Statistics for Computing MA4413

Midterm Examination 1

Type B

- Do not turn over the page until instructed to do so.
- Rough work pages are provided within.
- Useful formulae and Binomial tables are provided at the back.
- **Enter your answers (using an "X") in the table on the last page.**
- There are 15 questions in total. Each question answered $\begin{cases} \text{correctly} &= 1\%. \\ \text{incorrectly} &= -\frac{1}{3}\%. \end{cases}$
- For each question, only *one* answer is correct.
- Scientific calculators approved by the University of Limerick can be used.

Questions 1 - 5

Q1 The time it takes you to complete this exam is an example of which type of data?

(a) numeric discrete **(b)** categorical **(c)** numeric continuous **(d)** infinite

---

A large online retailer believes that its customers spend on average €55 per transaction. Each week over 100,000 transactions are made. Thus, a random sample of 500 transactions were selected and the sample average was found to be €63.20.

Q2 What is the parameter here?

(a) $\mu =$ unknown **(b)** $n = 100,000$ **(c)** $p =$ unknown **(d)** $\mu = 55$

Q3 What is the statistic here?

(a) $\hat{p} = \frac{63.20}{500}$ **(b)** $\hat{p} = \frac{500}{100,000}$ **(c)** $\mu = 55$ **(d)** $\bar{x} = 63.20$

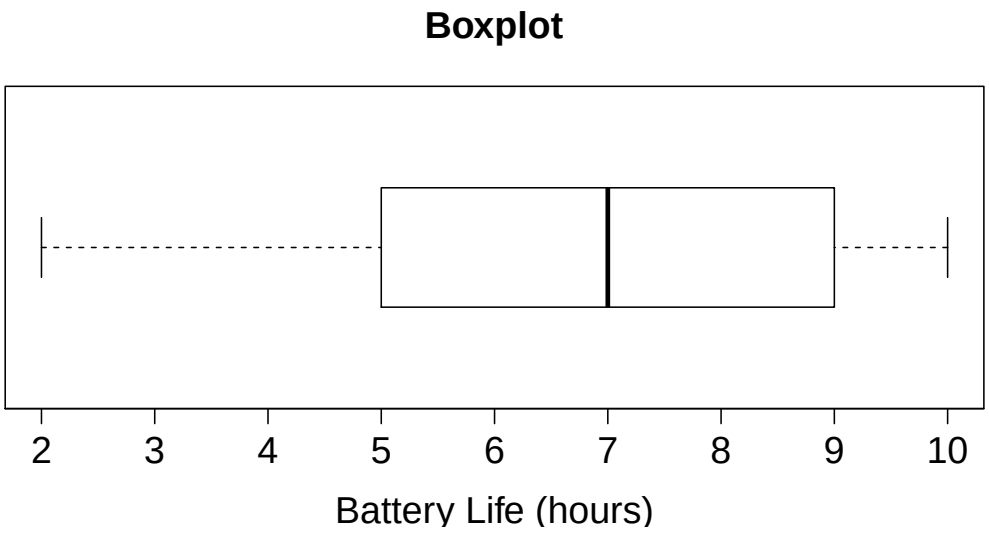
Consider the following sample of ages of mechanical components:

3	5	4	12	4	8

Q4 What is the value of the sample variance?

(a) 11.6 years^2 **(b)** 3.41 years **(c)** 8.3 years^2 **(d)** 11.6 years

---



Q5 Based on the above boxplot, what is the value of the IQR?

(a) 8 hours **(b)** 2 hours **(c)** 4 hours **(d)** 9 hours

Rough Work

Next page: Questions 6 - 10

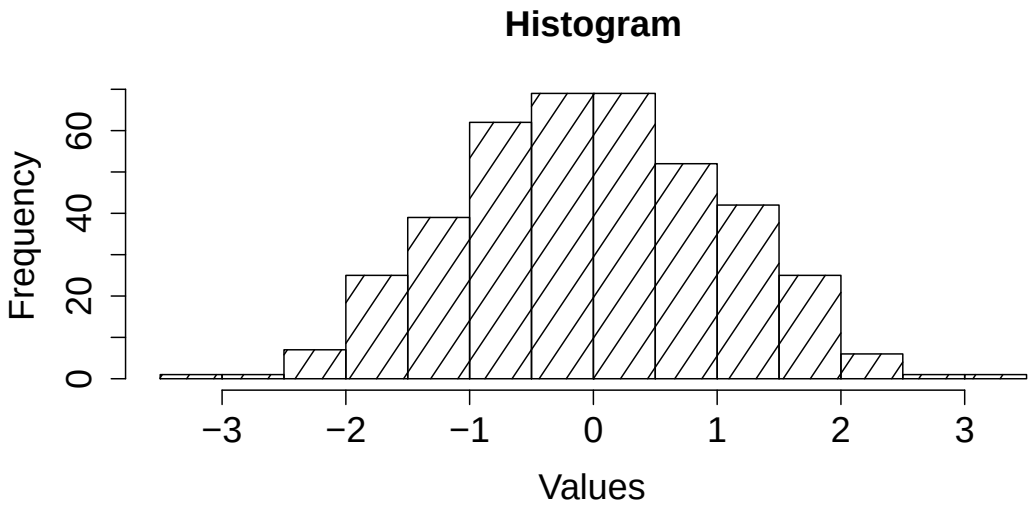
Questions 6 - 10

Consider the following set of numbers:

13	5	21	11	15	13	12	14	8

Q6 What is the value of the 1st quartile?

(a) 13 **(b)** 9.5 **(c)** 2.5 **(d)** 14.5



Q7 From the above histogram, the distribution of data is:

(a) frequent **(b)** right-skewed **(c)** left-skewed **(d)** symmetrical

A mobile application developer employs two coders (call them C_1 and C_2 for simplicity): 80% of all code is written by C_1 and the remaining 20% is written by C_2. Given that C_1 writes the code, there is 10% chance that it contains a bug. On the other hand, given that C_2 writes the code, there is 30% chance that it contains a bug.

(Note: answers below are given to two decimal places)

Q8 Let B represent the event that the code contains a bug. What is the value of $\Pr(B)$?

(a) 0.03 **(b)** 0.26 **(c)** 0.14 **(d)** 0.40

Q9 Some code is checked and found to be *bug-free*; what is the probability that C_2 wrote it?

(a) 0.70 **(b)** 0.84 **(c)** 0.16 **(d)** 0.43

Q10 You have 9 t-shirts. You're going on holidays and can only bring 4 of them. How many possible groups of 4 t-shirts are there if one of them is your "lucky" t-shirt and you *must* bring it?

(a) 336 **(b)** 56 **(c)** 84 **(d)** 126

Rough Work

Next page: Questions 11 - 15

Questions 11 - 15

Q11 Let $\Pr(A) = 0.6$, $\Pr(B) = 0.7$ and $\Pr(A \cap B) = 0.55$. What is the value of $\Pr(A^c \cap B^c)$?

(a) 0.75 **(b)** 0.25 **(c)** 0.45 **(d)** 0.42

Consider the following *joint* distribution for X and Y:

		X			
		1	2	3	$p(y)$
Y	1	0.25	0.10	0.10	0.45
	2	0.05	0.35	0.15	0.55
	$p(x)$	0.3	?	0.25	1

(Note: answers below are given to two decimal places)

Q12 What is the value of $E(X)$?

(a) 2.05 **(b)** 1.95 **(c)** 1.55 **(d)** 0.45

Q13 What is the value of $\Pr(X = 3 \,|\, Y = 1)$?

(a) 0.22 **(b)** 0.40 **(c)** 0.27 **(d)** 0.11

There is a 6% chance of pressing the wrong button on a keyboard and thus make a typographical error. Assuming these errors occur independently, the number of errors in typing n letters is $X \sim \text{Binomial}(n, p)$.

(Note: answers below are given to four decimal places)

Q14 You type 11 letters. What is the probability of making *less than* 2 errors?

(a) 0.3555 **(b)** 0.8618 **(c)** 0.1135 **(d)** 0.9752

Q15 You type 100 letters. What is $\Pr(X > 10)$?

(a) 0.0209 **(b)** 0.0399 **(c)** 0.0376 **(d)** 0.0775

Rough Work

Don't forget to enter your answers on the last page!

Useful Formulae: Page 1

Histogram:

- class width $= \dfrac{\max(x) - \min(x)}{\text{number of classes}}$

Numerical Summaries:

- $\bar{x} = \dfrac{\sum x_i}{n}$
- $s^2 = \dfrac{\sum x_i^2 - n\,\bar{x}^2}{n-1}$
- Position of Q_k : $\dfrac{n+1}{4} \times k$
- $IQR = Q_3 - Q_1$
- $LF = Q_1 - 1.5 \times IQR$
- $UF = Q_3 + 1.5 \times IQR$

Probability:

- $\Pr(A^c) = 1 - \Pr(A)$
- $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
- $\Pr(E_1 \cup E_2 \cup \cdots \cup E_k) = \Pr(E_1) + \Pr(E_2) + \cdots + \Pr(E_k)$ (if mutually exclusive)
- $\Pr(A \cap B) = \Pr(A)\ \Pr(B \mid A) = \Pr(B)\ \Pr(A \mid B)$
- $\Pr(E_1 \cap E_2 \cap \cdots \cap E_k) = \Pr(E_1)\ \Pr(E_2) \cdots \Pr(E_k)$ (if independent)
- $\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)} = \dfrac{\Pr(A)\ \Pr(B \mid A)}{\Pr(B)}$
- If $E_1, \ldots, E_k$ are mutually exclusive & exhaustive

