Question 1

a)
$$H_0: \quad \mu_1 - \mu_2 = 0$$
$$H_a: \quad \mu_1 - \mu_2 \neq 0$$

b)
$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_{01} - \mu_{02})}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(7.1 - 5.2) - 0}{\sqrt{\frac{10.1}{42} + \frac{16.1}{50}}} = \frac{1.9}{0.75} = 2.53.$$

c) Two tailed test:

$$\begin{aligned} \Rightarrow \text{p-value} &= 2 \cdot \Pr(Z > |2.54|) \\ &= 2 \cdot \Pr(Z > 2.54) \\ &= 2 \cdot (0.0057) = 0.0114. \end{aligned}$$

d) The evidence against H_0 is strong (almost at the 1% level). Thus, we reject the null hypothesis that there is no difference between population means.

Conclusion: There is a difference in the diet plans and, in particular, weight loss is greater with diet plan 1.

(Note: we have not said anything about how healthy the plans are).

Question 2

a) For the F test the hypotheses are:

$$H_0: \quad \sigma_1^2 = \sigma_2^2$$
$$H_a: \quad \sigma_1^2 \neq \sigma_2^2$$

A p-value of 0.7297 means that there is no evidence to reject H_0, i.e., we accept the hypothesis that the population variances are equal.

b)
$$H_0: \quad \mu_1 - \mu_2 = 0$$
$$H_a: \quad \mu_1 - \mu_2 \neq 0$$

c) As it is a two tailed test and the samples are small the critical values are $\pm t_{\nu, \alpha/2}$.

$\alpha = 0.1 \Rightarrow \alpha/2 = 0.05$ in each tail.

Since we can assume equal variances, $\nu = n_1 + n_2 - 2 = 5 + 3 - 2 = 6$.

Thus, the rejection region is outside of $\pm t_{6, 0.05} = \pm 1.943$.

d) We have standard deviations and require variances:

$s_1^2 = (1.7)^2 = 2.89$ and $s_1^2 = (1.9)^2 = 3.61$.

We need the pooled variance:

$$\begin{aligned} s_p^2 &= \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \\ &= \frac{(5-1)(2.89) + (3-1)(3.61)}{5 + 3 - 2} \\ &= 3.7317. \end{aligned}$$

Thus, the test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_{01} - \mu_{02})}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}} = \frac{(30.2 - 28.4) - 0}{\sqrt{\frac{3.7317}{5} + \frac{3.7317}{3}}} = \frac{1.8}{1.411} = 1.276.$$

The test statistic is within the acceptance region (i.e., within ± 1.943) $\Rightarrow$ we accept the hypothesis that there is no difference in the average salaries.

Conclusion: There is no evidence of gender inequality.

Question 3

a)

$$H_0: \quad \mu_1 - \mu_2 = 0$$
$$H_a: \quad \mu_1 - \mu_2 \neq 0$$

b) Two tailed test and the samples are small $\Rightarrow$ the critical values are $\pm t_{\nu, \alpha/2}$.

$\alpha = 0.05 \Rightarrow \alpha/2 = 0.025$ in each tail.

As we are not assuming equal variances we have to calculate:

$$a = \frac{s_1^2}{n_1} = \frac{3}{15} = 0.2 \qquad b = \frac{s_2^2}{n_2} = \frac{1.5}{15} = 0.1.$$

$$\Rightarrow \nu = \frac{(a+b)^2}{\frac{a^2}{n_1-1} + \frac{b^2}{n_2-1}} = \frac{(0.2+0.1)^2}{\frac{(0.2)^2}{15-1} + \frac{(0.1)^2}{15-1}} = \frac{0.09}{\frac{0.04}{14} + \frac{0.01}{14}} = \frac{0.09}{\frac{0.05}{14}} = \frac{0.09}{0.05} \times \frac{14}{1} = 25.2.$$

Need integer value for tables $\Rightarrow \nu = 25$.

