UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4413

SEMESTER: Autumn 2008

MODULE TITLE: Computer Mathematics 3

DURATION OF EXAM: 2.5 hours

LECTURER: Dr. Patricia Gunning

GRADING SCHEME:
Examination: 80%

INSTRUCTIONS TO CANDIDATES

Answer any **4** questions.
Calculators may be used. Statistical tables are available from the invigilators. A set of formulae is attached to this paper.

Question 1

(a) On completion of a programming project, four programmers from a team submit a collection of subroutines to an acceptance group. The following table shows the percentage of subroutines each programmer submitted and the probability that a subroutine submitted by each programmer will pass the certification test based on historical data.

	Programmer			
	1	2	3	4
Proportion of subroutines submitted	.10	.20	.40	.30
Probability of acceptance	.55	.60	.95	.75

(i) What is the proportion of subroutines that pass the acceptance test?
(ii) After the acceptance tests are completed, one of the subroutines is selected at random and found to have passed the test. What is the probability that it was written by Programmer 1?

(8 marks)

(b) If two events A and B have the following probabilities
$P(A) = .3$, $P(B) = .6$, $P(A \cap B) = .2$
(i) Are A and B independent? Justify your answer.
(ii) Are A and B mutually exclusive? Justify your answer.
(iii) Calculate $P(A \cup B)$.

(6 marks)

(c) Orders for a computer are summarised by the number of optional features that are requested as follows:

Number of optional features (X)	0	1	2	3	4
Probability P(X)	.35	.25	.20	.10	.10

Calculate the expected number of optional features and the variance.

(6 marks)

(d) The random variables X and Y have the following joint distribution

	Y = 1	Y = 2	Y = 3
X = 0	.20	.30	.20
X = 1	.04	.06	.20

(i) Calculate the marginal distribution of X.
(ii) Calculate the marginal distribution of Y.
(iii) Are X and Y independent? Justify your answer.

(5 marks)

Question 2

(a) State the four conditions to be satisfied for the Binomial probability distribution to apply.

(4 marks)

(b) When can the Poisson distribution be used as an approximation to the Binomial distribution?

(2 marks)

(c) An inspector of computer parts selects a random sample of components from a large batch to decide whether or not to audit the full batch.

(i) If 20% or more of the sample is defective, the entire batch is inspected. Calculate the probability of this happening if it is thought that the population contains 4% defective components and a sample of 20 is selected.

(ii) If 10% or more of the sample is defective, the entire batch is inspected. Calculate the probability of this happening if it is thought that the population contains 4% defective components and a sample of 50 is selected.

(10 marks)

(d) A model of an on-line computer system gives a mean time to retrieve a record from a direct access storage system device of 200 milliseconds with a standard deviation of 58 milliseconds. If it can be assumed that the data are normally distributed:

(i) What proportion of retrieval times will be greater than 75 milliseconds?

(ii) What proportion of retrieval times will be between 150 milliseconds and 250 milliseconds?

(iii) What is the retrieval time below which 10% of retrieval times will be?

(9 marks)

Question 3

(a) A source language has 5 symbols A, B, C, D and E. The associated probabilities of these symbols are given in the table below:

Symbol	Probability
A	.60
B	.30
C	.05
D	.03
E	.02

(i) Calculate the entropy of the source language. **(5 marks)**

(ii) Define a Huffman binary code for the source language. **(10 marks)**

(iii) Calculate the efficiency of the code in (ii) above. **(3 marks)**

(iv) Calculate the redundancy of the code in (ii) above. **(2 marks)**

(b) **Briefly** state what is meant by the Prefix Condition. **(2 marks)**

(c) For each of the following codes state whether they are

(i) non-singular

(ii) uniquely decodable

(iii) instantaneous

	Code 1	Code 2	Code 3
A	1	10	1
B	01	01	00
C	001	10	000

(3 marks)

Question 4

The frequency of 0 as an input to a binary channel is 0.6. If 0 is the input, then 0 is the output with probability 0.8. If 1 is the input, then 1 is the output with probability 0.9.

