1 Question 5 (Sample Variant 1)[25 marks]

(a) ***Huffman Coding (5 Marks)***
A discrete memoryless source X has five symbols $\{x_1, x_2, x_3, x_4, x_5\}$ with probabilities $P(x_1) = 0.45$, $P(x_2) = 0.20$, $P(x_3) = 0.16$, $P(x_4) = 0.14$ and $P(x_5) = 0.05$.

(i) (5 Marks) Construct a Huffman code for X.

(b) ***Rate of Information (6 Marks)***
A Computer monitor consists of about 2×10^4 picture elements (symbols) and 8 different brightness levels.

Pictures are repeated at a rate of 16 per second. All picture elements are assumed to be independent, and all levels have equal likelihood of occurrence.

(i) (6 Marks) Calculate the average rate of information conveyed by this TV picture source.

(c) ***Communication Channels (14 Marks)***
The input source to a noisy communication channel is a random variable X over the four symbols $\{a, b, c, d\}$. The output from this channel is a random variable Y over these same four symbols.
The joint distribution of these two random variables is as follows:

y=a	0.125	0.03125	0	0.015625
y=b	0	0.1875	0.125	0
y=c	0	0.015625	0.1875	0
y=d	0.0625	0	0	0.25

(i) (2 Marks) Write down the marginal distribution for X and compute the marginal entropy $H(X)$.

(ii) (2 Marks) Write down the marginal distribution for Y and compute the marginal entropy $H(Y)$.

(iii) (2 Marks) What is the joint entropy $H(X, Y)$ of the two random variables?

(iv) (4 marks) What is the conditional entropy $H(Y|X)$?

(v) (2 marks) What is the conditional entropy $H(X|Y)$?

(vi) (2 marks) What is the mutual information $I(X; Y)$ between the two random variables?