Attempt ALL questions

Q1. Sample Spaces (3 Mark)

Consider a coin toss game played by players A and B. Player A tosses a fair coin three times, with the number of heads being the score for player A. Player B tosses a coin four times, with the number of heads being the score for player B.

a. (1 mark) Write out the sample space for the outcomes for both players A and B.

b. (1 mark) Write out the sample space for the outcomes of C, where C is the difference of the two scores (i.e. B-A)

c. (1 mark) Are the sample points for the sample space of C equally probable? Provide a brief justification for your answer.

Q2. Probability (3 Marks)

The following contingency table illustrates 250 students in different university departments, categorized by home place (i.e. from Munster, or from outside Munster).

	Physics	Biology	Statistics
Munster	50	40	60
Outside Munster	20	50	30

a. (1 mark) What is the probability that a randomly chosen person from the sample is a Physics student?

b. (1 mark) What is the probability that a randomly chosen person from the sample is both from Munster and studying Biology?

c. (1 mark) Given that the student is from Munster, what is the probability that he or she is a Biology student?

Q3. Discrete Random Variables (4 Marks)

A die is rolled. If an odd number turns up, we lose an amount equal to this number; if an even number turns up, we win an amount equal to this number. For example, if a two turns up we win 2. If a three comes up we lose 3. We will use the variable name X to describe the score. You may assume the die is fair.

a. (1 Mark) Tabulate the probability distribution table for this game.

b. (1 Mark) What is the expected value of the outcome of each roll of the die?

c. (1 Mark) Is this game unfavourable, fair or favourable? Justify your answer.

d. (1 Mark) Given that $E(X^2) = 15.166$, compute the variance of X.

Q4. Binomial Distribution (2 Marks)

A biased coin yields 'Tails' on 51% of throws. Consider an experiment that consists of throwing this coin 9 times.

a. (1 Mark) Evaluate the following term ${}^{9}C_{3}$.

b. (1 Mark) Compute the probability of getting three 'Tails' in this experiment.

Q5. Poisson Distribution (2 Marks)

Suppose that an IT help-line receives 6 calls per hour during offices hours.

a. (1 Mark) Compute the value of m for a 40 minute period during office hours.

b. (1 Mark) Compute the probability of the help-line getting exactly two calls in a 40 minute period during office hours.

Q6. Normal Distribution (1 Marks)

Suppose X is a normally distributed random variable with mean $\mu = 500$ and $\sigma = 24$

a. (1 Mark) Compute the Z-score of an observed value of 464 for X.

Formulae

- Conditional probability:
$$P(B|A) = \frac{P\left(A \text{ and } B\right)}{P\left(A\right)}.$$
- Bayes' Theorem:
$$P(B|A) = \frac{P\left(A|B\right) \times P(B)}{P\left(A\right)}.$$
- Binomial probability distribution:
$$P(X = k) =^n C_k \times p^k \times (1-p)^{n-k} \qquad \left(\text{where} \qquad {}^nC_k = \frac{n!}{k!\,(n-k)!}.\right)$$
- Poisson probability distribution:
$$P(X = k) = \frac{m^k \mathrm{e}^{-m}}{k!}.$$