

# Entropy Calculations

Suppose we have a five letter source alphabet, with  $m=5$  symbols i.e.  $\{A, B, C, D, E\}$ . In each of the following 7 examples, we adjust the probabilities for each symbol. We compute the information for each symbol ( $I(x_i)$ ), and hence the entropy ( $H(X)$ ).

$$I(x_i) = -\log_2(p(x_i))$$

$$H(X) = \sum_{i=1}^{i=m} (I(x_i) \times p(x_i))$$

## Example 1

$x_i$	$P(x_i)$	$I(x_i)$	$P(x_i) \times I(x_i)$
A	0.990	0.0145	0.0144
B	0.005	7.6439	0.0382
C	0.002	8.9658	0.0179
D	0.002	8.9658	0.0179
E	0.001	9.9658	0.0100
	1.000		0.0984

- 99% of the time, the symbol  $A$  is transmitted
- Low information content.  $H(X) = 0.0984$  b/sym

## Example 2

$x_i$	$P(x_i)$	$I(x_i)$	$P(x_i) \times I(x_i)$
A	0.900	0.1520	0.1368
B	0.050	4.3219	0.2161
C	0.020	5.6439	0.1129
D	0.020	5.6439	0.1129
E	0.010	6.6439	0.0664
	1.000		0.6451

- 90% of the time, the symbol  $A$  is transmitted. Other symbols more common.
- Slightly higher information content.  $H(X) = 0.6451$  b/sym

### Example 3

$x_i$	$P(x_i)$	$I(x_i)$	$P(x_i) \times I(x_i)$
A	0.800	0.3219	0.2575
B	0.100	3.3219	0.3322
C	0.050	4.3219	0.2161
D	0.030	5.0589	0.1518
E	0.020	5.6439	0.1129
	1.000		1.0705

- 80% of the time, the symbol  $A$  is transmitted. Again, other symbols more common.
- Slightly higher information content.  $H(X) = 1.0705$  b/sym

### Example 4

- Continue process of equalizing symbol probabilities over next few examples.
- Entropy values consistently increase.

$x_i$	$P(x_i)$	$I(x_i)$	$P(x_i) \times I(x_i)$
A	0.650	0.6215	0.4040
B	0.110	3.1844	0.3503
C	0.100	3.3219	0.3322
D	0.080	3.6439	0.2915
E	0.060	4.0589	0.2435
	1.000		1.6215

- information content.  $H(X) = 1.6215$  b/sym

### Example 5

$x_i$	$P(x_i)$	$I(x_i)$	$P(x_i) \times I(x_i)$
A	0.400	1.3219	0.5288
B	0.250	2.0000	0.5000
C	0.150	2.7370	0.4105
D	0.120	3.0589	0.3671
E	0.080	3.6439	0.2915
	1.000		2.0979

- information content.  $H(X) = 2.0979$  b/sym

### Example 6

$x_i$	$P(x_i)$	$I(x_i)$	$P(x_i) \times I(x_i)$
A	0.300	1.7370	0.5211
B	0.250	2.0000	0.5000
C	0.200	2.3219	0.4644
D	0.130	2.9434	0.3826
E	0.120	3.0589	0.3671
	1.000		2.2352

- information content.  $H(X) = 2.2352$  b/sym

### Example 7

- Equal probability of each symbol.
- Maximum level of entropy.
- Remark, where  $m$  is the number of symbols in the source alphabet.

$$\log_2(m) = \log_2(5) = 2.3219$$

$x_i$	$P(x_i)$	$I(x_i)$	$P(x_i) \times I(x_i)$
A	0.200	2.3219	0.4644
B	0.200	2.3219	0.4644
C	0.200	2.3219	0.4644
D	0.200	2.3219	0.4644
E	0.200	2.3219	0.4644
	1.000		2.3219

- When each symbol is equally probably, we can say

$$H(x) = \log_2(m)$$