Question 1

a)
$$H_0: \quad \mu = 2.5.$$
$$H_a: \quad \mu \neq 2.5.$$

b) The significance level is $\alpha = 0.1 \Rightarrow \alpha/2 = 0.05$ in each tail.

Small sample $\Rightarrow t_{\nu,\alpha/2}$ required where the degrees of freedom are $\nu = n - 1 = 4 - 1 = 3$.

Thus, the critical values are $\pm t_{3,\,0.05} = \pm 2.353$.

c)

					$\sum$
x	2.53	2.55	2.54	2.24	9.86
x^2	6.4009	6.5025	6.4516	5.0176	24.3726

$$\bar{x} = \frac{\sum x}{n} = \frac{9.86}{4} = 2.465.$$

$$s^2 = \frac{\sum x^2 - n\,\bar{x}^2}{n-1} = \frac{24.3726 - 4(2.465^2)}{3}$$
$$= 0.02256.$$

$$s = \sqrt{0.02256} = 0.1502.$$

d) Since the critical values are t values, it is appropriate to label our test statistic t rather than z (although it is not essential).

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{2.465 - 2.5}{\frac{0.1502}{\sqrt{4}}}$$
$$= \frac{-0.035}{0.0751}$$
$$= -0.466.$$

e) Since $t = -0.466$ lies within the *acceptance region*, ± 2.353, there is not enough evidence to reject H_0 at the 10% level of significance, i.e., we accept that $\mu = 2.5$.

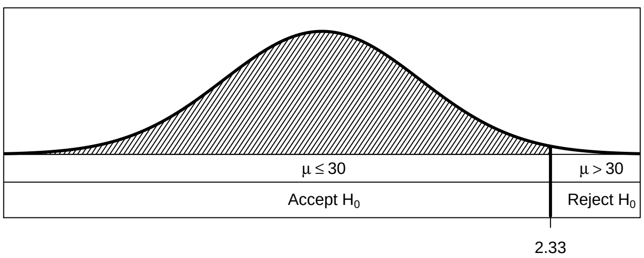
The evidence suggests that the CPUs are performing as expected.

Question 2

a)
$$H_0: \quad \mu \leq 30.$$
$$H_a: \quad \mu > 30.$$

b) The significance level is $\alpha = 0.01$. We do not divide by two since this is a one-tailed test; the rejection region is the *upper tail* ($H_a : \mu > 30$).

Since $n > 30$ the critical value is $z_{0.01} = 2.33$ $\Rightarrow$ the rejection region is the area above 2.33.



c) We have $s^2 = 10 \Rightarrow s = \sqrt{10}$.

$$\Rightarrow z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{32 - 30}{\frac{\sqrt{10}}{\sqrt{40}}}$$
$$= \frac{2}{0.5}$$
$$= 4.$$

d) Since $z = 4$ is greater than 2.33, we reject H_0 at the 1% level of significance, i.e., we accept the alternative hypothesis that $\mu > 30$.

The evidence suggests that the CPUs are running hotter than expected.

Question 3

a)

$$H_0: \quad p \leq 0.5.$$
$$H_a: \quad p > 0.5.$$

b) The significance level is $\alpha = 0.05$. Since this is a one-tailed test (and the alternative is pointing to the right tail) the critical value $z_{0.05} = 1.64$ $\Rightarrow$ the rejection region is the area above 1.64.

c) First calculate $\hat{p} = \frac{38}{65} = 0.5846$.

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0\,(1-p_0)}{n}}} = \frac{0.5846 - 0.5}{\sqrt{\frac{0.5\,(0.5)}{65}}}$$
$$= \frac{0.0846}{0.062}$$
$$= 1.36.$$

Note that p_0 is used in the standard error calculation.

d) Since $z = 1.36$ is within the acceptance region (i.e., it is less than 1.64), we accept H_0 at the 5% level of significance.

The evidence suggests that more people prefer the old system.

e) This is a one-tailed test with a ">" in the alternative hypothesis:

$$\Rightarrow \text{p-value } = \Pr(Z > 1.36) = 0.0869.$$

Therefore, while there is some evidence against H_0, it is not strong.

(note that we would reject H_0 at the 10% level)