This paper is not to be removed from the Examination Halls

UNIVERSITY OF LONDON **ST104A ZB**

BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diplomas in Economics and Social Sciences and Access Route

Statistics 1

Friday, 03 May 2013 : 10.00am to 12.00pm

Candidates should answer **THREE** of the following **FOUR** questions: **QUESTION 1** of Section A (50 marks) and **TWO** questions from Section B (25 marks each). **Candidates are strongly advised to divide their time accordingly.**

A list of formulae and extracts from statistical tables are provided after the final question on this paper.

Graph paper is provided at the end of this question paper. If used, it must be detached and fastened securely inside the answer book.

A calculator may be used when answering questions on this paper and it must comply in all respects with the specification given with your Admission Notice. The make and type of machine must be clearly stated on the front cover of the answer book.

PLEASE TURN OVER

SECTION A

Answer **all** parts of Question 1 (50 marks in total).

1. (a) Classify each one of the following variables as measurable (continuous) or categorical. If a variable is categorical, further classify it as nominal or ordinal. Justify your answer. (*Note that no marks will be awarded without justification.*)

   i. Rank of a university according to its reputation.
   ii. Country of residence.
   iii. Birth-weight of a baby.
   iv. Favourite pop group.

   [8 marks]

   (b) The table below contains the marks (out of 20) of all students taking an examination for the same course in two years:

2011	10	9	19	9	10	9			
2012	10	11	9	11	10	11	12	11	10

   i. Find the mean mark and the median mark for each year.
   ii. Calculate the range of the marks for each year and give an explanation for any differences you find.
   iii. Calculate the standard deviation of the marks for each year and give an explanation for any differences you find.
   iv. Comment on the differences in the mean and median for the two years that you found in part i. For this data set, which do you think would give a better description of the difference in marks: the mean or the median? Explain briefly.

   [12 marks]

   (c) Weekly household expenditure in country A is normally distributed with a mean of £300 per week and a standard deviation of £100 per week. In country B it is also normally distributed but with a mean of £240 per week and a standard deviation of £50 per week. Which country has a higher proportion of households spending less than £200? [4 marks]

   (d) We would like to start an internet service provider and need to estimate the average weekly internet usage of households for our business plan. Internet usage is measured in minutes. How many households must we randomly select to be 95 percent confident that the sample mean is within 2 minutes of the population mean? Assume that a previous survey of household usage has shown that the standard deviation of internet usage is 6.95 minutes. [3 marks]

(e) Suppose that $x_1 = 2$, $x_2 = -3$, $x_3 = 6$, $x_4 = 0$, $x_5 = 3$, and $y_1 = 3$, $y_2 = 2$, $y_3 = 1$, $y_4 = 0$, $y_5 = 1$. Calculate the following quantities:

i. $\sum_{i=1}^{5} x_i$ ii. $\sum_{i=2}^{5} 2x_i(y_i + 1)$ iii. $x_2^2 + \sum_{i=1}^{3}(x_i + y_i^3)$

[6 marks]

(f) In an introductory statistics class, the numbers of males and females are 17 and 23, respectively.

i. A student is selected randomly from the class. What is the probability the student is female?

ii. A student is selected at random and removed from the class. A second student is then selected. What is the probability that one of the students is male and the other is female?

iii. What is the probability that the second student is male, given that the first student is female and removed from the class?

iv. In previous years it was found that 80% of males pass the exam and 85% of females pass the examination. Based on the available information, find the probability that a student who passes the exam is female.

[8 marks]

(g) State whether the following are true or false and give a brief explanation. (*Note that no marks will be awarded for a simple true/false answer.*)

i. An important difference between an experimental design and an observational study is that in an observational study data are collected on units without any intervention.

ii. If two variables are correlated we can conclude that one causes the other.

iii. If a variable has a symmetric distribution, its mean and median are the same.

[6 marks]

(h) In the context of sampling, explain the difference between item non-response and unit non-response. [3 marks]

SECTION B

Answer **two** questions from this section (25 marks each).

2. (a) The 2006 General Social Survey in the United States asked subjects, 'Would you say that astrology is very scientific, sort of scientific, or not at all scientific?' The table below cross-classifies their responses with their highest level of education.

	Astrology is scientific			
Highest degree	Very	Sort of	Not at all	Total
Less than High school	23 (11%)	84 (41%)	98 (48%)	205 (100%)
High school	50 (5%)	286 (31%)	574 (63%)	910 (100%)
College or higher	16 (2%)	124 (18%)	538 (79%)	678 (100%)
Total	89 (5%)	494 (28%)	1210 (67%)	1793 (100%)

i. Based on the data in the table, and *without doing a significance test*, how would you describe the relationship between education and opinion on whether or not astrology is scientific? [4 marks]

ii. Calculate the χ^2 statistic and use it to test for independence, using a 5% significance level. What do you conclude? [9 marks]

(b) i. Define each of the following:

- Simple random sampling
- Stratified random sampling.

[4 marks]

ii. Why might a researcher prefer to take a stratified random sample rather than a simple random sample? Give two reasons. [3 marks]

iii. You have been asked to design a nation-wide survey in your country to find out about the smoking habits of adults. Give two stratification factors you might use, and explain why you have chosen them. [5 marks]

3. The level of infant mortality (y) is represented by the number of baby deaths for every 1000 births. For 12 areas these are shown in the following table. For each area, the percentage (x) of babies born into families earning at least £25,000 is also shown.

Area	A	B	C	D	E	F	G	H	I	J	K	L
Percentage (x)	19	5	9	20	11	35	5	18	25	12	20	15
Infant mortality (y)	3	15	14	6	13	3	23	10	9	9	5	10

The summary statistics for these data are:

Sum of x data: 194	Sum of the squares of x data: 3956
Sum of y data: 120	Sum of the squares of y data: 1560
Sum of the products of x and y data: 1504	

(a) i. Draw a scatter diagram of these data on the graph paper provided. Label the diagram carefully. [4 marks]

ii. Calculate the sample correlation coefficient. Interpret your findings. [3 marks]

iii. Calculate the least squares line of y on x and draw the line on the scatter diagram. [4 marks]

iv. Using the equation you found in iii., obtain the predicted infant mortality for an area where 34% of babies are born into families earning at least £25,000. Do you think this value is realistic? Justify your answer. [2 marks]

(b) A survey is conducted to compare public attitudes towards local policing. A number of people in two areas of interest are sampled, and asked if they are satisfied with their local police-community relationship. The results of this survey are shown in the following table.

	Sample size	Number satisfied
Area A	153	115
Area B	188	120

i. You are asked to consider an appropriate hypothesis test to determine whether there is a difference between the two areas in the proportion who are satisfied. Test at two appropriate significance levels and comment on your findings. Specify the test statistic you use and its distribution under the null hypothesis. [7 marks]

ii. State clearly any other assumptions you make. [2 marks]

iii. Give a 98% confidence interval for the proportion of people in Areas A and B combined who are satisfied, assuming the respective sample sizes are proportional to population sizes. [3 marks]

4. (a) i. Carefully construct a box plot on the graph paper provided to display the following yearly incomes of a group of people, measured in £1000:

3 2 4 8 7 19 2 5 3 4 10 12

[8 marks]

ii. Based on the shape of the box plot you have drawn, describe the distribution of the data. [2 marks]

iii. Name two other types of graphical displays that would be suitable to represent the data. Briefly explain your choices. [3 marks]

(b) A new fitness programme is devised for obese people. Each participant's weight in kg was measured before and after the program to see if the fitness program is effective in reducing their weights. The following data were obtained:

Before	After
145	143
116	120
120	118
133	130
119	119
133	134
125	128
126	123
140	141

i. Carry out an appropriate hypothesis test to determine whether the fitness programme is effective for reducing weight. State the test hypotheses, and specify your test statistic and its distribution under the null hypothesis. Comment on your findings. [6 marks]

ii. State any assumptions you made. [2 marks]

iii. Give an 80% confidence interval for the difference in means. [2 marks]

iv. On the basis of the data alone, would you recommend the programme to a friend who wants to lose weight? Explain why or why not. [2 marks]

END OF PAPER

ST104a **Statistics 1**

Examination Formula Sheet

Expected value of a discrete random variable:

$$\mu = E[X] = \sum_{i=1}^{N} p_i x_i$$

Standard deviation of a discrete random variable:

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{i=1}^{N} p_i (x_i - \mu)^2}$$

The transformation formula:

$$Z = \frac{X - \mu}{\sigma}$$

Finding Z for the sampling distribution of the sample mean:

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

Finding Z for the sampling distribution of the sample proportion:

$$Z = \frac{P - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$$

Confidence interval endpoints for a single mean (σ known):

$$\bar{x} \pm z \frac{\sigma}{\sqrt{n}}$$

Confidence interval endpoints for a single mean (σ unknown):

$$\bar{x} \pm t_{n-1} \frac{s}{\sqrt{n}}$$

Confidence interval endpoints for a single proportion:

$$p \pm z \sqrt{\frac{p(1-p)}{n}}$$

Sample size determination for a mean:

$$n \geq \frac{Z^2 \sigma^2}{e^2}$$

Sample size determination for a proportion:

$$n \geq \frac{Z^2 p(1-p)}{e^2}$$

Z-test of hypothesis for a single mean (σ known):

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

t-test of hypothesis for a single mean (σ unknown):

$$t = \frac{\bar{X} - \mu}{S / \sqrt{n}}$$

Z-test of hypothesis for a single proportion:

$$Z \cong \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$$

Z-test for the difference between two means (variances known):

