1 Single Sample Inference Procedures

Question 1

A machine is set to produce laptop sleeves with the following dimensions:
length $\times$ width $\times$ depth $= 40$ cm $\times$ 30 cm $\times$ 2 cm. A sample of 40 sleeves was selected and each was measured. The results were as follows:

	length	width	depth
$\bar{x}$	40.11	30.09	1.91
s	0.51	0.17	0.15

Test the following hypotheses (use the 5% level of significance in each case):

(a) The mean length is equal to 40cm. **(b)** The mean width is equal to 30cm.
(c) The mean depth is equal to 2cm. **(d)** Both the width and the depth of the sleeve need to be addressed here - which do you think is more urgent? **(d)** Calculate the p-values for the tests carried out in parts (a), (b) and (c).

Question 1

A random sample of 18 software engineers was selected and it was found that their average income was $40,000 with a standard deviation of $3,125.

Calculate the following:

(a) An 80% confidence interval for μ. **(b)** A 95% confidence interval for μ. **(c)** A 99% confidence interval for μ.

Question 1

A market researcher wishes to know the market share for Android devices. From a sample of 500 individuals, it was found that 359 use an Android device.

(a) What type of data has been collected here? **(b)** What is the parameter and its value?
(c) What is the statistic and its value? **(d)** Calculate a 95% confidence interval and interpret this interval. **(e)** How large a sample is required to reduce the *margin of error* in the previous confidence interval to ± 0.02?

Question 2

Google want to estimate the average amount (in dollars) that an individual spends after clicking on a particular Google Ad. In order to achieve this, 1000 people are randomly selected and the amount they spend is recorded. It is found that the average spend is $42.38 and the standard deviation is $16.80.

(a) What is the data type? **(b)** Write down the values of: p, $\hat{p}$, μ, $\bar{x}$, σ and s. **(c)** Calculate a 99% confidence interval for the average spend. **(d)** What value of n is needed to estimate the true average spend within $\pm$ 0.50 with 99% confidence? (i.e., $z_{0.005} \frac{s}{\sqrt{n}} = 0.5$)

Question 2

A matchbox is supposed to contain 100 matches. We wish to test this hypothesis.

(a) State the null and alternative hypotheses. **(b)** From a sample of 32 matchboxes, it is found that the average is 99.4 and the standard deviation is 2.1. Calculate the test statistic.

Question 3

An aircraft part is designed to last more than 500 hours. However, in the interest of safety, it will first be assumed that the part lasts *less than or equal to* 500 hours (i.e., this is the null hypothesis) unless there is firm evidence suggesting otherwise.

(a) State the null and alternative hypotheses.   (b) What is the critical value if $\alpha = 0.001$ and only 4 units will be run until wearout (due to the expense of wasting aircraft parts). (c) In this sample of size 4, it is found that the average is 566 hours and the variance is 83 hours^2. Calculate the test statistic.   (d) What is the conclusion?

Question 3

A manufacturer of aircraft parts wishes to estimate the operating life of a particular component. Thus, a sample of 45 components are used until failure. It is found that the average life is 671.23 hours and the variance is 400 hours-squared.

(a) What type of data was collected?

(b) What is the parameter and its value?

(c) What is the statistic and its value?

(d) Calculate / interpret the 99.9% confidence interval.

Question 4

The government are investigating the difference in proportions of people in rural and urban areas in support of a new policy. Researchers collected data on 154 individuals; of these, 38 lived in rural areas and 116 lived in urban areas. It was found that 52.63% of those in rural areas and 60.34% of those in urban areas support the policy.

(a) What is the true difference in proportions?   (b) Calculate a 90% confidence for this difference and comment on this interval.   (c) In the sample of 154 individuals, how many of them support the policy? (i.e., 52.63% of 38 plus 60.34% of 116)   (d) Based on the answer to part (c), estimate the overall proportion of individuals in support of the policy and construct a 90% confidence interval for the true proportion.

Question 4

A friend claims that he can pass a particular game in 4 hours or less (on average). We wish to test this hypothesis at the 10% level of significance. Your friend plays the game on 6 different occasions: his average completion time is 4.6 hours and the standard deviation is 0.5 hours.

(a) State the null and alternative hypotheses.   (b) What is the critical value?   (c) Calculate the test statistic and provide your conclusion.   (d) Between what two values does the p-value lie? (note: the p-value cannot be calculated exactly using the t-tables)

Question 5

A die is rolled 80 times and we count 18 sixes. We wish to test the hypothesis that the die is fair (note: if this is the case, the proportion of sixes is $p = \frac{1}{6}$).

(a) State the null and alternative hypotheses.   (b) If we wish to test at the 5% level of significance, what is the critical value?   (c) Calculate the test statistic and provide your

Question 5

Consider the following two types of heat sink used for the purposes of CPU cooling:

	Type 1	Type 2
number tested	50	50
mean CPU temperature	40.1	34.8
standard deviation	2.5	1.1

(a) Identify the parameter of interest. **(b)** Calculate a 95% confidence interval for the parameter and comment. **(c)** How large a sample is required to reduce the *margin of error* in the previous confidence interval to ± 0.4? (note: assume that $n_1 = n_2$)

Question 6

A group of 9 computer science students were randomly selected and asked how many hours they spent gaming last week. The average time was found to be 6.4 hours and the standard deviation was 2.2 hours.

(a) Calculate a 95% confidence interval for μ.

(b) Calculate a 99% confidence interval for μ.

Question 6

Assume that a particular brand dominates the market. More specifically, it is well-known that at least 60% of people use this brand (i.e., $p \geq 0.6$). However, in response to recent media claims that this brand is weakening, the company wish to test the hypothesis that $p \geq 0.6$.

(a) State the null and alternative hypotheses. **(b)** From a sample of 1000 people, it is found that 629 use this brand; calculate the test statistic and, hence, the p-value. **(c)** Based on the evidence, state your conclusion.

Question 7

Last year 30% of applicants to a graduate programme failed the aptitude test. This year 100 graduates applied - 25% of these failed the test.

(a) We wish to test the hypothesis that the quality of applicants has not changed since last year - what are the null and alternative hypotheses? **(b)** If we are testing at the 1% level, what is the rejection region? **(c)** Based on the data, calculate the test statistic and provide your conclusion.

Question 7

Guinness set their bottle-filling machine to put 33cl into each bottle. A sample of 5 bottles were selected at random and measured. The volumes in cl were as follows:

$$34.1 \quad 33.5 \quad 32.8 \quad 33.1 \quad 32.5$$

(a) Calculate the sample mean and standard deviation. **(b)** Calculate a 95% confidence interval. **(c)** Based on the confidence interval, does it appear that the machine is working correctly?

Question 8

A soft drinks company is working on a new recipe for its best-selling drink. The company intends to carry out a study where participants will taste both flavours (current and new) and then answer the question:

"Do you prefer the new flavour?"

It is assumed that the *current* recipe is superior, i.e., that *less than or equal to* 50% of people prefer the new drink $(p \leq 0.5)$.

We wish to test the hypothesis that $p \leq 0.5$.

(a) State the null and alternative hypotheses. **(b)** From a sample of 65 people, we find that 43 people prefer the new recipe. Calculate the test statistic and, hence, the p-value.
(c) Based on the evidence, state your conclusion.