# Discrete Memoryless Channels

- A communication channel is the path or medium through which the symbols flow to the receiver.
- A discrete memoryless channel (DMC) is a statistical model with an input X and an output Y. During each unit of the time, the channel accepts an input symbol from X, and in response it generates an output symbol from Y.
- The channel is "discrete" when the alphabets of X and Y are both finite.
- It is "memoryless" when the current output depends on only the current input and not on any of the previous inputs.

# DISCRETE MEMORYLESS CHANNELS

## A. Channel Representation:

- A communication channel is the path or medium through which the symbols flow to the receiver.
- A discrete memoryless channel (DMC) is a statistical model with an input X and an output Y. During each unit ofthe time (signaling interval), the channel accepts an input symbol from X, and iu response it generates an output symbol from Y.
- The channel is ”discrete” when the alphabets of X and Y are both finite.
- It is “memoryless” when the current output depends on only the current input and not on any of the previous inputs.

# Discrete memoryless channel - FIX THIS

- A diagram of a DMC with nt inputs and n outputs is illustrated in Fig. l0-1. The input X consists of input symbols ir, , X2, xm.
- The a priori probabilities of these source symbols P(.x,) are assumed to be known.
- The output $Y$ consists of output symbols $\{y_1, y_2, \ldots, y_i\}$
- Each possible input-to-output path is indicated along with a conditional probability $P(y_i|x_i)$, where $P(y_i|x_i)$ is the conditional probability of obtaining output $y_i$ given that the input is $x_i$, and is called a ***channel transition probability***.

# Channel Matrix - FIX THIS

A channel is completely specified by the complete set of transition probabilities. Accordingly, the channel of Fig. 10-l is ol‘ ten speciiied by the matrix of transition probabilities [P(YlX)l, given by
The matrix [P(YlX)] is called thc channel matrix. Since each input to the channel results in some output, each row of the channel matrix niust snm to unity. that is,

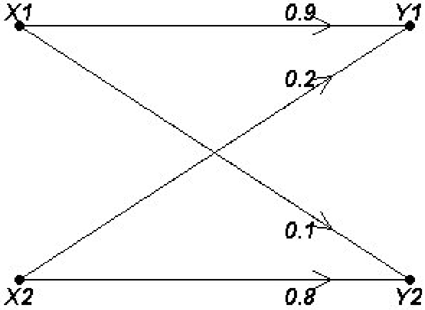
# Discrete memoryless channel

- A DMC can have any number of inputs and any number of outputs.
- For a DMC with "m" inputs and "n" outputs, the input X consists of input symbols $x_1, x_2, \ldots x_m$.
- The probabilities of these source symbols $P(x_i)$ are assumed to be known.
- The output Y consists of output symbols $\{y_1, y_2, \ldots, y_n\}$
- Each possible input-to-output path is indicated along with a conditional probability $P(y_i|x_i)$, where $P(y_i|x_i)$ is the conditional probability of obtaining output $y_i$ given that the input is $x_i$.
- $P(y_i|x_i)$ is called a ***channel transition probability***.

# Discrete memoryless channel

- On the next slide, we present a binary DMC, with the channel transition probabilities indicated.
- $P(Y_1|X_1) = 0.9$ and $P(Y_2|X_1) = 0.1$
- $P(Y_1|X_2) = 0.2$ and $P(Y_2|X_2) = 0.8$

# Discrete Memoryless Channels



# Channel Matrix

A channel is completely specified by the complete set of transition probabilities. Accordingly, a channel is specified by the matrix of transition probabilities $[P(YlX)]$, given by

$$[P(Y|X)] = \begin{bmatrix} P(y_1|x_1) & P(y_2|x_1) & \dots & P(y_n|x_1) \\ P(y_1|x_2) & P(y_2|x_2) & \dots & P(y_n|x_2) \\ \dots & \dots & \dots & \dots \\ P(y_1|x_m) & P(y_2|x_m) & \dots & P(y_n|x_n) \end{bmatrix}$$

The matrix $[P(YlX)]$ is called the ***channel matrix***.

# Channel Matrix

- Since each input to the channel results in some output, each row of the channel matrix must sum to unity (i.e. all rows must add up to 1. This condition is not necessary for columns).
- For the binary DMC presented previously, the channel matrix is

$$[P(Y|X)] = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

- (Remark: This is not a binary symmetric channel)

# Channel Matrix

- The input probabilities $P(X)$ are represented by the row matrix

$$[P(X)] = \left[ \begin{array}{cccc} P(x_1) & P(x_2) & \ldots & P(x_m) \end{array} \right]$$

- The input probabilities $P(Y)$ are represented by the row matrix

$$[P(Y)] = \left[ \begin{array}{cccc} P(y_1) & P(y_2) & \ldots & P(y_n) \end{array} \right]$$

- We can compute $[P(Y)]$ by the following formula:
  $[P(Y)] = [P(X)] \times [P(Y|X)]$
- (Note: Be mindful of the dimensions of each matrix).

# Channel Matrix

- Suppose for our Binary DMC that the input probabilities were given by $[P(X)] = [0.5\ 0.5]$.
- Compute $[P(Y)]$, given the channel matrix given in previous slides.

$$[P(Y)] = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \times \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

- Solving

$$[P(Y)] = \begin{bmatrix} (0.5 \times 0.9) + (0.5 \times 0.2) & (0.5 \times 0.1) + (0.5 \times 0.8) \end{bmatrix}$$

- Simplifying

$$[P(Y)] = \begin{bmatrix} 0.55 & 0.45 \end{bmatrix}$$

# Channel Matrix

- Let $[P(X)]$ is presented as a diagonal matrix , i.e.

$$[P(X)]_d = \begin{bmatrix} P(x_1) & 0 & \dots & 0 \\ 0 & P(x_2) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & P(x_m) \end{bmatrix}$$

- The ***joint probability matrix*** $[P(X,Y)]$can be computed as $[P(X,Y)] = [P(X)]_d \times [P(Y|X)]$

## Channel Matrix

- For the Binary DMC described in the previous example, compute the joint probability matrix.
- Diagonalize the input probabilities for $X$.

$$[P(X)]_d = \begin{bmatrix} 0.5 & 0.0 \\ 0.0 & 0.5 \end{bmatrix}$$

- Simplifying

$$[P(X,Y)] = \begin{bmatrix} (0.5\times 0.9)+(0\times 0.2) & (0.5\times 0.1)+(0\times 0.8) \\ (0\times 0.9)+(0.5\times 0.2) & (0\times 0.1)+(0.5\times 0.8) \end{bmatrix}$$

- Solving

$$[P(X,Y)] = \begin{bmatrix} 0.45 & 0.05 \\ 0.1 & 0.4 \end{bmatrix}$$

Notice the row and column totals.

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