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

t-test for the difference between two means (variances unknown):

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Confidence interval endpoints for the difference between two means:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{n_1+n_2-2} \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Pooled variance estimator:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

t-test for the difference in means in paired samples:

$$t = \frac{\bar{X}_d - \mu_d}{S_d/\sqrt{n}}$$

Confidence interval endpoints for the difference in means in paired samples:

$$\bar{x}_d \pm t_{n-1} \frac{s_d}{\sqrt{n}}$$

Z-test for the difference between two proportions:

$$Z = \frac{(P_1 - P_2) - (\pi_1 - \pi_2)}{\sqrt{P(1-P)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Pooled proportion estimator:

$$P = \frac{R_1 + R_2}{n_1 + n_2}$$

Confidence interval endpoints for the difference between two proportions:

$$(p_1 - p_2) \pm z \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

χ^2 test of association:

$$\sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Sample correlation coefficient:

$$r = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sqrt{\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)\left(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2\right)}}$$

Spearman rank correlation:

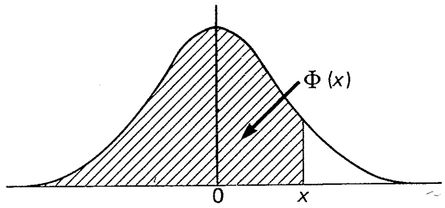
$$r_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2 - 1)}$$

Simple linear regression line estimates:

$$\begin{aligned} b &= \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2} \\ a &= \bar{y} - b\bar{x} \end{aligned}$$

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
3·075			0·9994		0·99990		0·99995
	0·9990	3·263		3·731		3·916	
3·105			0·9995		0·99991		0·99996
	0·9991	3·320		3·759		3·976	
3·138			0·9996		0·99992		0·99997
	0·9992	3·389		3·791		4·055	
3·174			0·9997		0·99993		0·99998
	0·9993	3·480		3·826		4·173	
3·215			0·9998		0·99994		0·99999
	0·9994	3·615		3·867		4·417	
			0·9999		0·99995		1·00000

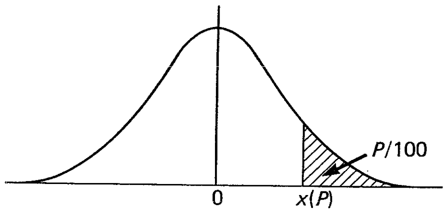
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \dfrac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\dfrac{1}{x^2}+\dfrac{3}{x^4}-\dfrac{15}{x^6}+\dfrac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

TABLE 7. THE χ^2-DISTRIBUTION FUNCTION

The function tabulated is

$$F_\nu(x) = \frac{1}{2^{\nu/2}\Gamma(\frac{\nu}{2})}\int_0^x t^{\frac{1}{2}\nu-1}\, e^{-\frac{1}{2}t}\, dt$$

for integer $\nu \leqslant 25$. $F_\nu(x)$ is the probability that a random variable X, distributed as χ^2 with ν degrees of freedom, will be less than or equal to x. Note that $F_1(x) = 2\Phi(x^{\frac{1}{2}})-1$ (cf. Table 4). For certain values of x and $\nu > 25$ use may be made of the following relation between the χ^2- and Poisson distributions:

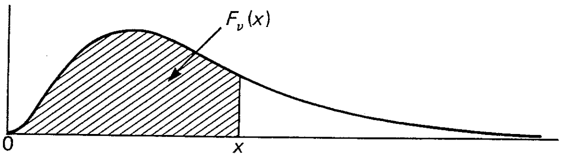
$$F_\nu(x) = 1 - F(\tfrac{1}{2}\nu - 1 \,|\, \tfrac{1}{2}x)$$

where $F(r|\mu)$ is the Poisson distribution function (see Table 2). If $\nu > 25$, X is approximately normally distributed with mean ν and variance 2ν. A better approximation is usually obtained by using the formula

$$F_\nu(x) \doteqdot \Phi(\sqrt{2x} - \sqrt{2\nu - 1})$$

where $\Phi(s)$ is the normal distribution function (see Table 4).

Omitted entries to the left and right of tabulated values are 1 and 0 respectively (to four decimal places).



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

$\nu =$	1	$\nu =$	1	$\nu =$	2	$\nu =$	2	$\nu =$	3	$\nu =$	3
$x =$ 0·0	0·0000	$x =$ 4·0	0·9545	$x =$ 0·0	0·0000	$x =$ 4·0	0·8647	$x =$ 0·0	0·0000	$x =$ 4·0	0·7385
·1	·2482	·1	·9571	·1	·0488	·1	·8713	·1	·0082	·2	·7593
·2	·3453	·2	·9596	·2	·0952	·2	·8775	·2	·0224	·4	·7786
·3	·4161	·3	·9619	·3	·1393	·3	·8835	·3	·0400	·6	·7965
·4	·4729	·4	·9641	·4	·1813	·4	·8892	·4	·0598	·8	·8130
0·5	0·5205	4·5	0·9661	0·5	0·2212	4·5	0·8946	0·5	0·0811	5·0	0·8282
·6	·5614	·6	·9680	·6	·2592	·6	·8997	·6	·1036	·2	·8423
·7	·5972	·7	·9698	·7	·2953	·7	·9046	·7	·1268	·4	·8553
·8	·6289	·8	·9715	·8	·3297	·8	·9093	·8	·1505	·6	·8672
·9	·6572	·9	·9731	·9	·3624	·9	·9137	·9	·1746	·8	·8782
1·0	0·6827	5·0	0·9747	1·0	0·3935	5·0	0·9179	1·0	0·1987	6·0	0·8884
·1	·7057	·1	·9761	·1	·4231	·1	·9219	·1	·2229	·2	·8977
·2	·7267	·2	·9774	·2	·4512	·2	·9257	·2	·2470	·4	·9063
·3	·7458	·3	·9787	·3	·4780	·3	·9293	·3	·2709	·6	·9142
·4	·7633	·4	·9799	·4	·5034	·4	·9328	·4	·2945	·8	·9214
1·5	0·7793	5·5	0·9810	1·5	0·5276	5·5	0·9361	1·5	0·3177	7·0	0·9281
·6	·7941	·6	·9820	·6	·5507	·6	·9392	·6	·3406	·2	·9342
·7	·8077	·7	·9830	·7	·5726	·7	·9422	·7	·3631	·4	·9398
·8	·8203	·8	·9840	·8	·5934	·8	·9450	·8	·3851	·6	·9450
·9	·8319	·9	·9849	·9	·6133	·9	·9477	·9	·4066	·8	·9497
2·0	0·8427	6·0	0·9857	2·0	0·6321	6·0	0·9502	2·0	0·4276	8·0	0·9540
·1	·8527	·1	·9865	·1	·6501	·2	·9550	·1	·4481	·2	·9579
·2	·8620	·2	·9872	·2	·6671	·4	·9592	·2	·4681	·4	·9616
·3	·8706	·3	·9879	·3	·6834	·6	·9631	·3	·4875	·6	·9649
·4	·8787	·4	·9886	·4	·6988	·8	·9666	·4	·5064	·8	·9679
2·5	0·8862	6·5	0·9892	2·5	0·7135	7·0	0·9698	2·5	0·5247	9·0	0·9707
·6	·8931	·6	·9898	·6	·7275	·2	·9727	·6	·5425	·2	·9733
·7	·8997	·7	·9904	·7	·7408	·4	·9753	·7	·5598	·4	·9756
·8	·9057	·8	·9909	·8	·7534	·6	·9776	·8	·5765	·6	·9777
·9	·9114	·9	·9914	·9	·7654	·8	·9798	·9	·5927	·8	·9797
3·0	0·9167	7·0	0·9918	3·0	0·7769	8·0	0·9817	3·0	0·6084	10·0	0·9814
·1	·9217	·1	·9923	·1	·7878	·2	·9834	·1	·6235	·2	·9831
·2	·9264	·2	·9927	·2	·7981	·4	·9850	·2	·6382	·4	·9845
·3	·9307	·3	·9931	·3	·8080	·6	·9864	·3	·6524	·6	·9859
·4	·9348	·4	·9935	·4	·8173	·8	·9877	·4	·6660	·8	·9871
3·5	0·9386	7·5	0·9938	3·5	0·8262	9·0	0·9889	3·5	0·6792	11·0	0·9883
·6	·9422	·6	·9942	·6	·8347	·2	·9899	·6	·6920	·2	·9893
·7	·9456	·7	·9945	·7	·8428	·4	·9909	·7	·7043	·4	·9903
·8	·9487	·8	·9948	·8	·8504	·6	·9918	·8	·7161	·6	·9911
·9	·9517	·9	·9951	·9	·8577	·8	·9926	·9	·7275	·8	·9919
4·0	0·9545	8·0	0·9953	4·0	0·8647	10·0	0·9933	4·0	0·7385	12·0	0·9926

