Attempt ALL questions

Q1. Sample Spaces (1 Mark)

Suppose one urn contains three balls; one red, one blue and one green, and a second urn contain three balls; numbered 1, 2, and 3. An experiment consists of two balls being drawn at random (i.e. one from each urn).

a. (1 mark) Write out the sample space for this experiment.

Q2. Probability (4 Marks)

The following contingency table illustrates the number of 200 students in different departments according to gender.

	Physics	Biology	Chemistry	Total
Males	30	20	50	100
Females	20	50	30	100
Total	50	70	80	200

a. (1 mark) What is the probability that a randomly chosen person from the sample is a Chemistry student?

b. (1 mark) What is the probability that a randomly chosen person from the sample is both female and studying Biology?

c. (1 mark) Given that the student is female, what is the probability that she is an Biology student?

d. (1 mark) Given that a student studies Biology, what is the probability that the student is female?

Q3. Discrete Random Variables (3 Marks)

The probability distribute of discrete random variable X is tabulated below. There are 5 possible outcome of X, i.e. 1, 2, 3, 4 and 5.

x_i	1	2	3	4	5
$p(x_i)$	0.30	0.20	k	0.10	0.20

a. (1 Mark) Compute the value of k.

b. (1 Mark) What is the expected value of X?

c. (1 Mark) Given that $E(X^2) = 9.5$, compute the variance of X.

Q4. Binomial Distribution (2 Marks)

A biased coin yields 'Tails' on 48% of throws. Consider an experiment that consists of throwing this coin 11 times.

a. (1 Mark) Evaluate the following term ${}^{11}C_2$.

b. (1 Mark) Compute the probability of getting two 'Tails' in this experiment.

Q5. Poisson Distribution (2 Marks)

Suppose that a telephone help-line receives 4 calls per hour during offices hours.

a. (1 Mark) Compute the value of m for a 30 minute period during office hours.

b. (1 Mark) Compute the probability of the help-line getting exactly one call in a 30 minute period during office hours.

Q6. Normal Distribution (3 Marks)

Suppose X is a normally distributed random variable with mean $\mu = 500$ and $\sigma = 24$

a. (1 Mark) Compute the value of $P(X \geq 518)$

b. (1 Mark) Compute the value of $P(X \leq 482)$

c. (1 Mark) Compute the value of $P(482 \leq X \leq 518)$

Formulae

- Conditional probability:
$$P(B|A) = \frac{P\left(A \text{ and } B\right)}{P\left(A\right)}.$$
- Bayes' Theorem:
$$P(B|A) = \frac{P\left(A|B\right) \times P(B)}{P\left(A\right)}.$$
- Binomial probability distribution:
$$P(X = k) =^{n} C_k \times p^k \times (1-p)^{n-k} \qquad \left(\text{where} \qquad {}^{n}C_k = \frac{n!}{k!\,(n-k)!}.\right)$$
- Poisson probability distribution:
$$P(X = k) = \frac{m^k \mathrm{e}^{-m}}{k!}.$$