9988	·9991	·9993	·9994	·9995	·9996
·3	·9937	·9961	·9975	·9982	·9987	·9990	·9992	·9994	·9995	·9996	·9996
·4	·9942	·9965	·9977	·9984	·9989	·9991	·9993	·9995	·9996	·9996	·9997
4·5	0·9946	0·9968	0·9979	0·9986	0·9990	0·9993	0·9994	0·9995	0·9996	0·9997	0·9998
·6	·9950	·9971	·9982	·9988	·9991	·9994	·9995	·9996	·9997	·9998	·9998
·7	·9953	·9973	·9983	·9989	·9992	·9994	·9996	·9997	·9997	·9998	·9998
·8	·9957	·9976	·9985	·9990	·9993	·9995	·9996	·9997	·9998	·9998	·9999
·9	·9960	·9978	·9986	·9991	·9994	·9996	·9997	·9998	·9998	·9999	·9999
5·0	0·9963	0·9979	0·9988	0·9992	0·9995	0·9996	0·9997	0·9998	0·9998	0·9999	0·9999

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TABLE 9. THE t-DISTRIBUTION FUNCTION

$\nu =$	15	16	17	18	19	20	24	30	40	60	∞
$t =$ 0·0	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000
·1	·5392	·5392	·5392	·5393	·5393	·5393	·5394	·5395	·5396	·5397	·5398
·2	·5779	·5780	·5781	·5781	·5782	·5782	·5784	·5786	·5788	·5789	·5793
·3	·6159	·6160	·6161	·6162	·6163	·6164	·6166	·6169	·6171	·6174	·6179
·4	·6526	·6528	·6529	·6531	·6532	·6533	·6537	·6540	·6544	·6547	·6554
0·5	0·6878	0·6881	0·6883	0·6884	0·6886	0·6887	0·6892	0·6896	0·6901	0·6905	0·6915
·6	·7213	·7215	·7218	·7220	·7222	·7224	·7229	·7235	·7241	·7246	·7257
·7	·7527	·7530	·7533	·7536	·7538	·7540	·7547	·7553	·7560	·7567	·7580
·8	·7819	·7823	·7826	·7829	·7832	·7834	·7842	·7850	·7858	·7866	·7881
·9	·8088	·8093	·8097	·8100	·8103	·8106	·8115	·8124	·8132	·8141	·8159
1·0	0·8334	0·8339	0·8343	0·8347	0·8351	0·8354	0·8364	0·8373	0·8383	0·8393	0·8413
·1	·8557	·8562	·8567	·8571	·8575	·8578	·8589	·8600	·8610	·8621	·8643
·2	·8756	·8762	·8767	·8772	·8776	·8779	·8791	·8802	·8814	·8826	·8849
·3	·8934	·8940	·8945	·8950	·8954	·8958	·8970	·8982	·8995	·9007	·9032
·4	·9091	·9097	·9103	·9107	·9112	·9116	·9128	·9141	·9154	·9167	·9192
1·5	0·9228	0·9235	0·9240	0·9245	0·9250	0·9254	0·9267	0·9280	0·9293	0·9306	0·9332
·6	·9348	·9354	·9360	·9365	·9370	·9374	·9387	·9400	·9413	·9426	·9452
