Statistics for Computing MA4413

Midterm Examination 2

Type A

- Do not turn over the page until instructed to do so.
- Rough work pages are provided within.
- Useful formulae and statistical tables are provided at the back.
- **Enter your answers (using an "X") in the table on the last page.**
- There are 15 questions in total: each correct answer = 1% with **no negative marks**.
- For each question, only *one* answer is correct.
- Scientific calculators approved by the University of Limerick can be used.

Questions 1 - 5

Q1 Identify the statement that is *true* (note: only one is true).

(a) The standard error tends to increase as the sample size increases.

(b) The population mean varies from sample to sample.

(c) A 99% confidence interval is wider than a 95% confidence interval.

(d) We always know the value of the parameter.

On average, flaws arise on a fibre optic cable at a rate of 0.5 per km (kilometer) according to a Poisson distribution.

Q2 What is the probability that there are *less than* 3 flaws in 8 km of this cable?

(a) 0.1465 **(b)** 0.2198 **(c)** 0.4335 **(d)** 0.2381

Q3 What is the standard deviation for the number of flaws in 100 km of cable?

(a) 0.141 **(b)** 0.020 **(c)** 7.071 **(d)** 50

Consider an $M/M/1$ system where customers arrive at a rate of 1.5/min and the average time spent in the *service node* is 15 seconds.

Q4 What is the utilisation factor?

(a) 4.00 **(b)** 0.250 **(c)** 0.375 **(d)** 2.667

Q5 On average, how many customers are in the *whole system*?

(a) 0.4 **(b)** 4.0 **(c)** 0.6 **(d)** 1.6

Rough Work

Next page: Questions 6 - 10

Questions 6 - 10

Assume that the time taken to complete this exam has an exponential distribution with parameter $\lambda = 0.025$, i.e., $T \sim \text{Exponential}(\lambda = 0.025)$.

Q6 What is the value of the probability $\Pr(30 < T < 50)$?

(a) 0.1353 **(b)** 0.6667 **(c)** 0.5 **(d)** 0.1859

Q7 A sample of 64 students are selected; let $\overline{T}$ be the *sample mean*, i.e., the mean calculated for this sample of students. What is the value of the probability $\Pr(\overline{T} > 48)$?

(a) 0.0548 **(b)** 0.3012 **(c)** 0.1446 **(d)** 0.0047

Patients with a particular disease can be treated using treatment A or B. It was of interest to estimate the *difference* in proportions of patients cured of the disease under the two treatment regimes, i.e., $p_A - p_B$. Hence, a sample of data was collected and the following 99% confidence interval for $p_A - p_B$ was calculated: $[0.12,\ 0.26]$.

Q8 What can we say based on the above confidence interval?
(note: a bigger cured proportion is better)

(a) Without any doubt, treatment A is better.

(b) There is no evidence of a difference in treatments.

(c) The evidence suggests that treatment A is better.

(d) The evidence suggests that treatment B is better.

A manufacturer wants to compare two designs of CPU in terms of their typical operating temperature (degrees Celsius). Two large samples are selected and the results are as follows:

	Design 1	Design 2
sample size	40	50
mean	38.7 °C	32.1 °C
variance	3.0 °C^2	8.0 °C^2

Q9 What type of data was collected on each CPU?

(a) numeric continuous **(b)** mean data **(c)** numeric discrete **(d)** categorical

Q10 The 88% confidence interval for $\mu_1 - \mu_2$ is:

(a) $[5.65,\ 7.55]$ **(b)** $[5.85,\ 7.35]$ **(c)** $[4.70,\ 8.50]$ **(d)** $[6.03,\ 7.17]$

Rough Work

Next page: Questions 11 - 15

Questions 11 - 15

The voting population of Ireland is approximately 3,000,000. It was of interest to gain insight into the voting intentions of the public prior to an upcoming referendum - more specifically, the *proportion* of voters who intended to vote "Yes" was of interest. Therefore, 985 individuals were contacted of which 670 said they would vote "Yes".

Q11 What is the sample size?

(a) 985 **(b)** unknown **(c)** $3,000,000$ **(d)** 670

Q12 What is the value of the statistic?

(a) $\hat{p} = 670$ **(b)** $\bar{x} = 0.68$ **(c)** $\bar{x} = 670$ **(d)** $\hat{p} = 0.68$

Let $X \sim \text{Normal}(\mu = 15, \sigma = 2)$.
(Note: answers below are given to two decimal places)

Q13 What is the value of $\Pr(X > 18.66)$?

