Statistics for Computing

MA4413 Lecture 11A

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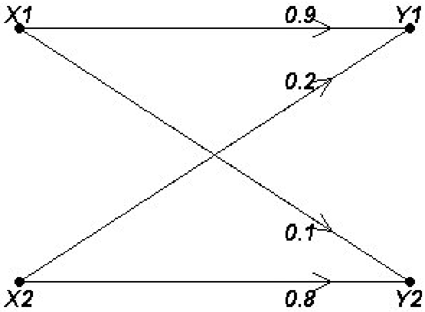
Discrete memoryless channel (from last lecture)

- For a DMC with "m" inputs and "n" outputs, the input X consists of input symbols $x_1, x_2, \ldots x_m$.
- The probabilities of these source symbols $P(x_i)$ are assumed to be known.
- The output Y consists of output symbols $\{y_1, y_2, \ldots, y_n\}$
- Each possible input-to-output path is indicated along with a conditional probability $P(y_j|x_i)$, where $P(y_j|x_i)$ is the conditional probability of obtaining output y_j given that the input is x_i.
- $P(y_j|x_i)$ is called a ***channel transition probability***.

Discrete memoryless channel

- On the next slide, we present a binary DMC, with the channel transition probabilities indicated.
- $P(y_1|x_1) = 0.9$ and $P(y_2|x_1) = 0.1$
- $P(y_1|x_2) = 0.2$ and $P(y_2|x_2) = 0.8$

Discrete Memoryless Channels



Types of Channel

1. Lossless Channel:

A channel described by a channel matrix with only one non-zero element in each column is called a lossless channel.

$$[P(Y|X)] = \begin{bmatrix} 3/4 & 1/4 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 2/3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

It can be shown that in the lossless channel, no source information is lost in transmission.

Types of Channel

1. Lossless Channel:

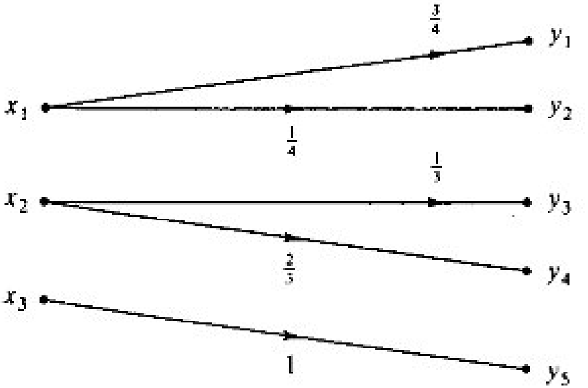


Figure:

Types of Channel

2. Deterministic Channel:
A channel described by a channel matrix with only one non-zero element in each row is called a deterministic channel.

$$[P(Y|X)] = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Note that since each row has only one non-zero element, this element must be 1. When a given source symbol is sent in a deterministic channel, it is clear which output symbol would be received.

Types of Channel

2. Deterministic Channel:

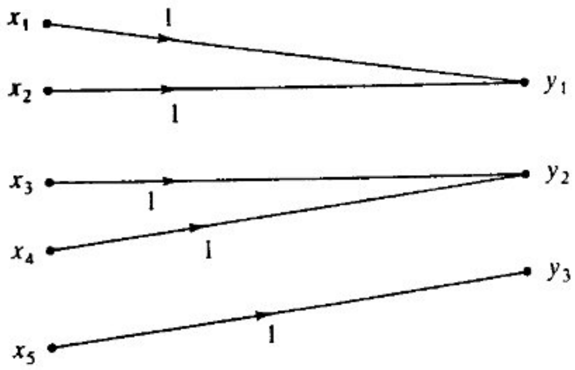


Figure:

Types of Channel

3. Noiseless Channel:
A channel is said to be ***noiseless*** if it is both lossless and deterministic. The channel matrix is the identity matrix: only one element in each row and each column, and each element is necessarily 1.

$$[P(Y|X)] = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

Note that the input and output alphabets have the same size , i.e. $m = n$.

Types of Channel

3. Noiseless Channel:

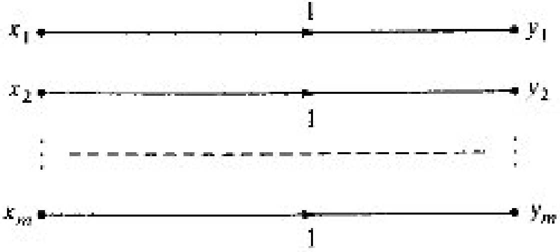


Figure:

Types of Channel

4. Binary Symmetric Channel:
The binary symmetric channel is defined by the following channel diagram (next slide) and the channel matrix is given by

$$[P(Y|X)] = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$$

The channel has two inputs and two outputs $(x_1 = 0, x_2 = 1)$ and two outputs $(x_1 = 0, x_2 = 1)$. The channel is symmetric because the probability of receiving a 1 if a 0 is sent is the same as the probability of receiving a 0 if a 1 was sent. This common probability is denoted p.

Binary Symmetric Channels