The critical values are $\pm t_{25,\, 0.025} = \pm 2.06$.

c) The test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_{01} - \mu_{02})}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(12.5 - 11.1) - 0}{\sqrt{\frac{3}{15} + \frac{1.5}{15}}} = \frac{1.4}{0.5477} = 2.556.$$

The test statistic is outside of $\pm 2.06 \Rightarrow$ we reject H_0 at the 5% level.

Conclusion: There is a difference in the average time and, in particular, the students of University B are quicker at completing the task.

d) Two-tailed test:

$$\Rightarrow \text{p-value} = 2 \cdot \Pr(T_{25} > |2.556|) = 2 \cdot \Pr(T_{25} > 2.556).$$

From the t-tables we have

$$2 \cdot \Pr(T_{25} > 2.485) = 2(0.01) = 0.02.$$
$$2 \cdot \Pr(T_{25} > 2.787) = 2(0.005) = 0.01.$$

Thus, the p-value is between 0.01 and 0.02, i.e., there is strong evidence against the null hypothesis.

Question 4

a)

$$H_0: \quad p_1 - p_2 = 0$$
$$H_a: \quad p_1 - p_2 \neq 0$$

Two tailed test and $\alpha = 0.05 \Rightarrow$ the critical values are $\pm z_{0.025} = \pm 1.96$.

From the data we have

$$\hat{p}_1 = \frac{20}{38} = 0.5263,$$

$$\hat{p}_2 = \frac{70}{116} = 0.6034.$$

We also need to calculate the overall combined proportion for the standard error:

$$\hat{p}_c = \frac{20+70}{38+116} = \frac{90}{154} = 0.5844.$$

The test statistic is

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_{01} - p_{02})}{\sqrt{\frac{\hat{p}_c\,(1-\hat{p}_c)}{n_1} + \frac{\hat{p}_c\,(1-\hat{p}_c)}{n_2}}} = \frac{(0.5263 - 0.6034) - 0}{\sqrt{\frac{0.5844\,(0.4156)}{38} + \frac{0.5844\,(0.4156)}{116}}} = \frac{-0.0771}{0.0921} = -0.837.$$

The test statistic is within $\pm 1.96 \Rightarrow$ we accept H_0.

Conclusion: There is no difference in the level of support of this policy in rural and urban areas.

Question 5

a) If there was no difference between the products then the probability of an individual preferring one of the 5 products would be $\frac{1}{5}$, i.e., an individual is equally likely to prefer any of the 5.

The expected frequencies are all the same:

$$e_i = \text{total} \times \frac{1}{5} = 100 \times \frac{1}{5} = 20$$

	Product #1	#2	#3	#4	#5	$\sum$
o_i	19	24	24	14	19	100
e_i	20	20	20	20	20	100
$\frac{(o_i-e_i)^2}{e_i}$	0.05	0.80	0.80	1.80	0.05	3.5

b)
H_0 : preference equally likely
H_a : preference not equally likely

We have that $\alpha = 0.05$ and $\nu = n_f - 1 - k$.

As no parameters have been estimated $k = 0$.

$\Rightarrow \nu = 5 - 1 - 0 = 4$. The critical value is therefore $\chi^2_{4,\,0.05} = 9.488$.

Since the test statistic, $\chi^2 = 3.5$, is below the critical value, we cannot reject H_0.

Conclusion: There appears to be no difference between these products.

Question 6

a) To obtain the expected frequencies, multiply each theoretical probability by the overall total (160).
(these are the frequencies we would expect to see if the data came from a normal distribution)

x	< 5	$5-7$	$7-9$	$9-11$	$11-13$	$13-15$	$15-17$	> 17	$\sum$
$160 \times p_i = e_i$	2.40	11.52	32.16	49.12	40.96	18.56	4.64	0.64	160