·7	·9451	·9458	·9463	·9468	·9473	·9477	·9490	·9503	·9516	·9528	·9554
·8	·9540	·9546	·9552	·9557	·9561	·9565	·9578	·9590	·9603	·9616	·9641
·9	·9616	·9622	·9627	·9632	·9636	·9640	·9652	·9665	·9677	·9689	·9713
2·0	0·9680	0·9686	0·9691	0·9696	0·9700	0·9704	0·9715	0·9727	0·9738	0·9750	0·9772
·1	·9735	·9740	·9745	·9750	·9753	·9757	·9768	·9779	·9790	·9800	·9821
·2	·9781	·9786	·9790	·9794	·9798	·9801	·9812	·9822	·9832	·9842	·9861
·3	·9819	·9824	·9828	·9832	·9835	·9838	·9848	·9857	·9866	·9875	·9893
·4	·9851	·9855	·9859	·9863	·9866	·9869	·9877	·9886	·9894	·9902	·9918
2·5	0·9877	0·9882	0·9885	0·9888	0·9891	0·9894	0·9902	0·9909	0·9917	0·9924	0·9938
·6	·9900	·9903	·9907	·9910	·9912	·9914	·9921	·9928	·9935	·9941	·9953
·7	·9918	·9921	·9924	·9927	·9929	·9931	·9937	·9944	·9949	·9955	·9965
·8	·9933	·9936	·9938	·9941	·9943	·9945	·9950	·9956	·9961	·9966	·9974
·9	·9945	·9948	·9950	·9952	·9954	·9956	·9961	·9965	·9970	·9974	·9981
3·0	0·9955	0·9958	0·9960	0·9962	0·9963	0·9965	0·9969	0·9973	0·9977	0·9980	0·9987
·1	·9963	·9966	·9967	·9969	·9971	·9972	·9976	·9979	·9982	·9985	·9990
·2	·9970	·9972	·9974	·9975	·9976	·9978	·9981	·9984	·9987	·9989	·9993
·3	·9976	·9977	·9979	·9980	·9981	·9982	·9985	·9988	·9990	·9992	·9995
·4	·9980	·9982	·9983	·9984	·9985	·9986	·9988	·9990	·9992	·9994	·9997
3·5	0·9984	0·9985	0·9986	0·9987	0·9988	0·9989	0·9991	0·9993	0·9994	0·9996	0·9998
·6	·9987	·9988	·9989	·9990	·9990	·9991	·9993	·9994	·9996	·9997	·9998
·7	·9989	·9990	·9991	·9992	·9992	·9993	·9994	·9996	·9997	·9998	·9999
·8	·9991	·9992	·9993	·9993	·9994	·9994	·9996	·9997	·9998	·9998	·9999
·9	·9993	·9994	·9994	·9995	·9995	·9996	·9997	·9997	·9998	·9999	
4·0	0·9994	0·9995	0·9995	0·9996	0·9996	0·9996	0·9997	0·9998	0·9999	0·9999	
·1	·9995	·9996	·9996	·9997	·9997	·9997	·9998	·9999	·9999	·9999	
·2	·9996	·9997	·9997	·9997	·9998	·9998	·9998	·9999	·9999		
·3	·9997	·9997	·9998	·9998	·9998	·9998	·9999	·9999	·9999		
·4	·9997	·9998	·9998	·9998	·9998	·9999	·9999	·9999			
4·5	0·9998	0·9998	0·9998	0·9999	0·9999	0·9999	0·9999				

