Technological Mathematics 4

MA4704 Lecture 10A

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Spring Semester 2013

Using R for Inference Procedures

1. Paired t-test
2. Single Sample Tests for Proportions
3. Two Sample Test for Proportions
4. Test for the equality of variances for two samples
5. Shapiro-Wilk Test for Normality
6. Graphical procedures for assessing normality

p-values using R

- In every inference procedure performed using R, a p-value is presented to the screen for the user to interpret.
- If the p-value is larger than a specified threshold α/k then the appropriate conclusion is a failure to reject the null hypothesis.
- Conversely, if the p-value is less than threshold, the appropriate conclusion is to reject the null hypothesis.
- In this module, we will use a significance level$\alpha = 0.05$ and almost always the procedures will be two tailed ($k = 2$). Therefore the threshold α/k will be 0.025.

Using Confidence Limits

- Alternatively, we can use the confidence interval to make a decision on whether or not we should reject or fail to reject the null hypothesis.
- If the null value is within the range of the confidence limits, we fail to reject the null hypothesis.
- If the null value is outside the range of the confidence limits, we reject the null hypothesis.
- Occasionally a conclusion based on this approach may differ from a conclusion based on the p-value. In such a case, remark upon this discrepancy.

The paired t-test (a)

- Previously we have seen the paired t-test. It is used to determine whether or not there is a significant difference between paired measurements. Equivalently whether or not the case-wise differences are zero.
- The mean and standard deviation of the case-wise differences are used to determine the test statistic.
- Under the null hypothesis, the expected value of the case-wise differences is zero (i.e $H_0 : \mu_d = 0$).
- The test statistic is computed as

$$TS = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

The Paired t-test (b)

- The calculation is dependent on the case-wise differences.
- Here the case-wise differences between paired measurements (e.g. "before" and "after").
- Under the null hypothesis, the mean of case-wise differences is zero.
- As a quick example, the mean, standard deviation and sample size are presented in the next slide.

The paired t-test (c)

- Observed Mean of Case-wise differences $\bar{d} = 8.21$,
- Expected Mean of Case-wise differences under $H_0 : \mu_d = 0$,
- Standard Deviation of Case-wise differences $S_d = 7.90$,
- Sample Size $n = 14$.

$$TS = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{8.21 - 0}{\frac{7.90}{\sqrt{14}}} = 3.881$$

The paired t-test (e)

- Procedure is two-tailed, and you can assume a significance level of 5%.
- It is also a small sample procedure (n=14, hence df = 13).
- The Critical value is determined from statistical tables (2.1603).
- Decision Rule: Reject Null Hypothesis ($|TS| > CV$).

The paired t-test (f)

Alternative Approach : using p-values.

- The p-values are determined from computer code. (We will use a software called `R`. Other types of software inlcude `SAS` and `SPSS`.)
- The null and alternative are as before.
- The computer software automatically generates the appropriate test statistic, and hence the corresponding p-value.
- The user then interprets the p-values. If p-value is small, reject the null hypothesis. If the p-value is large, the appropriate conclusion is a failure to reject H_0.
- The threshold for being considered small is less than α/k, (usually 0.0250). This is a very arbitrary choice of threshold.

The paired t-test (g)

Implementing the paired t-test using R for the example previously discussed.

```
> t.test(Before,After,paired=TRUE)

        Paired t-test

data:  Before and After
t = 3.8881, df = 13, p-value = 0.001868
alternative hypothesis: true difference in means is not 0
95 percent confidence interval:
  3.650075 12.778496
sample estimates:
mean of the differences
               8.214286
```

The paired t-test (h)

- The p-value (0.001868) is less than the threshold is less than the threshold 0.0250.
- We reject the null hypothesis (mean of case-wise differences being zero, i.e. expect no difference between "before" and "after").
- We conclude that there is a difference between 'before' and 'after'.
- That is to say, we can expected a difference between two paired measurements.

The paired t-test (i)

- We could also consider the confidence interval. We are 95% confident that the expected value of the case-wise difference is at least 3.65.
- Here the null value (i.e. 0) is not within the range of the confidence limits.
- Therefore we reject the null hypothesis.

```
> t.test(Before,After,paired=TRUE)
...
...
95 percent confidence interval:
  3.650075 12.778496
...
```

Test for Equality of Variance (a)

- In this procedure, we determine whether or not two data sets have the same variance.
- The assumption of equal variance underpins several inference procedures.
- We will not get into too much detail about that, but concentrate on how to assess equality of variance.
- The null and alternative hypotheses are as follows:

$$H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_1 : \sigma_1^2 \neq \sigma_2^2$$

Test for Equality of Variance (b)

- When using R it would be convenient to consider the null and alternative in terms of variance ratios.
- Two data sets have equal variance if the variance ration is 1.

