Entropies: Example

- The input source to a noisy communication channel is a random variable X over the four symbols $\{a, b, c, d\}$.
- The output from this channel is a random variable Y over these same four symbols.

Entropies: Example

The joint distribution of these two random variables is as follows:

	x=a	x=b	x=c	x=d
y=a	1/8	1/16	1/16	1/4
y=b	1/16	1/8	1/16	0
y=c	1/32	1/32	1/16	0
y=d	1/32	1/32	1/16	0

Entropies: Example

- Write down the marginal distribution for X and compute the marginal entropy $H(X)$.
- Write down the marginal distribution for Y and compute the marginal entropy $H(Y)$.
- (next class) What is the joint entropy $H(X,Y)$ of the two random variables?
- (next class) What is the conditional entropy $H(Y|X)$?
- (next class) What is the conditional entropy $H(X|Y)$?
- (next class) What is the mutual information $I(X;Y)$ between the two random variables?

Entropies: Example

The marginal distribution of these two random variables is as follows:

	x=a	x=b	x=c	x=d	P(Y)
y=a	1/8	1/16	1/16	1/4	0.50
y=b	1/16	1/8	1/16	0	0.25
y=c	1/32	1/32	1/16	0	0.125
y=d	1/32	1/32	1/16	0	0.125
P(X)	0.25	0.25	0.25	0.25	

Entropies: Example

- H(X) , the entropy of X, is computed as

$$H(X) = -\sum P(x_i)\log_2 P(x_i)$$

- $H(X) = (-0.25 \times -2) + (-0.25 \times -2) + (-0.25 \times -2) + (-0.25 \times -2)$
- $H(X) = 2\text{b}$

- H(X) , the entropy of Y, is computed as

$$H(Y) = -\sum P(y_j)\log_2 P(y_j)$$

- $H(Y) = (-0.5 \times -1) + (-0.25 \times -2) + (-0.125 \times -3) + (-0.125 \times -3)$
- $H(Y) = 1.75\text{b}$