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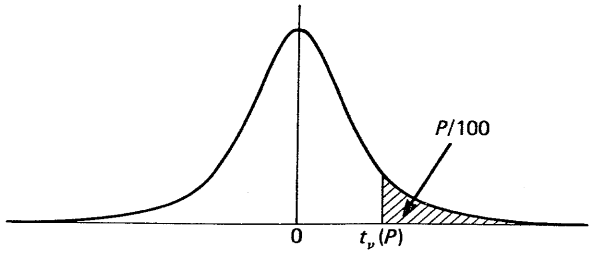
TABLE 10. PERCENTAGE POINTS OF THE t-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2-distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t-distribution with ν degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

P	40	30	25	20	15	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·3249	0·7265	1·0000	1·3764	1·963	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	0·2887	0·6172	0·8165	1·0607	1·386	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	0·2767	0·5844	0·7649	0·9785	1·250	1·638	2·353	3·182	4·541	5·841	10·21	12·92
4	0·2707	0·5686	0·7407	0·9410	1·190	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	0·2672	0·5594	0·7267	0·9195	1·156	1·476	2·015	2·571	3·365	4·032	5·893	6·869
6	0·2648	0·5534	0·7176	0·9057	1·134	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	0·2632	0·5491	0·7111	0·8960	1·119	1·415	1·895	2·365	2·998	3·499	4·785	5·408
8	0·2619	0·5459	0·7064	0·8889	1·108	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	0·2610	0·5435	0·7027	0·8834	1·100	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	0·2602	0·5415	0·6998	0·8791	1·093	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	0·2596	0·5399	0·6974	0·8755	1·088	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	0·2590	0·5386	0·6955	0·8726	1·083	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	0·2586	0·5375	0·6938	0·8702	1·079	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	0·2582	0·5366	0·6924	0·8681	1·076	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	0·2579	0·5357	0·6912	0·8662	1·074	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	0·2576	0·5350	0·6901	0·8647	1·071	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	0·2573	0·5344	0·6892	0·8633	1·069	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	0·2571	0·5338	0·6884	0·8620	1·067	1·330	1·734	2·101	2·552	2·878	3·610	3·922
19	0·2569	0·5333	0·6876	0·8610	1·066	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	0·2567	0·5329	0·6870	0·8600	1·064	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	0·2566	0·5325	0·6864	0·8591	1·063	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	0·2564	0·5321	0·6858	0·8583	1·061	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	0·2563	0·5317	0·6853	0·8575	1·060	1·319	1·714	2·069	2·500	2·807	3·485	3·768
24	0·2562	0·5314	0·6848	0·8569	1·059	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	0·2561	0·5312	0·6844	0·8562	1·058	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	0·2560	0·5309	0·6840	0·8557	1·058	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	0·2559	0·5306	0·6837	0·8551	1·057	1·314	1·703	2·052	2·473	2·771	3·421	3·690
28	0·2558	0·5304	0·6834	0·8546	1·056	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	0·2557	0·5302	0·6830	0·8542	1·055	1·311	1·699	2·045	2·462	2·756	3·396	3·659
30	0·2556	0·5300	0·6828	0·8538	1·055	1·310	1·697	2·042	2·457	2·750	3·385	3·646
32	0·2555	0·5297	0·6822	0·8530	1·054	1·309	1·694	2·037	2·449	2·738	3·365	3·622
34	0·2553	0·5294	0·6818	0·8523	1·052	1·307	1·691	2·032	2·441	2·728	3·348	3·601
36	0·2552	0·5291	0·6814	0·8517	1·052	1·306	1·688	2·028	2·434	2·719	3·333	3·582
38	0·2551	0·5288	0·6810	0·8512	1·051	1·304	1·686	2·024	2·429	2·712	3·319	3·566
40	0·2550	0·5286	0·6807	0·8507	1·050	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	0·2547	0·5278	0·6794	0·8489	1·047	1·299	1·676	2·009	2·403	2·678	3·261	3·496
60	0·2545	0·5272	0·6786	0·8477	1·045	1·296	1·671	2·000	2·390	2·660	3·232	3·460
120	0·2539	0·5258	0·6765	0·8446	1·041	1·289	1·658	1·980	2·358	2·617	3·160	3·373
∞	0·2533	0·5244	0·6745	0·8416	1·036	1·282	1·645	1·960	2·326	2·576	3·090	3·291

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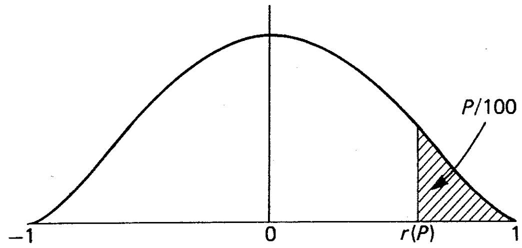
TABLE 13. PERCENTAGE POINTS OF THE CORRELATION COEFFICIENT r WHEN $\rho = 0$

The function tabulated is $r(P) = r(P|\nu)$ defined by the equation

$$\frac{\Gamma\left(\frac{\nu-1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{\nu-2}{2}\right)} \int_{r(P)}^{1} (1-r^2)^{\frac{\nu-4}{2}}\, dr = P/100.$$