(a) 0.83 **(b)** 0.18 **(c)** 0.03 **(d)** 0.97

Q14 What is the value of x such that $\Pr(X > x) = 0.8$?

(a) 18.37 **(b)** 13.32 **(c)** 16.68 **(d)** 11.63

Consider the following sample of ages of computers in an office block:

4.9	5.3	7.8

Q15 The 95% confidence interval for the mean is:

(a) $[3.350,\ 8.650]$ **(b)** $[2.096,\ 9.904]$ **(c)** $[-0.136,\ 12.136]$ **(d)** $[4.222,\ 7.778]$

Rough Work

Don't forget to enter your answers on the last page!

Useful Formulae: Page 1

Numerical Summaries:

- $\bar{x} = \dfrac{\sum x_i}{n}$

- $s^2 = \dfrac{\sum x_i^2 - n\,\bar{x}^2}{n-1}$

Probability:

- $\Pr(A^c) = 1 - \Pr(A)$

- $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

- $\Pr(E_1 \cup E_2 \cup \cdots \cup E_k) = \Pr(E_1) + \Pr(E_2) + \cdots + \Pr(E_k)$ (if mutually exclusive)

- $\Pr(A \cap B) = \Pr(A)\ \Pr(B \mid A) = \Pr(B)\ \Pr(A \mid B)$

- $\Pr(E_1 \cap E_2 \cap \cdots \cap E_k) = \Pr(E_1)\ \Pr(E_2)\ \cdots\ \Pr(E_k)$ (if independent)

- $\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)} = \dfrac{\Pr(A)\ \Pr(B \mid A)}{\Pr(B)}$

- If $E_1, \ldots, E_k$ are mutually exclusive & exhaustive

$$\begin{aligned} \Rightarrow \Pr(B) &= \Pr(B \cap E_1) + \Pr(B \cap E_2) + \cdots + \Pr(B \cap E_k) \\ &= \Pr(E_1)\ \Pr(B \mid E_1) + \Pr(E_2)\ \Pr(B \mid E_2) + \cdots + \Pr(E_k)\ \Pr(B \mid E_k) \end{aligned}$$

Distributions:

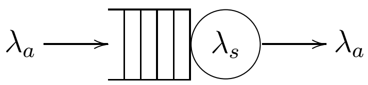
• $X \sim \text{Poisson}(\lambda)$	• $T \sim \text{Exponential}(\lambda)$	• $X \sim \text{Normal}(\mu, \sigma)$
• $\Pr(X = x) = \dfrac{\lambda^x}{x\,!}\, e^{-\lambda}$	• $\Pr(T > t) = e^{-\lambda t}$	• $\Pr(X > x) = \Pr\left(Z > \dfrac{x - \mu}{\sigma}\right)$
• $x \in \{0, 1, 2, \ldots, \infty\}$	• $t \in [0,\ \infty)$	• $x \in (-\infty,\ \infty)$
• $E(X) = \lambda$	• $E(T) = \dfrac{1}{\lambda}$	• $E(X) = \mu$
• $Var(X) = \lambda$	• $Var(T) = \dfrac{1}{\lambda^2}$	• $Var(X) = \sigma^2$

Useful Formulae: Page 2

Queueing Theory:

- $E(N) = \lambda_a \, E(T)$

- $\rho = \dfrac{\lambda_a}{\lambda_s}$

- $M/M/1$ System: 

$$\Rightarrow \; T \sim \text{Exponential}(\lambda_s - \lambda_a)$$

(where T is the total time in the system)

Normal Distribution:

- $\Pr(Z < -z) = \Pr(Z > z)$

- $\Pr(Z > -z) = \Pr(Z < z) = 1 - \Pr(Z > z)$

- $\Pr(X > x) = \Pr\left(Z > \dfrac{x - \mu}{\sigma}\right)$

- $(1 - \alpha)100\%$ of the Normal(μ, σ) distribution lies in $\mu \pm z_{\alpha/2}\, \sigma$

