Question 5

Let $X \sim \text{Normal}(\mu = 10, \sigma = 2)$. Calculate the following:

(a) $\Pr(X > 10)$. **(b)** $\Pr(X < 3)$. **(c)** $\Pr(X > 8.4)$. **(d)** $\Pr(6 < X < 14)$. **(e)** The value of x such that $\Pr(X > x) = 0.3$. **(f)** The value of x such that $\Pr(X > x) = 0.8$.

Question 6

Assume that speeds of cars on a motorway have a normal distribution with mean 115km/hr and standard deviation 4km/hr.

(a) Draw a rough sketch of the distribution. **(b)** $\Pr(X > 120) = ?$ **(c)** $\Pr(X < 100) = ?$ **(d)** $\Pr(100 < X < 110) = ?$ **(e)** 1% of drivers travel above what speed?

Question 7

For *any* normal variable $X \sim \text{Normal}(\mu, \sigma)$:

(a) Show that $\Pr(\mu - 3\,\sigma < X < \mu + 3\,\sigma) = 0.997$. **(b)** Find a value for k such that $\Pr(\mu - k\,\sigma < X < \mu + k\,\sigma) = 0.95$. **(c)** Find k such that $\Pr(\mu - k\,\sigma < X < \mu + k\,\sigma) = 0.99$. **(d)** Show that $\Pr(X > \mu + 1.64\,\sigma) = 0.05$.

Question 1

Assume that the number of weekly study hours for students at a certain university is approximately normally distributed with a mean of 22 and a standard deviation of 6.

- Find the probability that a randomly chosen student studies less than 12 hours.
- Estimate the percentage of students that study more than 37 hours.

1 Tutorial G - Normal Distribution

Question 1

Assume that a character in a game is programmed to have an attack power according to $X \sim \text{Normal}(\mu = 40, \sigma = 3)$.

(a) What is the probability that the attack is greater than 45? **(b)** What is the probability that the attack is between 32 and 42? **(c)** Let X_1 and X_2 be the first and second attacks. What is the probability that the *sum* of these two attacks is greater than 85 units?
(d) Calculate 99% limits for the sum of two attacks. **(e)** What is the probability that the *difference* in attacks is more than 5 units? Note that attack 2 can be 5 units more than attack 1 or attack 1 can be 5 units more than attack 2, i.e., $\Pr(|D| > 5) = \Pr(D < -5) + \Pr(D > 5)$.

Question 2

1. The income of a technician (in thousands) is $X_1 \sim \text{Normal}(\mu = 30, \sigma = 2)$. The income of an engineer is $X_2 \sim \text{Normal}(\mu = 40, \sigma = 3.5)$.

   (a) Calculate the probability that an engineer earns more than a technician. **(b)** Calculate 90% limits for the difference in their income. **(c)** For a group of 25 technicians, calculate the probability that the average wage is less than 30500, i.e., $\Pr(\overline{X}_1 < 30.5)$. **(d)** In a group of 10 engineers, what is the probability that *at least two* of them earn more than 45000? (hint: binomial with $p = \Pr(X_2 > 45)$) **(e)** For a sample of 30 technicians and 35 engineers, calculate the 80% limits for the difference in their average wages.

2. Assume that the number of weekly study hours for students at a certain university is approximately normally distributed with a mean of 22 and a standard deviation of 6.

i. Find the probability that a randomly chosen student studies less than 12 hours.
ii. Estimate the percentage of students that study more than 37 hours.

3. *Taken from MA4104 Business Statistics Examination paper, Spring 2008 Question 1 part A*

   A tyre manufacturer claims that under normal driving conditions, the tread life of a certain tyre follows a normal distribution with mean 50,000 miles and standard deviation 5000 miles.

   (i) If your tyres wear out at 45,000 miles, would you consider this unusual? Support your answer with an appropriate probability calculation using the normal curve.
   (ii) If the manufacturer sells 100,000 of these tyres and warrants them to last at least 40,000 miles, about how many tyres will wear out before the warranty expires?

4. Suppose X is a normally distributed random variable with mean $\mu = 500$ and $\sigma = 24$

   a. (1 Mark) Compute the value of $P(X \geq 518)$
   b. (1 Mark) Compute the value of $P(X \leq 482)$
   c. (1 Mark) Compute the value of $P(482 \leq X \leq 518)$

5. A character in a game deals a standard attack 75% of the time and a critical attack the rest of the time (call these events S and S^c). Given that it is a standard attack, the attack power is $X \mid S \sim \text{Normal}(\mu = 40, \sigma = 3)$. When the character deals a critical attack, a random fluctuation is added to this according to a $\text{Normal}(\mu = 5, \sigma = 1)$ distribution.