TABLE 7. THE χ^2-DISTRIBUTION FUNCTION

$\nu =$	4	5	6	7	8	9	10	11	12	13	14
$x =$ 0·5	0·0265	0·0079	0·0022	0·0006	0·0001						
1·0	·0902	·0374	·0144	·0052	·0018	0·0006	0·0002	0·0001			
1·5	·1734	·0869	·0405	·0177	·0073	·0029	·0011	·0004	0·0001		
2·0	·2642	·1509	·0803	·0402	·0190	·0085	·0037	·0015	·0006	0·0002	0·0001
2·5	0·3554	0·2235	0·1315	0·0729	0·0383	0·0191	0·0091	0·0042	0·0018	0·0008	0·0003
3·0	·4422	·3000	·1912	·1150	·0656	·0357	·0186	·0093	·0045	·0021	·0009
3·5	·5221	·3766	·2560	·1648	·1008	·0589	·0329	·0177	·0091	·0046	·0022
4·0	·5940	·4506	·3233	·2202	·1429	·0886	·0527	·0301	·0166	·0088	·0045
4·5	·6575	·5201	·3907	·2793	·1906	·1245	·0780	·0471	·0274	·0154	·0084
5·0	0·7127	0·5841	0·4562	0·3400	0·2424	0·1657	0·1088	0·0688	0·0420	0·0248	0·0142
5·5	·7603	·6421	·5185	·4008	·2970	·2113	·1446	·0954	·0608	·0375	·0224
6·0	·8009	·6938	·5768	·4603	·3528	·2601	·1847	·1266	·0839	·0538	·0335
6·5	·8352	·7394	·6304	·5173	·4086	·3110	·2283	·1620	·1112	·0739	·0477
7·0	·8641	·7794	·6792	·5711	·4634	·3629	·2746	·2009	·1424	·0978	·0653
7·5	0·8883	0·8140	0·7229	0·6213	0·5162	0·4148	0·3225	0·2427	0·1771	0·1254	0·0863
8·0	·9084	·8438	·7619	·6674	·5665	·4659	·3712	·2867	·2149	·1564	·1107
8·5	·9251	·8693	·7963	·7094	·6138	·5154	·4199	·3321	·2551	·1904	·1383
9·0	·9389	·8909	·8264	·7473	·6577	·5627	·4679	·3781	·2971	·2271	·1689
9·5	·9503	·9093	·8527	·7813	·6981	·6075	·5146	·4242	·3403	·2658	·2022
10·0	0·9596	0·9248	0·8753	0·8114	0·7350	0·6495	0·5595	0·4696	0·3840	0·3061	0·2378
10·5	·9672	·9378	·8949	·8380	·7683	·6885	·6022	·5140	·4278	·3474	·2752
11·0	·9734	·9486	·9116	·8614	·7983	·7243	·6425	·5567	·4711	·3892	·3140
11·5	·9785	·9577	·9259	·8818	·8251	·7570	·6801	·5976	·5134	·4310	·3536
12·0	·9826	·9652	·9380	·8994	·8488	·7867	·7149	·6364	·5543	·4724	·3937
12·5	0·9860	0·9715	0·9483	0·9147	0·8697	0·8134	0·7470	0·6727	0·5936	0·5129	0·4338
13·0	·9887	·9766	·9570	·9279	·8882	·8374	·7763	·7067	·6310	·5522	·4735
13·5	·9909	·9809	·9643	·9392	·9042	·8587	·8030	·7381	·6662	·5900	·5124
14·0	·9927	·9844	·9704	·9488	·9182	·8777	·8270	·7670	·6993	·6262	·5503
14·5	·9941	·9873	·9755	·9570	·9304	·8944	·8486	·7935	·7301	·6604	·5868
15·0	0·9953	0·9896	0·9797	0·9640	0·9409	0·9091	0·8679	0·8175	0·7586	0·6926	0·6218
15·5	·9962	·9916	·9833	·9699	·9499	·9219	·8851	·8393	·7848	·7228	·6551
16·0	·9970	·9932	·9862	·9749	·9576	·9331	·9004	·8589	·8088	·7509	·6866
16·5	·9976	·9944	·9887	·9791	·9642	·9429	·9138	·8764	·8306	·7768	·7162
17·0	·9981	·9955	·9907	·9826	·9699	·9513	·9256	·8921	·8504	·8007	·7438
17·5	0·9985	0·9964	0·9924	0·9856	0·9747	0·9586	0·9360	0·9061	0·8683	0·8226	0·7695
18·0	·9988	·9971	·9938	·9880	·9788	·9648	·9450	·9184	·8843	·8425	·7932
18·5	·9990	·9976	·9949	·9901	·9822	·9702	·9529	·9293	·8987	·8606	·8151
19·0	·9992	·9981	·9958	·9918	·9851	·9748	·9597	·9389	·9115	·8769	·8351
19·5	·9994	·9984	·9966	·9932	·9876	·9787	·9656	·9473	·9228	·8916	·8533
20	0·9995	0·9988	0·9972	0·9944	0·9897	0·9821	0·9707	0·9547	0·9329	0·9048	0·8699
21	·9997	·9992	·9982	·9962	·9929	·9873	·9789	·9666	·9496	·9271	·8984
22	·9998	·9995	·9988	·9975	·9951	·9911	·9849	·9756	·9625	·9446	·9214
23	·9999	·9997	·9992	·9983	·9966	·9938	·9893	·9823	·9723	·9583	·9397
24	·9999	·9998	·9995	·9989	·9977	·9957	·9924	·9873	·9797	·9689	·9542
25	0·9999	0·9999	0·9997	0·9992	0·9984	0·9970	0·9947	0·9909	0·9852	0·9769	0·9654
26		·9999	·9998	·9995	·9989	·9980	·9963	·9935	·9893	·9830	·9741
27		·9999	·9999	·9997	·9993	·9986	·9974	·9954	·9923	·9876	·9807
28			·9999	·9998	·9995	·9990	·9982	·9968	·9945	·9910	·9858
29			·9999	·9999	·9997	·9994	·9988	·9977	·9961	·9935	·9895
30				0·9999	0·9998	0·9996	0·9991	0·9984	0·9972	0·9953	0·9924

TABLE 7. THE χ^2-DISTRIBUTION FUNCTION

$\nu =$	15	16	17	18	19	20	21	22	23	24	25
$x = 3$	0·0004	0·0002	0·0001								
4	·0023	·0011	·0005	0·0002	0·0001						
5	0·0079	0·0042	0·0022	0·0011	0·0006	0·0003	0·0001	0·0001			
6	·0203	·0119	·0068	·0038	·0021	·0011	·0006	·0003	0·0001	0·0001	
7	·0424	·0267	·0165	·0099	·0058	·0033	·0019	·0010	·0005	·0003	0·0001
8	·0762	·0511	·0335	·0214	·0133	·0081	·0049	·0028	·0016	·0009	·0005
9	·1225	·0866	·0597	·0403	·0265	·0171	·0108	·0067	·0040	·0024	·0014
10	0·1803	0·1334	0·0964	0·0681	0·0471	0·0318	0·0211	0·0137	0·0087	0·0055	0·0033
11	·2474	·1905	·1434	·1056	·0762	·0538	·0372	·0253	·0168	·0110	·0071
12	·3210	·2560	·1999	·1528	·1144	·0839	·0604	·0426	·0295	·0201	·0134
13	·3977	·3272	·2638	·2084	·1614	·1226	·0914	·0668	·0480	·0339	·0235
14	·4745	·4013	·3329	·2709	·2163	·1695	·1304	·0985	·0731	·0533	·0383
15	0·5486	0·4754	0·4045	0·3380	0·2774	0·2236	0·1770	0·1378	0·1054	0·0792	0·0586
16	·6179	·5470	·4762	·4075	·3427	·2834	·2303	·1841	·1447	·1119	·0852
17	·6811	·6144	·5456	·4769	·4101	·3470	·2889	·2366	·1907	·1513	·1182
18	·7373	·6761	·6112	·5443	·4776	·4126	·3510	·2940	·2425	·1970	·1576
19	·7863	·7313	·6715	·6082	·5432	·4782	·4149	·3547	·2988	·2480	·2029
20	0·8281	0·7798	0·7258	0·6672	0·6054	0·5421	0·4787	0·4170	0·3581	0·3032	0·2532
21	·8632	·8215	·7737	·7206	·6632	·6029	·5411	·4793	·4189	·3613	·3074
22	·8922	·8568	·8153	·7680	·7157	·6595	·6005	·5401	·4797	·4207	·3643
23	·9159	·8863	·8507	·8094	·7627	·7112	·6560	·5983	·5392	·4802	·4224
24	·9349	·9105	·8806	·8450	·8038	·7576	·7069	·6528	·5962	·5384	·4806
25	0·9501	0·9302	0·9053	0·8751	0·8395	0·7986	0·7528	0·7029	0·6497	0·5942	0·5376
26	·9620	·9460	·9255	·9002	·8698	·8342	·7936	·7483	·6991	·6468	·5924
27	·9713	·9585	·9419	·9210	·8953	·8647	·8291	·7888	·7440	·6955	·6441
28	·9784	·9684	·9551	·9379	·9166	·8906	·8598	·8243	·7842	·7400	·6921
29	·9839	·9761	·9655	·9516	·9340	·9122	·8860	·8551	·8197	·7799	·7361
30	0·9881	0·9820	0·9737	0·9626	0·9482	0·9301	0·9080	0·8815	0·8506	0·8152	0·7757
31	·9912	·9865	·9800	·9712	·9596	·9448	·9263	·9039	·8772	·8462	·8110
32	·9936	·9900	·9850	·9780	·9687	·9567	·9414	·9226	·8999	·8730	·8420
33	·9953	·9926	·9887	·9833	·9760	·9663	·9538	·9381	·9189	·8959	·8689
34	·9966	·9946	·9916	·9874	·9816	·9739	·9638	·9509	·9348	·9153	·8921
35	0·9975	0·9960	0·9938	0·9905	0·9860	0·9799	0·9718	0·9613	0·9480	0·9316	0·9118
36	·9982	·9971	·9954	·9929	·9894	·9846	·9781	·9696	·9587	·9451	·9284
37	·9987	·9979	·9966	·9948	·9921	·9883	·9832	·9763	·9675	·9562	·9423
38	·9991	·9985	·9975	·9961	·9941	·9911	·9871	·9817	·9745	·9653	·9537
39	·9994	·9989	·9982	·9972	·9956	·9933	·9902	·9859	·9802	·9727	·9632
40	0·9995	0·9992	0·9987	0·9979	0·9967	0·9950	0·9926	0·9892	0·9846	0·9786	0·9708
41	·9997	·9994	·9991	·9985	·9976	·9963	·9944	·9918	·9882	·9833	·9770
42	·9998	·9996	·9993	·9989	·9982	·9972	·9958	·9937	·9909	·9871	·9820
43	·9998	·9997	·9995	·9992	·9987	·9980	·9969	·9953	·9931	·9901	·9860
44	·9999	·9998	·9997	·9994	·9991	·9985	·9977	·9965	·9947	·9924	·9892
45	0·9999	0·9999	0·9998	0·9996	0·9993	0·9989	0·9983	0·9973	0·9960	0·9942	0·9916
46	·9999	·9999	·9998	·9997	·9995	·9992	·9987	·9980	·9970	·9956	·9936
47		·9999	·9999	·9998	·9996	·9994	·9991	·9985	·9978	·9967	·9951
48			·9999	·9998	·9997	·9996	·9993	·9989	·9983	·9975	·9963
49			·9999	·9999	·9998	·9997	·9995	·9992	·9988	·9981	·9972
50				0·9999	0·9999	0·9998	0·9996	0·9994	0·9991	0·9986	0·9979