- If $X_1 \sim \text{Normal}(\mu_1, \sigma_1)$ and $X_2 \sim \text{Normal}(\mu_2, \sigma_2)$

$$\Rightarrow \quad \text{Sum:} \quad X_1 + X_2 \sim \text{Normal}\left(\mu_1 + \mu_2, \; \sqrt{\sigma_1^2 + \sigma_2^2}\right)$$

$$\Rightarrow \quad \text{Difference:} \quad X_1 - X_2 \sim \text{Normal}\left(\mu_1 - \mu_2, \; \sqrt{\sigma_1^2 + \sigma_2^2}\right)$$

- For $X_1, \ldots, X_n \sim$ any distribution with $\mu = E(X)$ and $\sigma = Sd(X) = \sqrt{Var(X)}$

$$\Rightarrow \quad \text{Sample mean:} \quad \overline{X} \sim \text{Normal}\left(\mu, \; \frac{\sigma}{\sqrt{n}}\right) \quad \text{if } n > 30$$

Useful Formulae: Page 3

Confidence Intervals:

- Large sample: statistic $\pm\ z_{\alpha/2} \times$ standard error

- Small sample: statistic $\pm\ t_{\nu,\alpha/2} \times$ standard error

Parameter	Statistic	Standard Error	Samples	D. of. F.
μ	$\bar{x}$	$\dfrac{s}{\sqrt{n}}$	large / small	$\nu = n - 1$
p	$\hat{p}$	$\sqrt{\dfrac{\hat{p}\,(1-\hat{p})}{n}}$	large	n/a
$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$	$\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$	large / small	$\nu = \dfrac{(a+b)^2}{\frac{a^2}{n_1-1} + \frac{b^2}{n_2-1}}$ $a = \dfrac{s_1^2}{n_1},\ \ b = \dfrac{s_2^2}{n_2}$
		$\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}$ where $s_p^2 = \dfrac{(n_1-1)\,s_1^2 + (n_2-1)\,s_2^2}{n_1+n_2-2}$	small	$\nu = n_1 + n_2 - 2$ assuming $\sigma_1^2 = \sigma_2^2$
$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$	$\sqrt{\dfrac{\hat{p}_1\,(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2\,(1-\hat{p}_2)}{n_2}}$	large	n/a

- $F = \dfrac{\text{larger variance}}{\text{smaller variance}} = \dfrac{s^2_{\text{larger}}}{s^2_{\text{smaller}}}$

  $\nu_1 =$ top sample size $- 1$

  $\nu_2 =$ bottom sample size $- 1$

Table 2 Cumulative Poisson Probabilities

The table gives the probability that r *or more* random events are contained in an interval when the average number of such events per interval is m, i.e.

$$\sum_{x=r}^{\infty} e^{-m} \frac{m^x}{x!}$$

Where there is no entry for a particular pair of values of r and m, this indicates that the appropriate probability is less than 0.000 05. Similarly, except for the case $r = 0$ when the entry is exact, a tabulated value of 1.0000 represents a probability greater than 0.999 95.

$m =$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.0952	.1813	.2592	.3297	.3935	.4512	.5034	.5507	.5934	.6321
2	.0047	.0175	.0369	.0616	.0902	.1219	.1558	.1912	.2275	.2642
3	.0002	.0011	.0036	.0079	.0144	.0231	.0341	.0474	.0629	.0803
4		.0001	.0003	.0008	.0018	.0034	.0058	.0091	.0135	.0190
5				.0001	.0002	.0004	.0008	.0014	.0023	.0037
6							.0001	.0002	.0003	.0006
7										.0001

$m =$	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.6671	.6988	.7275	.7534	.7769	.7981	.8173	.8347	.8504	.8647
2	.3010	.3374	.3732	.4082	.4422	.4751	.5068	.5372	.5663	.5940
3	.0996	.1205	.1429	.1665	.1912	.2166	.2428	.2694	.2963	.3233
4	.0257	.0338	.0431	.0537	.0656	.0788	.0932	.1087	.1253	.1429
5	.0054	.0077	.0107	.0143	.0186	.0237	.0296	.0364	.0441	.0527
6	.0010	.0015	.0022	.0032	.0045	.0060	.0080	.0104	0132	.0166
7	.0001	.0003	.0004	.0006	.0009	.0013	.0019	.0026	.0034	.0045
8			.0001	.0001	.0002	.0003	.0004	.0006	.0008	.0011
9							.0001	.0001	.0002	.0002