Let r be a partial correlation coefficient, after s variables have been eliminated, in a sample of size n from a multivariate normal population with corresponding true partial correlation coefficient $\rho = 0$, and let $\nu = n - s$. This table gives upper P per cent points of r; the corresponding lower P per cent points are given by $-r(P)$, and the tabulated values are also upper $2P$ per cent points of $|r|$. For $s = 0$ we have $\nu = n$ and r is the ordinary correlation coefficient. When $\nu > 130$ use the results that r is approximately normally distributed with zero mean and variance $\frac{1}{\nu-1}$, or (more accurately) that $z = \tanh^{-1} r$ is approximately normally distributed with zero mean and variance $\frac{1}{\nu-3}$ (cf. Tables 16 and 17).



(This shape applies for $\nu \geqslant 5$ only. When $\nu = 4$ the distribution is uniform and when $\nu = 3$ the probability density function is U-shaped.)

Tables of the distribution of r for various values of ρ are given by, for example, F. N. David, *Tables of the Ordinates and Probability Integral of the Distribution of the Correlation Coefficient in Small Samples*, Cambridge University Press (1954), and R. E. Odeh, 'Critical values of the sample product–moment correlation coefficient in the bivariate normal distribution', *Commun. Statist. – Simula Computa.* **11** (1) (1982), pp. 1–26. The z-transformation may also be used (cf. Tables 16 and 17).

P	5	2·5	1	0·5	0·1
$\nu = 3$	0·9877	0·9969	0·9995	0·9999	0·999995
4	·9000	·9500	·9800	·9900	·9980
5	0·8054	0·8783	0·9343	0·9587	0·9859
6	·7293	·8114	·8822	·9172	·9633
7	·6694	·7545	·8329	·8745	·9350
8	·6215	·7067	·7887	·8343	·9049
9	·5822	·6664	·7498	·7977	·8751
10	0·5494	0·6319	0·7155	0·7646	0·8467
11	·5214	·6021	·6851	·7348	·8199
12	·4973	·5760	·6581	·7079	·7950
13	·4762	·5529	·6339	·6835	·7717
14	·4575	·5324	·6120	·6614	·7501
15	0·4409	0·5140	0·5923	0·6411	0·7301
16	·4259	·4973	·5742	·6226	·7114
17	·4124	·4821	·5577	·6055	·6940
18	·4000	·4683	·5425	·5897	·6777
19	·3887	·4555	·5285	·5751	·6624
20	0·3783	0·4438	0·5155	0·5614	0·6481
21	·3687	·4329	·5034	·5487	·6346
22	·3598	·4227	·4921	·5368	·6219
23	·3515	·4132	·4815	·5256	·6099
24	·3438	·4044	·4716	·5151	·5986
25	0·3365	0·3961	0·4622	0·5052	0·5879
26	·3297	·3882	·4534	·4958	·5776
27	·3233	·3809	·4451	·4869	·5679
28	·3172	·3739	·4372	·4785	·5587
29	·3115	·3673	·4297	·4705	·5499
30	0·3061	0·3610	0·4226	0·4629	0·5415
31	·3009	·3550	·4158	·4556	·5334
32	·2960	·3494	·4093	·4487	·5257
33	·2913	·3440	·4032	·4421	·5184
34	·2869	·3388	·3972	·4357	·5113
35	0·2826	0·3338	0·3916	0·4296	0·5045
36	·2785	·3291	·3862	·4238	·4979
37	·2746	·3246	·3810	·4182	·4916
38	·2709	·3202	·3760	·4128	·4856
39	·2673	·3160	·3712	·4076	·4797

