Question 1

$\Pr(A) = \frac{3}{12}$, $\Pr(B) = \frac{6}{12}$ and $\Pr(C) = \frac{2}{12}$.
$\Pr(A \cap B) = \frac{3}{12}$, $\Pr(A \cap C) = \frac{0}{12}$ and $\Pr(B \cap C) = \frac{1}{12}$.

a)
$$\Pr(B \mid A) = \frac{\Pr(A \cap B)}{\Pr(A)} = \frac{\frac{3}{12}}{\frac{3}{12}} = \frac{12}{3} \times \frac{3}{12} = 1.$$
$$\Pr(C \mid A) = \frac{\Pr(A \cap C)}{\Pr(A)} = \frac{\frac{0}{12}}{\frac{3}{12}} = \frac{12}{3} \times \frac{0}{12} = 0.$$
$$\Pr(B \mid C) = \frac{\Pr(B \cap C)}{\Pr(C)} = \frac{\frac{1}{12}}{\frac{2}{12}} = \frac{12}{2} \times \frac{1}{12} = \frac{1}{2}.$$
$$\Pr(C \mid B) = \frac{\Pr(B \cap C)}{\Pr(C)} = \frac{\frac{1}{12}}{\frac{6}{12}} = \frac{12}{6} \times \frac{1}{12} = \frac{1}{6}.$$

b) $\Pr(B \mid A) = 1$.
We *know* we have $A =$ "head & even" $\Rightarrow$ we are certain that we have $B =$ "coin shows head".

$\Pr(C \mid A) = 0$.
We *know* we have $A =$ "head & even" $\Rightarrow C =$ "die show five" is *impossible*.

$\Pr(B \mid C) = \frac{1}{2} = \Pr(B)$.
Knowing that the die shows five does not alter our prior probability for $B =$ "coin shows head". The die cannot inform us about the coin.

$\Pr(C \mid B) = \frac{1}{6} = \Pr(C)$.
Knowing that the coin shows a head does not alter our prior probability for $C =$ "die shows a five". The coin cannot inform us about the die.

c) $\Pr(B \mid A) = 1 \neq \Pr(B) = \frac{1}{2}$
$\Rightarrow A$ and B are *dependent*.

$\Pr(C \mid A) = 0 \neq \Pr(C) = \frac{1}{6}$
$\Rightarrow A$ and C are *dependent*.

$\Pr(B \mid C) = \frac{1}{2} = \Pr(B) = \frac{1}{2}$
$\Rightarrow B$ and C are *independent*.

d) The events A and C are *mutually exclusive* since $\Pr(A \cap C) = 0$.

Question 2

a) i) $\Pr(S_H) = \frac{145}{955} \approx 0.152$.
ii) $\Pr(S_A) = \frac{670}{955} \approx 0.702$.
iii) $\Pr(S_L) = \frac{140}{955} \approx 0.147$.

b) $\Pr(B) = \frac{115}{955} \approx 0.12$.

c) $\Pr(S_H \cap B) = \frac{5}{955} \approx 0.005$, $\Pr(S_A \cap B) = \frac{70}{955} \approx 0.073$ and $\Pr(S_L \cap B) = \frac{40}{955} \approx 0.042$.

i) $\Pr(B \mid S_H) = \frac{\Pr(S_H \cap B)}{\Pr(S_H)} = \frac{0.005}{0.152} = 0.033$.

ii) $\Pr(B \mid S_A) = \frac{\Pr(S_A \cap B)}{\Pr(S_A)} = \frac{0.073}{0.702} = 0.104$.

(iii) $\Pr(B \mid S_L) = \frac{\Pr(S_L \cap B)}{\Pr(S_L)} = \frac{0.042}{0.147} = 0.286$.

d) The *prior* probability of a bug is $\Pr(B) = 0.12$. The presence of bugs is *not* independent of skill level as it changes for different skill levels. In Particular $\Pr(B \mid S_H) < \Pr(B \mid S_A) < \Pr(B \mid S_L)$. As we might expect more skill $\Rightarrow$ less bugs.

e) $\Pr(S_A \mid B) = \frac{\Pr(S_A \cap B)}{\Pr(B)} = \frac{0.073}{0.12} = 0.609$.

$\Pr(S_L \mid B) = \frac{\Pr(S_L \cap B)}{\Pr(B)} = \frac{0.042}{0.12} = 0.35$.

Given that a bug is present, it is more likely to have been the work of a programmer who has average skill simply because *most of the code* is written by these individuals, i.e., $\Pr(S_A) = 0.702$.