$m =$	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.8775	.8892	.8997	.9093	.9179	.9257	.9328	.9392	.9450	.9502
2	.6204	.6454	.6691	.6916	.7127	.7326	.7513	.7689	.7854	.8009
3	.3504	.3773	.4040	.4303	.4562	.4816	.5064	.5305	.5540	.5768
4	.1614	.1806	.2007	.2213	.2424	.2640	.2859	.3081	.3304	.3528
5	.0621	.0725	.0838	.0959	.1088	.1226	.1371	.1523	.1682	.1847
6	.0204	.0249	.0300	.0357	.0420	.0490	.0567	.0651	.0742	.0839
7	.0059	.0075	.0094	.0116	.0142	.0172	.0206	.0244	.0287	.0335
8	.0015	.0020	.0026	.0033	.0042	.0053	.0066	.0081	.0099	.0119
9	.0003	.0005	.0006	.0009	.0011	.0015	.0019	.0024	.0031	.0038
10	.0001	.0001	.0001	.0002	.0003	.0004	.0005	.0007	.0009	.0011
11					.0001	.0001	.0001	.0002	.0002	.0003
12									.0001	.0001

Table 2 Cumulative Poisson Probabilities – continued

$m =$	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9550	.9592	.9631	.9666	.9698	.9727	.9753	.9776	.9798	.9817
2	.8153	.8288	.8414	.8532	.8641	.8743	.8838	.8926	.9008	.9084
3	.5988	.6201	.6406	.6603	.6792	.6973	.7146	.7311	.7469	.7619
4	.3752	.3975	.4197	.4416	.4634	.4848	.5058	.5265	.5468	.5665
5	.2018	.2194	.2374	.2558	.2746	.2936	.3128	.3322	.3516	.3712
6	.0943	.1054	.1171	.1295	.1424	.1559	.1699	.1844	.1994	.2149
7	.0388	.0446	.0510	.0579	.0653	.0733	.0818	.0909	.1005	.1107
8	.0142	.0168	.0198	.0231	.0267	.0308	.0352	.0401	.0454	.0511
9	.0047	.0057	.0069	.0083	.0099	.0117	.0137	.0160	.0185	.0214
10	.0014	.0018	.0022	.0027	.0033	.0040	.0048	.0058	.0069	.0081
11	.0004	.0005	.0006	.0008	.0010	.0013	.0016	.0019	.0023	.0028
12	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0006	.0007	.0009
13				.0001	.0001	.0001	.0001	.0002	.0002	.0003
14									.0001	.0001

$m =$	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9834	.9850	.9864	.9877	.9889	.9899	.9909	.9918	.9926	.9933
2	.9155	.9220	.9281	.9337	.9389	.9437	.9482	.9523	.9561	.9596
3	.7762	.7898	.8026	.8149	.8264	.8374	.8477	.8575	.8667	.8753
4	.5858	.6046	.6228	.6406	.6577	.6743	.6903	.7058	.7207	.7350
5	.3907	.4102	.4296	.4488	.4679	.4868	.5054	.5237	.5418	.5595
6	.2307	.2469	.2633	.2801	.2971	.3142	.3316	.3490	.3665	.3840
7	.1214	.1325	.1442	.1564	.1689	.1820	.1954	.2092	.2233	.2378
8	.0573	.0639	.0710	.0786	.0866	.0951	.1040	.1133	.1231	.1334
9	.0245	.0279	.0317	.0358	.0403	.0451	.0503	.0558	.0618	.0681
10	.0095	.0111	.0129	.0149	.0171	.0195	.0222	.0251	.0283	.0318
11	.0034	.0041	.0048	.0057	.0067	.0078	.0090	.0104	.0120	.0137
12	.0011	.0014	.0017	.0020	.0024	.0029	.0034	.0040	.0047	.0055
13	.0003	.0004	.0005	.0007	.0008	.0010	.0012	.0014	.0017	.0020
14	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005	.0006	.0007
15				.0001	.0001	.0001	.0001	.0001	.0002	.0002
16									.0001	.0001