P	5	2·5	1	0·5	0·1
$\nu = 40$	0·2638	0·3120	0·3665	0·4026	0·4741
42	·2573	·3044	·3578	·3932	·4633
44	·2512	·2973	·3496	·3843	·4533
46	·2455	·2907	·3420	·3761	·4439
48	·2403	·2845	·3348	·3683	·4351
50	0·2353	0·2787	0·3281	0·3610	0·4267
52	·2306	·2732	·3218	·3542	·4188
54	·2262	·2681	·3158	·3477	·4114
56	·2221	·2632	·3102	·3415	·4043
58	·2181	·2586	·3048	·3357	·3976
60	0·2144	0·2542	0·2997	0·3301	0·3912
62	·2108	·2500	·2948	·3248	·3850
64	·2075	·2461	·2902	·3198	·3792
66	·2042	·2423	·2858	·3150	·3736
68	·2012	·2387	·2816	·3104	·3683
70	0·1982	0·2352	0·2776	0·3060	0·3632
72	·1954	·2319	·2737	·3017	·3583
74	·1927	·2287	·2700	·2977	·3536
76	·1901	·2257	·2664	·2938	·3490
78	·1876	·2227	·2630	·2900	·3447
80	0·1852	0·2199	0·2597	0·2864	0·3405
82	·1829	·2172	·2565	·2830	·3364
84	·1807	·2146	·2535	·2796	·3325
86	·1786	·2120	·2505	·2764	·3287
88	·1765	·2096	·2477	·2732	·3251
90	0·1745	0·2072	0·2449	0·2702	0·3215
92	·1726	·2050	·2422	·2673	·3181
94	·1707	·2028	·2396	·2645	·3148
96	·1689	·2006	·2371	·2617	·3116
98	·1671	·1986	·2347	·2591	·3085
100	0·1654	0·1966	0·2324	0·2565	0·3054
105	·1614	·1918	·2268	·2504	·2983
110	·1576	·1874	·2216	·2446	·2915
115	·1541	·1832	·2167	·2393	·2853
120	·1509	·1793	·2122	·2343	·2794
125	0·1478	0·1757	0·2079	0·2296	0·2738
130	·1449	·1723	·2039	·2252	·2686

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TABLE 14. PERCENTAGE POINTS OF SPEARMAN'S S
TABLE 15. PERCENTAGE POINTS OF KENDALL'S K

Spearman's S and Kendall's K are both used to measure the degree of association between two rankings of n objects. Let d_i $(1 \leqslant i \leqslant n)$ be the difference in the ranks of the ith object; Spearman's S is defined as $\sum_{i=1}^{n} d_i^2$. To define Kendall's K, re-order the pairs of ranks so that the first set is in natural order from left to right, and let m_i $(1 \leqslant i \leqslant n)$ be the number of ranks greater than i in the second ranking which are to the right of rank i. Kendall's K is defined as $\sum_{i=1}^{n} m_i$.

For Table 14 the tabulated value $x(P)$ is the lower percentage point, i.e. the largest value x such that, in independent rankings, $\Pr(S \leqslant x) \leqslant P/100$; in Table 15, K replaces S and the upper percentage point is given. A dash indicates that there is no value with the required property. The distributions are symmetric about means $\frac{1}{6}(n^3-n)$ for S and $\frac{1}{4}n(n-1)$ for K, with maxima equal to twice the means; hence the upper percentage points of S are $\frac{1}{3}(n^3-n)-x(P)$ and the lower percentage points of K are $\frac{1}{2}n(n-1)-x(P)$. The variances are $\frac{1}{36}n^2(n+1)^2(n-1)$ for S and $\frac{1}{72}n(n-1)(2n+5)$ for K, and when $n > 40$ both statistics are approximately normally distributed; more accurately, the distribution function of $X = [S-\frac{1}{6}(n^3-n)]/[\frac{1}{6}n(n+1)\sqrt{n-1}]$ is approximately equal to $\Phi(x) - \frac{\gamma}{24\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(x^3-3x)$, where $\gamma = \frac{-0{\cdot}04(19n^2+5n-36)}{\frac{1}{6}(n^3-n)}$ and $\Phi(x)$ is the normal distribution function (see Table 4).

