Chemometrics

MA4605

Week 10. Lecture 19. Latin square design

November 8, 2011

Latin squares

A **Latin square** is an $n \times n$ table filled with n different symbols (Latin letters) such that each symbol occurs exactly once in each row and exactly once in each column.

Examples of 3×3 Latin squares

A	B	C
C	A	B
B	C	A

A	B	C
B	C	A
C	A	B

B	A	C
C	B	A
A	C	B

C	B	A
A	C	B
B	A	C

How many possible Latin squares exist? There are many possible Latin squares of a given size *n*.

- For n=2 (A,B) there are 2.
- For n=3 (A,B,C) there are 12 possible Latin square permutations.
- For n=4 (A,B,C,D) there are 576 possible Latin square permutations.

There is no simple way to count all possible Latin squares for a given size *n* and for any Latin square, swapping two rows or two columns will lead to another Latin square.
Sudoku puzzles are a special case of Latin squares; any solution to a Sudoku puzzle is a Latin square.

- In the design of experiments, Latin squares are a special case of row-column designs for two blocking factors.
- In designs with two factors: A and B, where the number of treatments(k) and blocks(b) are equal (k=b), it is possible to use the Latin square experimental design in which we can account for an additional factor without requiring extra data.
- The additional factor must have k=b levels.

Example In an experiment to compare the efficiency of different chelating agents in extracting a metal ion from aqueous solution, the following results were obtained. On each day a fresh solution was prepared.
The following results were obtained:

Day	Agent1=A	Agent2=B	Agent3=C
1	84	80	83
2	79	77	80
3	83	78	80

An uncontrollable factor not taken into account is the time of day at which measurements are made. The levels of the new factor are : Morning, Noon, Afternoon.

- We have 2 blocking variables: Day and Time, which represent the new rows and columns.
- There is only one treatment: Agent.
- Day, Time and Agent have 3 levels each.
- Agent has 3 levels labelled A,B and C, and the blocking variables, called the "row effect" and "column effect" have each 3 levels 1,2 and 3.
- Only one single treatment(Agent) is applied within each combination of blocking variables.

Day	Morning=1	Noon=2	Afternoon=3
1	A=84	B=80	C= 83
2	C=80	A=79	B= 77
3	B=78	C=80	A= 83

```
y <- c(84, 80, 83, 80, 79, 77, 78, 80, 83)
day <- c(1,1,1,2,2,2,3,3,3)
time <- c(1,2,3,1,2,3,1,2,3)
agent <- c("A","B","C","C","A","B","B","C","A")
```

```
day<- factor(day)
time<- factor(time)
agent<-factor(agent)
model <- lm(y ~ day+time+agent)
anova(model)
Analysis of Variance Table
Response: y
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
day	2	20.2222	10.1111	13.0000	0.07143
time	2	2.8889	1.4444	1.8571	0.35000
agent	2	21.5556	10.7778	13.8571	0.06731
Residuals	2	1.5556	0.7778		

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