Statistics for Computing MA4413

Midterm Examination 2

Type A

- Do not turn over the page until instructed to do so.

- Rough work pages are provided within.

- Useful formulae and statistical tables are provided at the back.

- **Enter your answers (using an "X") in the table on the last page.**

- There are 15 questions in total: each correct answer = 1% (*there are no negative marks*).

- For each question, only *one* answer is correct.

- Scientific calculators approved by the University of Limerick can be used.

Questions 1 - 5

Q1 Identify the statement that is *false* (note: only one is false).

  (a) Increasing the confidence level leads to an increase in the confidence interval width.

  (b) The standard error tends to increase as the sample size increases.

  (c) The sample mean varies from sample to sample.

  (d) The population mean does not vary from sample to sample.

On average, potholes arise on a road at a rate of 2 per mile according to a Poisson distribution.

Q2 What is the probability that there are 3 potholes on a 2 mile stretch of road?

  (a) 0.3233   **(b)** 0.7619   **(c)** 0.1804   **(d)** 0.1954

Q3 What is the average distance (in miles) between potholes?

  (a) $\frac{1}{2}$ **(b)** 4   **(c)** $\frac{1}{4}$ **(d)** 2

Consider an $M/M/1$ system where the arrival rate is 8/hour and the service rate is 12/hour.

Q4 On average, how long (in hours) does a customer spend in this system?

  (a) 0.050   **(b)** 0.667   **(c)** 0.250   **(d)** 4.00

Q5 On average, how many customers are in the *queue* component?

  (a) 0.167   **(b)** 1.333   **(c)** 2.00   **(d)** 0.667

Rough Work

Questions 6 - 10

The lifetime of a particular component has an exponential distribution (i.e., $T \sim \text{Exponential}(\lambda)$) with an average life of 6 months.

Q6 What is the probability that the component lasts more than 12 months?

(a) 0.1146   (b) 0.0025   (c) 0.1353   (d) 0.0088

Q7 Let T_1 and T_2 represent the lifetimes for two of the components described above. Assuming these components work/fail independently, what is $\Pr(T_1 < 12 \cup T_2 < 12)$?

(a) 0.9817   (b) 0.0183   (c) 0.8647   (d) 0.7477

A 99% confidence interval for the average salary (in thousands) of a computer programmer was calculated as [30.5, 45.8].

Q8 What does the "99%" refer to?

The probability that:

(a) a randomly chosen programmer has a salary in the interval;

(b) the average salary for the population lies in the interval;

(c) a computer science student gets this question correct;

(d) the average salary for our sample lies in the interval.

A manufacturer wants to compare two designs of CPU in terms of their typical operating temperature (degrees Celsius). Two small samples are selected and the results are as follows:

	Design A	Design B
sample size	6	8
mean	38.7 °C	30.1 °C
variance	3.0 °C^2	12.0 °C^2

Q9 If we wish to assume equal variances (i.e., $\sigma_1^2 = \sigma_2^2$) when constructing a confidence interval for $\mu_1 - \mu_2$, we must carry out the F-test first. In the case of the data shown above, what is the critical value for the F test?

(a) 5.29   (b) 4.00   (c) 6.85   (d) 5.60

Q10 If we choose *not* to assume equal variances when constructing the confidence interval for $\mu_1 - \mu_2$ here, what is the value of ν needed to look up the appropriate $t_{\nu, \alpha/2}$ value in the tables?

(a) 14   (b) 13   (c) 12   (d) 11

Rough Work

Questions 11 - 15

A particular Android app has 10,000 users overall. The developers think that at least 70% of these users like a new feature. To investigate this claim, 600 users are contacted: 400 respond and it is found that 350 of them like the new feature.

Q11 What is the value of the statistic here?

(a) 0.875   (b) 0.583   (c) 0.035   (d) 0.667

Q12 What is the parameter here?

(a) $\mu = 70$ (b) $\mu = $ unknown   (c) $p = 0.7$ (d) $p = $ unknown

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Let $X \sim \text{Normal}(\mu = 15, \sigma = 4)$.