Due to e_i values less than 5, we combine the < 5 and $5-7$ classes and also the $15-17$ and > 17 classes:

x	< 7	$7-9$	$9-11$	$11-13$	$13-15$	> 15	$\sum$
o_i	13	23	62	39	14	9	160
e_i	13.92	32.16	49.12	40.96	18.56	5.28	160
$\frac{(o_i-e_i)^2}{e_i}$	0.061	2.609	3.377	0.094	1.120	2.621	9.882

b) The test statistic is $\chi^2 = \sum \frac{(o_i-e_i)^2}{e_i} = 9.882$.

c)
H_0 : The normal distribution fits the data
H_a : The normal distribution does not fit the data

Note that $k = 2$ parameters (μ and σ) were estimated in order to calculate the theoretical probabilities and there are $n_f = 6$ frequencies in the above table $\Rightarrow \nu = n_f - 1 - k = 6 - 1 - 2 = 3$.

Thus, the critical value for the 5% level is $\chi^2_{3,\,0.05} = 7.815$. Since $9.882 > 7.815$ we can reject H_0 at the 5% level $\Rightarrow$ the evidence suggests that the data does not come from a normal distribution.

d) From the chi-squared tables we see that $\Pr(\chi^2_3 > 9.837) = 0.02$ and $\Pr(\chi^2_3 > 11.345) = 0.01$.

Hence p-value $= \Pr(\chi^2_3 > 9.882)$ is between 0.01 and 0.02 $\Rightarrow$ strong evidence against H_0.

Question 7

a) The observed frequencies are:

Observed		Languages 1	2	3	4+	$\sum$
University	A	16	38	39	7	100
	B	18	29	41	12	100
	C	28	31	38	3	100
	$\sum$	62	98	118	22	300

Hence, using the formula $e_{ij} = \frac{r_i \times c_j}{\text{total}}$, the expected frequencies are:

Expected		Languages 1	2	3	4+	$\sum$
University	A	$\frac{100(62)}{300}$ =20.67	$\frac{100(98)}{300}$ =32.67	$\frac{100(118)}{300}$ =39.33	$\frac{100(22)}{300}$ =7.33	100
	B	$\frac{100(62)}{300}$ =20.67	$\frac{100(98)}{300}$ =32.67	$\frac{100(118)}{300}$ =39.33	$\frac{100(22)}{300}$ =7.33	100
	C	$\frac{100(62)}{300}$ =20.67	$\frac{100(98)}{300}$ =32.67	$\frac{100(118)}{300}$ =39.33	$\frac{100(22)}{300}$ =7.33	100
	$\sum$	62	98	118	22	300

$\frac{(o_i - e_i)^2}{e_i}$		Languages 1	2	3	4+
University	A	1.055	0.870	0.003	0.015
	B	0.345	0.412	0.071	2.975
	C	2.599	0.085	0.045	2.558

b) The test statistic is $\chi^2 = \sum \frac{(o_i - e_i)^2}{e_i} = 11.033$.

c) Since $\nu = (n_r - 1)(n_c - 1) = (3-1)(4-1) = (2)(3) = 6$, the p-value $= \Pr(\chi^2_6 > 11.033)$.
From the chi-squared tables we see that $\Pr(\chi^2_6 > 10.645) = 0.1$ and $\Pr(\chi^2_6 > 12.592) = 0.05$.

Therefore, p-value $= \Pr(\chi^2_6 > 11.033)$ is between 0.05 and 0.1 which suggests that there is some evidence against H_0 but it is not strong (i.e., we would not reject at the 5% level).

Conclusion: The number of programming languages that a graduate is competent in *may* depend on the university (see comments below but bear in mind that the evidence is not strong).

d) The raw difference scores are:

$o_i - e_i$	Languages 1	2	3	4+
A	-4.67	5.33	-0.33	-0.33
B	-2.67	-3.67	1.67	4.67
C	7.33	-1.67	-1.33	-4.33

Compared with what we would expect:

- Uni-A has less students with only 1 language but more with 2 languages.
- Uni-B has are more students with a better skill-base.
- Uni-C has more students with a limited skill-base.