Q.6 (a)

(i) $E[T_S] = 0.025$ hrs $\Rightarrow \lambda_S = \frac{1}{0.025} = 40$ / hr.

(ii) $T \sim Exp(\lambda = \lambda_S - \lambda_a)$
$= 40 - 30$
$= 10.$

$\Rightarrow E[T] = \frac{1}{10}$ hrs $= 6$ mins.

$E[N] = \lambda_a E[T] = 30\left(\frac{1}{10}\right) = 3$ customers.

$E[T_q] = E[T] - E[T_S]$
$= 0.1 - 0.025$
$= 0.075$ hrs $= 4.5$ mins.

$E[N_q] = \lambda_a E[T_q] = 30(0.075) = 2.25$ customers

(iii) $\rho = \frac{\lambda_a}{\lambda_S} = \frac{30}{40} = 0.75 \Rightarrow 75\%.$

The service counter is busy 75% of the time.
(and idle 25% of the time).

(iv) 12 minutes $= \frac{12}{60} = 0.2$ hrs.

$Pr(T > 0.2) = e^{-\lambda t} = e^{-10(0.2)} = e^{-2} = 0.1353.$

$\Rightarrow Pr(T < 0.2) = 1 - 0.1353$
$= 0.8647$

(v) Exit rate is $\lambda_a = 30$ / hr. (same as arrival).
$\Rightarrow \lambda_a = 30\left(\frac{1}{6}\right) = 5$ / 10 mins i.e. $\frac{1}{6}$ of an hr.

$Pr(X > 7) = Pr(X \geq 8) = 0.1334.$