Q13 What is the value of $\Pr(X > 19.2)$? (rounded to 2 decimal places)

(a) 1.05   (b) 0.15   (c) 0.07   (d) 0.44

Q14 In a group of size $n = 40$, what is the value of $\Pr(\overline{X} > 14.7)$? (rounded to 2 decimal places)

(a) 0.47   (b) 0.32   (c) 0.53   (d) 0.68

Consider the following sample of ages of computers in an office block:

$$\boxed{4.3 \quad 5.1 \quad 2.6}$$

Q15 The 95% confidence interval for the mean is:

(a) [1.65, 6.35]   (b) [1.85, 6.15]   (c) [2.56, 5.44]   (d) [0.83, 7.17]

Rough Work

Don't forget to enter your answers on the last page!

Useful Formulae: Page 1

Numerical Summaries:

$$\bullet \quad \bar{x} = \frac{\sum x_i}{n}$$

$$\bullet \quad s^2 = \frac{\sum x_i^2 - n\,\bar{x}^2}{n-1}$$

Probability:

- $\Pr(A^c) = 1 - \Pr(A)$

- $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

- $\Pr(E_1 \cup E_2 \cup \cdots \cup E_k) = \Pr(E_1) + \Pr(E_2) + \cdots + \Pr(E_k)$ (if mutually exclusive)

- $\Pr(A \cap B) = \Pr(A)\,\Pr(B \,|\, A) = \Pr(B)\,\Pr(A \,|\, B)$

- $\Pr(E_1 \cap E_2 \cap \cdots \cap E_k) = \Pr(E_1)\,\Pr(E_2)\,\cdots\,\Pr(E_k)$ (if independent)

- $\Pr(A \,|\, B) = \dfrac{\Pr(A \cap B)}{\Pr(B)} = \dfrac{\Pr(A)\,\Pr(B \,|\, A)}{\Pr(B)}$

- If $E_1, \ldots, E_k$ are mutually exclusive & exhaustive
$$\Rightarrow \Pr(B) = \Pr(B \cap E_1) + \Pr(B \cap E_2) + \cdots + \Pr(B \cap E_k)$$
$$= \Pr(E_1)\,\Pr(B \,|\, E_1) + \Pr(E_2)\,\Pr(B \,|\, E_2) + \cdots + \Pr(E_k)\,\Pr(B \,|\, E_k)$$

Distributions:

$\bullet\ X \sim \text{Poisson}(\lambda)$	$\bullet\ T \sim \text{Exponential}(\lambda)$	$\bullet\ X \sim \text{Normal}(\mu, \sigma)$
$\bullet\ \Pr(X = x) = \dfrac{\lambda^x}{x!}\,e^{-\lambda}$	$\bullet\ \Pr(T > t) = e^{-\lambda t}$	$\bullet\ \Pr(X > x) = \Pr\left(Z > \dfrac{x - \mu}{\sigma}\right)$
$\bullet\ x \in \{0, 1, 2, \ldots, \infty\}$	$\bullet\ t \in [0, \infty)$	$\bullet\ x \in (-\infty, \infty)$
$\bullet\ E(X) = \lambda$	$\bullet\ E(T) = \dfrac{1}{\lambda}$	$\bullet\ E(X) = \mu$
$\bullet\ Var(X) = \lambda$	$\bullet\ Var(T) = \dfrac{1}{\lambda^2}$	$\bullet\ Var(X) = \sigma^2$

Useful Formulae: Page 2

Queueing Theory:

- $E(N) = \lambda_a \, E(T)$

- $\rho = \dfrac{\lambda_a}{\lambda_s}$

- $M/M/1$ System: $\lambda_a \longrightarrow$ ⬚⬚⬚⬚$\left(\lambda_s\right)\longrightarrow \lambda_a$

$$\Rightarrow \ T \sim \text{Exponential}(\lambda_s - \lambda_a)$$

(where T is the total time in the system)

Normal Distribution:

- $\Pr(Z < -z) = \Pr(Z > z)$

- $\Pr(Z > -z) = \Pr(Z < z) = 1 - \Pr(Z > z)$

- $\Pr(X > x) = \Pr\left(Z > \dfrac{x - \mu}{\sigma}\right)$

- $(1 - \alpha)100\%$ of the Normal(μ, σ) distribution lies in $\mu \pm z_{\alpha/2}\, \sigma$

- If $X_1 \sim \text{Normal}(\mu_1, \sigma_1)$ and $X_2 \sim \text{Normal}(\mu_2, \sigma_2)$

$$\Rightarrow \quad \text{Sum:} \quad X_1 + X_2 \sim \text{Normal}\left(\mu_1 + \mu_2, \ \sqrt{\sigma_1^2 + \sigma_2^2}\right)$$

$$\Rightarrow \quad \text{Difference:} \quad X_1 - X_2 \sim \text{Normal}\left(\mu_1 - \mu_2, \ \sqrt{\sigma_1^2 + \sigma_2^2}\right)$$

- For $X_1, \ldots, X_n \sim$ any distribution with $\mu = E(X)$ and $\sigma = Sd(X) = \sqrt{Var(X)}$

$$\Rightarrow \quad \text{Sample mean:} \quad \overline{X} \sim \text{Normal}\left(\mu, \ \dfrac{\sigma}{\sqrt{n}}\right) \quad \text{if } n > 30$$

Useful Formulae: Page 3

Confidence Intervals:

- Large sample: statistic $\pm\ z_{\alpha/2}\ \times$ standard error

- Small sample: statistic $\pm\ t_{\nu,\,\alpha/2}\ \times$ standard error

Parameter	Statistic	Standard Error	Samples	D. of. F.
μ	$\bar{x}$	$\dfrac{s}{\sqrt{n}}$	large / small	$\nu = n - 1$
p	$\hat{p}$	$\sqrt{\dfrac{\hat{p}\,(1-\hat{p})}{n}}$	large	n/a
$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$	$\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$	large / small	$\nu = \dfrac{(a+b)^2}{\frac{a^2}{n_1-1} + \frac{b^2}{n_2-1}}$ $\quad a = \dfrac{s_1^2}{n_1},\ \ b = \dfrac{s_2^2}{n_2}$
		$\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}$ where $\ s_p^2 = \dfrac{(n_1-1)\,s_1^2 + (n_2-1)\,s_2^2}{n_1 + n_2 - 2}$	small	$\nu = n_1 + n_2 - 2$ assuming $\sigma_1^2 = \sigma_2^2$
$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$	$\sqrt{\dfrac{\hat{p}_1\,(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2\,(1-\hat{p}_2)}{n_2}}$	large	n/a

- $F = \dfrac{\text{larger variance}}{\text{smaller variance}} = \dfrac{s_{\text{larger}}^2}{s_{\text{smaller}}^2}$

$\nu_1 = \text{top sample size} - 1$

$\nu_2 = \text{bottom sample size} - 1$

Table 2 Cumulative Poisson Probabilities

The table gives the probability that *r or more* random events are contained in an interval when the average number of such events per interval is *m*, i.e.

$$\sum_{x=r}^{\infty} e^{-m}\frac{m^x}{x!}$$

Where there is no entry for a particular pair of values of *r* and *m*, this indicates that the appropriate probability is less than 0.000 05. Similarly, except for the case *r* = 0 when the entry is exact, a tabulated value of 1.0000 represents a probability greater than 0.999 95.

m =	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
r = 0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.0952	.1813	.2592	.3297	.3935	.4512	.5034	.5507	.5934	.6321
2	.0047	.0175	.0369	.0616	.0902	.1219	.1558	.1912	.2275	.2642
3	.0002	.0011	.0036	.0079	.0144	.0231	.0341	.0474	.0629	.0803
4		.0001	.0003	.0008	.0018	.0034	.0058	.0091	.0135	.0190
5				.0001	.0002	.0004	.0008	.0014	.0023	.0037
6							.0001	.0002	.0003	.0006
7										.0001

