Question 1

A machine is set to produce laptop sleeves with the following dimensions:
length × width × depth = 40 cm × 30 cm × 2 cm. A sample of 40 sleeves was selected and each was measured. The results were as follows:

	length	width	depth
$\bar{x}$	40.11	30.09	1.91
s	0.51	0.17	0.15

Test the following hypotheses (use the 5% level of significance in each case):

(a) The mean length is equal to 40cm. **(b)** The mean width is equal to 30cm. **(c)** The mean depth is equal to 2cm. **(d)** Both the width and the depth of the sleeve need to be addressed here - which do you think is more urgent? **(d)** Calculate the p-values for the tests carried out in parts (a), (b) and (c).

Question 2

A matchbox is supposed to contain 100 matches. We wish to test this hypothesis.

(a) State the null and alternative hypotheses. **(b)** From a sample of 32 matchboxes, it is found that the average is 99.4 and the standard deviation is 2.1. Calculate the test statistic. **(c)** Provide your conclusion based on the p-value.

Question 3

An aircraft part is designed to last more than 500 hours. However, in the interest of safety, it will first be assumed that the part lasts *less than or equal to* 500 hours (i.e., this is the null hypothesis) unless there is firm evidence suggesting otherwise.

(a) State the null and alternative hypotheses. **(b)** What is the critical value if $\alpha = 0.001$ and only 4 units will be run until wearout (due to the expense of wasting aircraft parts). **(c)** In this sample of size 4, it is found that the average is 566 hours and the variance is 83 hours^2. Calculate the test statistic. **(d)** What is the conclusion?

Question 4

A friend claims that he can pass a particular game in 4 hours or less (on average). We wish to test this hypothesis at the 10% level of significance. Your friend plays the game on 6 different occasions: his average completion time is 4.6 hours and the standard deviation is 0.5 hours.

(a) State the null and alternative hypotheses. **(b)** What is the critical value? **(c)** Calculate the test statistic and provide your conclusion. **(d)** Between what two values does the p-value lie? (note: the p-value cannot be calculated exactly using the t-tables)

Question 5

A die is rolled 80 times and we count 18 sixes. We wish to test the hypothesis that the die is fair (note: if this is the case, the proportion of sixes is $p = \frac{1}{6}$).

(a) State the null and alternative hypotheses. **(b)** If we wish to test at the 5% level of significance, what is the critical value? **(c)** Calculate the test statistic and provide your conclusion.

Question 6

Assume that a particular brand dominates the market. More specifically, it is well-known that at least 60% of people use this brand (i.e., $p \geq 0.6$). However, in response to recent media claims that this brand is weakening, the company wish to test the hypothesis that $p \geq 0.6$.

(a) State the null and alternative hypotheses. **(b)** From a sample of 1000 people, it is found that 629 use this brand; calculate the test statistic and, hence, the p-value. **(c)** Based on the evidence, state your conclusion.

Question 7

Last year 30% of applicants to a graduate programme failed the aptitude test. This year 100 graduates applied - 25% of these failed the test.

(a) We wish to test the hypothesis that the quality of applicants has not changed since last year - what are the null and alternative hypotheses? **(b)** If we are testing at the 1% level, what is the rejection region? **(c)** Based on the data, calculate the test statistic and provide your conclusion.

Question 8

A soft drinks company is working on a new recipe for its best-selling drink. The company intends to carry out a study where participants will taste both flavours (current and new) and then answer the question:

> "Do you prefer the new flavour?"

It is assumed that the *current* recipe is superior, i.e., that *less than or equal to* 50% of people prefer the new drink ($p \leq 0.5$).

We wish to test the hypothesis that $p \leq 0.5$.

(a) State the null and alternative hypotheses. **(b)** From a sample of 65 people, we find that 43 people prefer the new recipe. Calculate the test statistic and, hence, the p-value.
(c) Based on the evidence, state your conclusion.