$m =$	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0
$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9945	.9955	.9963	.9970	.9975	.9980	.9983	.9986	.9989	.9991
2	.9658	.9711	.9756	.9794	.9826	.9854	.9877	.9897	.9913	.9927
3	.8912	.9052	.9176	.9285	.9380	.9464	.9537	.9600	.9656	.9704
4	.7619	.7867	.8094	.8300	.8488	.8658	.8811	.8948	.9072	.9182
5	.5939	.6267	.6579	.6873	.7149	.7408	.7649	.7873	.8080	.8270
6	.4191	.4539	.4881	.5217	.5543	.5859	.6163	.6453	.6730	.6993
7	.2676	.2983	.3297	.3616	.3937	.4258	.4577	.4892	.5201	.5503
8	.1551	.1783	.2030	.2290	.2560	.2840	.3127	.3419	.3715	.4013
9	.0819	.0974	.1143	.1328	.1528	.1741	.1967	.2204	.2452	.2709
10	.0397	.0488	.0591	.0708	.0839	.0984	.1142	.1314	.1498	.1695
11	.0177	.0225	.0282	.0349	.0426	.0514	.0614	.0726	.0849	.0985
12	.0073	.0096	.0125	.0160	.0201	.0250	.0307	.0373	.0448	.0534
13	.0028	.0038	.0051	.0068	.0088	.0113	.0143	.0179	.0221	.0270
14	.0010	.0014	.0020	.0027	.0036	.0048	.0063	.0080	.0102	.0128
15	.0003	.0005	.0007	.0010	.0014	.0019	.0026	.0034	.0044	.0057
16	.0001	.0002	.0002	.0004	.0005	.0007	.0010	.0014	.0018	.0024
17		.0001	.0001	.0001	.0002	.0003	.0004	.0005	.0007	.0010
18					.0001	.0001	.0001	.0002	.0003	.0004
19								.0001	.0001	.0001

Table 2 Cumulative Poisson Probabilities – continued

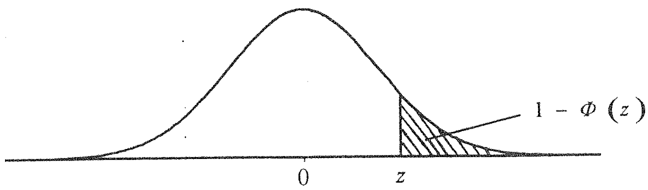
m =	7.2	7.4	7.6	7.8	8.0	8.2	8.4	8.6	8.8	9.0
r = 0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9993	.9994	.9995	.9996	.9997	.9997	.9998	.9998	.9998	.9999
2	.9939	.9949	.9957	.9964	.9970	.9975	.9979	.9982	.9985	.9988
3	.9745	.9781	.9812	.9839	.9862	.9882	.9900	.9914	.9927	.9938
4	.9281	.9368	.9446	.9515	.9576	.9630	.9677	.9719	.9756	.9788
5	.8445	.8605	.8751	.8883	.9004	.9113	.9211	.9299	.9379	.9450
6	.7241	.7474	.7693	.7897	.8088	.8264	.8427	.8578	.8716	.8843
7	.5796	.6080	.6354	.6616	.6866	.7104	.7330	.7543	.7744	.7932
8	.4311	.4607	.4900	.5188	.5470	.5746	.6013	.6272	.6522	.6761
9	.2973	.3243	.3518	.3796	.4075	.4353	.4631	.4906	.5177	.5443
10	.1904	.2123	.2351	.2589	.2834	.3085	.3341	.3600	.3863	.4126
11	.1133	.1293	.1465	.1648	.1841	.2045	.2257	.2478	.2706	.2940
12	.0629	.0735	.0852	.0980	.1119	.1269	.1429	.1600	.1780	.1970
13	.0327	.0391	.0464	.0546	.0638	.0739	.0850	.0971	.1102	.1242
14	.0159	.0195	.0238	.0286	.0342	.0405	.0476	.0555	.0642	.0739
15	.0073	.0092	.0114	.0141	.0173	.0209	.0251	.0299	.0353	.0415
16	.0031	.0041	.0052	.0066	.0082	.0102	.0125	.0152	.0184	.0220
17	.0013	.0017	.0022	.0029	.0037	.0047	.0059	.0074	.0091	.0111
18	.0005	.0007	.0009	.0012	.0016	.0021	.0027	.0034	.0043	.0053
19	.0002	.0003	.0004	.0005	.0006	.0009	.0011	.0015	.0019	.0024
20	.0001	.0001	.0001	.0002	.0003	.0003	.0005	.0006	.0008	.0011
21				.0001	.0001	.0001	.0002	.0002	.0003	.0004
22							.0001	.0001	.0001	.0002
23										.0001