A test of the null hypothesis of independent rankings is provided by rejecting at the P per cent level if $S \leqslant x(P)$, or $K \geqslant x(P)$, when the alternative is contrary rankings. The other points are similarly used when the alternative is similar rankings. To cover both alternatives reject at the $2P$ per cent level if S, or K, lies in either tail. Spearman's rank correlation coefficient r_S is defined as $1-6S/(n^3-n)$, and has upper and lower P per cent points $1-6x(P)/(n^3-n)$ and $-[1-6x(P)/(n^3-n)]$ respectively. Kendall's rank correlation coefficient r_K is defined as $4K/[n(n-1)]-1$, and has upper and lower P per cent points $4x(P)/[n(n-1)]-1$ and $-\{4x(P)/[n(n-1)]-1\}$ respectively.

SPEARMAN'S S

P	5	2·5	1	0·5	0·1	$\frac{1}{6}(n^3-n)$
$n = 4$	0	—	—	—	—	**10**
5	2	0	0	—	—	**20**
6	6	4	2	0	—	**35**
7	16	12	6	4	0	**56**
8	30	22	14	10	4	**84**
9	48	36	26	20	10	**120**
10	72	58	42	34	20	**165**
11	102	84	64	54	34	**220**
12	142	118	92	78	52	**286**
13	188	160	128	108	76	**364**
14	244	210	170	146	104	**455**
15	310	268	222	194	140	**560**
16	388	338	284	248	184	**680**
17	478	418	354	312	236	**816**
18	580	512	436	388	298	**969**
19	694	616	530	474	370	**1140**
20	824	736	636	572	452	**1330**
21	970	868	756	684	544	**1540**
22	1132	1018	890	808	650	**1771**
23	1310	1182	1040	948	768	**2024**
24	1508	1364	1206	1102	900	**2300**
25	1724	1566	1388	1272	1048	**2600**
26	1958	1784	1588	1460	1210	**2925**
27	2214	2022	1806	1664	1388	**3276**
28	2492	2282	2044	1888	1584	**3654**
29	2794	2564	2304	2132	1796	**4060**
30	3118	2866	2584	2396	2028	**4495**
31	3466	3194	2884	2682	2280	**4960**
32	3840	3544	3210	2988	2552	**5456**
33	4240	3920	3558	3318	2844	**5984**
34	4666	4322	3930	3672	3160	**6545**
35	5120	4750	4330	4050	3498	**7140**
36	5604	5206	4754	4454	3858	**7770**
37	6118	5692	5206	4884	4244	**8436**
38	6662	6206	5686	5342	4656	**9139**
39	7238	6750	6196	5826	5092	**9880**
40	7846	7326	6736	6342	5556	**10660**

KENDALL'S K

P	5	2·5	1	0·5	0·1	$\frac{1}{4}n(n-1)$
$n = 4$	6	—	—	—	—	**3**
5	9	10	10	—	—	**5**
6	13	14	14	15	—	**7·5**
7	17	18	19	20	21	**10·5**
8	22	23	24	25	26	**14**
9	27	28	30	31	33	**18**
10	33	34	36	37	40	**22·5**
11	39	41	43	44	47	**27·5**
12	46	48	51	52	55	**33**
13	53	56	59	61	64	**39**
14	62	64	67	69	73	**45·5**
15	70	73	77	79	83	**52·5**
16	79	83	86	89	94	**60**
17	89	93	97	100	105	**68**
18	99	103	108	111	117	**76·5**
19	110	114	119	123	129	**85·5**
20	121	126	131	135	142	**95**
21	133	138	144	148	156	**105**
22	146	151	157	161	170	**115·5**
23	159	164	171	176	184	**126·5**
24	172	178	185	190	200	**138**
25	186	193	200	205	216	**150**
26	201	208	216	221	232	**162·5**
27	216	223	232	238	249	**175·5**
28	232	239	248	254	267	**189**
29	248	256	266	272	285	**203**
30	265	273	283	290	303	**217·5**
31	282	291	301	308	323	**232·5**
32	300	309	320	328	342	**248**
33	318	328	340	347	363	**264**
34	337	347	359	368	384	**280·5**
35	356	367	380	388	405	**297·5**
36	376	388	401	410	428	**315**
37	397	409	422	432	450	**333**
38	418	430	444	454	473	**351·5**
39	440	452	467	477	497	**370·5**
40	462	475	490	501	522	**390**

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