m =	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
r = 0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.6671	.6988	.7275	.7534	.7769	.7981	.8173	.8347	.8504	.8647
2	.3010	.3374	.3732	.4082	.4422	.4751	.5068	.5372	.5663	.5940
3	.0996	.1205	.1429	.1665	.1912	.2166	.2428	.2694	.2963	.3233
4	.0257	.0338	.0431	.0537	.0656	.0788	.0932	.1087	.1253	.1429
5	.0054	.0077	.0107	.0143	.0186	.0237	.0296	.0364	.0441	.0527
6	.0010	.0015	.0022	.0032	.0045	.0060	.0080	.0104	0132	.0166
7	.0001	.0003	.0004	.0006	.0009	.0013	.0019	.0026	.0034	.0045
8			.0001	.0001	.0002	.0003	.0004	.0006	.0008	.0011
9							.0001	.0001	.0002	.0002

m =	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
r = 0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.8775	.8892	.8997	.9093	.9179	.9257	.9328	.9392	.9450	.9502
2	.6204	.6454	.6691	.6916	.7127	.7326	.7513	.7689	.7854	.8009
3	.3504	.3773	.4040	.4303	.4562	.4816	.5064	.5305	.5540	.5768
4	.1614	.1806	.2007	.2213	.2424	.2640	.2859	.3081	.3304	.3528
5	.0621	.0725	.0838	.0959	.1088	.1226	.1371	.1523	.1682	.1847
6	.0204	.0249	.0300	.0357	.0420	.0490	.0567	.0651	.0742	.0839
7	.0059	.0075	.0094	.0116	.0142	.0172	.0206	.0244	.0287	.0335
8	.0015	.0020	.0026	.0033	.0042	.0053	.0066	.0081	.0099	.0119
9	.0003	.0005	.0006	.0009	.0011	.0015	.0019	.0024	.0031	.0038
10	.0001	.0001	.0001	.0002	.0003	.0004	.0005	.0007	.0009	.0011
11					.0001	.0001	.0001	.0002	.0002	.0003
12									.0001	.0001

Table 2 Cumulative Poisson Probabilities – continued

m =	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
r = 0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9550	.9592	.9631	.9666	.9698	.9727	.9753	.9776	.9798	.9817
2	.8153	.8288	.8414	.8532	.8641	.8743	.8838	.8926	.9008	.9084
3	.5988	.6201	.6406	.6603	.6792	.6973	.7146	.7311	.7469	.7619
4	.3752	.3975	.4197	.4416	.4634	.4848	.5058	.5265	.5468	.5665
5	.2018	.2194	.2374	.2558	.2746	.2936	.3128	.3322	.3516	.3712
6	.0943	.1054	.1171	.1295	.1424	.1559	.1699	.1844	.1994	.2149
7	.0388	.0446	.0510	.0579	.0653	.0733	.0818	.0909	.1005	.1107
8	.0142	.0168	.0198	.0231	.0267	.0308	.0352	.0401	.0454	.0511
9	.0047	.0057	.0069	.0083	.0099	.0117	.0137	.0160	.0185	.0214
10	.0014	.0018	.0022	.0027	.0033	.0040	.0048	.0058	.0069	.0081
11	.0004	.0005	.0006	.0008	.0010	.0013	.0016	.0019	.0023	.0028
12	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0006	.0007	.0009
13				.0001	.0001	.0001	.0001	.0002	.0002	.0003
14									.0001	.0001

m =	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
r = 0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9834	.9850	.9864	.9877	.9889	.9899	.9909	.9918	.9926	.9933
2	.9155	.9220	.9281	.9337	.9389	.9437	.9482	.9523	.9561	.9596
3	.7762	.7898	.8026	.8149	.8264	.8374	.8477	.8575	.8667	.8753
4	.5858	.6046	.6228	.6406	.6577	.6743	.6903	.7058	.7207	.7350
5	.3907	.4102	.4296	.4488	.4679	.4868	.5054	.5237	.5418	.5595
6	.2307	.2469	.2633	.2801	.2971	.3142	.3316	.3490	.3665	.3840
7	.1214	.1325	.1442	.1564	.1689	.1820	.1954	.2092	.2233	.2378
8	.0573	.0639	.0710	.0786	.0866	.0951	.1040	.1133	.1231	.1334
9	.0245	.0279	.0317	.0358	.0403	.0451	.0503	.0558	.0618	.0681
10	.0095	.0111	.0129	.0149	.0171	.0195	.0222	.0251	.0283	.0318
11	.0034	.0041	.0048	.0057	.0067	.0078	.0090	.0104	.0120	.0137
12	.0011	.0014	.0017	.0020	.0024	.0029	.0034	.0040	.0047	.0055
13	.0003	.0004	.0005	.0007	.0008	.0010	.0012	.0014	.0017	.0020
14	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005	.0006	.0007
15				.0001	.0001	.0001	.0001	.0001	.0002	.0002
16									.0001	.0001