m =	9.2	9.4	9.6	9.8	10.0	11.0	12.0	13.0	14.0	15.0
r = 0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.9990	.9991	.9993	.9994	.9995	.9998	.9999	1.0000	1.0000	1.0000
3	.9947	.9955	.9962	.9967	.9972	.9988	.9995	.9998	.9999	1.0000
4	.9816	.9840	.9862	.9880	.9897	.9951	.9977	.9990	.9995	.9998
5	.9514	.9571	.9622	.9667	.9707	.9849	.9924	.9963	.9982	.9991
6	.8959	.9065	.9162	.9250	.9329	.9625	.9797	.9893	.9945	.9972
7	.8108	.8273	.8426	.8567	.8699	.9214	.9542	.9741	.9858	.9924
8	.6990	.7208	.7416	.7612	.7798	.8568	.9105	.9460	.9684	.9820
9	.5704	.5958	.6204	.6442	.6672	.7680	.8450	.9002	.9379	.9626
10	.4389	.4651	.4911	.5168	.5421	.6595	.7576	.8342	.8906	.9301
11	.3180	.3424	.3671	.3920	.4170	.5401	.6528	.7483	.8243	.8815
12	.2168	.2374	.2588	.2807	.3032	.4207	.5384	.6468	.7400	.8152
13	.1393	.1552	.1721	.1899	.2084	.3113	.4240	.5369	.6415	.7324
14	.0844	.0958	.1081	.1214	.1355	.2187	.3185	.4270	.5356	.6368
15	.0483	.0559	.0643	.0735	.0835	.1460	.2280	.3249	.4296	.5343
16	.0262	.0309	.0362	.0421	.0487	.0926	.1556	.2364	.3306	.4319
17	.0135	.0162	.0194	.0230	.0270	.0559	.1013	.1645	.2441	.3359
18	.0066	.0081	.0098	.0119	.0143	.0322	.0630	.1095	.1728	.2511
19	.0031	.0038	.0048	.0059	.0072	.0177	.0374	.0698	.1174	.1805
20	.0014	.0017	.0022	.0028	.0035	.0093	.0213	.0427	.0765	.1248
21	.0006	.0008	.0010	.0012	.0016	.0047	.0116	.0250	.0479	.0830
22	.0002	.0003	.0004	.0005	.0007	.0023	.0061	.0141	.0288	.0531
23	.0001	.0001	.0002	.0002	.0003	.0010	.0030	.0076	.0167	.0327
24			.0001	.0001	.0001	.0005	.0015	.0040	.0093	.0195
25						.0002	.0007	.0020	.0050	.0122
26						.0001	.0003	.0010	.0026	.0062
27							.0001	.0005	.0013	.0033
28							.0001	.0002	.0006	.0017
29								.0001	.0003	.0009
30									.0001	.0004
31									.0001	.0002
32										.0001

Table 3 Areas in Upper Tail of the Normal Distribution

The function tabulated is $1 - \Phi(z)$ where $\Phi(z)$ is the cumulative distribution function of a standardised Normal variable, z.

Thus $1 - \Phi(z) = \frac{1}{\sqrt{2\pi}} \int_z^{\infty} e^{-z^2/2}$ is the probability that a standardised Normal variate selected at random will be greater than a value of $z \left(= \frac{x-\mu}{\sigma} \right)$



$\frac{x-\mu}{\sigma}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
3.7	.000108	.000104	.000100	.000096	.000092	.000088	.000085	.000082	.000078	.000075
3.8	.000072	.000069	.000067	.000064	.000062	.000059	.000057	.000054	.000052	.000050
3.9	.000048	.000046	.000044	.000042	.000041	.000039	.000037	.000036	.000034	.000033
4.0	.000032									

5.0 → 0.000 000 286 7 5.5 → 0.000 000 019 0 6.0 → 0.000 000 001 0

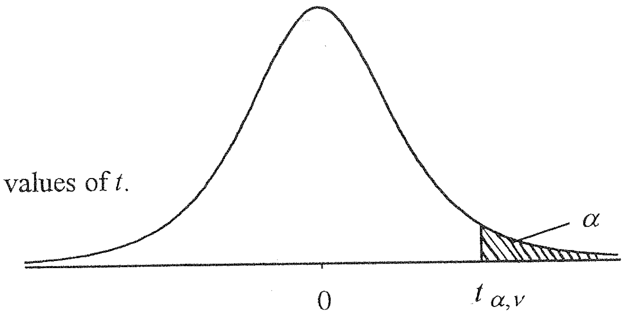
Table 7 Percentage Points of the t Distribution

The table gives the value of $t_{\alpha,\nu}$ – the 100α percentage point of the t distribution for ν degrees of freedom.