   (a) What is the distribution of $X \mid S^c$? **(b)** Calculate $\Pr(X < 43 \mid S)$ and $\Pr(X < 43 \mid S^c)$. **(c)** Calculate $\Pr(X < 43)$. (hint: law of total probability) **(d)** If the character deals less than 43 damage points, what is the probability that the attack was a critical attack?

6. A model of an online computer system gives a mean time to retrieve a record from a direct access storage system device of 200 milliseconds with a standard deviation of 58 milliseconds. If it can be assumed that the data are normally distributed:

   (i) What proportion of retrieval times will be greater than 75 milliseconds?
   (ii) What proportion of retrieval times will be between 150 milliseconds and 250 milliseconds?
   (iii) What is the retrieval time below which 10% of retrieval times will be?

7. A machine produces components whose thicknesses are normally distributed with a mean of 0.40 cm and a standard deviation of 0.02 cm. Components are rejected if they have a thickness outside the range 0.38 cm to 0.41 cm.

   (i) What is the probability that a component will have a thickness exceeding 0.41 cm? (4 marks)
   (ii) What is the probability that a component will have a thickness between 0.38 cm and 0.41 cm? (4 marks)
   (iii) What is the thickness below which 25% of the components will be? (4 marks)

8. A charity believes that when it puts out an appeal for charitable donations the donations it receives will normally distributed with a mean £50 and standard deviation £6, and it is assumed that donations will be independent of each other.

   (i) Find the probability that the first donation it receives will be greater than £40.
   (ii) Find the probability that it will be between £55 and £60.
   (iii) Find the value x such that 5% of donations are more than £x.

9. Assume that the number of weekly study hours for students at a certain university is approximately normally distributed with a mean of 22 and a standard deviation of 6.

i. Find the probability that a randomly chosen student studies less than 12 hours.
ii. Estimate the percentage of students that study more than 37 hours.

10. A scientific publishing house produces assembly manuals for kit cars. The number of manuals sold every year is known to be normally distributed with a mean of 500 and a standard deviation of 50.

    a. (2 marks) What is the probability that the number of manuals sold will exceed 600?
    b. (2 marks) What is the probability that the number of manuals sold will be less than 300?
    c. (2 marks) What is the probability that the number of manuals sold will be between 450 and 550?
    d. (2 marks) What is the minimum number of manuals that the company must print such that that 90% of the demand is satisfied?

11. The lifetime of an electrical component is known to follow a normal distribution with a mean of 2,000 hours and a standard deviation of 200 hours.

    Compute the probability that a randomly selected component will last 1. more than 2,220 hours, 2. between 2,000 and 2,400 hours.

12. The amount of time required for routine automobile transmission service is normally distributed with the mean 45 minutes and the standard deviation 8.0 minutes.

    The service manager plans to have work begin on the transmission of a customer's car 10 min after the car is dropped off, and the customer is told that the car will be ready within 1 hour total time (i.e. after the car is dropped off).

    What is the probability that the service manager will be wrong?

13. The mass of shire horses is assumed to have a normal distribution with mean 1000kg and standard deviation 50kg. I. Calculate the probability that the mass of a shire horse is more than 975kgs. II. Calculate the probability that the mass of a shire horse is between 945kg and 1032kg. III. What weight is exceeded by 2.5

14. IQ is defined to have a normal distribution with mean 100 and standard deviation 15. a) Calculate the probability that a person's IQ is i) greater than 130 ii) less than 110 iii) between 82 and 120

    b) Calculate the IQ that is exceeded by 15% of the population.

15. An elevator can lift 600kg. 4 men and 4 women are in the lift. The mean mass of males is 80kg with a standard deviation of 20kg, the mean mass of females is 65kg with a standard deviation of 15kg. Assuming the weights of these individuals are independent and approximately normally distributed, estimate the probability that the elevator will lift these passengers.

    Note: 1) Use the results regarding the sum of independent, normally distributed random variables. 2) The sum of the masses of the males is the sum of 4 random variables.

16. The lengths of Padraig Harrington's drives are normally distributed with mean of 250m and standard deviation of 15m. The lengths of Rory McIlroy's drives are normally distributed with a mean of 245m and a standard deviation of 20m. Calculate the probability that Rory drives further than Padraig.