**Binomial Distribution - Formula**

**Distributions**

$X \sim$ Binomial $(n, p)$: $\quad P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$E(X) = np; \qquad Var(X) = np(1-p).$$

$X \sim$ Poisson$(\lambda)$: $\quad P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$.

**Exercise 1**

The probability that a component produced in a certain factory is defective is 0.01. A batch contains 200 components.

$n = 200$

$p = 0.01$

1) Calculate the probability that none of the components are faulty.

$$P(x=0) = \binom{200}{0}(0.01)^0(0.99)^{200}$$

$$\binom{200}{0} = \frac{200!}{(200!)0!} = 1$$

$$P(x=0) = (1)\times(1)\times(0.134)$$

$$P(x=0) = 0.134$$

2) Calculate the probability that at least two of the components are faulty.

$$P(x \geq 2) = 1 - P(x \leq 1) = 1 - [P(x=0) + P(x=1)]$$

$$P(x=1) = \binom{200}{1}(0.01)^1(0.99)^{199}$$

$$\binom{200}{1} = \frac{200!}{(1!)199!} = 200$$

$$P(x=1) = (200)\times(0.01)\times(0.1353)$$

$$P(x=1) = 0.270$$

$$P(x \leq 1) = 0.134 + 0.270$$

$$P(x \leq 1) = 0.404$$

$$P(x \geq 2) = 1 - 0.404$$
$$P(x \geq 2) = 0.596$$

**<u>Binomial formula</u>**

$$P(x = k) = \binom{n}{k}(p)^k (1-p)^{n-k}$$

**<u>"From n choose k"</u>**

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n \times (n-1) \times (n-2) \times .. \times (n-k)!}{k \times (k-1) \times (k-2) \times ... \times (n-k)!}$$

**<u>Example</u>**

$n = 10$

$p = 0.25$

$$P(x = 4) = \binom{10}{4}(0.25)^4 (0.75)^6$$
$$\binom{10}{4} = \frac{10!}{(4!)6!} = \frac{10 \times 9 \times 8 \times 7 \times (6!)}{4 \times 3 \times 2 \times 1 \times (6!)} = 210$$
$$P(x = 4) = (210) \times (0.0039) \times (0.17797)$$
$$P(x = 4) = 0.146$$