m =	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0
r = 0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9945	.9955	.9963	.9970	.9975	.9980	.9983	.9986	.9989	.9991
2	.9658	.9711	.9756	.9794	.9826	.9854	.9877	.9897	.9913	.9927
3	.8912	.9052	.9176	.9285	.9380	.9464	.9537	.9600	.9656	.9704
4	.7619	.7867	.8094	.8300	.8488	.8658	.8811	.8948	.9072	.9182
5	.5939	.6267	.6579	.6873	.7149	.7408	.7649	.7873	.8080	.8270
6	.4191	.4539	.4881	.5217	.5543	.5859	.6163	.6453	.6730	.6993
7	.2676	.2983	.3297	.3616	.3937	.4258	.4577	.4892	.5201	.5503
8	.1551	.1783	.2030	.2290	.2560	.2840	.3127	.3419	.3715	.4013
9	.0819	.0974	.1143	.1328	.1528	.1741	.1967	.2204	.2452	.2709
10	.0397	.0488	.0591	.0708	.0839	.0984	.1142	.1314	.1498	.1695
11	.0177	.0225	.0282	.0349	.0426	.0514	.0614	.0726	.0849	.0985
12	.0073	.0096	.0125	.0160	.0201	.0250	.0307	.0373	.0448	.0534
13	.0028	.0038	.0051	.0068	.0088	.0113	.0143	.0179	.0221	.0270
14	.0010	.0014	.0020	.0027	.0036	.0048	.0063	.0080	.0102	.0128
15	.0003	.0005	.0007	.0010	.0014	.0019	.0026	.0034	.0044	.0057
16	.0001	.0002	.0002	.0004	.0005	.0007	.0010	.0014	.0018	.0024
17			.0001	.0001	.0002	.0003	.0004	.0005	.0007	.0010
18						.0001	.0001	.0002	.0003	.0004
19								.0001	.0001	.0001

Table 2 Cumulative Poisson Probabilities – continued

m =	7.2	7.4	7.6	7.8	8.0	8.2	8.4	8.6	8.8	9.0
r = 0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9993	.9994	.9995	.9996	.9997	.9997	.9998	.9998	.9998	.9999
2	.9939	.9949	.9957	.9964	.9970	.9975	.9979	.9982	.9985	.9988
3	.9745	.9781	.9812	.9839	.9862	.9882	.9900	.9914	.9927	.9938
4	.9281	.9368	.9446	.9515	.9576	.9630	.9677	.9719	.9756	.9788
5	.8445	.8605	.8751	.8883	.9004	.9113	.9211	.9299	.9379	.9450
6	.7241	.7474	.7693	.7897	.8088	.8264	.8427	.8578	.8716	.7932
7	.5796	.6080	.6354	.6616	.6866	.7104	.7330	.7543	.7744	.7932
8	.4311	.4607	.4900	.5188	.5470	.5746	.6013	.6272	.6522	.6761
9	.2973	.3243	.3518	.3796	.4075	.4353	.4631	.4906	.5177	.5443
10	.1904	.2123	.2351	.2589	.2834	.3085	.3341	.3600	.3863	.4126
11	.1133	.1293	.1465	.1648	.1841	.2045	.2257	.2478	.2706	.2940
12	.0629	.0735	.0852	.0980	.1119	.1269	.1429	.1600	.1780	.1970
13	.0327	.0391	.0464	.0546	.0638	.0739	.0850	.0971	.1102	.1242
14	.0159	.0195	.0238	.0286	.0342	.0405	.0476	.0555	.0642	.0739
15	.0073	.0092	.0114	.0141	.0173	.0209	.0251	.0299	.0353	.0415
16	.0031	.0041	.0052	.0066	.0082	.0102	.0125	.0152	.0184	.0220
17	.0013	.0017	.0022	.0029	.0037	.0047	.0059	.0074	.0091	.0111
18	.0005	.0007	.0009	.0012	.0016	.0021	.0027	.0034	.0043	.0053
19	.0002	.0003	.0004	.0005	.0006	.0009	.0011	.0015	.0019	.0024
20	.0001	.0001	.0001	.0002	.0003	.0003	.0005	.0006	.0008	.0011
21				.0001	.0001	.0001	.0002	.0002	.0003	.0004
22							.0001	.0001	.0001	.0002
23										.0001