(i) Calculate the information per bit contained in the input. **(4 marks)**

(ii) Calculate the probability that the output is 0. **(2 marks)**

(iii) Calculate the probability that the output is 1. **(2 marks)**

(iv) Calculate the probability that the input is 0 given that the output is 0. **(2 marks)**

(v) Calculate the probability that the input is 1given that the output is 1. **(2 marks)**

(vi) Calculate the probability that the input is 1 given that the output is 0. **(2 marks)**

(vii) Calculate the probability that the input is 0 given that the output is 1. **(2 marks)**

(viii) Calculate the amount of information transmitted by the channel. **(6 marks)**

(ix) Derive the globally optimal reconstruction rule. **(3 marks)**

Question 5

(a) A manufacturer of a common cold cure claims that the product provides relief for 70% of people who use it. In a test of 400 people, it was found that 300 people said the treatment provided relief.

(i) Calculate a 95% confidence interval for the true proportion of people who would get relief from the product.

(4 marks)

(ii) Suppose the manufacturer wishes to be 95% confident that the prediction is correct to within 2% of the true proportion. What sample size is needed?

(4 marks)

(iii) Using a significance level of 5%, test the hypothesis that more than 70% of people who use the product find relief. Clearly state your null and alternative hypotheses and your conclusion.

(7 marks)

(b) ABC Software has 125 programmers divided into two groups with 75 in Group A and 50 in Group B. In order to compare the efficiencies of the two groups, the programmers are observed for one day. The 75 programmers of Group A averaged 76.21 lines of code with a standard deviation of 10.37. The 50 programmers of Group B averaged 72.72 lines of code with a standard deviation of 10.07. Using a significance level of 5%, test the hypothesis that there is no difference between the two groups versus the alternative that there is a difference. Clearly state your null and alternative hypotheses and your conclusion.

(10 marks)

Formulae

1. Conditional probability: $P(B|A) = \dfrac{P(A \text{ and } B)}{P(A)}$

2. Binomial probability function: $P(X = x) = \binom{n}{x} p^x q^{n-x}$ where $\binom{n}{x} = \dfrac{n!}{x!(n-x)!}$

3. Poisson probability function $P(X = x) = e^{-\lambda} \dfrac{\lambda^x}{x!}$, $x = 0, 1, \ldots\ldots, \infty$

4. Hypergeometric probability function $P(X = x) = \dfrac{\binom{M}{x}\binom{L}{n-x}}{\binom{N}{n}}$

   where M are the number of successes and L the number of failures

5. Geometric probability function $P(X = x) = q^{x-1}\,p$ and $P(X \le x) = 1 - q^x$

6. Uniform distribution p.d.f $f(x) = \begin{cases} 0 & \text{elsewhere} \\ \dfrac{1}{b-a} & a \le x \le b \end{cases}$

7. Standard Error of $\bar{x} = \dfrac{s}{\sqrt{n}}$

8. Standard Error of $\bar{x}_1 - \bar{x}_2 = \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$ (two means – large samples)

9. Standard Error of $\bar{d}$ (differences) $= \dfrac{s_d}{\sqrt{n}}$

10. Standard Error of $\hat{p}$ (proportion) $= \sqrt{\hat{p}(1-\hat{p})/n}$ (confidence interval)

    or $\sqrt{p_0(1-p_0)/n}$ (hypothesis testing)

11. Standard Error of $\hat{p}_1 - \hat{p}_2 = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ (confidence interval)

(two proportions)

or

$\sqrt{\bar{p}(1-\bar{p})\left[\frac{1}{n1} + \frac{1}{n2}\right]}$ (hypothesis testing)

12. $I(p) = -\log_2(p) = \log_2(1/p)$

13. $I(pq) = I(p) + I(q)$

14. $H = -\sum_{i=1}^{M} p_i \log_2(p_i)$

15. $E(L) = \sum_{i=1}^{M} l_i\, p_i$

16. Efficiency = H / E(L)

17. $I(X,Y) = H(X) - H(X|Y)$

18. $P(C[r]) = \sum_{j=1}^{m} P(C[r]|Y=d_j)P(Y=d_j)$