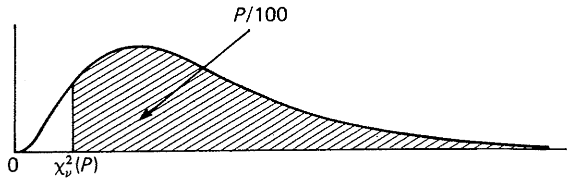
TABLE 8. PERCENTAGE POINTS OF THE χ^2-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})} \int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1}\, e^{-\frac{1}{2}x}\, dx.$$

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu-1}$ and unit variance.



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

P	99·95	99·9	99·5	99	97·5	95	90	80	70	60
$\nu = 1$	$0{\cdot}0^{6}3927$	$0{\cdot}0^{5}1571$	$0{\cdot}0^{4}3927$	$0{\cdot}0^{3}1571$	$0{\cdot}0^{3}9821$	0·003932	0·01579	0·06418	0·1485	0·2750
2	0·001000	0·002001	0·01003	0·02010	0·05064	0·1026	0·2107	0·4463	0·7133	1·022
3	0·01528	0·02430	0·07172	0·1148	0·2158	0·3518	0·5844	1·005	1·424	1·869
4	0·06392	0·09080	0·2070	0·2971	0·4844	0·7107	1·064	1·649	2·195	2·753
5	0·1581	0·2102	0·4117	0·5543	0·8312	1·145	1·610	2·343	3·000	3·655
6	0·2994	0·3811	0·6757	0·8721	1·237	1·635	2·204	3·070	3·828	4·570
7	0·4849	0·5985	0·9893	1·239	1·690	2·167	2·833	3·822	4·671	5·493
8	0·7104	0·8571	1·344	1·646	2·180	2·733	3·490	4·594	5·527	6·423
9	0·9717	1·152	1·735	2·088	2·700	3·325	4·168	5·380	6·393	7·357
10	1·265	1·479	2·156	2·558	3·247	3·940	4·865	6·179	7·267	8·295
11	1·587	1·834	2·603	3·053	3·816	4·575	5·578	6·989	8·148	9·237
12	1·934	2·214	3·074	3·571	4·404	5·226	6·304	7·807	9·034	10·18
13	2·305	2·617	3·565	4·107	5·009	5·892	7·042	8·634	9·926	11·13
14	2·697	3·041	4·075	4·660	5·629	6·571	7·790	9·467	10·82	12·08
15	3·108	3·483	4·601	5·229	6·262	7·261	8·547	10·31	11·72	13·03
16	3·536	3·942	5·142	5·812	6·908	7·962	9·312	11·15	12·62	13·98
17	3·980	4·416	5·697	6·408	7·564	8·672	10·09	12·00	13·53	14·94
18	4·439	4·905	6·265	7·015	8·231	9·390	10·86	12·86	14·44	15·89
19	4·912	5·407	6·844	7·633	8·907	10·12	11·65	13·72	15·35	16·85
20	5·398	5·921	7·434	8·260	9·591	10·85	12·44	14·58	16·27	17·81
21	5·896	6·447	8·034	8·897	10·28	11·59	13·24	15·44	17·18	18·77
22	6·404	6·983	8·643	9·542	10·98	12·34	14·04	16·31	18·10	19·73
23	6·924	7·529	9·260	10·20	11·69	13·09	14·85	17·19	19·02	20·69
24	7·453	8·085	9·886	10·86	12·40	13·85	15·66	18·06	19·94	21·65
25	7·991	8·649	10·52	11·52	13·12	14·61	16·47	18·94	20·87	22·62
26	8·538	9·222	11·16	12·20	13·84	15·38	17·29	19·82	21·79	23·58
27	9·093	9·803	11·81	12·88	14·57	16·15	18·11	20·70	22·72	24·54
28	9·656	10·39	12·46	13·56	15·31	16·93	18·94	21·59	23·65	25·51
29	10·23	10·99	13·12	14·26	16·05	17·71	19·77	22·48	24·58	26·48
30	10·80	11·59	13·79	14·95	16·79	18·49	20·60	23·36	25·51	27·44
32	11·98	12·81	15·13	16·36	18·29	20·07	22·27	25·15	27·37	29·38
34	13·18	14·06	16·50	17·79	19·81	21·66	23·95	26·94	29·24	31·31
36	14·40	15·32	17·89	19·23	21·34	23·27	25·64	28·73	31·12	33·25
38	15·64	16·61	19·29	20·69	22·88	24·88	27·34	30·54	32·99	35·19
40	16·91	17·92	20·71	22·16	24·43	26·51	29·05	32·34	34·87	37·13
50	23·46	24·67	27·99	29·71	32·36	34·76	37·69	41·45	44·31	46·86
60	30·34	31·74	35·53	37·48	40·48	43·19	46·46	50·64	53·81	56·62
70	37·47	39·04	43·28	45·44	48·76	51·74	55·33	59·90	63·35	66·40
80	44·79	46·52	51·17	53·54	57·15	60·39	64·28	69·21	72·92	76·19
90	52·28	54·16	59·20	61·75	65·65	69·13	73·29	78·56	82·51	85·99
100	59·90	61·92	67·33	70·06	74·22	77·93	82·36	87·95	92·13	95·81

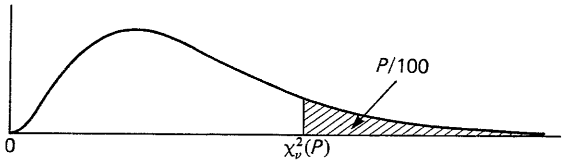
TABLE 8. PERCENTAGE POINTS OF THE χ^2-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})}\int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1}\,e^{-\frac{1}{2}x}\,dx.$$

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu-1}$ and unit variance.