m =	9.2	9.4	9.6	9.8	10.0	11.0	12.0	13.0	14.0	15.0
r = 0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.9990	.9991	.9993	.9994	.9995	.9998	.9999	1.0000	1.0000	1.0000
3	.9947	.9955	.9962	.9967	.9972	.9988	.9995	.9998	.9999	1.0000
4	.9816	.9840	.9862	.9880	.9897	.9951	.9977	.9990	.9995	.9998
5	.9514	.9571	.9622	.9667	.9707	.9849	.9924	.9963	.9982	.9991
6	.8959	.9065	.9162	.9250	.9329	.9625	.9797	.9893	.9945	.9972
7	.8108	.8273	.8426	.8567	.8699	.9214	.9542	.9741	.9858	.9924
8	.6990	.7208	.7416	.7612	.7798	.8568	.9105	.9460	.9684	.9820
9	.5704	.5958	.6204	.6442	.6672	.7680	.8450	.9002	.9379	.9626
10	.4389	.4651	.4911	.5168	.5421	.6595	.7576	.8342	.8906	.9301
11	.3180	.3424	.3671	.3920	.4170	.5401	.6528	.7483	.8243	.8815
12	.2168	.2374	.2588	.2807	.3032	.4207	.5384	.6468	.7400	.8152
13	.1393	.1552	.1721	.1899	.2084	.3113	.4240	.5369	.6415	.7324
14	.0844	.0958	.1081	.1214	.1355	.2187	.3185	.4270	.5356	.6368
15	.0483	.0559	.0643	.0735	.0835	.1460	.2280	.3249	.4296	.5343
16	.0262	.0309	.0362	.0421	.0487	.0926	.1556	.2364	.3306	.4319
17	.0135	.0162	.0194	.0230	.0270	.0559	.1013	.1645	.2441	.3359
18	.0066	.0081	.0098	.0119	.0143	.0322	.0630	.1095	.1728	.2511
19	.0031	.0038	.0048	.0059	.0072	.0177	.0374	.0698	.1174	.1805
20	.0014	.0017	.0022	.0028	.0035	.0093	.0213	.0427	.0765	.1248
21	.0006	.0008	.0010	.0012	.0016	.0047	.0116	.0250	.0479	.0830
22	.0002	.0003	.0004	.0005	.0007	.0023	.0061	.0141	.0288	.0531
23	.0001	.0001	.0002	.0002	.0003	.0010	.0030	.0076	.0167	.0327
24			.0001	.0001	.0001	.0005	.0015	.0040	.0093	.0195
25						.0002	.0007	.0020	.0050	.0122
26						.0001	.0003	.0010	.0026	.0062
27							.0001	.0005	.0013	.0033
28								.0002	.0006	.0017
29								.0001	.0003	.0009
30									.0001	.0004
31										.0002
32										.0001

Table 3 Areas in Upper Tail of the Normal Distribution

The function tabulated is $1 - \Phi(z)$ where $\Phi(z)$ is the cumulative distribution function of a standardised Normal variable, z.