The values of t are obtained by solution of the equation:

$$\alpha = \Gamma[\tfrac{1}{2}(\nu+1)][\Gamma(\tfrac{1}{2}\nu)]^{-1}(\nu\pi)^{-1/2}\int_t^{\infty}(1+x^2/\nu)^{-(\nu+1)/2}\,dx$$

Note: The tabulation is for one tail only, that is, for positive values of t.
For $|t|$ the column headings for α should be doubled.



$\alpha =$	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
$\nu = 1$	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

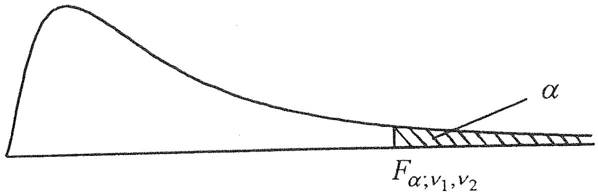
Table 9 Percentage Points of the F Distribution

The table gives the values of $F_{\alpha;\nu_1,\nu_2}$ the 100α percentage point of the F distribution having ν_1 degrees of freedom in the numerator and ν_2 degrees of freedom in the denominator. For each pair of values of ν_1 and ν_2, $F_{\alpha;\nu_1,\nu_2}$ is tabulated for $\alpha = 0.05$, 0.025, 0.01, 0.001, the 0.025 values being bracketed.

The lower percentage points of the distribution may be obtained from the relation:

$F_{1-\alpha;\nu_1,\nu_2} = {}^{1}/F_{\alpha;\nu_2,\nu_1}$

Example: $F_{.95,12,8} = {}^{1}/F_{.05,8,12} = {}^{1}/_{2.85} = \underline{0.351}$



