1. A standard pack of 52 playing cards consists of 4 suits (clubs, diamonds, hearts and spades), each consisting of 13 cards numbered $2, 3, 4, \ldots, 10$, Jack, Queen, King, Ace (their face values). In the game of poker, a hand of 5 cards is drawn without replacement from a well-shuffled pack.

   (a) How many different poker hands are possible?

   (b) A poker hand consisting of a pair of cards with the same face value and three other cards with the same face value (different from that of the pair) is called a full house. Find the probability that a poker hand drawn from a well-shuffled pack is a full house. Express your answer either as a fraction in lowest terms or as a decimal correct to 3 significant figures.

   (c) A poker hand consisting of a pair of cards with the same face value and three other cards with face values different from each other and from that of the pair is called a pair. Find the probability that a poker hand drawn from a wellshuffled pack is a pair. Express your answer either as a fraction in lowest terms or as a decimal correct to 3 significant figures.

2. Clay pots made at a pottery are subject to three types of defect. It is found that 10% of pots show brittle fracture (B), 4% have cracked glazing (C) and 10% are discoloured (D).

   (a) Assuming that all three types of defect occur independently, what is the probability that a randomly chosen pot has no defects?

   (b) Experience has shown that the three types of defect do not all occur independently. 20% of pots with brittle fracture also have cracked glazing, but both of these defects occur independently of discoloration.

      (i) Find the probability that a pot has both brittle fracture and cracked glazing, and hence find the probability that a pot has either or both of these defects.

      (ii) What is now the probability that a randomly chosen pot has no defects? (5)

      (iii) Suppose that a pot does not have cracked glazing and is not discoloured. Find the probability that it has brittle fracture. (6)

3. In Newtopia, the weather on any day is dry with probability 23 and wet with probability 13, the weather on different days being independent.

   (a) Find the probability that the next three days are dry.

   (b) Find the probability that exactly two of the next three days are wet.

   (c) A Newtopian resident walks his dog with probability 0.9 when it is dry but with probability 0.6 when it is wet. If it is known that he walked his dog last Tuesday, what is the probability that last Tuesday was dry in Newtopia?

4. Suppose now that the weather on different days is not independent but that

$$P(\text{next day is dry}|\text{today is dry}) = 0.8$$

and

$$P(\text{next day is wet}|\text{today is wet}) = 0.6$$

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(a) Given that today is dry, what is the probability that the next three days are dry?

(b) Given that today is wet, what is the probability that exactly two of the next three days are wet?

(c) Let p denote the overall probability that any day is dry. Explain clearly why p must satisfy the equation

$$0.8p + 0.4(1p) = p,$$

and deduce the value of p.

5. Let A and B be two events with $P(B) > 0$.

   (a) Write down an expression for the conditional probability of A given B.

   (b) Determine the conditional probability of A given B in the following cases.

      (i) A and B are independent.

      (ii) A and B are mutually exclusive.

      (iii) $B \subset A$.

6. A diagnostic test for a disease gives a positive result with probability 0.98 for people who have the disease, and a negative result with probability 0.99 for people who do not have the disease. Suppose 3% of the population have the disease.

   (a) A person is selected at random from the population and given the test. If the result is positive, what is the probability that this person has the disease?

   (b) Suppose a person, initially selected at random from the population, is given the test once and the result is positive. This person is then given the test, independently, a second time and the result is again positive. What is the probability that this person has the disease?

7. Two football teams M and C each have one game left to play (not against each other) in the season. If M wins and C does not win, or if M draws and C loses, then M wins the championship. Otherwise C wins the championship. The probabilities that M wins, draws or loses the last game are 1/2 ,1/6 and 1/3 respectively. The probabilities that C wins, draws or loses the last game are 2/3, 1/6 and 1/6 respectively.

   (a) What is the probability that M wins the championship?

   (b) What is the probability that C has drawn the last game given that M has won the championship?