  $\Rightarrow \Pr(B) = \Pr(B \cap E_1) + \Pr(B \cap E_2) + \cdots + \Pr(B \cap E_k)$

  $= \Pr(E_1)\ \Pr(B \mid E_1) + \Pr(E_2)\ \Pr(B \mid E_2) + \cdots + \Pr(E_k)\ \Pr(B \mid E_k)$

Useful Formulae: Page 2

Counting Techniques:

- $n\,! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$
- $\binom{n}{k} = \dfrac{n\,!}{k\,!(n-k)\,!}$

Random Variables:

- $E(X) = \sum x_i \; p(x_i)$
- $E(X^2) = \sum x_i^2 \; p(x_i)$
- $Var(X) = E(X^2) - [E(X)]^2$
- $Sd(X) = \sqrt{Var(X)}$

Binomial Distribution:

- $X \sim \text{Binomial}(n, p)$
- $\Pr(X = x) = \binom{n}{x} p^x \, (1-p)^{n-x}$
- $x \in \{0, 1, 2, \ldots, n\}$
- $E(X) = n\,p$
- $Var(X) = n\,p\,(1-p)$

Table 1 Cumulative Binomial Probabilities

p = probability of success in a single trial; n = number of trials. The table gives the probability of obtaining r *or more* successes in n independent trials. That is

$$\sum_{x=r}^{n} \binom{n}{x} p^x (1-p)^{n-x}$$

When there is no entry for a particular pair of values of r and p, this indicates that the appropriate probability is less than 0.000 05. Similarly, except for the case $r = 0$, when the entry is exact, a tabulated value of 1.0000 represents a probability greater than 0.999 95.

	p =	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$n = 2$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.0199	.0396	.0591	.0784	.0975	.1164	.1351	.1536	.1719
	2	.0001	.0004	.0009	.0016	.0025	.0036	.0049	.0064	.0081
$n = 5$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.0490	.0961	.1413	.1846	.2262	.2661	.3043	.3409	.3760
	2	.0010	.0038	.0085	.0148	.0226	.0319	.0425	.0544	.0674
	3		.0001	.0003	.0006	.0012	.0020	.0031	.0045	.0063
	4						.0001	.0001	.0002	.0003
$n = 10$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.0956	.1829	.2626	.3352	.4013	.4614	.5160	.5656	.6106
	2	.0043	.0162	.0345	.0582	.0861	.1176	.1517	.1879	.2254
	3	.0001	.0009	.0028	.0062	.0115	.0188	.0283	.0401	.0540
	4			.0001	.0004	.0010	.0020	.0036	.0058	.0088
	5					.0001	.0002	.0003	.0006	.0010
	6									.0001
$n = 20$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.1821	.3324	.4562	.5580	.6415	.7099	.7658	.8113	.8484
	2	.0169	.0599	.1198	.1897	.2642	.3395	.4131	.4831	.5484
	3	.0010	.0071	.0210	.0439	.0755	.1150	.1610	.2121	.2666
	4		.0006	.0027	.0074	.0159	.0290	.0471	.0706	.0993
	5			.0003	.0010	.0026	.0056	.0107	.0183	.0290
	6				.0001	.0003	.0009	.0019	.0038	.0068
	7						.0001	.0003	.0006	.0013
	8								.0001	.0002
$n = 50$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.3950	.6358	.7819	.8701	.9231	.9547	.9734	.9845	.9910
	2	.0894	.2642	.4447	.5995	.7206	.8100	.8735	.9173	.9468
	3	.0138	.0784	.1892	.3233	.4595	.5838	.6892	.7740	.8395
	4	.0016	.0178	.0628	.1391	.2396	.3527	.4673	.5747	.6697
	5	.0001	.0032	.0168	.0490	.1036	.1794	.2710	.3710	.4723
	6		.0005	.0037	.0144	.0378	.0776	.1350	.2081	.2928
	7		.0001	.0007	.0036	.0118	.0289	.0583	.1019	.1596
	8			.0001	.0008	.0032	.0094	.0220	.0438	.0768
	9				.0001	.0008	.0027	.0073	.0167	.0328
	10					.0002	.0007	.0022	.0056	.0125
	11						.0002	.0006	.0017	.0043
	12							.0001	.0005	.0013
	13								.0001	.0004
	14									.0001

