

1. 100 men are surveyed as to whether they play cricket, tennis or golf. It is found that
- 10 play none of these sports
 - 5 play all three of these sports
 - 88 play cricket or tennis or both
 - 78 play cricket or golf or both
 - 30 play golf and tennis but not cricket
 - 38 play golf
 - 74 play tennis.

Using a Venn diagram, or otherwise, find the following.

- (a) The number of the men who play at least one of these sports.
 - (b) The number of the men who play exactly one of these sports.
 - (c) The number of the men who play exactly two of these sports.
 - (d) Of those who do not play golf, the proportion who play cricket.
 - (e) The mean number of sports played by these men.
2. A Personal Identification Number (PIN) consists of four digits in order, each of which may be any one of $0, 1, 2, \dots, 9$.

- (a) Find the number of PINs satisfying each of the following requirements.
 - (a) All four digits are different.
 - (b) There are exactly three different digits.
 - (c) There are two different digits, each of which occurs twice.
 - (d) There are exactly three digits the same.
- (b) Two PINs are chosen independently and at random, and you are given that each PIN consists of four different digits. Let X be the random variable denoting the number of digits that the two PINs have in common.

- (a) Explain clearly why

$$P(X = k) = \frac{\binom{4}{k} \binom{6}{4-k}}{\binom{10}{k}},$$

for $k = 0, 1, 2, 3, 4$.

- (b) Hence write down the values of the probability mass function of X , and find its mean.
3. A combination lock consists of four rings each labelled with the digits 1, 2, 3, 4, 5, 6. The rings may be rotated individually and independently, so that all 4-digit combinations of the digits $1, \dots, 6$ (with repetition) can be shown. A customer buys such a lock. The instructions that come with the lock give the correct combination for opening the lock and state that this combination has been chosen at random from all possible combinations.
- (a) Evaluate k , the total number of combinations that can be shown.
 - (b) Find the probability that the purchased lock has a combination
 - (i) with all digits equal,

- (ii) with all digits different,
- (iii) with a pair of digits equal, the other two digits being different from each other and from the pair, (6)
- (iv) with exactly three digits equal,
- (v) with two pairs of equal digits (but not all four digits the same).