Thus $1 - \Phi(z) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{z}^{\infty} e^{-z^2/2}$ is the probability that a standardised Normal variate selected at random will be greater than a

value of $z \left(= \dfrac{x - \mu}{\sigma} \right)$
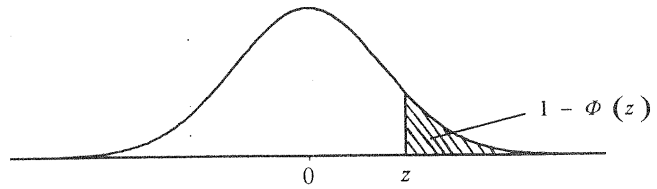
$\dfrac{x-\mu}{\sigma}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
3.7	.000108	.000104	.000100	.000096	.000092	.000088	.000085	.000082	.000078	.000075
3.8	.000072	.000069	.000067	.000064	.000062	.000059	.000057	.000054	.000052	.000050
3.9	.000048	.000046	.000044	.000042	.000041	.000039	.000037	.000036	.000034	.000033
4.0	.000032									

$5.0 \rightarrow 0.000\ 000\ 286\ 7$　　　　$5.5 \rightarrow 0.000\ 000\ 019\ 0$　　　　$6.0 \rightarrow 0.000\ 000\ 001\ 0$
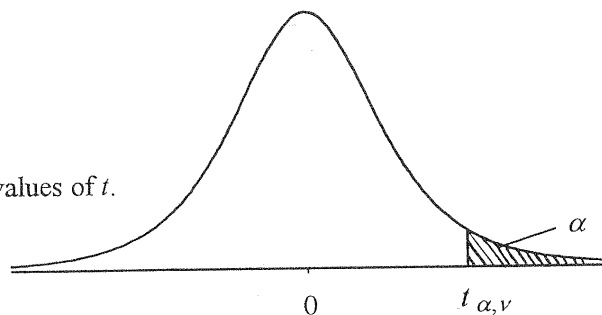
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Table 7 Percentage Points of the *t* Distribution

The table gives the value of $t_{\alpha,v}$ – the 100α percentage point of the *t* distribution for v degrees of freedom.

The values of *t* are obtained by solution of the equation:

$$\alpha = \Gamma[\tfrac{1}{2}(v+1)][\Gamma(\tfrac{1}{2}v)]^{-1}(v\pi)^{-1/2}\int_{t}^{\infty}(1+x^2/v)^{-(v+1)/2}\,dx$$

Note: The tabulation is for one tail only, that is, for positive values of *t*.
For $|t|$ the column headings for α should be doubled.

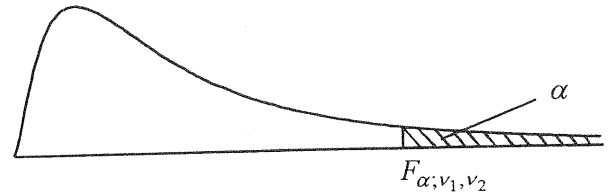| $\alpha =$	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
$v = 1$	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Table 9 Percentage Points of the F Distribution

The table gives the values of $F_{\alpha;\nu_1,\nu_2}$ the 100α percentage point of the F distribution having ν_1 degrees of freedom in the numerator and ν_2 degrees of freedom in the denominator. For each pair of values of ν_1 and ν_2, $F_{\alpha;\nu_1,\nu_2}$ is tabulated for $\alpha = 0.05$, 0.025, 0.01, 0.001, the 0.025 values being bracketed.
The lower percentage points of the distribution may be obtained from the relation:

$$F_{1-\alpha;\nu_1,\nu_2} = 1/F_{\alpha;\nu_2,\nu_1}$$

Example: $F_{.95,12,8} = 1/F_{.05,8,12} = 1/2.85 = \underline{0.351}$

ν_1	1	2	3	4	5	6	7	8	10	12	24	∞
ν_2												
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	241.9	243.9	249.0	254.3
	(648)	(800)	(864)	(900)	(922)	(937)	(948)	(957)	(969)	(977)	(997)	(1018)
	4052	5000	5403	5625	5764	5859	5928	5981	6056	6106	6235	6366
	4053*	5000*	5405*	5625*	5764*	5859*	5929*	5981*	6056*	6107*	6235*	6366*
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.5	19.5
	(38.5)	(39.0)	(39.2)	(39.2)	(39.3)	(39.3)	(39.4)	(39.4)	(39.4)	(39.4)	(39.5)	(39.5)
	98.5	99.0	99.2	99.2·	99.3	99.3	99.4	99.4	99.4	99.4	99.5	99.5
	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	999.4	999.5	999.5
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.79	8.74	8.64	8.53
	(17.4)	(16.0)	(15.4)	(15.1)	(14.9)	(14.7)	(14.6)	(14.5)	(14.4)	(14.3)	(14.1)	(13.9)
	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.2	27.1	26.6	26.1
	167.0	148.5	141.1	137.1	134.6	132.8	131.5	130.6	129.2	128.3	125.9	123.5
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	5.96	5.91	5.77	5.63
	(12.22)	(10.65)	(9.98)	(9.60)	(9.36)	(9.20)	(9.07)	(8.98)	(8.84)	(8.75)	(8.51)	(8.26)
	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.5	14.4	13.9	13.5
	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.05	47.41	45.77	44.05
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.74	4.68	4.53	4.36
	(10.01)	(8.43)	(7.76)	(7.39)	(7.15)	(6.98)	(6.85)	(6.76)	(6.62)	(6.52)	(6.28)	(6.02)
	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.05	9.89	9.47	9.02
	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	26.92	26.42	25.14	23.79
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.06	4.00	3.84	3.67
	(8.81)	(7.26)	(6.60)	(6.23)	(5.99)	(5.82)	(5.70)	(5.60)	(5.46)	(5.37)	(5.12)	(4.85)
	13.74	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.87	7.72	7.31	6.88
	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.41	17.99	16.90	15.75
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.64	3.57	3.41	3.23
	(8.07)	(6.54)	(5.89)	(5.52)	(5.29)	(5.12)	(4.99)	(4.90)	(4.76)	(4.67)	(4.42)	(4.14)
	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.62	6.47	6.07	5.65
	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.08	13.71	12.73	11.70
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.35	3.28	3.12	2.93
	(7.57)	(6.06)	(5.42)	(5.05)	(4.82)	(4.65)	(4.53)	(4.43)	(4.30)	(4.20)	(3.95)	(3.67)
	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.81	5.67	5.28	4.86
	25.42	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.54	11.19	10.30	9.34
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.14	3.07	2.90	2.71
	(7.21)	(5.71)	(5.08)	(4.72)	(4.48)	(4.32)	(4.20)	(4.10)	(3.96)	(3.87)	(3.61)	(3.33)
	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.26	5.11	4.73	4.31
	22.86	16.39	13.90	12.56	11.71	11.13	10.69	10.37	9.87	9.57	8.72	7.81
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.98	2.91	2.74	2.54
	(6.94)	(5.46)	(4.83)	(4.47)	(4.24)	(4.07)	(3.95)	(3.85)	(3.72)	(3.62)	(3.37)	(3.08)
	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.85	4.71	4.33	3.91
	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.74	8.44	7.64	6.76
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.85	2.79	2.61	2.40
	(6.72)	(5.26)	(4.63)	(4.28)	(4.04)	(3.88)	(3.76)	(3.66)	(3.53)	(3.43)	(3.17)	(2.88)
	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.54	4.40	4.02	3.60
	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	7.92	7.63	6.85	6.00
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.75	2.69	2.51	2.30
	(6.55)	(5.10)	(4.47)	(4.12)	(3.89)	(3.73)	(3.61)	(3.51)	(3.37)	(3.28)	(3.02)	(2.72)
	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.30	4.16	3.78	3.36
	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.29	7.00	6.25	5.42

* Entries marked thus must be multiplied by 100

Answer Sheet

Name: _____

ID Number: _____

Enter your answers with an "X' in the table below.

Do not enter the "X" until you have made your *final decision* to avoid scribbling out.

	A	B	C	D
Q1				
Q2				
Q3				
Q4				
Q5				

	A	B	C	D
Q6				
Q7				
Q8				
Q9				
Q10				

	A	B	C	D
Q11				
Q12				
Q13				
Q14				
Q15				