ν_2 \ ν_1	1	2	3	4	5	6	7	8	10	12	24	∞
1	161.4 (648) 4052 4053*	199.5 (800) 5000 5000*	215.7 (864) 5403 5405*	224.6 (900) 5625 5625*	230.2 (922) 5764 5764*	234.0 (937) 5859 5859*	236.8 (948) 5928 5929*	238.9 (957) 5981 5981*	241.9 (969) 6056 6056*	243.9 (977) 6106 6107*	249.0 (997) 6235 6235*	254.3 (1018) 6366 6366*
2	18.5 (38.5) 98.5 998.5	19.0 (39.0) 99.0 999.0	19.2 (39.2) 99.2 999.2	19.2 (39.2) 99.2 999.2	19.3 (39.3) 99.3 999.3	19.3 (39.3) 99.3 999.3	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.5 (39.5) 99.5 999.5	19.5 (39.5) 99.5 999.5
3	10.13 (17.4) 34.1 167.0	9.55 (16.0) 30.8 148.5	9.28 (15.4) 29.5 141.1	9.12 (15.1) 28.7 137.1	9.01 (14.9) 28.2 134.6	8.94 (14.7) 27.9 132.8	8.89 (14.6) 27.7 131.5	8.85 (14.5) 27.5 130.6	8.79 (14.4) 27.2 129.2	8.74 (14.3) 27.1 128.3	8.64 (14.1) 26.6 125.9	8.53 (13.9) 26.1 123.5
4	7.71 (12.22) 21.2 74.14	6.94 (10.65) 18.0 61.25	6.59 (9.98) 16.7 56.18	6.39 (9.60) 16.0 53.44	6.26 (9.36) 15.5 51.71	6.16 (9.20) 15.2 50.53	6.09 (9.07) 15.0 49.66	6.04 (8.98) 14.8 49.00	5.96 (8.84) 14.5 48.05	5.91 (8.75) 14.4 47.41	5.77 (8.51) 13.9 45.77	5.63 (8.26) 13.5 44.05
5	6.61 (10.01) 16.26 47.18	5.79 (8.43) 13.27 37.12	5.41 (7.76) 12.06 33.20	5.19 (7.39) 11.39 31.09	5.05 (7.15) 10.97 29.75	4.95 (6.98) 10.67 28.83	4.88 (6.85) 10.46 28.16	4.82 (6.76) 10.29 27.65	4.74 (6.62) 10.05 26.92	4.68 (6.52) 9.89 26.42	4.53 (6.28) 9.47 25.14	4.36 (6.02) 9.02 23.79
6	5.99 (8.81) 13.74 35.51	5.14 (7.26) 10.92 27.00	4.76 (6.60) 9.78 23.70	4.53 (6.23) 9.15 21.92	4.39 (5.99) 8.75 20.80	4.28 (5.82) 8.47 20.03	4.21 (5.70) 8.26 19.46	4.15 (5.60) 8.10 19.03	4.06 (5.46) 7.87 18.41	4.00 (5.37) 7.72 17.99	3.84 (5.12) 7.31 16.90	3.67 (4.85) 6.88 15.75
7	5.59 (8.07 12.25 29.25	4.74 (6.54) 9.55 21.69	4.35 (5.89) 8.45 18.77	4.12 (5.52) 7.85 17.20	3.97 (5.29) 7.46 16.21	3.87 (5.12) 7.19 15.52	3.79 (4.99) 6.99 15.02	3.73 (4.90) 6.84 14.63	3.64 (4.76) 6.62 14.08	3.57 (4.67) 6.47 13.71	3.41 (4.42) 6.07 12.73	3.23 (4.14) 5.65 11.70
8	5.32 (7.57) 11.26 25.42	4.46 (6.06) 8.65 18.49	4.07 (5.42) 7.59 15.83	3.84 (5.05) 7.01 14.39	3.69 (4.82) 6.63 13.48	3.58 (4.65) 6.37 12.86	3.50 (4.53) 6.18 12.40	3.44 (4.43) 6.03 12.05	3.35 (4.30) 5.81 11.54	3.28 (4.20) 5.67 11.19	3.12 (3.95) 5.28 10.30	2.93 (3.67) 4.86 9.34
9	5.12 (7.21) 10.56 22.86	4.26 (5.71) 8.02 16.39	3.86 (5.08) 6.99 13.90	3.63 (4.72) 6.42 12.56	3.48 (4.48) 6.06 11.71	3.37 (4.32) 5.80 11.13	3.29 (4.20) 5.61 10.69	3.23 (4.10) 5.47 10.37	3.14 (3.96) 5.26 9.87	3.07 (3.87) 5.11 9.57	2.90 (3.61) 4.73 8.72	2.71 (3.33) 4.31 7.81
10	4.96 (6.94) 10.04 21.04	4.10 (5.46) 7.56 14.91	3.71 (4.83) 6.55 12.55	3.48 (4.47) 5.99 11.28	3.33 (4.24) 5.64 10.48	3.22 (4.07) 5.39 9.93	3.14 (3.95) 5.20 9.52	3.07 (3.85) 5.06 9.20	2.98 (3.72) 4.85 8.74	2.91 (3.62) 4.71 8.44	2.74 (3.37) 4.33 7.64	2.54 (3.08) 3.91 6.76
11	4.84 (6.72) 9.65 19.69	3.98 (5.26) 7.21 13.81	3.59 (4.63) 6.22 11.56	3.36 (4.28) 5.67 10.35	3.20 (4.04) 5.32 9.58	3.09 (3.88) 5.07 9.05	3.01 (3.76) 4.89 8.66	2.95 (3.66) 4.74 8.35	2.85 (3.53) 4.54 7.92	2.79 (3.43) 4.40 7.63	2.61 (3.17) 4.02 6.85	2.40 (2.88) 3.60 6.00
12	4.75 (6.55) 9.33 18.64	3.89 (5.10) 6.93 12.97	3.49 (4.47) 5.95 10.80	3.26 (4.12) 5.41 9.63	3.11 (3.89) 5.06 8.89	3.00 (3.73) 4.82 8.38	2.91 (3.61) 4.64 8.00	2.85 (3.51) 4.50 7.71	2.75 (3.37) 4.30 7.29	2.69 (3.28) 4.16 7.00	2.51 (3.02) 3.78 6.25	2.30 (2.72) 3.36 5.42

* Entries marked thus must be multiplied by 100

Answer Sheet

Name: ____

ID Number: ____

Enter your answers with an "X' in the table below.

Do not enter the "X" until you have made your *final decision* to avoid scribbling out.

	A	B	C	D
Q1				
Q2				
Q3				
Q4				
Q5				
Q6				
Q7				
Q8				
Q9				
Q10				
Q11				
Q12				
Q13				
Q14				
Q15				