Question 1

a)
$$E(T) = \frac{1}{\lambda} = 5$$

$$\Rightarrow \lambda = \frac{1}{E(T)} = \frac{1}{5} = 0.2 \text{ cust. / minute.}$$

Thus, the exponential probability function is $\Pr(T > t) = e^{-\lambda t} = e^{-0.2\,t}$.

b)
$$\Pr(T > 15) = e^{-0.2(15)} = e^{-3} = 0.0498.$$

c)
$$\begin{aligned}\Pr(T < 1) &= 1 - \Pr(T > 1) \\ &= 1 - e^{-0.2(1)} \\ &= 1 - 0.8187 \\ &= 0.1813.\end{aligned}$$

d) This is the *number of customers* (Poisson) rather than the time between (exponential).

$X \sim \text{Poisson}(\lambda = 0.2)$ for a 1 minute period.

For a *one hour* period we have a λ value of $0.2(60) = 0.2(60) = 12$ customers / hour.

$X \sim \text{Poisson}(12)$ for a 1 hour period. Thus:

$$\begin{aligned}E(X) &= \lambda = 12 \text{ customers.} \\ Var(X) &= \lambda = 12 \text{ customers}^2. \\ Sd(X) &= \sqrt{Var(X)} = \sqrt{12} = 3.46 \text{ customers.}\end{aligned}$$

e) 1 hour period $\Rightarrow \lambda = 12$ again.

$$\Pr(X \geq 15) = 0.2280.$$

(found using the Poisson tables ($m = \lambda = 12$) since using the probability function here is very laborious.)