Question 3

a) $$\begin{aligned}\Pr(B) &= \Pr(B \cap S_H) + \Pr(B \cap S_A) + \Pr(B \cap S_L)\\ &= 0.005 + 0.073 + 0.042\\ &= 0.12.\end{aligned}$$

(as we had before from the table)

b) For this we need
$$\Pr(S_H \cap B^c) = \frac{140}{955} = 0.147.$$
$$\Pr(S_A \cap B^c) = \frac{600}{955} = 0.628.$$
$$\Pr(S_L \cap B^c) = \frac{100}{955} = 0.105.$$

Now we can calculate
$$\begin{aligned}\Pr(S_H) &= \Pr(S_H \cap B) + \Pr(S_H \cap B^c)\\ &= 0.005 + 0.147.\\ &= 0.152.\end{aligned}$$
$$\begin{aligned}\Pr(S_A) &= \Pr(S_A \cap B) + \Pr(S_A \cap B^c)\\ &= 0.073 + 0.628\\ &= 0.701.\end{aligned}$$
$$\begin{aligned}\Pr(S_L) &= \Pr(S_L \cap B) + \Pr(S_L \cap B^c)\\ &= 0.042 + 0.105\\ &= 0.147.\end{aligned}$$
(as we had before from the table)

Question 4

Define the event A_1 = "processor comes from A_1" and similarly A_2 and A_3. We also let D = "defective processor".

The information we are given is as follows:
$$\Pr(A_1) = 0.2 \qquad \Pr(D \mid A_1) = 0.1$$
$$\Pr(A_2) = 0.55 \qquad \Pr(D \mid A_2) = 0.04$$
$$\Pr(A_3) = 0.25 \qquad \Pr(D \mid A_3) = 0.01$$

We can also calculate:
$$\begin{aligned}\Pr(D \cap A_1) &= \Pr(A_1)\Pr(D \mid A_1)\\ &= 0.2(0.1) = 0.02.\end{aligned}$$
$$\begin{aligned}\Pr(D \cap A_2) &= \Pr(A_2)\Pr(D \mid A_2)\\ &= 0.55(0.04) = 0.022.\end{aligned}$$
$$\begin{aligned}\Pr(D \cap A_3) &= \Pr(A_3)\Pr(D \mid A_3)\\ &= 0.25(0.01) = 0.0025.\end{aligned}$$

a) $$\begin{aligned}\Pr(D) &= \Pr(D \cap A_1) + \Pr(D \cap A_2) + \Pr(D \cap A_3)\\ &= 0.02 + 0.022 + 0.0025\\ &= 0.0445.\end{aligned}$$

b) We know the processor is defective, i.e., *given* it is defective:
$$\begin{aligned}\Pr(A_1 \mid D) &= \frac{\Pr(D \cap A_1)}{\Pr(D)}\\ &= \frac{0.02}{0.0445} = 0.449.\end{aligned}$$
$$\begin{aligned}\Pr(A_2 \mid D) &= \frac{\Pr(D \cap A_2)}{\Pr(D)}\\ &= \frac{0.022}{0.0445} = 0.494.\end{aligned}$$
$$\begin{aligned}\Pr(A_3 \mid D) &= \frac{\Pr(D \cap A_3)}{\Pr(D)}\\ &= \frac{0.0025}{0.0445} = 0.056.\end{aligned}$$
$\Rightarrow$ It most likely came from A_2.

c) $$\begin{aligned}\Pr(D^c \mid A_1) &= 1 - \Pr(D \mid A_1)\\ &= 1 - 0.1 = 0.9.\end{aligned}$$

d) We will need
$$\begin{aligned}\Pr(D^c) &= 1 - \Pr(D)\\ &= 1 - 0.0445 = 0.9555.\end{aligned}$$
$$\begin{aligned}\Rightarrow \Pr(A_1 \mid D^c) &= \frac{\Pr(D^c \cap A_1)}{\Pr(D^c)}\\ &= \frac{\Pr(A_1)\Pr(D^c \mid A_1)}{\Pr(D^c)}\\ &= \frac{0.2(0.9)}{0.9555} = 0.188.\end{aligned}$$

e) If all stock came from A_3 then $\Pr(D) = \Pr(D \mid A_3) = 0.01$.

As all stock comes from A_3, $\Pr(A_3 \mid D) = 1$.

As no stock comes from A_1, $\Pr(A_1 \mid D) = 0$.