Table 1 Cumulative Binomial Probabilities – continued

	p =	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$n = 100$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.6340	.8674	.9524	.9831	.9941	.9979	.9993	.9998	.9999
	2	.2642	.5967	.8054	.9128	.9629	.9848	.9940	.9977	.9991
	3	.0794	.3233	.5802	.7679	.8817	.9434	.9742	.9887	.9952
	4	.0184	.1410	.3528	.5705	.7422	.8570	.9256	.9633	.9827
	5	.0034	.0508	.1821	.3711	.5640	.7232	.8368	.9097	.9526
	6	.0005	.0155	.0808	.2116	.3840	.5593	.7086	.8201	.8955
	7	.0001	.0041	.0312	.1064	.2340	.3936	.5557	.6968	.8060
	8		.0009	.0106	.0475	.1280	.2517	.4012	.5529	.6872
	9		.0002	.0032	.0190	.0631	.1463	.2660	.4074	.5506
	10			.0009	.0068	.0282	.0775	.1620	.2780	.4125
	11			.0002	.0022	.0115	.0376	.0908	.1757	.2882
	12				.0007	.0043	.0168	.0469	.1028	.1876
	13				.0002	.0015	.0069	.0224	.0559	.1138
	14					.0005	.0026	.0099	.0282	.0645
	15					.0001	.0009	.0041	.0133	.0341
	16						.0003	.0016	.0058	.0169
	17						.0001	.0006	.0024	.0078
	18							.0002	.0009	.0034
	19							.0001	.0003	.0014
	20								.0001	.0005
	21									.0002
	22									.0001

	p =	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 2$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.1900	.2775	.3600	.4375	.5100	.5775	.6400	.6975	.7500
	2	.0100	.0225	.0400	.0625	.0900	.1225	.1600	.2025	.2500
$n = 5$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.4095	.5563	.6723	.7627	.8319	.8840	.9222	.9497	.9688
	2	.0815	.1648	.2627	.3672	.4718	.5716	.6630	.7438	.8125
	3	.0086	.0266	.0579	.1035	.1631	.2352	.3174	.4069	.5000
	4	.0005	.0022	.0067	.0156	.0308	.0540	.0870	.1312	.1875
	5		.0001	.0003	.0010	.0024	.0053	.0102	.0185	.0313
$n = 10$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.6513	.8031	.8926	.9437	.9718	.9865	.9940	.9975	.9990
	2	.2639	.4557	.6242	.7560	.8507	.9140	.9536	.9767	.9893
	3	.0702	.1798	.3222	.4744	.6172	.7384	.8327	.9004	.9453
	4	.0128	.0500	.1209	.2241	.3504	.4862	.6177	.7430	.8281
	5	.0016	.0099	.0328	.0781	.1503	.2485	.3669	.4956	.6230
	6	.0001	.0014	.0064	.0197	.0473	.0949	.1662	.2616	.3770
	7		.0001	.0009	.0035	.0106	.0260	.0548	.1020	.1719
	8			.0001	.0004	.0016	.0048	.0123	.0274	.0547
	9					.0001	.0005	.0017	.0045	.0107
	10							.0001	.0003	.0010
$n = 20$	$r = 0$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	.8784	.9612	.9885	.9968	.9992	.9998	1.0000	1.0000	1.0000
	2	.6083	.8244	.9308	.9757	.9924	.9979	.9995	.9999	1.0000
	3	.3231	.5951	.7939	.9087	.9645	.9879	.9964	.9991	.9998
	4	.1330	.3523	.5886	.7748	.8929	.9556	.9840	.9951	.9987
	5	.0432	.1702	.3704	.5852	.7625	.8818	.9490	.9811	.9941
	6	.0113	.0673	.1958	.3828	.5836	.7546	.8744	.9447	.9793
	7	.0024	.0219	.0867	.2142	.3920	.5834	.7500	.8701	.9423
	8	.0004	.0059	.0321	.1018	.2277	.3990	.5841	.7480	.8684
	9	.0001	.0013	.0100	.0409	.1133	.2376	.4044	.5857	.7483
	10		.0002	.0026	.0139	.0480	.1218	.2447	.4086	.5881
	11			.0006	.0039	.0171	.0532	.1275	.2493	.4119
	12			.0001	.0009	.0051	.0196	.0565	.1308	.2517
	13				.0002	.0013	.0060	.0210	.0580	.1316
	14					.0003	.0015	.0065	.0214	.0577
	15						.0003	.0016	.0064	.0207
	16							.0003	.0015	.0059
	17								.0003	.0013
	18									.0002

Answer Sheet

Name: _____

ID Number: _____

Enter your answers with an "X' in the table below.

Do not enter the "X" until you have made your *final decision* to avoid scribbling out.

	A	B	C	D
Q1				
Q2				
Q3				
Q4				
Q5				

Q6				
Q7				
Q8				
Q9				
Q10				

Q11				
Q12				
Q13				
Q14				
Q15				