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

P	50	40	30	20	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·4549	0·7083	1·074	1·642	2·706	3·841	5·024	6·635	7·879	10·83	12·12
2	1·386	1·833	2·408	3·219	4·605	5·991	7·378	9·210	10·60	13·82	15·20
3	2·366	2·946	3·665	4·642	6·251	7·815	9·348	11·34	12·84	16·27	17·73
4	3·357	4·045	4·878	5·989	7·779	9·488	11·14	13·28	14·86	18·47	20·00
5	4·351	5·132	6·064	7·289	9·236	11·07	12·83	15·09	16·75	20·52	22·11
6	5·348	6·211	7·231	8·558	10·64	12·59	14·45	16·81	18·55	22·46	24·10
7	6·346	7·283	8·383	9·803	12·02	14·07	16·01	18·48	20·28	24·32	26·02
8	7·344	8·351	9·524	11·03	13·36	15·51	17·53	20·09	21·95	26·12	27·87
9	8·343	9·414	10·66	12·24	14·68	16·92	19·02	21·67	23·59	27·88	29·67
10	9·342	10·47	11·78	13·44	15·99	18·31	20·48	23·21	25·19	29·59	31·42
11	10·34	11·53	12·90	14·63	17·28	19·68	21·92	24·72	26·76	31·26	33·14
12	11·34	12·58	14·01	15·81	18·55	21·03	23·34	26·22	28·30	32·91	34·82
13	12·34	13·64	15·12	16·98	19·81	22·36	24·74	27·69	29·82	34·53	36·48
14	13·34	14·69	16·22	18·15	21·06	23·68	26·12	29·14	31·32	36·12	38·11
15	14·34	15·73	17·32	19·31	22·31	25·00	27·49	30·58	32·80	37·70	39·72
16	15·34	16·78	18·42	20·47	23·54	26·30	28·85	32·00	34·27	39·25	41·31
17	16·34	17·82	19·51	21·61	24·77	27·59	30·19	33·41	35·72	40·79	42·88
18	17·34	18·87	20·60	22·76	25·99	28·87	31·53	34·81	37·16	42·31	44·43
19	18·34	19·91	21·69	23·90	27·20	30·14	32·85	36·19	38·58	43·82	45·97
20	19·34	20·95	22·77	25·04	28·41	31·41	34·17	37·57	40·00	45·31	47·50
21	20·34	21·99	23·86	26·17	29·62	32·67	35·48	38·93	41·40	46·80	49·01
22	21·34	23·03	24·94	27·30	30·81	33·92	36·78	40·29	42·80	48·27	50·51
23	22·34	24·07	26·02	28·43	32·01	35·17	38·08	41·64	44·18	49·73	52·00
24	23·34	25·11	27·10	29·55	33·20	36·42	39·36	42·98	45·56	51·18	53·48
25	24·34	26·14	28·17	30·68	34·38	37·65	40·65	44·31	46·93	52·62	54·95
26	25·34	27·18	29·25	31·79	35·56	38·89	41·92	45·64	48·29	54·05	56·41
27	26·34	28·21	30·32	32·91	36·74	40·11	43·19	46·96	49·64	55·48	57·86
28	27·34	29·25	31·39	34·03	37·92	41·34	44·46	48·28	50·99	56·89	59·30
29	28·34	30·28	32·46	35·14	39·09	42·56	45·72	49·59	52·34	58·30	60·73
30	29·34	31·32	33·53	36·25	40·26	43·77	46·98	50·89	53·67	59·70	62·16
32	31·34	33·38	35·66	38·47	42·58	46·19	49·48	53·49	56·33	62·49	65·00
34	33·34	35·44	37·80	40·68	44·90	48·60	51·97	56·06	58·96	65·25	67·80
36	35·34	37·50	39·92	42·88	47·21	51·00	54·44	58·62	61·58	67·99	70·59
38	37·34	39·56	42·05	45·08	49·51	53·38	56·90	61·16	64·18	70·70	73·35
40	39·34	41·62	44·16	47·27	51·81	55·76	59·34	63·69	66·77	73·40	76·09
50	49·33	51·89	54·72	58·16	63·17	67·50	71·42	76·15	79·49	86·66	89·56
60	59·33	62·13	65·23	68·97	74·40	79·08	83·30	88·38	91·95	99·61	102·7
70	69·33	72·36	75·69	79·71	85·53	90·53	95·02	100·4	104·2	112·3	115·6
80	79·33	82·57	86·12	90·41	96·58	101·9	106·6	112·3	116·3	124·8	128·3
90	89·33	92·76	96·52	101·1	107·6	113·1	118·1	124·1	128·3	137·2	140·8
100	99·33	102·9	106·9	111·7	118·5	124·3	129·6	135·8	140·2	149·4	153·2

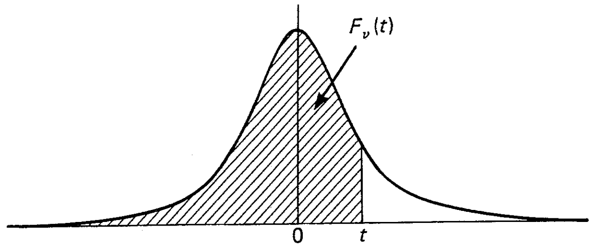
TABLE 9. THE t-DISTRIBUTION FUNCTION

The function tabulated is

$$F_\nu(t) = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{-\infty}^{t} \frac{ds}{(1+s^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

$F_\nu(t)$ is the probability that a random variable, distributed as t with ν degrees of freedom, will be less than or equal to t. When $t < 0$ use $F_\nu(t) = 1-F_\nu(-t)$, the t distribution being symmetric about zero.

The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance (see Table 4). When ν is large interpolation in ν should be harmonic.



Omitted entries to the right of tabulated values are 1 (to four decimal places).

$\nu =$	1	$\nu =$	1	$\nu =$	2	$\nu =$	2	$\nu =$	3	$\nu =$	3
$t =$ 0·0	0·5000	$t =$ 4·0	0·9220	$t =$ 0·0	0·5000	$t =$ 4·0	0·9714	$t =$ 0·0	0·5000	$t =$ 4·0	0·9860
·1	·5317	4·2	·9256	·1	·5353	·1	·9727	·1	·5367	·1	·9869
·2	·5628	4·4	·9289	·2	·5700	·2	·9739	·2	·5729	·2	·9877
·3	·5928	4·6	·9319	·3	·6038	·3	·9750	·3	·6081	·3	·9884
·4	·6211	4·8	·9346	·4	·6361	·4	·9760	·4	·6420	·4	·9891
0·5	0·6476	5·0	0·9372	0·5	0·6667	4·5	0·9770	0·5	0·6743	4·5	0·9898
·6	·6720	5·5	·9428	·6	·6953	·6	·9779	·6	·7046	·6	·9903
·7	·6944	6·0	·9474	·7	·7218	·7	·9788	·7	·7328	·7	·9909
·8	·7148	6·5	·9514	·8	·7462	·8	·9796	·8	·7589	·8	·9914
·9	·7333	7·0	·9548	·9	·7684	·9	·9804	·9	·7828	·9	·9919
1·0	0·7500	7·5	0·9578	1·0	0·7887	5·0	0·9811	1·0	0·8045	5·0	0·9923
·1	·7651	8·0	·9604	·1	·8070	·1	·9818	·1	·8242	·1	·9927
·2	·7789	8·5	·9627	·2	·8235	·2	·9825	·2	·8419	·2	·9931
·3	·7913	9·0	·9648	·3	·8384	·3	·9831	·3	·8578	·3	·9934
·4	·8026	9·5	·9666	·4	·8518	·4	·9837	·4	·8720	·4	·9938
1·5	0·8128	10·0	0·9683	1·5	0·8638	5·5	0·9842	1·5	0·8847	5·5	0·9941
·6	·8222	10·5	·9698	·6	·8746	·6	·9848	·6	·8960	·6	·9944
·7	·8307	11·0	·9711	·7	·8844	·7	·9853	·7	·9062	·7	·9946
·8	·8386	11·5	·9724	·8	·8932	·8	·9858	·8	·9152	·8	·9949
·9	·8458	12·0	·9735	·9	·9011	·9	·9862	·9	·9232	·9	·9951
2·0	0·8524	12·5	0·9746	2·0	0·9082	6·0	0·9867	2·0	0·9303	6·0	0·9954
·1	·8585	13·0	·9756	·1	·9147	·1	·9871	·1	·9367	·1	·9956
·2	·8642	13·5	·9765	·2	·9206	·2	·9875	·2	·9424	·2	·9958
·3	·8695	14·0	·9773	·3	·9259	·3	·9879	·3	·9475	·3	·9960
·4	·8743	14·5	·9781	·4	·9308	·4	·9882	·4	·9521	·4	·9961
2·5	0·8789	15	0·9788	2·5	0·9352	6·5	0·9886	2·5	0·9561	6·5	0·9963
·6	·8831	16	·9801	·6	·9392	·6	·9889	·6	·9598	·6	·9965
·7	·8871	17	·9813	·7	·9429	·7	·9892	·7	·9631	·7	·9966
·8	·8908	18	·9823	·8	·9463	·8	·9895	·8	·9661	·8	·9967
·9	·8943	19	·9833	·9	·9494	·9	·9898	·9	·9687	·9	·9969
3·0	0·8976	20	0·9841	3·0	0·9523	7·0	0·9901	3·0	0·9712	7·0	0·9970
·1	·9007	21	·9849	·1	·9549	·1	·9904	·1	·9734	·1	·9971
·2	·9036	22	·9855	·2	·9573	·2	·9906	·2	·9753	·2	·9972
·3	·9063	23	·9862	·3	·9596	·3	·9909	·3	·9771	·3	·9973
·4	·9089	24	·9867	·4	·9617	·4	·9911	·4	·9788	·4	·9974
3·5	0·9114	25	0·9873	3·5	0·9636	7·5	0·9913	3·5	0·9803	7·5	0·9975
·6	·9138	30	·9894	·6	·9654	·6	·9916	·6	·9816	·6	·9976
·7	·9160	35	·9909	·7	·9670	·7	·9918	·7	·9829	·7	·9977
·8	·9181	40	·9920	·8	·9686	·8	·9920	·8	·9840	·8	·9978
·9	·9201	45	·9929	·9	·9701	·9	·9922	·9	·9850	·9	·9979
4·0	0·9220	50	0·9936	4·0	0·9714	8·0	0·9924	4·0	0·9860	8·0	0·9980

