0.0.1 Joint Probability Tables

Find $E[X|Y = 2]$

	X=0	X=1	X=2	
Y=1	0.15	0.2	0.25	P(Y=1) = 0.6
Y=2	0.05	0.15	0.20	P(Y=2) = 0.4
	P(X=0) = 0.2	P(X=1) = 0.35	P(X=2)=0.45	

Solution

$$\frac{(0 \times 0.05) + (1 \times 0.15) + (2 \times 0.2)}{0.4} = \frac{0.55}{0.4}$$

$E[X|Y = 2] = 1.375$

0.0.2 Combined Probability : Worked Example

- Suppose an electronics assembly subcontractor recieves resistors from two suppliers A and B
- Supplier A supplies 80% of the resistors
- P(A) = 0.80 probability that a randomly chosen resistor comes from A
- Supplier B supplies 20% of the resistors
- P(B) = 0.20 probability that a randomly chosen resistor comes from B
- 1% of the resistors supplied by A are faulty (i.e. resistor fails the final test)
- 3% of the resistors supplied by B are faulty

Question:
What is the probability that a randomly selected resistor fails the final test?
Compute P(F)

$$P(F) = P(F \text{ and } A) + P(F \text{ and } B)$$

0.0.3 Question 1

A doctor treating a patient issues a prescription for antibiotics and provides for two repeat prescriptions. The probability that the infection will be cleared by the first prescription is p1 =0.6. The probability that successive treatments are successful, given that previous prescriptions were not successful are p2 = 0.5, p3 = 0.4. Calculate the probability that

1. the patient is still infected after the third prescription
2. the patient is cured by the second prescription.
3. the patient is cured by the second prescription, given that the patient is eventually cured.

0.0.4 Question 2

A driver passes through 3 traffic lights. The chance he/she will stop at the first is 1/2 , at the second 1/3 and at the third independently of what happens at any of the other lights.

What is the probability that 4. the driver makes the whole journey without being stopped at any of the lights 5. the driver is only stopped at the first and third lights 6. the driver is stopped at just one set of lights. 7. the driver stopped at the second set of lights, given he/she stopped at one set of lights.

0.0.5 Question 1 : Probability Distribution

Introduction

Consider playing a game in which you are winning when a ***fair die*** is showing 'six' and losing otherwise.

0.0.6 Part 1

If you play three such games in a row, find the probability mass function (pmf) of the number X of times you have won.

- Firstly: what type of probability distribution is this?
- Is this the distribution ***discrete*** or ***continuous***?
- The outcomes are whole numbers - so the answer is discrete.
- So which type of discrete distribution? (We have two to choose from. See first page of formulae)
- **Binomial:** characterizing the number of ***successes*** in a series of n ***independent trials***, with the ***probability of a success*** in each trial being p.
- **Poisson:** characterizing the ***number of occurrences*** in a ***unit space*** (i.e. a unit length, unit area or unit volume, or a unit period in time), where λ is the the number of occurrences per unit space.

0.0.7 Standardisation Formula

$$Z = \frac{X - \mu}{\sigma} \tag{1}$$

Question 3

The masses of 30 human males and 30 arabian stallions were observed. Their masses (in lbs) are given below

Humans 106, 120, 130, 138, 145, 151, 156, 161, 166, 171 176, 180, 185, 189, 194, 198, 203, 208, 212, 217 223, 228, 234, 240, 247, 255, 264, 276, 290, 313

Stallions 808, 824, 835, 843, 851, 857, 862, 868, 872, 877 881, 886, 890, 894, 898, 902, 906, 910, 914, 919 923, 928, 932, 938, 943, 949, 957, 965, 976, 992

a) Draw histograms for these samples and compare them with respect to shape, centrality and relative dispersion. b) Calculate the medians of these samples (from the raw data).

Question 4

The following data give the marks of 10 students in a test (out of 20 marks). Calculate i) the median ii) the mean iii) the range iv) the standard deviation v) The Inter-Quartile Range

12, 17, 7, 11, 18, 6, 14, 15, 11, 9.

The Addition Rule for Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\binom{8}{2} = \frac{8!}{2!(8-2)!} = \frac{8 \times 7 \times 6!}{2! \times 6!} = \frac{8 \times 7}{2 \times 1} = \frac{56}{2} = 28$$

0.0.8 Independent Events

$$P(AandB) = P(A)P(B)$$

$$P(A)P(B) = 0.98 \times 0.95 = 0.931$$

$$P(D) = P(C \text{ and } D) + P(M \text{ and } D) + P(L \text{ and } D)$$

$$P(D) = P(D|C)P(C) + P(D|M)P(M) + P(D|L)P(L)$$

0.0.9 Example

For a particular Java assembler interface, the operand stack size has the following probabilities:

Stack Size	0	1	2	3	4
Probability	.15	.05	.10	.20	.50

- Calculate the expected stack size.
- Calculate the variance of the stack size.

Probability

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Probability

Part 1) A total of 2 or 6

$$E_1 = \{(1,1), (1,5), (5,1), (4,2), (2,4), (3,3)\}$$

$$P(E_1) = \frac{6}{36}$$

Probability

Part 2) A total greater than 9

$$E_2 = \{(4,6),(5,5),(6,4),(6,5),(6,6),(3,3)\}$$

$$P(E_2) = \frac{6}{36}$$

Probability

Part 3) A total which is three times as great as other possible totals.

These totals are 6, 9 and 12.

$$E_3 = \{(1,5),(2,4),(4,2),(1,5),(3,3),(6,3),(4,5)(5,4),(6,6)\}$$

$$P(E_3) = \frac{10}{36}$$

Example

In the above example where the die is thrown repeatedly, lets work out $P(X \leq t)$ for some values of t.

P(X $\leq$ 1) is the probability that the number of throws until we get a 6 is less than or equal to 1. So it is either 0 or 1.

- P(X = 0) = 0
- $P(X = 1) = 1/6$.
- Hence $P(X \leq 1) = 1/6$

Similarly, $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$
= 0 + 1/6 + 5/36 = 11/36