$$H_0 : \sigma_1^2/\sigma_2^2 = 1$$
$$H_1 : \sigma_1^2/\sigma_2^2 \neq 1$$

Test for Equality of Variance(c)

You would be required to compute the test statistic for this procedure. The test statistic is the ratio of the variances for both data sets.

$$TS = \frac{s_x^2}{s_y^2}$$

The standard deviations would be provided in the `R` code.

```
> sd(x)
[1] 3.40098
> sd(y)
[1] 4.630815
```

To compute the test statistic.

$$TS = \frac{3.40^2}{4.63^2} = \frac{11.56}{21.43} = 0.5394$$

Variance Test (d)

```
> var.test(x,y)

        F test to compare two variances

data:  x and y
F = 0.5394, num df = 9, denom df = 8, p-value = 0.3764
alternative hypothesis: true ratio of variances is not equal t
95 percent confidence interval:
 0.1237892 2.2125056
sample estimates:
ratio of variances
         0.5393782
```

Variance Test (e)

- The p-value is 0.3764, above the threshold level of 0.0250.
- We fail to reject the null hypothesis.
- We can assume that there is no significant difference in sample size.
- Additionally the 95% confidence interval (0.1237, 2.2125) contains the null values i.e. 1.

Shapiro-Wilk Test(a)

- We will often be required to determine whether or not a data set is normally distributed.
- Again, this assumption underpins many statistical models.
- The null hypothesis is that the data set is normally distributed.
- The alternative hypothesis is that the data set is not normally distributed.
- One procedure for testing these hypotheses is the Shapiro-Wilk test, implemented in `R` using the command `shapiro.test()`.
- (Remark: You will not be required to compute the test statistic for this test.)

Shapiro Wilk Test(b)

For the data set used previously; x and y, we use the Shapiro-Wilk test to determine that both data sets are normally distributed.

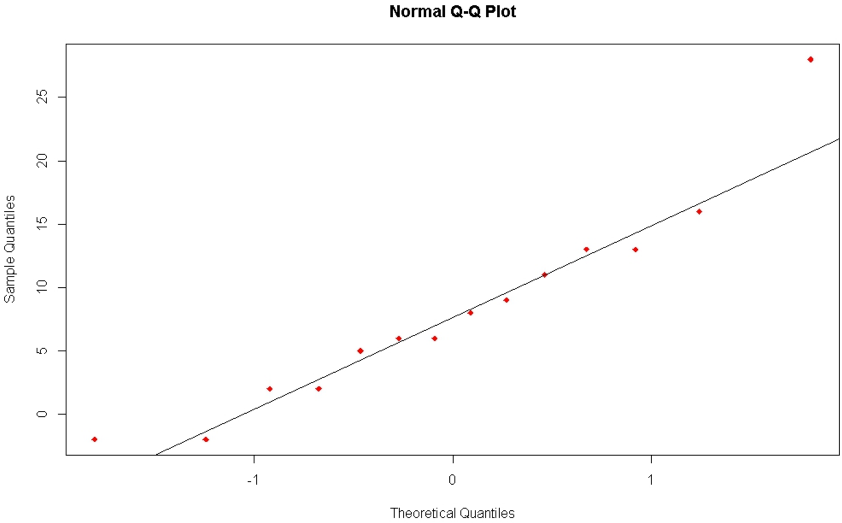
```
> shapiro.test(x)

        Shapiro-Wilk normality test

data:  x
W = 0.9474, p-value = 0.6378

> shapiro.test(y)

        Shapiro-Wilk normality test

data:  y
W = 0.9347, p-value = 0.5273
```

Graphical Procedures for assessing Normality

- The Q-Q plot is a very useful tool for determining whether or not a data set is normally distributed
- Interpretation is simple. If the points follow the trendline (provided by the second line of `R` code `qqline`).
- One should expect minor deviations. Numerous major deviations would lead the analyst to conclude that the data set is not normally distributed.
- The Q-Q plot is best used in conjunction with a formal procedure such as the Shapiro-Wilk test.

```
>qqnorm(CWdiff)
>qqline(CWdiff)
```

Graphical Procedures for Assessing Normality



Graphical Procedures for Determining an Outlier

The Grubbs test is used to determine if there are any outliers in a data set. The definition of outlier used for this procedure is a value that unusually distance from the rest of the values. Consider the following data set: is the lowest value 4.01 an outlier.

```
6.98 8.49 7.97 6.64
8.80 8.48 5.94 6.94
6.89 7.47 7.32 4.01
```

Under the null hypothesis, there are no outliers present in the data set. We reject this hypothesis if the p-value is sufficiently small.

Graphical Procedures for Determining an Outlier

```
> grubbs.test(x, two.sided=T)
Grubbs test for one outlier
data: x
G = 2.4093, U = 0.4243, p-value = 0.05069
alternative hypothesis: lowest value 4.01 is an outlier
```