TABLE 9. THE *t*-DISTRIBUTION FUNCTION

ν =	4	5	6	7	8	9	10	11	12	13	14
t = 0·0	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000
·1	·5374	·5379	·5382	·5384	·5386	·5387	·5388	·5389	·5390	·5391	·5391
·2	·5744	·5753	·5760	·5764	·5768	·5770	·5773	·5774	·5776	·5777	·5778
·3	·6104	·6119	·6129	·6136	·6141	·6145	·6148	·6151	·6153	·6155	·6157
·4	·6452	·6472	·6485	·6495	·6502	·6508	·6512	·6516	·6519	·6522	·6524
0·5	0·6783	0·6809	0·6826	0·6838	0·6847	0·6855	0·6861	0·6865	0·6869	0·6873	0·6876
·6	·7096	·7127	·7148	·7163	·7174	·7183	·7191	·7197	·7202	·7206	·7210
·7	·7387	·7424	·7449	·7467	·7481	·7492	·7501	·7508	·7514	·7519	·7523
·8	·7657	·7700	·7729	·7750	·7766	·7778	·7788	·7797	·7804	·7810	·7815
·9	·7905	·7953	·7986	·8010	·8028	·8042	·8054	·8063	·8071	·8078	·8083
1·0	0·8130	0·8184	0·8220	0·8247	0·8267	0·8283	0·8296	0·8306	0·8315	0·8322	0·8329
·1	·8335	·8393	·8433	·8461	·8483	·8501	·8514	·8526	·8535	·8544	·8551
·2	·8518	·8581	·8623	·8654	·8678	·8696	·8711	·8723	·8734	·8742	·8750
·3	·8683	·8748	·8793	·8826	·8851	·8870	·8886	·8899	·8910	·8919	·8927
·4	·8829	·8898	·8945	·8979	·9005	·9025	·9041	·9055	·9066	·9075	·9084
1·5	0·8960	0·9030	0·9079	0·9114	0·9140	0·9161	0·9177	0·9191	0·9203	0·9212	0·9221
·6	·9076	·9148	·9196	·9232	·9259	·9280	·9297	·9310	·9322	·9332	·9340
·7	·9178	·9251	·9300	·9335	·9362	·9383	·9400	·9414	·9426	·9435	·9444
·8	·9269	·9341	·9390	·9426	·9452	·9473	·9490	·9503	·9515	·9525	·9533
·9	·9349	·9421	·9469	·9504	·9530	·9551	·9567	·9580	·9591	·9601	·9609
2·0	0·9419	0·9490	0·9538	0·9572	0·9597	0·9617	0·9633	0·9646	0·9657	0·9666	0·9674
·1	·9482	·9551	·9598	·9631	·9655	·9674	·9690	·9702	·9712	·9721	·9728
·2	·9537	·9605	·9649	·9681	·9705	·9723	·9738	·9750	·9759	·9768	·9774
·3	·9585	·9651	·9694	·9725	·9748	·9765	·9779	·9790	·9799	·9807	·9813
·4	·9628	·9692	·9734	·9763	·9784	·9801	·9813	·9824	·9832	·9840	·9846
2·5	0·9666	0·9728	0·9767	0·9795	0·9815	0·9831	0·9843	0·9852	0·9860	0·9867	0·9873
·6	·9700	·9759	·9797	·9823	·9842	·9856	·9868	·9877	·9884	·9890	·9895
·7	·9730	·9786	·9822	·9847	·9865	·9878	·9888	·9897	·9903	·9909	·9914
·8	·9756	·9810	·9844	·9867	·9884	·9896	·9906	·9914	·9920	·9925	·9929
·9	·9779	·9831	·9863	·9885	·9901	·9912	·9921	·9928	·9933	·9938	·9942
3·0	0·9800	0·9850	0·9880	0·9900	0·9915	0·9925	0·9933	0·9940	0·9945	0·9949	0·9952
·1	·9819	·9866	·9894	·9913	·9927	·9936	·9944	·9949	·9954	·9958	·9961
·2	·9835	·9880	·9907	·9925	·9937	·9946	·9953	·9958	·9962	·9965	·9968
·3	·9850	·9893	·9918	·9934	·9946	·9954	·9960	·9965	·9968	·9971	·9974
·4	·9864	·9904	·9928	·9943	·9953	·9961	·9966	·9970	·9974	·9976	·9978
3·5	0·9876	0·9914	0·9936	0·9950	0·9960	0·9966	0·9971	0·9975	0·9978	0·9980	0·9982
·6	·9886	·9922	·9943	·9956	·9965	·9971	·9976	·9979	·9982	·9984	·9986
·7	·9896	·9930	·9950	·9962	·9970	·9975	·9979	·9982	·9985	·9987	·9988
·8	·9904	·9937	·9955	·9966	·9974	·9979	·9983	·9985	·9987	·9989	·9990
·9	·9912	·9943	·9960	·9971	·9977	·9982	·9985	·9988	·9989	·9991	·9992
4·0	0·9919	0·9948	0·9964	0·9974	0·9980	0·9984	0·9987	0·9990	0·9991	0·9992	0·9993
·1	·9926	·9953	·9968	·9977	·9983	·9987	·9989	·9991	·9993	·9994	·9995
·2	·9932	·9958	·9972	·9980	·9985	·9988	·9991	·9993	·9994	·9995	·9996
·3	·9937	·9961	·9975	·9982	·9987	·9990	·9992	·9994	·9995	·9996	·9996
·4	·9942	·9965	·9977	·9984	·9989	·9991	·9993	·9995	·9996	·9996	·9997
4·5	0·9946	0·9968	0·9979	0·9986	0·9990	0·9993	0·9994	0·9995	0·9996	0·9997	0·9998
·6	·9950	·9971	·9982	·9988	·9991	·9994	·9995	·9996	·9997	·9998	·9998
·7	·9953	·9973	·9983	·9989	·9992	·9994	·9996	·9997	·9997	·9998	·9998
·8	·9957	·9976	·9985	·9990	·9993	·9995	·9996	·9997	·9998	·9998	·9999
·9	·9960	·9978	·9986	·9991	·9994	·9996	·9997	·9998	·9998	·9999	·9999
5·0	0·9963	0·9979	0·9988	0·9992	0·9995	0·9996	0·9997	0·9998	0·9998	0·9999	0·9999

TABLE 9. THE *t*-DISTRIBUTION FUNCTION

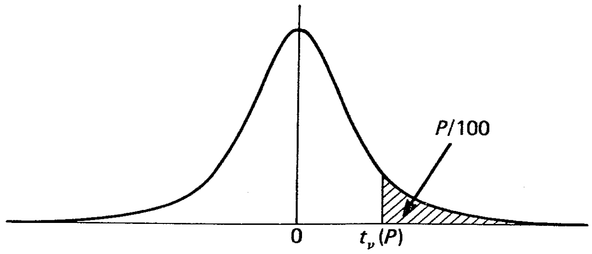
$\nu =$	15	16	17	18	19	20	24	30	40	60	∞
$t =$ 0·0	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000
·1	·5392	·5392	·5392	·5393	·5393	·5393	·5394	·5395	·5396	·5397	·5398
·2	·5779	·5780	·5781	·5781	·5782	·5782	·5784	·5786	·5788	·5789	·5793
·3	·6159	·6160	·6161	·6162	·6163	·6164	·6166	·6169	·6171	·6174	·6179
·4	·6526	·6528	·6529	·6531	·6532	·6533	·6537	·6540	·6544	·6547	·6554
0·5	0·6878	0·6881	0·6883	0·6884	0·6886	0·6887	0·6892	0·6896	0·6901	0·6905	0·6915
·6	·7213	·7215	·7218	·7220	·7222	·7224	·7229	·7235	·7241	·7246	·7257
·7	·7527	·7530	·7533	·7536	·7538	·7540	·7547	·7553	·7560	·7567	·7580
·8	·7819	·7823	·7826	·7829	·7832	·7834	·7842	·7850	·7858	·7866	·7881
·9	·8088	·8093	·8097	·8100	·8103	·8106	·8115	·8124	·8132	·8141	·8159
1·0	0·8334	0·8339	0·8343	0·8347	0·8351	0·8354	0·8364	0·8373	0·8383	0·8393	0·8413
·1	·8557	·8562	·8567	·8571	·8575	·8578	·8589	·8600	·8610	·8621	·8643
·2	·8756	·8762	·8767	·8772	·8776	·8779	·8791	·8802	·8814	·8826	·8849
·3	·8934	·8940	·8945	·8950	·8954	·8958	·8970	·8982	·8995	·9007	·9032
·4	·9091	·9097	·9103	·9107	·9112	·9116	·9128	·9141	·9154	·9167	·9192
1·5	0·9228	0·9235	0·9240	0·9245	0·9250	0·9254	0·9267	0·9280	0·9293	0·9306	0·9332
·6	·9348	·9354	·9360	·9365	·9370	·9374	·9387	·9400	·9413	·9426	·9452
·7	·9451	·9458	·9463	·9468	·9473	·9477	·9490	·9503	·9516	·9528	·9554
·8	·9540	·9546	·9552	·9557	·9561	·9565	·9578	·9590	·9603	·9616	·9641
·9	·9616	·9622	·9627	·9632	·9636	·9640	·9652	·9665	·9677	·9689	·9713
2·0	0·9680	0·9686	0·9691	0·9696	0·9700	0·9704	0·9715	0·9727	0·9738	0·9750	0·9772
·1	·9735	·9740	·9745	·9750	·9753	·9757	·9768	·9779	·9790	·9800	·9821
·2	·9781	·9786	·9790	·9794	·9798	·9801	·9812	·9822	·9832	·9842	·9861
·3	·9819	·9824	·9828	·9832	·9835	·9838	·9848	·9857	·9866	·9875	·9893
·4	·9851	·9855	·9859	·9863	·9866	·9869	·9877	·9886	·9894	·9902	·9918
2·5	0·9877	0·9882	0·9885	0·9888	0·9891	0·9894	0·9902	0·9909	0·9917	0·9924	0·9938
·6	·9900	·9903	·9907	·9910	·9912	·9914	·9921	·9928	·9935	·9941	·9953
·7	·9918	·9921	·9924	·9927	·9929	·9931	·9937	·9944	·9949	·9955	·9965
·8	·9933	·9936	·9938	·9941	·9943	·9945	·9950	·9956	·9961	·9966	·9974
·9	·9945	·9948	·9950	·9952	·9954	·9956	·9961	·9965	·9970	·9974	·9981
3·0	0·9955	0·9958	0·9960	0·9962	0·9963	0·9965	0·9969	0·9973	0·9977	0·9980	0·9987
·1	·9963	·9966	·9967	·9969	·9971	·9972	·9976	·9979	·9982	·9985	·9990
·2	·9970	·9972	·9974	·9975	·9976	·9978	·9981	·9984	·9987	·9989	·9993
·3	·9976	·9977	·9979	·9980	·9981	·9982	·9985	·9988	·9990	·9992	·9995
·4	·9980	·9982	·9983	·9984	·9985	·9986	·9988	·9990	·9992	·9994	·9997
3·5	0·9984	0·9985	0·9986	0·9987	0·9988	0·9989	0·9991	0·9993	0·9994	0·9996	0·9998
·6	·9987	·9988	·9989	·9990	·9990	·9991	·9993	·9994	·9996	·9997	·9998
·7	·9989	·9990	·9991	·9992	·9992	·9993	·9994	·9996	·9997	·9998	·9999
·8	·9991	·9992	·9993	·9993	·9994	·9994	·9996	·9997	·9998	·9998	·9999
·9	·9993	·9994	·9994	·9995	·9995	·9996	·9997	·9997	·9998	·9999	
4·0	0·9994	0·9995	0·9995	0·9996	0·9996	0·9996	0·9997	0·9998	0·9999	0·9999	
·1	·9995	·9996	·9996	·9997	·9997	·9997	·9998	·9999	·9999	·9999	
·2	·9996	·9997	·9997	·9997	·9998	·9998	·9998	·9999	·9999		
·3	·9997	·9997	·9998	·9998	·9998	·9998	·9999	·9999	·9999		
·4	·9997	·9998	·9998	·9998	·9998	·9999	·9999	·9999			
4·5	0·9998	0·9998	0·9998	0·9999	0·9999	0·9999	0·9999				

