Hypothesis Testing : Tutorial Sheet C

1. An IT competency test, used for staff recruitment, is devised so as to give a normal distribution of scores with a mean of 100. A random sample of 49 experienced IT users who are given the test achieve a mean score of 121 with a standard deviation of 14.

   (a) Perform a hypothesis test to assess whether this group of IT Users is unusual (i.e. have a different mean from the general population).

   (b) Compute a 95% confidence interval for the group.

2. A claim has been made that the mean body temperature of healthy adults is equal to 98.6 degrees. A sample of 106 people has produced a mean body temperature of 98.2 degrees and a standard deviation of 0.62. Test the claim using a 5% significance.

3. A manufacturer of computer monitors has for many years used a process giving a mean life of 4700 hours and a standard deviation of 1460 hours. A new process is tried to see if it will increase the life significantly. A sample of 100 monitors gave a mean life of 5000 hours. Does the new process make a difference at the 5% level of significance?

4. In a study of store checkout scanners, 1234 items were checked and 20 of them were overcharges. Use a 5% significance level to test the claim that with scanners, 1% of sales are overcharges.

5. The average height of a sample of 16 students was 173cm with a variance of 144cm^2. The average height of the Irish population is 169cm.

   (a) Can it be stated at a significance level of 5% that students are on average taller than the population as a whole?

   (b) What assumption is used to carry out this test? Is this assumption reasonable?

6. The average mass of a sample of 64 Irish teenagers (Let say - 18 year old males) was 73.5kg with a variance of 100kg^2. The average mass of an equivalent sample of 81 Japanese teenagers was 68.5kg with a variance of 81 kg^2.

   (a) Test the hypothesis that Irish students are larger (in terms of mass) than Japanese teenagers.

   (b) By calculating the appropriate $p-$value, test the null hypothesis that the mean mass of all Irish students is 70kg at significance levels of 5%.

   (c) Using the appropriate confidence interval, test the hypotheses that the average mass of all Irish students is a) 70kg, b) 72kg, c) 75kg at a significance level of 5%.

7. A study was carried out in which researchers collected crime data. Of those convicted of arson, 50 were drinkers and 43 abstained. Of those convicted of fraud, 63 were drinkers and 144 abstained. Use a 0.01 level of significance to test the claim that the proportion of drinkers among convicted arsonists is greater than the proportion of drinkers convicted of fraud.

8. A coal-fired power plant is considering two different systems for pollution abatement. The first system has reduced the emission of pollutants to acceptable levels 68% of the time, as determined from 200 air samples. The second, more expensive system has reduced the emission of pollutants to acceptable levels 70% of the time, as determined from 250 air samples. lf the expen sive system is significantly more eilective than the inexpensive system in reducing the pollutants to acceptable levels, then the management of the power plant will install the expensive system.

   (a) Which system will be installed if management uses a significance level of 0.05 in making its decision?

   (b) Construct a 95% confidence interval for the difference in the two proportions. Interpret this interval.

9. The quality control manager at the Telektronic Company considers the production of telephone answering machines to be out of control when the overall rate of defects exceeds 4%. Testing of a random sample of 150 machines revealed that 9 are defective. The production manager claims that production is not out of control and no corrective action is necessary. Use a 0.05 significance level to test the production managers claim.

10. It is generally assumed that older people are more likely to vote for the Conservatives than younger people. In a survey, 160 of 400 people over 40 and 120 of 400 people under 40 stated they would vote Conservative.

    (a) Do the data support this hypothesis at a significance level of 5%?

    (b) Calculate a 95% confidence interval for the difference between the proportion of people over 40 voting Conservative and the proportion of people below 40 voting Conservative.

11. The following are measurements (in mm) of a critical dimension on a sample of engine crankshafts:

224.120	224.017	223.976	223.961
224.089	223.982	223.980	223.989
223.960	223.902	223.987	224.001

    (a) Calculate the mean and standard deviation for these data.

    (b) The process mean is supposed to be $\mu = 224$mm. Is this the case? Give reasons for your answer.

    (c) Construct a 95% confidence interval for these data and interpret.

    (d) Check that the normality assumption is valid using 2 suitable plots.

12. Answer the following theory questions on hypothesis testing.

    (a) In the context of hypothesis testing, explain what a p-value is, and how it is used. Support your answer with a simple example.

(b) What is meant by Type I error and Type II error?

13. The working lifetimes of 100 of both of two different types of batteries were observed. The mean lifetime for the sample of type 1 batteries was 25 hrs with a standard deviation of 4hrs. The mean lifetime for the sample of type 1 batteries was 23 hrs with a standard deviation of 3hrs.

Type 1	Type 2
$n_1 = 100$	$n_2 = 100$
$x_1 = 25$ hours	$x_1 = 23$ hours
$s_1 = 4$ hours	$s_1 = 3$ hours

(a) Test the hypothesis that the mean working lifetimes of these batteries do not differ at a significance level of 5% .

(b) Calculate a 95% confidence interval for the difference between the average working lifetimes of these batteries.

(c) Using this confidence interval, test the hypothesis that battery 1 on average works for 3 hours longer than battery 2.