Question 1

a)

H_0 : The exponential model fits the data

H_a : The exponential model does not fit the data

b)

Class	<3	3 - 6	6 - 9	9 - 12	12 - 15	>15	$\sum$
o_i	56	24	12	6	1	1	100
p_i	0.577	0.244	0.103	0.044	0.018	0.014	
$100 \times p_i = e_i$	57.7	24.4	10.3	4.4	1.8	1.4	100

c) We need to combine the last three columns so that all expected frequencies are bigger than 5.

Class	<3	3 - 6	6 - 9	>9	$\sum$
o_i	56	24	12	8	100
e_i	57.7	24.4	10.3	7.6	100
$\frac{(o_i-e_i)^2}{e_i}$	0.050	0.007	0.281	0.021	0.359

d) There are 4 frequencies in the above table and we estimated λ to calculate the exponential probabilities $\Rightarrow \nu = n_f - 1 - k = 4 - 1 - 1 = 2$.

Thus, for the 5% level the critical value is $\chi^2_{\nu,\alpha} = \chi^2_{2,\,0.05} = 5.991$.

e) Since the test statistic $\chi^2 = \sum \frac{(o_i-e_i)^2}{e_i} = 0.359$ is below the critical value, we accept the null hypothesis.

Conclusion: the exponential model fits this data.

Question 2

a)

H_0 : The examiner and grade awarded are *independent*

H_a : The examiner and grade awarded are *dependent*

Observed	Examiner 1	Examiner 2	Examiner 3	$\sum$
Grade A	9	6	10	25
Grade B	11	20	26	57
Grade C	25	19	74	118
$\sum$	45	45	110	200

We need to calculate the expected frequencies using the formula: $e_{ij} = \frac{r_i \times c_j}{\text{total}}$.

For example, $\frac{25(45)}{200} = 5.625$ corresponds to row one, column one.

Question 2 Continued

a) The expected frequencies are:

Observed	Examiner 1	Examiner 2	Examiner 3	$\sum$
Grade A	$\frac{25(45)}{200}$ =5.625	$\frac{25(45)}{200}$ =5.625	$\frac{25(110)}{200}$ =13.75	25
Grade B	$\frac{57(45)}{200}$ =12.825	$\frac{57(45)}{200}$ =12.825	$\frac{57(110)}{200}$ =31.35	57
Grade C	$\frac{118(45)}{200}$ =26.550	$\frac{118(45)}{200}$ =26.550	$\frac{118(110)}{200}$ =64.90	118
$\sum$	45	45	110	200

To calculate the test statistic we need the following:

$\frac{(o_i-e_i)^2}{e_i}$	Examiner 1	Examiner 2	Examiner 3
Grade A	2.02	0.02	1.02
Grade B	0.26	4.01	0.91
Grade C	0.09	2.15	1.28

$\Rightarrow \chi^2 = \sum \frac{(o_i-e_i)^2}{e_i} = 11.77.$

Note that $\nu = (n_r - 1) \times (n_c - 1) = (3-1)(3-1) = (2)(2) = 4$ and $\alpha = 0.05 \Rightarrow \chi^2_{4,\,0.05} = 9.488$.

The test statistic $\chi^2 = 11.77$ lies above the critical value $\chi^2_{4,\,0.05} = 9.488$ and, hence, we reject the hypothesis that there is no relationship between the two variables.

Conclusion: The grade awarded depends on the specific examiner.

b) The raw difference scores are:

$o_i - e_i$	Examiner 1	Examiner 2	Examiner 3
Grade A	3.38	0.38	−3.75
Grade B	−1.83	7.17	−5.35
Grade C	−1.55	−7.55	9.10

The main points are:

- Examiner 1 gives more As than expected but less Bs and Cs.
- Examiner 2 gives more Bs than expected but less Cs.
- Examiner 3 gives less As and Bs but more Cs.

It is better if Examiner 1 or 2 mark the exam.