TABLE 10. PERCENTAGE POINTS OF THE t-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2-distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t-distribution with ν degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

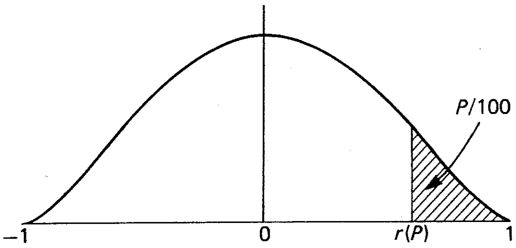
P	40	30	25	20	15	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·3249	0·7265	1·0000	1·3764	1·963	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	0·2887	0·6172	0·8165	1·0607	1·386	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	0·2767	0·5844	0·7649	0·9785	1·250	1·638	2·353	3·182	4·541	5·841	10·21	12·92
4	0·2707	0·5686	0·7407	0·9410	1·190	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	0·2672	0·5594	0·7267	0·9195	1·156	1·476	2·015	2·571	3·365	4·032	5·893	6·869
6	0·2648	0·5534	0·7176	0·9057	1·134	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	0·2632	0·5491	0·7111	0·8960	1·119	1·415	1·895	2·365	2·998	3·499	4·785	5·408
8	0·2619	0·5459	0·7064	0·8889	1·108	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	0·2610	0·5435	0·7027	0·8834	1·100	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	0·2602	0·5415	0·6998	0·8791	1·093	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	0·2596	0·5399	0·6974	0·8755	1·088	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	0·2590	0·5386	0·6955	0·8726	1·083	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	0·2586	0·5375	0·6938	0·8702	1·079	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	0·2582	0·5366	0·6924	0·8681	1·076	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	0·2579	0·5357	0·6912	0·8662	1·074	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	0·2576	0·5350	0·6901	0·8647	1·071	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	0·2573	0·5344	0·6892	0·8633	1·069	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	0·2571	0·5338	0·6884	0·8620	1·067	1·330	1·734	2·101	2·552	2·878	3·610	3·922
19	0·2569	0·5333	0·6876	0·8610	1·066	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	0·2567	0·5329	0·6870	0·8600	1·064	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	0·2566	0·5325	0·6864	0·8591	1·063	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	0·2564	0·5321	0·6858	0·8583	1·061	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	0·2563	0·5317	0·6853	0·8575	1·060	1·319	1·714	2·069	2·500	2·807	3·485	3·768
24	0·2562	0·5314	0·6848	0·8569	1·059	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	0·2561	0·5312	0·6844	0·8562	1·058	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	0·2560	0·5309	0·6840	0·8557	1·058	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	0·2559	0·5306	0·6837	0·8551	1·057	1·314	1·703	2·052	2·473	2·771	3·421	3·690
28	0·2558	0·5304	0·6834	0·8546	1·056	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	0·2557	0·5302	0·6830	0·8542	1·055	1·311	1·699	2·045	2·462	2·756	3·396	3·659
30	0·2556	0·5300	0·6828	0·8538	1·055	1·310	1·697	2·042	2·457	2·750	3·385	3·646
32	0·2555	0·5297	0·6822	0·8530	1·054	1·309	1·694	2·037	2·449	2·738	3·365	3·622
34	0·2553	0·5294	0·6818	0·8523	1·052	1·307	1·691	2·032	2·441	2·728	3·348	3·601
36	0·2552	0·5291	0·6814	0·8517	1·052	1·306	1·688	2·028	2·434	2·719	3·333	3·582
38	0·2551	0·5288	0·6810	0·8512	1·051	1·304	1·686	2·024	2·429	2·712	3·319	3·566
40	0·2550	0·5286	0·6807	0·8507	1·050	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	0·2547	0·5278	0·6794	0·8489	1·047	1·299	1·676	2·009	2·403	2·678	3·261	3·496
60	0·2545	0·5272	0·6786	0·8477	1·045	1·296	1·671	2·000	2·390	2·660	3·232	3·460
120	0·2539	0·5258	0·6765	0·8446	1·041	1·289	1·658	1·980	2·358	2·617	3·160	3·373
∞	0·2533	0·5244	0·6745	0·8416	1·036	1·282	1·645	1·960	2·326	2·576	3·090	3·291

TABLE 13. PERCENTAGE POINTS OF THE CORRELATION COEFFICIENT r WHEN $\rho = 0$

The function tabulated is $r(P) = r(P|\nu)$ defined by the equation

$$\frac{\Gamma\left(\frac{\nu-1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{\nu-2}{2}\right)} \int_{r(P)}^{1} (1-r^2)^{\frac{\nu-4}{2}}\, dr = P/100.$$

Let r be a partial correlation coefficient, after s variables have been eliminated, in a sample of size n from a multivariate normal population with corresponding true partial correlation coefficient $\rho = 0$, and let $\nu = n - s$. This table gives upper P per cent points of r; the corresponding lower P per cent points are given by $-r(P)$, and the tabulated values are also upper $2P$ per cent points of $|r|$. For $s = 0$ we have $\nu = n$ and r is the ordinary correlation coefficient. When $\nu > 130$ use the results that r is approximately normally distributed with zero mean and variance $\frac{1}{\nu-1}$, or (more accurately) that $z = \tanh^{-1} r$ is approximately normally distributed with zero mean and variance $\frac{1}{\nu-3}$ (cf. Tables 16 and 17).



(This shape applies for $\nu \geq 5$ only. When $\nu = 4$ the distribution is uniform and when $\nu = 3$ the probability density function is U-shaped.)

Tables of the distribution of r for various values of ρ are given by, for example, F. N. David, *Tables of the Ordinates and Probability Integral of the Distribution of the Correlation Coefficient in Small Samples*, Cambridge University Press (1954), and R. E. Odeh, 'Critical values of the sample product–moment correlation coefficient in the bivariate normal distribution', *Commun. Statist. – Simula Computa.* **11** (1) (1982), pp. 1–26. The z-transformation may also be used (cf. Tables 16 and 17).

