Question 1

Assume that a character in a game is programmed to have an attack power according to $X \sim \text{Normal}(\mu = 40, \sigma = 3)$.

(a) What is the probability that the attack is greater than 45? **(b)** What is the probability that the attack is between 32 and 42? **(c)** Let X_1 and X_2 be the first and second attacks. What is the probability that the *sum* of these two attacks is greater than 85 units?
(d) Calculate 99% limits for the sum of two attacks. **(e)** What is the probability that the *difference* in attacks is more than 5 units? Note that attack 2 can be 5 units more than attack 1 or attack 1 can be 5 units more than attack 2, i.e., $\Pr(|D| > 5) = \Pr(D < -5) + \Pr(D > 5)$.

Question 2

A character in a game deals a standard attack 75% of the time and a critical attack the rest of the time (call these events S and S^c). Given that it is a standard attack, the attack power is $X \mid S \sim \text{Normal}(\mu = 40, \sigma = 3)$. When the character deals a critical attack, a random fluctuation is added to this according to a $\text{Normal}(\mu = 5, \sigma = 1)$ distribution.

(a) What is the distribution of $X \mid S^c$? **(b)** Calculate $\Pr(X < 43 \mid S)$ and $\Pr(X < 43 \mid S^c)$. **(c)** Calculate $\Pr(X < 43)$. (hint: law of total probability) **(d)** If the character deals less than 43 damage points, what is the probability that the attack was a critical attack?

Question 3

The income of a technician (in thousands) is $X_1 \sim \text{Normal}(\mu = 30, \sigma = 2)$. The income of an engineer is $X_2 \sim \text{Normal}(\mu = 40, \sigma = 3.5)$.

(a) Calculate the probability that an engineer earns more than a technician. **(b)** Calculate 90% limits for the difference in their income. **(c)** For a group of 25 technicians, calculate the probability that the average wage is less than 30500, i.e., $\Pr(\overline{X}_1 < 30.5)$. **(d)** In a group of 10 engineers, what is the probability that *at least two* of them earn more than 45000? (hint: binomial with $p = \Pr(X_2 > 45)$) **(e)** For a sample of 30 technicians and 35 engineers, calculate the 80% limits for the difference in their average wages.

Question 4

Let $X \sim \text{Exponential}(\lambda = 0.02)$. Calculate the following:

(a) $\Pr(\overline{X} > 55)$ in a group of 100. **(b)** $\Pr(\overline{X} < 53)$ in a group of 40. **(c)** The value of $\bar{x}$ such that $\Pr(\overline{X} > \bar{x}) = 0.1$ when $n = 65$. **(c)** The value of n if $\Pr(\overline{X} < 49) = 0.1$.