

Statistics and Probability

Probability Functions

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Probability Functions

The random variable X has probability density function given by

$$f_X(x) = 3e^{-3x}$$

defined over the region $x > 0$. Let $Y = e^x$.

1. Compute $E(Y)$ (i.e. Compute $E(e^x)$).
2. Compute $\text{Var}(Y)$.

Remarks

- $e^0 = 1$
- $e^{-\infty} = 0$

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Question 1: Compute $E(Y)$:

$$E(X) = \int_0^{\infty} x f_X(x) dx$$

$$E(Y) = \int_0^{\infty} Y 3e^{-3x} dx$$

$$E(e^x) = \int_0^{\infty} e^x 3e^{-3x} dx$$

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$$E(e^x) = \int_0^{\infty} e^x 3e^{-3x} dx$$

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Question 2: Compute $\text{Var}(Y)$.

$$\text{Var}(Y) = E(Y^2) - E(Y)^2$$

$$\text{N.B: } Y^2 = (e^x)^2 = e^{2x}$$

$$E(Y^2) = \int_0^{\infty} Y^2 \cdot 3e^{-3x} dx$$

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$$E(e^{2x}) = \int_0^{\infty} e^{2x} 3e^{-3x} dx$$

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$$\text{Var}(Y) = E(Y^2) - E(Y)^2$$