P	5	2·5	1	0·5	0·1
ν = 3	0·9877	0·9969	0·9995	0·9999	0·999995
4	·9000	·9500	·9800	·9900	·9980
5	0·8054	0·8783	0·9343	0·9587	0·9859
6	·7293	·8114	·8822	·9172	·9633
7	·6694	·7545	·8329	·8745	·9350
8	·6215	·7067	·7887	·8343	·9049
9	·5822	·6664	·7498	·7977	·8751
10	0·5494	0·6319	0·7155	0·7646	0·8467
11	·5214	·6021	·6851	·7348	·8199
12	·4973	·5760	·6581	·7079	·7950
13	·4762	·5529	·6339	·6835	·7717
14	·4575	·5324	·6120	·6614	·7501
15	0·4409	0·5140	0·5923	0·6411	0·7301
16	·4259	·4973	·5742	·6226	·7114
17	·4124	·4821	·5577	·6055	·6940
18	·4000	·4683	·5425	·5897	·6777
19	·3887	·4555	·5285	·5751	·6624
20	0·3783	0·4438	0·5155	0·5614	0·6481
21	·3687	·4329	·5034	·5487	·6346
22	·3598	·4227	·4921	·5368	·6219
23	·3515	·4132	·4815	·5256	·6099
24	·3438	·4044	·4716	·5151	·5986
25	0·3365	0·3961	0·4622	0·5052	0·5879
26	·3297	·3882	·4534	·4958	·5776
27	·3233	·3809	·4451	·4869	·5679
28	·3172	·3739	·4372	·4785	·5587
29	·3115	·3673	·4297	·4705	·5499
30	0·3061	0·3610	0·4226	0·4629	0·5415
31	·3009	·3550	·4158	·4556	·5334
32	·2960	·3494	·4093	·4487	·5257
33	·2913	·3440	·4032	·4421	·5184
34	·2869	·3388	·3972	·4357	·5113
35	0·2826	0·3338	0·3916	0·4296	0·5045
36	·2785	·3291	·3862	·4238	·4979
37	·2746	·3246	·3810	·4182	·4916
38	·2709	·3202	·3760	·4128	·4856
39	·2673	·3160	·3712	·4076	·4797

P	5	2·5	1	0·5	0·1
ν = 40	0·2638	0·3120	0·3665	0·4026	0·4741
42	·2573	·3044	·3578	·3932	·4633
44	·2512	·2973	·3496	·3843	·4533
46	·2455	·2907	·3420	·3761	·4439
48	·2403	·2845	·3348	·3683	·4351
50	0·2353	0·2787	0·3281	0·3610	0·4267
52	·2306	·2732	·3218	·3542	·4188
54	·2262	·2681	·3158	·3477	·4114
56	·2221	·2632	·3102	·3415	·4043
58	·2181	·2586	·3048	·3357	·3976
60	0·2144	0·2542	0·2997	0·3301	0·3912
62	·2108	·2500	·2948	·3248	·3850
64	·2075	·2461	·2902	·3198	·3792
66	·2042	·2423	·2858	·3150	·3736
68	·2012	·2387	·2816	·3104	·3683
70	0·1982	0·2352	0·2776	0·3060	0·3632
72	·1954	·2319	·2737	·3017	·3583
74	·1927	·2287	·2700	·2977	·3536
76	·1901	·2257	·2664	·2938	·3490
78	·1876	·2227	·2630	·2900	·3447
80	0·1852	0·2199	0·2597	0·2864	0·3405
82	·1829	·2172	·2565	·2830	·3364
84	·1807	·2146	·2535	·2796	·3325
86	·1786	·2120	·2505	·2764	·3287
88	·1765	·2096	·2477	·2732	·3251
90	0·1745	0·2072	0·2449	0·2702	0·3215
92	·1726	·2050	·2422	·2673	·3181
94	·1707	·2028	·2396	·2645	·3148
96	·1689	·2006	·2371	·2617	·3116
98	·1671	·1986	·2347	·2591	·3085
100	0·1654	0·1966	0·2324	0·2565	0·3054
105	·1614	·1918	·2268	·2504	·2983
110	·1576	·1874	·2216	·2446	·2915
115	·1541	·1832	·2167	·2393	·2853
120	·1509	·1793	·2122	·2343	·2794
125	0·1478	0·1757	0·2079	0·2296	0·2738
130	·1449	·1723	·2039	·2252	·2686

TABLE 14. PERCENTAGE POINTS OF SPEARMAN'S *S*
TABLE 15. PERCENTAGE POINTS OF KENDALL'S *K*

Spearman's S and Kendall's K are both used to measure the degree of association between two rankings of n objects. Let d_i $(1 \leqslant i \leqslant n)$ be the difference in the ranks of the ith object; Spearman's S is defined as $\sum_{i=1}^{n} d_i^2$. To define Kendall's K, re-order the pairs of ranks so that the first set is in natural order from left to right, and let m_i $(1 \leqslant i \leqslant n)$ be the number of ranks greater than i in the second ranking which are to the right of rank i. Kendall's K is defined as $\sum_{i=1}^{n} m_i$.

For Table 14 the tabulated value $x(P)$ is the lower percentage point, i.e. the largest value x such that, in independent rankings, $\Pr(S \leqslant x) \leqslant P/100$; in Table 15, K replaces S and the upper percentage point is given. A dash indicates that there is no value with the required property. The distributions are symmetric about means $\frac{1}{6}(n^3-n)$ for S and $\frac{1}{4}n(n-1)$ for K, with maxima equal to twice the means; hence the upper percentage points of S are $\frac{1}{3}(n^3-n)-x(P)$ and the lower percentage points of K are $\frac{1}{2}n(n-1)-x(P)$. The variances are $\frac{1}{36}n^2(n+1)^2(n-1)$ for S and $\frac{1}{72}n(n-1)(2n+5)$ for K, and when $n > 40$ both statistics are approximately normally distributed; more accurately, the distribution function of $X = [S-\frac{1}{6}(n^3-n)]/[\frac{1}{6}n(n+1)\sqrt{n-1}]$ is approximately equal to $\Phi(x) - \frac{\gamma}{24\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(x^3-3x)$, where $\gamma = \frac{-0{\cdot}04(19n^2+5n-36)}{\frac{1}{6}(n^3-n)}$ and $\Phi(x)$ is the normal distribution function (see Table 4).

A test of the null hypothesis of independent rankings is provided by rejecting at the P per cent level if $S \leqslant x(P)$, or $K \geqslant x(P)$, when the alternative is contrary rankings. The other points are similarly used when the alternative is similar rankings. To cover both alternatives reject at the $2P$ per cent level if S, or K, lies in either tail. Spearman's rank correlation coefficient r_S is defined as $1-6S/(n^3-n)$, and has upper and lower P per cent points $1-6x(P)/(n^3-n)$ and $-[1-6x(P)/(n^3-n)]$ respectively. Kendall's rank correlation coefficient r_K is defined as $4K/[n(n-1)]-1$, and has upper and lower P per cent points $4x(P)/[n(n-1)]-1$ and $-\{4x(P)/[n(n-1)]-1\}$ respectively.

SPEARMAN'S *S*

P	5	2·5	1	0·5	0·1	$\frac{1}{6}(n^3-n)$
$n=4$	0	—	—	—	—	**10**
5	2	0	0	—	—	**20**
6	6	4	2	0	—	**35**
7	16	12	6	4	0	**56**
8	30	22	14	10	4	**84**
9	48	36	26	20	10	**120**
10	72	58	42	34	20	**165**
11	102	84	64	54	34	**220**
12	142	118	92	78	52	**286**
13	188	160	128	108	76	**364**
14	244	210	170	146	104	**455**
15	310	268	222	194	140	**560**
16	388	338	284	248	184	**680**
17	478	418	354	312	236	**816**
18	580	512	436	388	298	**969**
19	694	616	530	474	370	**1140**
20	824	736	636	572	452	**1330**
21	970	868	756	684	544	**1540**
22	1132	1018	890	808	650	**1771**
23	1310	1182	1040	948	768	**2024**
24	1508	1364	1206	1102	900	**2300**
25	1724	1566	1388	1272	1048	**2600**
26	1958	1784	1588	1460	1210	**2925**
27	2214	2022	1806	1664	1388	**3276**
28	2492	2282	2044	1888	1584	**3654**
29	2794	2564	2304	2132	1796	**4060**
30	3118	2866	2584	2396	2028	**4495**
31	3466	3194	2884	2682	2280	**4960**
32	3840	3544	3210	2988	2552	**5456**
33	4240	3920	3558	3318	2844	**5984**
34	4666	4322	3930	3672	3160	**6545**
35	5120	4750	4330	4050	3498	**7140**
36	5604	5206	4754	4454	3858	**7770**
37	6118	5692	5206	4884	4244	**8436**
38	6662	6206	5686	5342	4656	**9139**
39	7238	6750	6196	5826	5092	**9880**
40	7846	7326	6736	6342	5556	**10660**

KENDALL'S *K*

P	5	2·5	1	0·5	0·1	$\frac{1}{4}n(n-1)$
$n=4$	6	—	—	—	—	**3**
5	9	10	10	—	—	**5**
6	13	14	14	15	—	**7·5**
7	17	18	19	20	21	**10·5**
8	22	23	24	25	26	**14**
9	27	28	30	31	33	**18**
10	33	34	36	37	40	**22·5**
11	39	41	43	44	47	**27·5**
12	46	48	51	52	55	**33**
13	53	56	59	61	64	**39**
14	62	64	67	69	73	**45·5**
15	70	73	77	79	83	**52·5**
16	79	83	86	89	94	**60**
17	89	93	97	100	105	**68**
18	99	103	108	111	117	**76·5**
19	110	114	119	123	129	**85·5**
20	121	126	131	135	142	**95**
21	133	138	144	148	156	**105**
22	146	151	157	161	170	**115·5**
23	159	164	171	176	184	**126·5**
24	172	178	185	190	200	**138**
25	186	193	200	205	216	**150**
26	201	208	216	221	232	**162·5**
27	216	223	232	238	249	**175·5**
28	232	239	248	254	267	**189**
29	248	256	266	272	285	**203**
30	265	273	283	290	303	**217·5**
31	282	291	301	308	323	**232·5**
32	300	309	320	328	342	**248**
33	318	328	340	347	363	**264**
34	337	347	359	368	384	**280·5**
35	356	367	380	388	405	**297·5**
36	376	388	401	410	428	**315**
37	397	409	422	432	450	**333**
38	418	430	444	454	473	**351·5**
39	440	452	467	477	497	**370·5**
40	462	475	490	501	522	**390**